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Application of Stable Parameter Identification and Control Scheme for the Classical Lur'e Problem (PART I)

古典ルーリエ問題への安定なパラメータ同定と制御則の応用（その1）

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Summary

Numerous works have been done on the absolute stability problem proposed by Lur'e and Postnikov in the area of nonlinear system stability. This type of the problem often appears when total linearization is not possible, but partial linearization is often profitable. On the basis of the above results; we can show the stable parameter identification and control scheme for the Lur'e problem in the literature of the Model Reference Adaptive System (MRAS) via the Lyapunov method.

This is ready to be applied to the two temperature feedback model of nuclear reactor systems for the purpose of adaptive parameter identifications and controls.

1. Introduction

There have been many papers published on the adaptive parameter identification and control of linear time invariant systems so far[1),2),3),4)], but few of nonlinear systems, due to the difficulty of the stability of nonlinear systems[5)]. Among the nonlinear systems, the Lur'e problem on the absolute stability is one which has been most extensively studied and whose results are well known as the Lur'e resolving equations, the Popov's method[6)], and the circle criterion[7)]. Therefore we focus on the adaptive problem ot Lur'e type nonlinear systems.

In the following section, we show the adaptive parameter identification and control of the above mentioned nonlinear system are stable, regardless the stability of a plant or a model. This can be considered as a step forward to tackle adaptive problems of nonlinear systems.

In the field of a nuclear reactor system, suppose there are two kinds of feedback, fast and slow feedback, the existence of nonlinear oscillations or limit cycles has been pointed out as one of the harmful sources for reactor operations[8),9)]. The proposed adaptive scheme is expected effective and useful to know the stability margin of a plant or eliminate the oscillations.

2. Formulation

The Lur'e problem has been traditionally classified into two types of problems in the past, which are called direct control and indirect control. Direct control is described by eq. (2.1), while indirect control described by eq. (2.2).

$$\dot{X}=AX+bu,\ \sigma_0=h^TX+\delta u,\ u=v+\tau,\ \tau=-g(\sigma_0,t) \tag{2.1}$$

$$\dot{X}=AX+bu,\ \sigma_0=h^TX+\delta u,\ u=v+\tau,\ \dot{\tau}=-g(\sigma_0,t) \tag{2.2}$$

where X is a n-dimensional vector, u a scalor, v an input, and the triple (h,A,b) has corresponding dimensions.

These equations, however, turn out to be equivalent if the linear part of eq. (2.1) has an eigenvalue $s=0$. Therefore we consider eq. (2.1) throughout this report without loss of generality.

The problems dealt with are clearly stated as follows:

I. "For the parameter identification, find out the stable parameter identification rule for the unknown triple (h,A,b)."

II. "For the adaptive control, find out the stable control rule for the unknown triple (h,A,b)."

For the sake of simplicity, set $\delta=0$ in the following, and the linear part of eq. (2.1) is also assumed to be completely controllable and completely observable. Then any linear time invariant, completely controllable and completely observable,

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single input-single output system may be represented in the following nonminimal realization form.

$$\begin{cases} \dfrac{d}{dt}\begin{pmatrix} x_1 \\ \overline{x}^1 \\ \overline{x}^2 \end{pmatrix} = \begin{bmatrix} -a_1 & -\overline{a}^1 & \overline{b}^1 \\ \overline{l} & \Lambda & 0 \\ 0 & 0 & \Lambda \end{bmatrix}\begin{pmatrix} x_1 \\ \overline{x}^1 \\ \overline{x}^2 \end{pmatrix} + \begin{pmatrix} b_1 \\ 0 \\ \overline{l} \end{pmatrix} u \\ \sigma_0 = h^T x = (1\ 0 \cdots\cdots 0)\, x \end{cases} \quad (2.3)$$

