特集　10

A Discrete Analysis on Dynamic Collapse of a Beam under Impulsive Transverse Load

横衝撃荷重を受ける梁の動的崩壊に関する一離散化解析法

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Summary

Using a rigid bar − spring model of beam bending recently proposed by one of authors, dynamic collapse of a beam under impulsive transverse load is studied.

Dynamic response analysis (elastic as well as inelastic) of a long simply supported beam under a lateral concentrated impact load was made successfuly within a very short computing time and yet the results obtained are in good agreement with those of the previous authors.

1. Solution procedure

A simply supported beam as shown in Fig. 1 is considered. Using a new physical model of beam bending, this beam is replaced by an equivalent
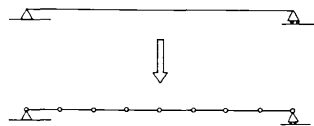

Fig.1　Discrete model of a simply supported beam

system or rigid bars connected with rotational springs at each joint.

In this model, the beam bending stiffness is lumped so that its stiffness matrix is (2 × 2) which is 1/2 of the conventional finite element stiffness matrix and concept of the plastic hinge can be easily introduced in case of inelastic deformation. Except the stiffness and mass matrices used, the solution procedure is the same as in case of the standard finite element method.

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Newmark's β method is applied to integration of the equation of motion derived. Since a sufficiently small time increment is chosen no equilibrium check is made.

1.1 Element matrices used

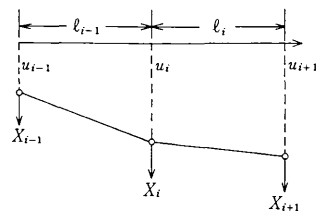A typical element is shown in Fig. 2. The stiffness and mass matrices are given as follows:

Fig.2　A trypical element of beam bending

(a) Stiffness matrix

$$
\left\{ \begin{array}{c} X_{i-1} \\ X_i \\ X_{i+1} \end{array} \right\} = k_i \left[\begin{array}{ccc} \dfrac{1}{l_{i-1}^2} & & \text{SYM} \\ -\dfrac{1}{l_i}(\dfrac{1}{l_{i-1}}+\dfrac{1}{l_i}) & (\dfrac{1}{l_{i-1}}+\dfrac{1}{l_i})^2 & \\ \dfrac{1}{l_{i-1}l_i} & -\dfrac{1}{l_i}(\dfrac{1}{l_{i-1}}+\dfrac{1}{l_i}) & \dfrac{1}{l_i^2} \end{array} \right] \left\{ \begin{array}{c} u_{i-1} \\ u_i \\ u_{i+1} \end{array} \right\} \quad (1)
$$

where the spring constant k_i can be calculated by the following formula

$$
k_i = \frac{EI}{\dfrac{1}{2}(l_{i-1} + l_i)} \quad (2)
$$

(b) Mass matrix

$$
\left\{ \begin{array}{c} X_{i-1} \\ X_i \\ X_{i+1} \end{array} \right\} = \frac{rA}{6g} \left[\begin{array}{ccc} 2\,l_{i-1} & & \text{SYM} \\ l_{i-1} & 2\,(l_{i-1}+l_i) & \\ 0 & l_i & 2\,l_i \end{array} \right] \left\{ \begin{array}{c} \ddot{u}_{i-1} \\ \ddot{u}_i \\ \ddot{u}_{i+1} \end{array} \right\} \quad (3)
$$

where $\dfrac{r}{g}$ is the density of material under consideration and A is the cross sectional area of a given

beam

1.2 Time integration scheme

Using the above mentioned element matrices, the equation of motion for this discrete system is expressed in the incremental form as follows:

$$[M]\{\Delta \ddot{u}\} + [K]\{\Delta u\} = \{\Delta F(t)\} \quad (4)$$

where $[M]$, $[K]$ are mass and stiffness matrices of a whole structure, $\{\Delta u\}$, $\{\Delta F\}$ are displacement and external force increments respectively.

