研 究 速 報

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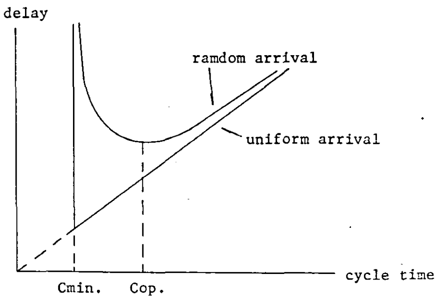
OPTIMIZATION OF CYCLE TIME FOR COMPUTERIZED TRAFFIC SIGNAL SYSTEMS

交通信号の電子計算機制御におけるサイクル長の最適化

by Masaki KOSHI*
越 正 毅

1. Introduction

Delay and number of stops at a stop line of a signalized intersection depend obviously on cycle time. In case of an isolated intersection with fixed green time percentage, delay at a stop line vs. cycle time relationship is expressed as shown in Fig. 1. Delay increases monotonously as cycle time increases in the range over the optimum cycle length. The formulae of the optimum cycle time for isolated intersections have been developed by Webster[1] and others.



Cmin. is the minimun cycle time.
Cop. is the optimun cycle time.

Fig. 1 Delay vs. cycle time at a stop line of an isolated intersection.

For coordinated control of a group of signals, however, the delay vs. cycle time relationship is not as simple as that for an isolated intersection. Let us think of the simplst example of two traffic signals connected by a two-way link. Assume that (1) the effective green times and the saturation flow rates of the stop lines are the same, (2) the proportion of the green times to the cycle time is constant, (3) there is no turning traffic, (4) the green times are just saturated and (5) there is no platoon dispersion. In this single saturated rectangular wave model, the delay vs. cycle time relationship is expressed in a relatively regular way as shown in Fig. 2. The relationship would become more complicated in the real field since the above assumptions no more hold true and many links of different length are included in a system to which a common cycle time should be applied. It is nevertheless understandable from Figs. 1 and 2 that there may exist the optimum value of cycle time for coordination of a traffic signal system in the different sense from that for an isolated intersection control.

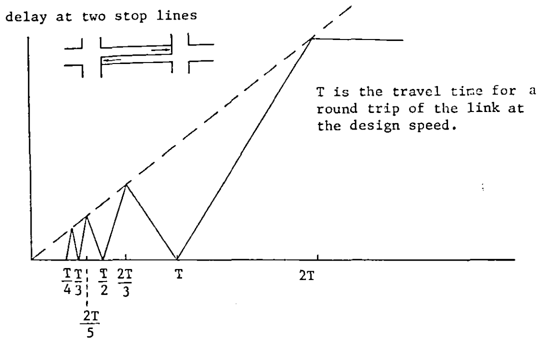


Fig. 2 Delay vs. cycle time of single a link based on single saturated rectangular arrival model.

The theory and the method presented here are for obtaining differntial coefficient of delay or number of stops with respect to cycle time on-line based on detector pulses. Once it is obtained, it is clear that feedback optimization of cycle time becomes possible.

* Dept. of Building and Civil Engineering, Inst. of Industrial Science, Univ. of Tokyo.

2. Formulation of the Problem

The author has already pointed out[2] that differential coefficient of delay at a stop line with respect to relative offset of the link can be obtained on-line based on the detector informations and that on-line feedback optimazation of offsets is practically possible.

Let us take a link with signalized stop lines A and B at the ends and define the relative offset of the link as time from the start of green of A till the start of green of B. Let f_A and f_B be the differential coefficients of delay per cycle at stop lines A and B, respectively, with respect to the relative offset of the link.

Let us assume, just for easy understanding, that $f_A>0$, $f_B<0$. That $f_A>0$ means that delay at stop line A would be decreased by f_A if the green at stop line A started one second earlier and that $f_B<0$ means that delay at stop line B would be decreased by $-f_B$ if the green at stop line B started also one second earlier. It is impossible, of course, to give one second earlier start to both greens at the same time only through offset variation, but only if cycle time is shortened by two seconds, it becomes possible to give one second earlier start to both greens and then link delay decrease by f_A-f_B per cycle looks to be obtained. f_A-f_B is, in this sense, a measure of delay decrease caused by offset improvement which is enabled by cycle time shortening. Therefore, let us now name $\frac{1}{2}(f_A-f_B)$ "an apparent differential coefficient of delay of a link in a cycle with respect to cycle time".

There is another general effect of cycle time on delay that delay in a cycle is proportional to to the square of cycle time if the arrival pattern is similar. Even if f_A-f_B is equal to zero, for instane, delay per cycle would still increase as the cycle time increases.