where $(\overline{l}, \Lambda)$ is any obsevable pair, x is a scalor, $\overline{x^1}$, $\overline{x^2}$ are $(n-1)$ dimensional vectors. And the initial condition is $x^T = (x_{10}, \overline{x}_0^{1T}, \overline{0}^T)$.
Then the plant can be assumed to be represented by the following differential equation,

$$\text{PLANT:} \begin{cases} \dot{x}_1 = -a_1 x_1 - \overline{a}^{1T}\overline{x}^1 + b_1 u + \overline{b}^{1T}\overline{x}^2 \\ \dot{\overline{x}}^1 = \Lambda\,\overline{x}^1 + \overline{l} x_1 \\ \dot{\overline{x}}^2 = \Lambda\,\overline{x}^2 + \overline{l} u \\ u = r - g(x_1, t) \end{cases} \quad (2.4)$$

with initial conditions,

$$x_1(0) = x_{10},\quad \overline{x}^1(0) = \overline{x}_0^1,\quad \overline{x}^2(0) = \overline{0}$$

where $g(x_1, t)$ is a known nonlinear function, and r is a reference input.

3. Identification

We can now set up a model for the parameter identification purpose, associated with eq. (2.4):

$$\text{MODEL:} \begin{cases} \dot{\hat{x}}_1 = -\hat{a}_1 x_1 - \hat{\overline{a}}^{1T}\hat{\overline{x}}^1 + \hat{b}_1 u + \hat{\overline{b}}^{1T}\hat{\overline{x}}^2 - \lambda_1(\hat{x}_1 - x_1) \\ \dot{\hat{\overline{x}}}^1 = \Lambda\hat{\overline{x}}^1 + \overline{l} x_1 \\ \dot{\hat{\overline{x}}}^2 = \Lambda\hat{\overline{x}}^2 + \overline{l} u \\ u = r - g(x_1, t) \qquad \lambda_1 > 0 \end{cases} \quad (3.1)$$

with the initial conditions,

$$\hat{x}_1(0) = 0,\quad \hat{\overline{x}}^1(0) = \hat{\overline{x}}^2(0) = \overline{0}$$

Define errors,

$$e_1 = \hat{x}_1 - x_1,\quad \overline{e}^1 = \hat{\overline{x}}^1 - \overline{x}^1,\quad \overline{e}^2 = \hat{\overline{x}}^2 - \overline{x}^2$$

Then substruction from eq. (3.1) to eq. (2.4) leads to the following error equation.

$$\begin{cases} \dot{e}_1 = -\lambda_1 e_1 - \overline{a}^{1T}\overline{e}^1 + \overline{b}^{1T}\overline{e}^2 + \Phi^T\overline{V} \\ \dot{\overline{e}}^1 = \Lambda\overline{e}^1 \\ \dot{\overline{e}}^2 = \Lambda\overline{e}^2. \end{cases} \quad (3.2)$$

with $e_1(0) = e_{10}$, $\overline{e}^1(0) = \overline{e}_0^1$, $\overline{e}^2(0) = \overline{0}$, where Φ and $\overline{V}$ are 2n dimensional vectors,

$$\Phi^T = (-\hat{a}_1 + a_1,\ -(\hat{\overline{a}}^1 - \overline{a}^1)^T,\ \hat{b}_1 - b_1,\ (\hat{\overline{b}}^1 - \overline{b}^1)^T)$$
$$\overline{V}^T = (x_1,\ \hat{\overline{x}}^{1T},\ u,\ \hat{\overline{x}}^{2T})$$

From the initial conditions, note $\overline{e}^2(t) = \overline{0}$ for all t, which yields the simple error equation written in a vector matrix form,

$$\begin{cases} \dot{e} = Ae + d\Phi^T\overline{V} \\ \varepsilon_1 = h^T e \end{cases} \quad (3.3)$$

where $e^T = (e_1, \overline{e}^{1T})$, $A = \begin{bmatrix} -\lambda_1 & -\overline{a}^{1T} \\ 0 & \Lambda \end{bmatrix}$, $A^T = (1, \overline{0}^T)$, $h^T = (1, \overline{0}^T)$.

