研 究 速 報
UDC 62-503.2

Application of Stable Parameter Identification and Control Scheme for the Classical Lur'e Problem (PART II)

古典ルーリエ問題への安定なパラメータ同定と制御則の応用（そのII）

Toshio FUKUDA*
福 田 敏 男

Summary

We have previously reported the stable parameter identification and control scheme for the classical Lur'e problem, in which we dealt with the case such that control inputs are additive to the nonlinear feedback signals, and the system is treated as the single input-single output system. We also showed the proposed identification and control scheme are stable via the Lyapunov method and dealt with those problems within the framework of the Model Reference Adaptive System (MRAS).

In this report we consider the case such that control inputs are additive to the nonlinear feedback signals but the system is treated as the multiple input-single output system. And so there should increase the number of parameters to be identified, either coefficients of inputs or coefficients of nonlinear feedback signals.

1. Introduction

We consider the case such that inputs are additive to the nonlinear feedback signals and the system can be treated as the system of the multiple input-single output, based on the previous report in which inputs are also additive to nonlinear feedback signals, but the system is treated as the single input-single output system. This report has wider applications when total linearization is not possible, but partial linearization is often profitable, such as nuclear reactor systems[1), 2)].

2. Representation of the system

Fig. 1 shows the previously reported case, while

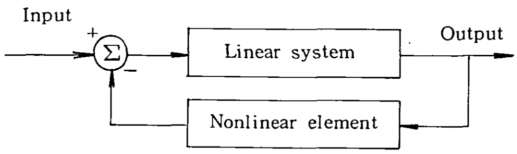

Fig. 1 A single input-single output system

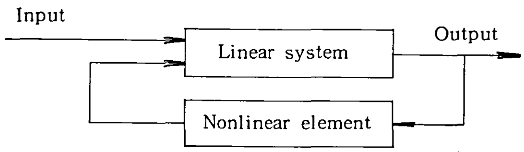

Fig. 2 A multiple input-single output system

Fig. 2 does the case dealt with in this report.

The n dimensional system shown in Fig. 1 is described by the following differential equations as was reported in the part I :

$$\dot{X} = AX + b(u+\tau), \quad \sigma_0 = h^T X$$
$$\tau = -g(\sigma_0, t) \qquad (2.1)$$

while the system shown in Fig. 2 can be described by the followings :

$$\dot{X} = AX + b_1 u + b_2 \tau, \quad \sigma_0 = h^T X$$
$$\tau = -g(\sigma_0, t) \qquad (2.2)$$

Then eq. (2.2) can be transformed to the equivalent system from the input-output relationship as follows :

$$\begin{cases} \dot{x}_1 = -a_1 x_1 - \bar{a}^{1T}\bar{x}^1 + {}_1b_1 u + {}_2b_1 \tau + {}_1\bar{b}^{1T}{}_1\bar{x}^2 + {}_2\bar{b}^{1T}{}_2\bar{x}^2 \\ \dot{\bar{x}}^1 = \Lambda \bar{x}^1 + \bar{l} x_1 \\ {}_1\dot{\bar{x}}^2 = \Lambda\, {}_1\bar{x}^2 + \bar{l} u \\ {}_2\dot{\bar{x}}^2 = \Lambda\, {}_2\bar{x}^2 + \bar{l}\tau \\ y = (1\ 0 \cdots 0)\, x, \quad x^T = (x_1, \bar{x}^{1T}, {}_1\bar{x}^{2T}, {}_2\bar{x}^{2T}) \end{cases} \qquad (2.3)$$

Initial conditions can be chosen as :

$$x_1(0) = x_{10}, \quad \bar{x}^1(0) = \bar{x}_0^1, \quad {}_1\bar{x}^2(0) = \bar{0}, \quad {}_2\bar{x}^2(0) = \bar{0}$$

and $\Lambda = \begin{bmatrix} -\lambda_2 & & & 0 \\ & -\lambda_3 & & \\ & & \ddots & \\ 0 & & & -\lambda_n \end{bmatrix}$, $\lambda_i > 0$, $\lambda_i \neq \lambda_j$ if $i \neq j$

$$\bar{l}^T = (1\ 1 \cdots 1)$$

* Graduate Student, Plant Eng'g. Lab.(Shibata), Department of Mechanical Eng'g. and Naval Architecture.

