研　究　速　報

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Transient Response Analysis of an Elasto-Viscoplastic Beam Subjected to Transverse Impact

— Application of a New Beam Bending Element for Elasto-Viscoplastic Analysis —

横衝撃を受ける弾／粘塑性梁の過度応答解析
—— 弾／粘塑性解析のための新しい梁の曲げ要素の応用 ——

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Summary

A new beam bending element for elasto-viscoplastic analysis is proposed in this note. This element consists of rigid bars which are connected by a rheological elasto-viscoplastic model composed of three elements, i.e. a spring, a slider, and a dashpot. As a numerical example transient response analysis of an elasto-viscoplastic beam subjected to transverse impact is carried out.

1. New Beam Bending Element for Elasto-Viscoplastic Analysis

The material of a beam is assumed to possess mechanical properties described by a one-dimensional rheological elasto-viscoplastic model shown in Fig. 1. In case of perfectly plastic materials stress-strain relation in this model is given as follows:

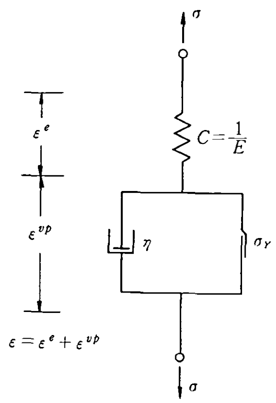
$$E\dot{\varepsilon} = \dot{\sigma} + k(\sigma - \sigma_Y \operatorname{sign} \sigma) \tag{1}$$

where

$$k \equiv \frac{E}{\eta} = 0 \quad \text{if} \ |\sigma| < \sigma_Y \tag{2}$$

Consider two rigid bars which are connected by an elasto-viscoplastic model resisting rotational motion as shown in Fig. 2. Relation between bending moment and relative rotational angle is expressed as follows:

$$\dot{\theta} = C_b \dot{M} + \frac{1}{\eta_b}(M - M_p \operatorname{sign} M) \tag{3}$$



σ_Y : static yield stress

Fig. 1 One-dimensional rheological elasto-viscoplastic model

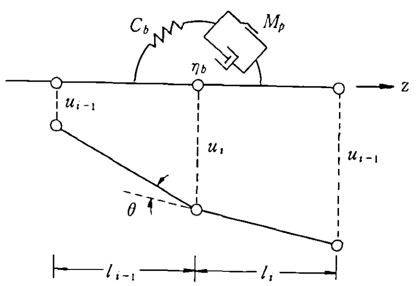


Fig. 2 A new beam bending element for elasto-viscoplastic analysis

where M_p is static plastic moment. Two parameters in Eq. (3) can be determined by using finite difference approximation of curvature as follows:

$$\left.\begin{aligned} \frac{1}{C_b} &= \frac{1}{\frac{1}{2}(l_{i-1}+l_i)C} \\ \eta_b &= \frac{\eta}{\frac{1}{2}(l_{i-1}+l_i)} \end{aligned}\right\} \tag{4}$$

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Solving Eq.(3), the increment of rotational angle is calculated as

$$\Delta\theta = C_b \Delta M + \frac{1}{\eta_b}\int_t^{t+\Delta t} \{M(\tau) - M_p \,\mathrm{sign}\, M(\tau)\}\, d\tau \tag{5}$$

Time variation of bending moment is assumed to be linear during the time section $[t \;\; t+\Delta t]$, that is

$$M(\tau) = M(t) + \frac{\Delta M}{\Delta t}(\tau - t) \tag{6}$$

Substituting Eq.(6) into Eq.(5), the following equation is obtained :

$$\Delta\theta = C_b \Delta M + \frac{1}{\eta_b}\{M(t) + \frac{1}{2}\Delta M - M_p \,\mathrm{sign}\, M(t)\}\Delta t \tag{7}$$

From Eq.(7) the following relation between the increment of bending moment and relative rotational angle is obtained:

$$\Delta M = \frac{1}{C_B(t)}\{\Delta\theta - \Delta\theta^{vp}\} \tag{8}$$

where

$$C_B(t) = C_b + \frac{\Delta t}{2\eta_b} \tag{9}$$

$$\Delta\theta^{vp} = \frac{1}{\eta_b}\{M(t) - M_p \,\mathrm{sign}\, M(t)\}\Delta t \tag{10}$$