Now we can set a Lyapunov function candidate as follows:

$$V = \frac{1}{2} e^T P e + \frac{1}{2}\Phi^T G^{-1}\Phi \quad (3.4)$$

where $P = P^T > 0$, $G = G^T > 0$.
It is clear that V is bounded from the above and below by the norm in the $e-\Phi$ plane. The differentiation of eq. (3.4) with respect to time, leads to the following:

$$\dot{V} = \frac{1}{2} e^T(A^T P + PA)e + \Phi^T(G^{-1}\dot{\Phi} + \overline{V}e^T Pd)$$

We invoke the Kalman-Yakubovich lemma[7] here, such that there exist $P = P^T > 0$ and q, satisfying the following algebraic equations for sufficiently small ε_0 and a given positive definite matrix $L_0 = L_0^T > 0$,

$$\begin{cases} A^T P + PA = -qq^T - \varepsilon_0 L_0 \\ Pd - h = \overline{0} \end{cases} \quad (3.5)$$

because the transfer function of eq. (3.3) is

$$h^T(sI - A)^{-1} d = \frac{1}{s + \lambda_1} \in \{S.P.R.\} \quad (3.6)$$

which is a strictly positive real function. If we choose the adaptive rule as,

$$\dot{\Phi} = -\varepsilon_1 G\overline{V} \quad (3.7)$$

then

$$\dot{V} = -\frac{1}{2}|e^T q|^2 - \varepsilon_0 e^T L_0 e$$

From the Lyapunov invariant set theorem, the origin in the $e-\Phi$ plane, $e = \overline{0}$ and $\Phi = \overline{0}$, is clearly only one invariant set. From $\dot{\Phi} = \overline{0}$ and $\Phi^T\overline{V} = 0$, $\Phi = \overline{0}$ can be obtained if r is modulated so that each component of $\overline{V}$ is linearly independent.

If we choose G such that $G = \begin{bmatrix} g_1\overline{G}^1 & 0 \\ 0 & g_2\overline{G}^2 \end{bmatrix}$

then eq. (3.7) becomes

$$\begin{cases} \dot{\hat{a}}_1 = \varepsilon_1 g_1 x_1, & \dot{\hat{b}}_1 = -\varepsilon_1 g_2 u \\ \dot{\hat{\overline{a}}}^1 = \varepsilon_1 \overline{G}^1\hat{\overline{x}}^1, & \dot{\hat{\overline{b}}}^1 = -\varepsilon_1\overline{G}^2\hat{\overline{x}}^2 \end{cases} \quad (3.8)$$

with any arbitary initial conditions.

4. Adaptive Control

we can set up a model with parameters, α_1, $\overline{\alpha}^1$, β_1, and $\overline{\beta}^1$, which correspond to those of the plant. During the transient period of adaptation, the controller model can be represented as follows:

$$\begin{cases} \dot{y}_1 = -\alpha_1 x_1 - \bar{a}^{1T}\bar{y}^1 + \beta_1 u_r + \bar{\beta}^{1T}\bar{y}^2 - \lambda_1(y_1 - x_1) \\ \dot{\bar{y}}^1 = \Lambda\bar{y}^1 + \bar{l}x_1 \\ \dot{\bar{y}}^2 = \Lambda\bar{y}^2 + \bar{l}u_r \\ u_r = r - g(x_1, t) \end{cases} \quad (4.1)$$

and the control input to the plant should be written as

$$u_p = v - g(x_1, t) \quad (4.2)$$

Then similary to the identification problem, the substruction from eq. (4.1) to eq. (2.4) leads to the following differential error equations,