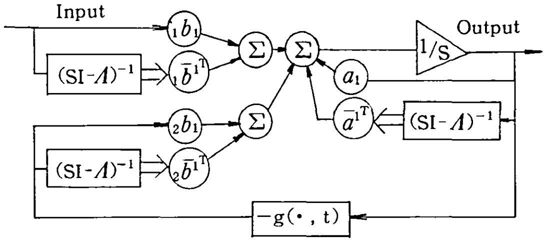

Fig. 3 Representation of the plant

The block diagram of the dynamical system described by eq. (2.3) can be depicted as is shown in Fig. 3.

From now on, we can assume that the linear part of the system is both completely controllable and completely observable and that the plant can be described by eq. (2.3).

3. Formulation and results

We deal with the problems of parameter identification and control separately in the following sections. In both cases, the nonlinearity of the plant can be assumed a priori known.

3.1 Identification

The model for identification can be set up, associated with eq. (2.3) as follows ;

$$\begin{cases} \dot{\hat{x}}_1 = -\hat{a}_1 x_1 - \hat{\bar{a}}^{1T}\hat{\bar{x}}^1 + {}_1\hat{b}_1 u + {}_2\hat{b}_1 \tau + {}_1\hat{\bar{b}}^{1T}{}_1\hat{\bar{x}}^2 + {}_2\hat{\bar{b}}^{1T}{}_2\hat{\bar{x}}^2 \\ \quad - \lambda_1(\hat{x}_1 - x_1) \\ \dot{\hat{\bar{x}}}^1 = \Lambda \hat{\bar{x}}^1 + \bar{l}\, x_1 \\ {}_1\dot{\hat{\bar{x}}}^2 = \Lambda\, {}_1\hat{\bar{x}}^2 + \bar{l}\, u \\ {}_2\dot{\hat{\bar{x}}}^2 = \Lambda\, {}_2\hat{\bar{x}}^2 + \bar{l}\, \tau \quad , \quad \tau = -g(y, t) \end{cases} \tag{3.1}$$

where $\lambda_1 > 0$ and initial conditions are

$$\hat{x}_1(0) = \hat{x}_{10} ; \quad \hat{\bar{x}}^1(0) = \hat{\bar{x}}^1_0, \quad {}_1\hat{\bar{x}}^2(0) = {}_2\hat{\bar{x}}^2(0) = \bar{0}$$

Define errors : $e_1 = \hat{x}_1 - x_1$, $\bar{e}^1 = \hat{\bar{x}}^1 - \bar{x}^1$, ${}_1\bar{e}^2 = {}_1\hat{\bar{x}}^2 - {}_1\bar{x}^2$, ${}_2\bar{e}^2 = {}_2\hat{\bar{x}}^2 - {}_2\bar{x}^2$ Then substruction from eq. (3.1) to eq. (2.3) leads to the following error differential equations :

$$\begin{cases} \dot{e}_1 = -\lambda_1 e_1 - \bar{a}^{1T}\bar{e}^1 + {}_1\bar{b}^{1T}{}_1\bar{e}^2 + {}_2\bar{b}^{1T}{}_2\bar{e}^2 + \Phi^T V \\ \dot{\bar{e}}^1 = \Lambda\, \bar{e}^1 \\ {}_1\dot{\bar{e}}^2 = \Lambda\, {}_1\bar{e}^2 \\ {}_2\dot{\bar{e}}^2 = \Lambda\, {}_2\bar{e}^2 \end{cases} \tag{3.2}$$

where

$$\Phi^T = (-\hat{a}_1 + a_1 , \ -(\hat{\bar{a}}^1 - \bar{a}^1)^T, \ {}_1\hat{b}_1 - {}_1b_1 , \ {}_2\hat{b}_1 - {}_2b_1 , \ ({}_1\hat{\bar{b}}^1 - {}_1\bar{b}^1)^T, ({}_2\hat{\bar{b}}^1 - {}_2\bar{b}^1)^T)$$

$$V^T = (x_1 , \hat{\bar{x}}^{1T}, \ u, \tau, {}_1\hat{\bar{x}}^{2T}, {}_2\hat{\bar{x}}^{2T})$$

and initial conditions are

$$e_1(0) = e_{10}, \ \bar{e}^1(0) = \bar{e}^1_0, \ {}_1\bar{e}^2(0) = {}_2\bar{e}^2(0) = \bar{0}$$