The virtual work equation is expressed as

$$\delta\Delta\theta \cdot \Delta M - \{\delta\Delta u\}^t\{\Delta F\} = \delta\Delta\theta \cdot \frac{1}{C_B(t)}(\Delta\theta - \Delta\theta^{vp}) - \{\delta\Delta u\}^t\{\Delta F\} = 0 \tag{11}$$

where

$$\left.\begin{aligned} &\theta = \lfloor R \rfloor\{u\} \quad \{u\}^t = \lfloor u_{i-1} u_i \; u_{i+1} \rfloor \\ &\lfloor R \rfloor = \left\lfloor \frac{1}{l_{i-1}} \;\; -\left(\frac{1}{l_{i-1}} + \frac{1}{l_i}\right) \;\; \frac{1}{l_i} \right\rfloor \end{aligned}\right\} \tag{12}$$

From Eq.(11) the following equilibrium equation is derived :

$$[K]\{\Delta u\} = \{\Delta F\} + \frac{1}{C_B(t)}\Delta\theta^{vp}\{R\} \tag{13}$$

where

$$[K] = \frac{1}{C_B(t)}\begin{bmatrix} \frac{1}{l_{i-1}^2} & & \text{sym.} \\ -\frac{1}{l_{i-1}}\left(\frac{1}{l_{i-1}} + \frac{1}{l_i}\right) & \left(\frac{1}{l_{i-1}} + \frac{1}{l_i}\right)^2 & \\ \frac{1}{l_{i-1} l_i} & -\frac{1}{l_i}\left(\frac{1}{l_{i-1}} + \frac{1}{l_i}\right) & \frac{1}{l_i^2} \end{bmatrix} \tag{14}$$

2. Results of Numerical Analysis

As a numerical example transient response of a simply supported beam subjected to concentrated transverse impact at the midspan is analized. Consistent mass matrix is used and Newmark's β method with $\beta = 1/6$ is applied to numerical time integration. Numerical analysis is made by using the following input data :

- Dimension $= 50 \times 0.953 \times 2.54$ cm
- Young's modulus $= 2.11 \times 10^6$ kg/cm^2
- Static plastic moment $= 0.922 \times 10^4$ kg·cm
- Density $= 0.8 \times 10^{-5}$ kg·sec^2/cm^4
- Impact velocity $= 30.5$ m/sec
- Total number of elements $= 40$
- Time increment $= 0.25\,\mu$sec

As for the value of k, the following three cases are analized :

$$\left.\begin{aligned} &\text{i)} \quad k = 1.0 \\ &\text{ii)} \quad k = 0.5 \\ &\text{iii)} \quad k = 0.1 \end{aligned}\right\} \times 10^6\ \mathrm{sec}^{-1}$$

The loading/unloading criterion used in present analysis is as follows :

$$\left.\begin{aligned} &|M| > M_p \quad \text{for loading} \\ &|M| \leq M_p \quad \text{for unloading} \end{aligned}\right\} \tag{15}$$

Fig. 3 − 7 show results of numerical analysis compared with those of elastic or elasto-plastic analysis neglecting strain rate effect. The variation of deflection curves with time is shown in Fig. 3, from which it can be seen that two types of visco-plastic hinges will be formed. Fig. 4 shows the bending moment-curvature variation at the impact point. The maximum value of bending moment increases with decreasing value of k. In case of $k = 0.1 \times 10^6$ sec^{-1}, it is about twice as large as the value of static plastic moment. Time variation of the position of moving hinges is shown in Fig. 5. It is seen from Fig. 5 that the velocity of moving hinges increases with decreasing value of k. Deflection curves and bending moment distribution near the impact point at $t = 30\,\mu$sec are shown in Fig. 6 and 7 respectively. Results of elasto-viscoplastic analysis are between the corresponding curves of elastic analysis and of elasto-plastic analysis respectively.

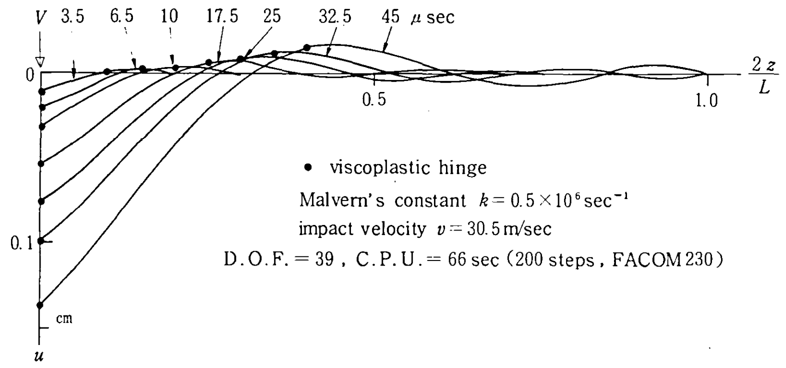

Fig. 3　Transient response of an elastic-viscoplastic beam subjected to transverse impact (deflected shapes)