$$\begin{cases} \dot{e}_1 = -\lambda_1 e_1 - \bar{a}^{1T}\bar{e}^1 + \beta_1(u_r - u_p) + \bar{\beta}^{1T}\bar{e}^2 + \Phi^T\bar{V} \\ \dot{\bar{e}}^1 = \Lambda\bar{e}^1 \\ \dot{\bar{e}}^2 = \Lambda\bar{e}^2 + \bar{l}(u_r - u_p) \end{cases} \quad (4.3)$$

with the initial conditions,

$$e_1(0) = e_{10},\ \bar{e}^1(0) = \bar{e}_0^1,\ \bar{e}_0^2(0) = \bar{0}$$

and Φ and $\bar{V}$ are 2n dimensional vectors such that

$$\Phi^T = (-\alpha_1 + a_1, -(\bar{\alpha}^1 - \bar{a}^1)^T, \beta_1 - b_1, (\bar{\beta}_1^1 - \bar{b}^1)^T)$$

$$\bar{V}^T = (x_1, \bar{y}^{1T}, u_p, \hat{\bar{x}}^{2T})$$

where $\hat{\bar{x}}^2$ can be generated by the equation,

$$\begin{cases} \dot{\hat{\bar{x}}}^2 = \Lambda\hat{\bar{x}}^2 + \bar{l}u_p \\ \hat{\bar{x}}^2(0) = \bar{0} \end{cases} \quad (4.4)$$

and so $\bar{x}^2(t) = \hat{\bar{x}}^2(t)$ for all t.

Therefore we can choose the control input to the plant as

$$u_r - u_p = G_p[K_1 G_t^{-1} x_1 + \bar{K}^1[G_t]^{-1}\bar{x}^1 + K_2 G_t^{-1} u_p + \bar{K}^{2T}[G_t]^{-1}\hat{\bar{x}}^2]$$

$$\therefore v = r - G_p[K_1 G_t^{-1} x_1 + \bar{K}^{1T}[G_t]^{-1}\bar{y}^1 + K_2 G_t^{-1} u_p + \bar{K}^{2T}[G_t]^{-1}\hat{\bar{x}}^2] \quad (4.5)$$

where K_1, $\bar{K}^1$, K_2, $\bar{K}^2$ are control gains, and

$$G_p = (s+\lambda_2)(s+\lambda_3)\cdots(s+\lambda_n)$$

$$G_{p,i} = (s+\lambda_2)(s+\lambda_3)\cdots(s+\lambda_{i-1})(s+\lambda_{i+1})\cdots(s+\lambda_n)$$

$$G_t = G_p + \bar{\beta}^2 G_{p,2} + \cdots + \bar{\beta}^n G_{p,n} \quad (\bar{\beta}^i = \beta^i/\beta_1)$$

$$\bar{l}^T = (1\ 1 \cdots\cdots 1)$$

$$\Lambda = \begin{bmatrix} -\lambda_2 & & 0 \\ & -\lambda_3 \ddots & \\ 0 & & -\lambda_n \end{bmatrix}$$

The stable gain adjustment rules are given by a set of differential equations,

$$\begin{cases} \dot{K}_1 = -\frac{g_1}{\beta_1}\eta_1 x_1 \\ \dot{\bar{K}}_1 = -\frac{1}{\beta_1}\bar{G}^1\eta_1\bar{y}^1 \\ \dot{K}_2 = -\frac{1}{\beta_1}g_2\eta_1 u_P \\ \dot{\bar{K}}^2 = -\frac{1}{\beta_1}\bar{G}^2\eta_1\hat{\bar{x}}^2 \end{cases} \quad (4.6)$$

where $\eta_1 = \varepsilon_1 + \xi_1$

and ξ_1 shoud be generated by the following differential eguation.

$$\dot{\xi}_1 = -\lambda_1\xi_1 - \beta_1 f(\dot{K}_1, \dot{\bar{K}}^1, \dot{K}_2, \dot{\bar{K}}^2) \quad (4.7)$$

where $f(\cdot)$ is a function of the derivatives of control gains, arising from the term consisting of all derivatives in eq. (4.5).