From the initial conditions, note that

$${}_1\bar{e}^2(t) = {}_2\bar{e}^2(t) = \bar{0}$$

for all t. Then eq. (3.2) can be written in a vector-matrix form ;

$$\begin{cases} \dot{e} = Ae + d\,\Phi^T V \\ e_1 = h^T e \end{cases} \tag{3.3}$$

where

$$e^T = (e_1 , \bar{e}^{1T}), \ h^T = (1\ 0 \cdots 0),$$

$$A = \begin{bmatrix} -\lambda_1 & -\bar{a}^{1T} \\ 0 & \Lambda \end{bmatrix}, \ d^T = (1\ 0 \cdots 0)$$

Note also that the transfer function of eq. (3.3) is

$$h^T(sI - A)^{-1} d = \frac{1}{s + \lambda_1} \tag{3.4}$$

which is a strictly positive real function and is essential for the proof of stability later on.

Now set up a Lyapunov function candidate as follows :

$$L = \frac{1}{2} e^T P e + \frac{1}{2} \Phi^T G^{-1} \Phi \tag{3.5}$$

where $P = P^T > 0$ and $G = G^T > 0$

Eq. (3.5) clearly satisfies all conditions as a Lyapunov function candidate. With differentiation of eq. (3.5) with respect to time t, we obtain the followings :

$$\dot{L} = \frac{1}{2} e^T(A^T P + PA)e + (e^T P d\, V + G^{-1}\dot{\Phi})^T \Phi \tag{3.6}$$

We now invoke the Kalman-Yakubovich lemma, that is to say, there exist a positive difinite symmetric matrix P and a vector q for a sufficiently small number $\varepsilon_0 > 0$ and a given positive definite symmetric matrix L_0, which satisfy the following algebraic equations :

$$\begin{cases} A^T P + PA = -q^T q - \varepsilon_0 L_0 \\ Pd = h \end{cases}$$

Then eq. (3.6) turns out to be

$$\dot{L} = -\frac{1}{2} e^T q^T q e - \varepsilon_0 \frac{1}{2} e^T L_0 e \leq 0$$

if we choose

$$\dot{\Phi} = -G e_1 V \tag{3.7}$$

Eq. (3.7) is the identification scheme and can be written in terms of parameters :

$$\begin{cases} \dot{\hat{a}}_1 = g_1 e_1 x_1, & \dot{\hat{\bar{a}}}^1 = G^1 e_1 \hat{\bar{x}}^1 \\ {}_1\dot{\hat{b}}_1 = -{}_1g_2 e_1 u, & {}_1\dot{\hat{\bar{b}}}^1 = -{}_1G^2 e_1 \, {}_1\hat{\bar{x}}^2 \\ {}_2\dot{\hat{b}}_1 = -{}_2g_2 e_1 \tau, & {}_2\dot{\hat{\bar{b}}}^1 = -{}_2G^2 e_1 \, {}_2\hat{\bar{x}}^2 \end{cases} \quad (3.8)$$

where the adaptation gain matrix is chosen in a diagonal form :

$$G = \begin{bmatrix} g_1 & & & & & \\ & G^1 & & & 0 & \\ & & {}_1g_2 & & & \\ & & & {}_2g_2 & & \\ & 0 & & & {}_1G^2 & \\ & & & & & {}_2G^2 \end{bmatrix}$$

The identification rule, eq. (3.8), is stable and if u can be modulated so that each component of V should be linearly independent, then $\Phi = \bar{0}$ can be brought about from $\dot{\Phi} = \bar{0}$ and $\Phi^T V = 0$. This concludes that the identification rule eq. (3.7) or eq. (3.8) is asymptotically stable in the large.

3.2 Control

${}_1\beta_1 \neq 0$ is assumed in this section and then the adaptive controller model can be set up analogously as the case of the identification problem :

$$\begin{cases} \dot{y}_1 = -\alpha_1 y_1 - \bar{\alpha}^{1T}\bar{y}^1 + {}_1\beta_1 r + {}_2\beta_1 \tau + {}_1\bar{\beta}^{1T}{}_1\bar{y}^2 + {}_2\bar{\beta}^{1T}{}_2\bar{y}^2 \\ \qquad -\lambda_1 (y_1 - x_1) \\ \dot{\bar{y}}^1 = \Lambda \bar{y}^1 + \bar{l} x_1 \\ {}_1\dot{\bar{y}}^2 = \Lambda \, {}_1\bar{y}^2 + \bar{l} r \\ {}_2\dot{\bar{y}}^2 = \Lambda \, {}_2\bar{y}^2 + \bar{l} \tau \end{cases} \quad (3.9)$$

where the initial conditions are

$y_1(0) = y_{10}$, $\bar{y}^1(0) = \bar{y}_0^1$, ${}_1\bar{y}^2(0) = {}_2\bar{y}^2(0) = \bar{0}$

and r is a reference input to the model, while u is a control input to the plant.