Denoting the displacement $\{u\}_n$, the velocity $\{\dot{u}\}_n$ and the acceleration $\{\ddot{u}\}_n$ at $t = t_n$, the corresponding values at $t = t_{n+1}$ are given in Newmark's β method as follows:

$$\left.\begin{array}{l} \{\dot{u}\}_{n+1} = \{\dot{u}\}_n + \Delta t(\{\ddot{u}\}_n + \{\ddot{u}\}_{n+1})/2 \\[2mm] \{u\}_{n+1} = \{u\}_n + \Delta t\{\dot{u}\}_n + \Delta t^2\{\ddot{u}\}_n/2 \\[2mm] \qquad + \beta \Delta t^2(\{\ddot{u}\}_{n+1} - \{\ddot{u}\}_n) \end{array}\right\} (5)$$

where

$$\Delta t = t_{n-1} - t_n$$

From which the following incremental accelecration and velocity are obtained.

$$\left.\begin{array}{l} \{\Delta \ddot{u}\} = \{\ddot{u}\}_{n+1} - \{\ddot{u}\}_n = 6\{\Delta u\}/\Delta t^2 \\[1mm] \qquad - 6\{\dot{u}\}_n/\Delta t - 3\{\ddot{u}\}_n \\[2mm] \{\Delta \dot{u}\} = \{\dot{u}\}_{n+1} - \{\dot{u}\}_n = 3\{\Delta u\}/\Delta t \\[1mm] \qquad - 3\{\dot{u}\}_n - \Delta t\{\ddot{u}\}_n/2 \end{array}\right\} (6)$$

where

$$\beta = \frac{1}{6} \quad \text{is assumed}$$

Substituting eq. (6) into eq.(4) the following equation is obtained:

$$\left(\frac{6}{\Delta t^2}[M] + [K]\right)\{\Delta u\}$$
$$= \frac{6}{\Delta t}[M]\{\dot{u}\}_n + 3[M]\{\ddot{u}\}_n + \{\Delta F\}$$
$$(7)$$

Solving eq. (7), incremental displacements can be determined.

2. Results of some numerical analysis

As for numerical examples, dynamic behavior of a sufficiently long beam under impact loading is analized. The beam is assumed that both ends are simply supported and its middle point will move with a constant velocity for a certain period from the initial state. Using the HITAC 8700−8800 which is comparable to the IBM 370−158, numerical

analysis is made by using the following input data:

Dimension = $305 \times 0.953 \times 2.54$ cm

Young's modulus = 2.11×10^7 kg /cm^2

Plastic Moment = 0.922×10^4 kg ·cm

Density = 0.8×10^{-5} kg ·sec^2 / cm^4

Impact velocity = 30.5 m / sec

Duration of impact = 0.87 msec (t_d)

Total number of elements = 40

Total degrees of freedom = 40

Time increment $= t_d/20$ (elastic analysis)
$= t_d/40$ (elasto-plastic analysis)

2.1 Elastic analysis

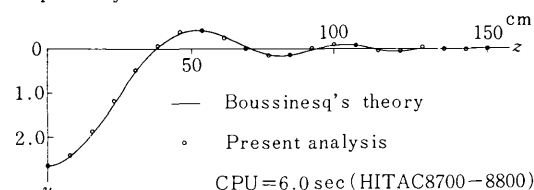The deflection curve and the bending moment diagram at $t = t_d$ are shown in Fig.3 and Fig.4 respectively.

CPU = 6.0 sec (HITAC8700−8800)
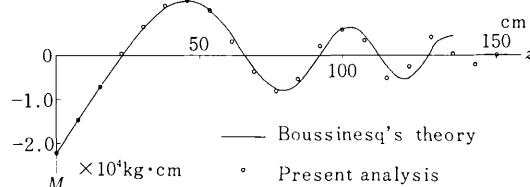
Fig.3   Deflection curve ($t = t_d$, EL)

Fig.4   Bending moment diagram ($t = t_d$, EL)

Good agreement between the authors' results and J. Boussinesq's theory was confirmed except in the vicinity of the supported ends.