Eq. (4.5) can be derived as follows:

From eq. (4.3), the error equation can be written

$$(s+\lambda_1)e_1 + \bar{a}^{1T}\bar{e}^1 = \beta_1 G_P^{-1} G_t(u_r - u_P) + \Phi^T\bar{V} \quad (4.8)$$

If we choose the control input of eq. (4.5), then eq. (4.8) turns out to be

$$(s+\lambda_1)e_1 + \bar{a}^{1T}\bar{e}^1 = \beta_1 G_t\{K_1 G_t^{-1} x_1 + \bar{K}^{1T}[G_t]^{-1}\bar{y}^1 + K_2 G_t^{-1} u_P + \bar{K}^{2T}[G_t]^{-1}\hat{\bar{x}}^2\} + \Phi^T\bar{V}$$

$$\therefore \dot{e}_1 = -\lambda_1 e_1 - \bar{a}^{1T}\bar{e}^1 + \beta_1(K_1 x_1 + \bar{K}^{1T}\bar{y}^1 + K_2 u_P + \bar{K}^{2T}\hat{\bar{x}}^2) + \beta_1 f(\dot{K}_1, \dot{\bar{K}}^1, \dot{K}_2, \dot{\bar{K}}^2) + \Phi^T\bar{V}$$

From eq. (4.7), the total error equation becomes as follows:

$$\begin{cases} \dot{\eta}_1 = -\lambda_1\eta_1 - \bar{a}^{1T}\bar{e}^1 + \{(\beta_1 K_1 - \alpha_1 + a_1)x_1 + (\beta_1\bar{K}^1 - \bar{\alpha}^1 + \bar{a}^{1T})\bar{x}^1 + (\beta_1 K_2 + \beta_1 - b_1)u_P + (\beta_1\bar{K}^2 + \bar{\beta}^1 - \bar{b}^{1T})\bar{x}^2\} \\ \dot{\bar{e}}^1 = \Lambda\bar{e}^1 \end{cases} \quad (4.9)$$

which is equivalent to eq. (4.10).

$$\begin{cases} \dot{\eta} = A\eta + d\Psi^T\bar{V} \\ \eta_1 = h^T\eta \end{cases} \quad (4.10)$$

where

$$\eta^T = (\eta_1, \bar{e}^{1T}),\ d^T = (1\ \bar{o}^T),\ h^T = (1\ \bar{o}^T)$$

$$A = \begin{bmatrix} -\lambda_1 & -\bar{a}^{1T} \\ o & \Lambda \end{bmatrix}$$

$$\Psi^T = \Phi^T + \beta_1(K_1, \bar{K}^{1T}, K_2, \bar{K}^{2T})$$

Since the transfer function of eq. (4.10) is

$$h^T(sI - A)^{-1}d = \frac{1}{s+\lambda_1}$$

which is strictly positive real, the differentiation of the following Lyapunov function candidate,

$$V = \frac{1}{2}\eta^T P\eta + \frac{1}{2}\Psi^T G^{-1}\Psi \quad (4.11)$$

leads to negative semi-definite with the aid of the Kalman Yakubovich lemma;

$$\dot{V} = -\frac{1}{2}|\eta^T q|^2 - \frac{1}{2}\varepsilon_0\eta^T L_0\eta \leq 0$$

with substitution of eq. (4.6), where $P = P^T > 0$ and $G = G^T > 0$.

Note that control gains converge to some values at the end of the adaptation, and so $f(\cdot)$ converges to zero. Then $\Psi = \bar{0}$ and $\eta = \bar{0}$ can be brought about from the invariant set theorem if the reference input r can be modulated so that each component of $\bar{V}$

is linearly independent as the case of the identification problem. This guarantees that the adaptive control rule of eq. (4.5) and (4.6), is asymptotically stable in the large.

This can be easily extended to the case of multiple inputs without any difficulties.

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