Substructing eq. (2.3) from eq. (3.9), we obtain the error equations :

$$\begin{cases} \dot{e}_1 = -\lambda_1 e_1 - \bar{a}^{1T}\bar{e}^1 + {}_1\bar{\beta}^{1T}{}_1\bar{e}^2 + {}_2\bar{\beta}^{1T}{}_2\bar{e}^2 \\ \qquad + {}_1\beta_1 (r-u) + \Phi^T V \\ \dot{\bar{e}}^1 = \Lambda \bar{e}^1 \\ {}_1\dot{\bar{e}}^2 = \Lambda \, {}_1\bar{e}^2 + \bar{l}(r-u) \\ {}_2\dot{\bar{e}}^2 = \Lambda \, {}_2\bar{e}^2 \end{cases} \quad (3.10)$$

where

$$\Phi^T = (-(\alpha_1 - a_1), -(\bar{\alpha}^1 - \bar{a}^1)^T, {}_1\beta_1 - {}_1b_1, {}_2\beta_1 - {}_2b_1, ({}_1\bar{\beta}^1 - {}_1\bar{b}^1)^T, ({}_2\bar{\beta}^1 - {}_2\bar{b}^1)^T)$$

$$V^T = (x_1, \bar{y}^1, u, \tau, {}_1\hat{\bar{x}}^2, {}_2\hat{\bar{x}}^2)$$

From the initial condition, ${}_2\bar{e}^2 = 0$ and so, ${}_2\bar{y}^2 = {}_2\hat{\bar{x}}^2$ for all time. The auxiliary signal ${}_1\hat{\bar{x}}^2$ can be generated by

$${}_1\dot{\hat{\bar{x}}}^2 = \Lambda \, {}_1\hat{\bar{x}}^2 + \bar{l} u \quad (3.11)$$

with the initial condition, ${}_1\hat{\bar{x}}^2(0) = \bar{0}$.

Then ${}_1\hat{\bar{x}}^2 = {}_1\bar{x}^2$ for all time.

The error differential equation becomes

$$\begin{cases} \dot{e}_1 = -\lambda_1 e_1 - \bar{a}^{1T}\bar{e}_1^1 + \{G_t/G_p\}(r-u) + \Phi^T V \\ \dot{\bar{e}}^1 = \Lambda \bar{e}^1 \end{cases} \quad (3.12)$$

where G_t and G_p are the numerator and the denominator of the following transfer function,

$$1 + \frac{{}_1\bar{\beta}^{1T}}{{}_1\beta_1} (sI - \Lambda)^{-1} \bar{l} \quad (3.13)$$

respectively.

Choose a control input as follows :

$$u = r - G_p[K_1 G_t^{-1} x_1 + \bar{K}^{1T}[G_t^{-1}]\bar{y}^1 + {}_1K_2 G_t^{-1} u + {}_2K_2 G_t^{-1}\tau + {}_1\bar{K}^{2T}[G_t^{-1}]\,{}_1\hat{\bar{x}}^2 + {}_2\bar{K}^{2T}[G_t^{-1}]\,{}_2\bar{y}^2] \quad (3.14)$$

and the control gain adjustment rules are given by a set of differential equations :

$$\begin{cases} \dot{K}_1 = -\frac{1}{{}_1\beta_1} g_1 \eta_1 x_1 \\ \dot{\bar{K}}^1 = -\frac{1}{{}_1\beta_1} G^1 \eta_1 \bar{y}^1 \\ {}_1\dot{K}_2 = -\frac{1}{{}_1\beta_1} {}_1g_2 \eta_1 u \\ {}_2\dot{K}_2 = -\frac{1}{{}_1\beta_1} {}_2g_2 \eta_1 \tau \\ {}_1\dot{\bar{K}}^2 = -\frac{1}{{}_1\beta_1} {}_1G^2 \eta_1 \, {}_1\hat{\bar{x}}^2 \\ {}_2\dot{\bar{K}}^2 = -\frac{1}{{}_1\beta_1} {}_2G^2 \eta_1 \, {}_2\bar{y}^2 \end{cases} \quad (3.15)$$