2.2 Elasto-plastic analysis

Using the same model, elasto-plastic dynamic analysis was made and the results obtained were compared with the theoretical and experimental study made by H. F. Bohnenblust. The variation of deflection curves with time is shown in Fig.5 from which it can be seen that two types of plastic hinges will be formed, i.e, the fixed plastic hinge at the midspan and the moving plastic hinges. The position of plastic hinges was shown in Fig. 6. The calculated deflection curve at $t = t_d$ was compared with Bohnenblust's theoretical and experimental results in Fig. 7.   Good agreement
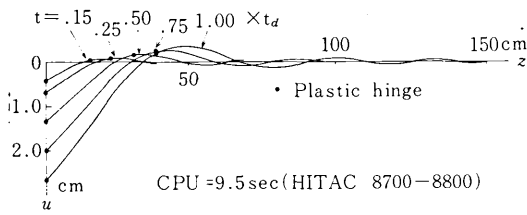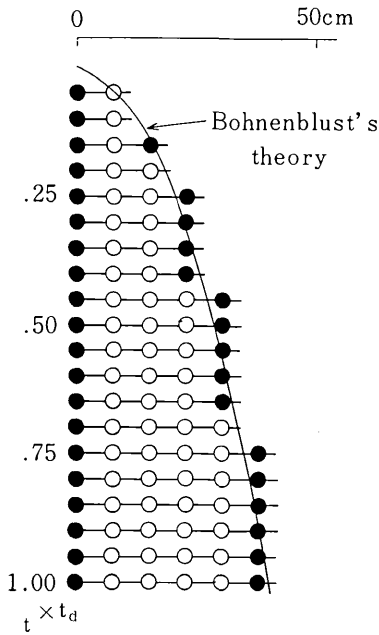
研　究　速　報

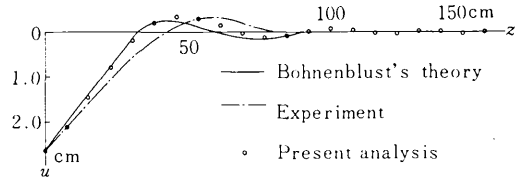Fig.5　Deflection curves　（EL-PL）

Fig.6　Plastic hinge(•) position

Fig.7　Deflection curve　($t = t_d$, EL-PL)

between the present analysis and Bohnenblust's study was again observed excluding in the neighbourhood of supported ends.

4. Conclusion

Dynamic collapse analysis of a beam under a concentrated lateral impact was analysed by using a new beam bending element. The result obtained duly justitied validity of the present method in nonlinear vibration problems of the framed structures.　　　(Manuscript received January 6, 1977)

References

1) S. Kawashima, "Dynamic Response Analysis", Baihukan, 1972 (in Japanese)
2) W. Johnson, "Impact Strength of Marerials", Edward Arnold (Publishers) LTD, 1972
3) P. E. Duwez, D. S. Clark and H. F. Bohnenblust, "The Behavior of Long Beams under Impact Loading", J. Appl. Mech., 17, 1950
4) N. M. Newmark, "A Method of Computation for Structural Dynamics", J. Eng. Mech. Div. ASCE, 85, 1959

正　誤　表　　　（4月号）

頁	段	行	種　　別	正	誤
205	左	↓10	本 文	削　除	can
〃			Table 1.	（表の外に）× $1/l_{35}^2$	脱　落
206	左	↓9	(11)式	$\int_{-\frac{l_{35}}{2}}^{\frac{l_{35}}{2}} \frac{k_d}{l_{35}}\ (s\varphi)\ sds$	$\int_{-\frac{l_{35}}{2}}^{\frac{l_{35}}{2}} k_d\ (s\varphi)\ sds$
〃	右	↓12	本 文	(3) h/b $\geq$ 8.74	(3) h/b = 1
〃	〃	↓16	〃	IBM （370 − 158）	IBM （360 − 195）
207	左		Fig.8の表題	of a	of aa
213	〃	↑3	数式	$(\zeta + \frac{\dot{x}_0}{\omega x_0})]$	$(\zeta + \frac{\dot{x}_0}{x_0})]$
〃	右	↑5	〃	$\frac{d}{\sqrt{(1-\nu^2)^2 + 4\zeta^2\nu^2}}e$	$\frac{d^2}{\sqrt{(1-\nu^2)^2 + 4\zeta^2\nu^2}}e$