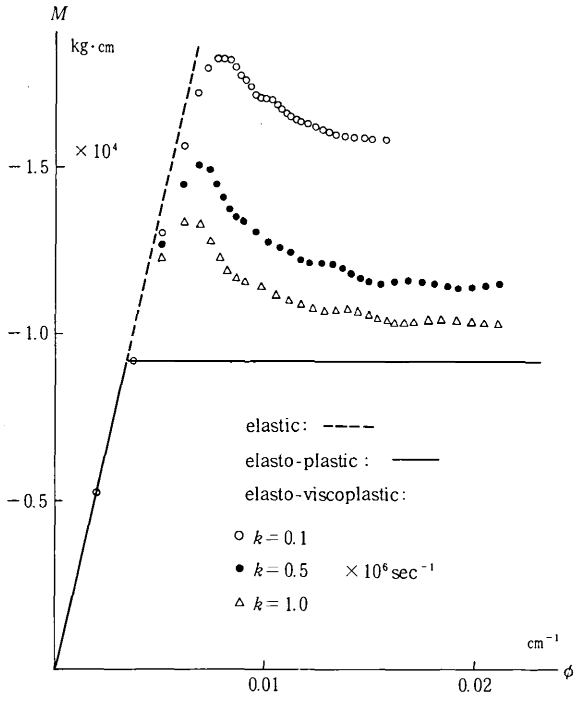

Fig. 4　Bending moment-curvature variation (at midspan)

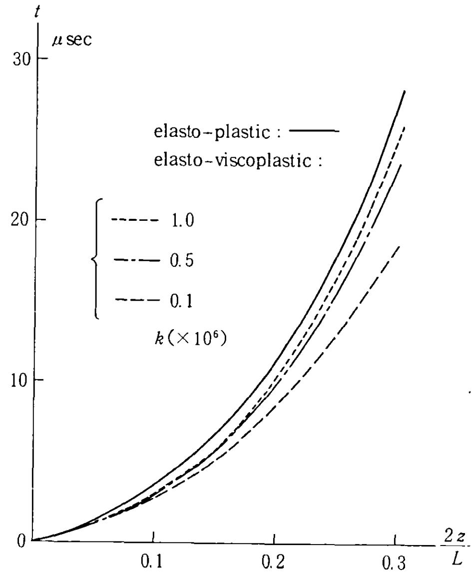

Fig. 5　Time variation of the position of moving hinges

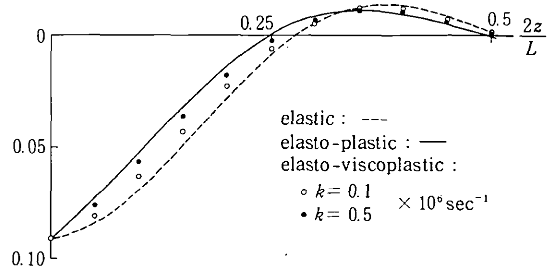

Fig. 6　Deflection curves at $t = 30\,\mu$sec

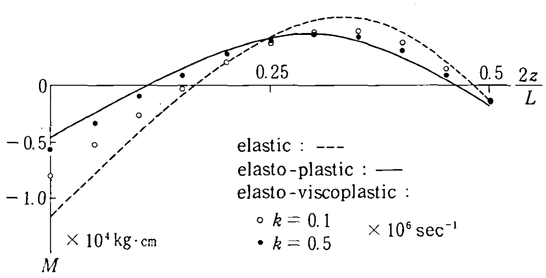

Fig. 7　Bending moment distribution at $t = 30\,\mu$sec

3. Conclusion

A new beam bending element for elasto-visco-plastic analysis was proposed in this note. In this element the material property is lumped, so that the size of the stiffness matrix is 1/2 of the conventional finite element and concept of the visco-plastic hinge is easily introduced.

Transient response analysis of an elasto-visco-plastic beam subjected to transverse impact was made successfully with very short computing time by using this element. The result obtained duly justified validity of the present method in elasto-viscoplastic dynamic analysis of framed structures.

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