where

$\eta_1 = e_1 + \xi_1$

and ξ_1 is given by

$$\dot{\xi}_1 = -\lambda_1 \xi_1 - {}_1\beta_1 f(\dot{K}_1, \dot{\bar{K}}^1, {}_1\dot{K}_2, {}_2\dot{K}_2, {}_1\dot{\bar{K}}^2, {}_2\dot{\bar{K}}^2)$$

where $f(\cdot)$ is a function of the derivatives of K in eq. (3.12) after substitution of eq. (3.14) into eq. (3.12).

Therefore at the end of the adaptation all derivatives go to zero and so do $f(\cdot)$ and ξ_1, too.

The proof of eq. (3.14) is as same as the previous case and so is omitted here.

4. Conclusions

In the previous section, the stable adaptive schemes have been derived in the framework of the model reference adaptive scheme in case there are some memoryless nonlinear elements in a feedback loop.

The proposed schemes can be ready to be applied for some nonlinear systems. Among them, the conventional neutron dynamics in a reactor core can be described by nonlinear differential equations when there exist two temperature feedbacks, due to fuel and coolant flow temperatures[1]. The feedback of fuel temperature has much shorter time constant than that of coolant flow. Then stability problems arise when the sign of feedback coefficients becomes positive or negative. That is to say, it is well known that there exist nonlinear oscillations or limit cycles in a reactor core with fast and slow mode feedbacks[1),2)]. It is imperative both to stabilize and control a reactor, and to assess stability margins for the safe operation of a nuclear reactor. This kind of problem can be successfully tackled as a special case of this report[3)].

Acknowledgements

I would like to express my deep and sincere gratitude to Professor Heki Shibata and Associate Professor Takafumi Fujita, the Institute of Industrial Science, University of Tokyo, for helpful advices throughout this work.

(Manuscript received, September, 25.1976)

References

(1) S.A.Vreeke and G.M.Sandquist, Phase space analysis of reactor kinetics, N.S.E. 42−3, Dec. 1970, p.295.
(2) J.P.Keener and D.S.Cohen, Nonlinear oscillations in a reactor with two temperature coefficients, N.S.E. 56−4, Nov. 1975, p.354.
(3) T.Fukuda, Parameter identification and control of a nuclear reactor with two temperature feedbacks-Application of nonlinear adaptive observer and controller, J.S.M.E.Preprint № 760−14, Oct. 1976, p.220 (in Japanese)

東京大学生産技術研究所報告刊行予告

第26巻　第3号

勝田　高司・村上　周三・吉野　博著

住宅設備の性能評価に関する研究（和文）

— 主としてエネルギー消費と住い方の観点から —

本報告は，暖冷房・給湯・換気設備を中心とする住宅設備について，エネルギー消費量や住い方などの生活実態の面から，性能評価を試みたものである．1章では，モデルとしての実験住宅を用いた性能評価の方法について述べ，その評価法に基づき，各種の観測によって，住宅を構成する機能要素－住宅設備，室内環境，屋外環境，シェルター，住い方，エネルギー消費－の相互関係を明らかにし，住宅設備の性能を明らかにしている．2章では，エネルギー消費量が設備の使用状況と密接な関連を持ち，住宅の質的内容を示す数値と考えられることから，一般の住宅におけるエネルギー消費実態を詳細に把握し，エネルギー消費と設備構成，シェルター性能，室内環境等との関連を明らかにし，エネルギー消費グレードが住宅設備性能の総合的指標になることを示している．また，実験住宅を用いて明らかにされた機能要素の相互関係を一般の住宅に適用する場合に，このエネルギー消費グレードが利用できることを述べている．3章では，住宅設備の導入を検討するに当たって，現時点の住宅設備に対する居住者の要求度を調べ，設備性能の評価項目を明らかにし，その項目に基づいて，設備方式を機能や生活維持，管理の面から評価している．最後に，既に明らかにした実験住宅における設備の評価やエネルギー消費実態も考慮して，近い将来の住宅生産における設備構成の標準を提案し，その標準に対応するエネルギー消費グレードを明らかにしている．（1977年2月中旬刊行予定）