From these preliminary considerations, the problem can now be formulated as follows:

$$\left.\begin{aligned} G(C+\Delta C)&=\{G(C)+f(C)\cdot\Delta C\}\left(\frac{C+\Delta C}{C}\right)^2 \\ f(C)&=\frac{1}{2}\{f_A(C)-f_B(C)\} \end{aligned}\right\} \quad (1)$$

where C is cycle time,

$G(C)$ is delay of the link in a cycle,

$f(C)$ is apparent differential coefficient of delay of the link in a cycle with respect to cycle time, and

$f_A(C)$ and $f_B(C)$ are differential coefficients of delay at stop lines A and B, respectively, with respect to relative offset of the link between the two stop lines where relative offset is so defined as time from the start of green at stop line A till the start of green at stop line B.

There are general equations as follows, as well:

$$g(C)=\frac{G(C)}{C} \quad (2)$$

$$g(C+\Delta C)-g(C)=\frac{G(C+\Delta C)}{C+\Delta C}-\frac{G(C)}{C} \quad (3)$$

where $g(C)$ is delay per unit time.

From Eqs. 1, 2 and 3, we obtain

$$\begin{aligned} \frac{G(C+\Delta C)}{C+\Delta C}&=\{G(C)+f(C)\cdot\Delta C\}\frac{C+\Delta C}{C^2} \\ &=\frac{G(C)}{C}+G(C)\frac{\Delta C}{C^2} \\ &\quad+f(C)\cdot\Delta C\frac{C+\Delta C}{C^2} \end{aligned}$$

$$\begin{aligned} \frac{G(C+\Delta C)}{C+\Delta C}-\frac{G(C)}{C}&=\frac{G(C)}{C^2}\cdot\Delta C \\ &\quad+f(C)\cdot\Delta C\frac{C+\Delta C}{C^2} \end{aligned} \quad (4)$$

$$\frac{g(C+\Delta C)-g(C)}{\Delta C}=\frac{G(C)}{C^2}+f(C)\frac{C+\Delta C}{C^2} \quad (4)$$

Then we obtain differential coefficient of delay of the link per unit time with respect to cycle time.

$$\begin{aligned} \frac{d}{dc}g(C)&=\lim_{\Delta C\to 0}\left\{\frac{g(C+\Delta C)-g(C)}{C}\right\} \\ &=\frac{G(C)}{C^2}+\frac{f(C)}{C} \end{aligned} \quad (6)$$

or,

$$=\frac{g(C)+f(C)}{C} \quad (7)$$

The similar discussion leads to differential coefficient of number of stops with respect to cycle time as follows:

$$S(C+\Delta C)=\{S(C)+h(C)\cdot\Delta C\}\frac{C+\Delta C}{C} \tag{8}$$

$$h(C)=\frac{1}{2}\{h_A(C)-h_B(C)\}$$

$$s(C)=\frac{S(C)}{C} \tag{9}$$

where $S(C)$ is number of stops of the link in a cycle,
$h(C)$ is apparent differential coefficient of number of stops of the link in a cycle with respect to cycle time,
$h_A(C)$ and $h_B(C)$ are differential coefficients of delay at stop lines A and B, respectively, with respect to relative offset of the link with the same definitions of the link and relative offset as those for Eq. 1, and
$s(C)$ is number of stops per unit time.

$$\frac{S(C+\Delta C)}{C+\Delta C}=\frac{S(C)}{C}+\frac{h(C)\cdot\Delta C}{C} \tag{10}$$

$$\frac{s(C+\Delta C)-s(C)}{\Delta C}=\frac{h(C)}{C} \tag{11}$$

$$\frac{d}{dc}s(C)=\lim_{\Delta C\to 0}\left\{\frac{s(C+\Delta C)-s(C)}{\Delta C}\right\}=\frac{h(C)}{C} \tag{12}$$

When there are n links in a system, Eqs. 7 and 12 become followings:

$$\frac{d}{dc}\sum^{n} g_i(C)=\frac{1}{C}\{\sum^{n} g_i(C)+\sum^{n} f_i(C)\} \tag{13}$$

$$\frac{d}{dc}\sum^{n} s_i(C)=\frac{1}{C}\sum^{n} h_i(C) \tag{14}$$

where i denotes link number.

3. On-Line Measurement of Differential Coefficients

It has already been indicated[2] that $f_A(C)$, $f_B(C)$, $h_A(C)$ and $h_B(C)$ in Eqs. 1 and 8 can be measured on-line with adequately located vehicle detectors. If it is allowed to assume that saturation flow rate is a certain fixed value, only one vehicle detector located 100 to 150 meters upstream of a stop line for detecting input flow is enough to obtain the differential coefficients of the stop line. This holds true also for obtaining delay value at a stop line and the same upstream detector is enough. When saturation flow rate is not consistent enough to be assumed constant, another detector located very close to the stop line is necessary in order to measure output flow.

4. Field Experiments

A small-scale experimental traffic signal control system in Tokyo was used for the tests. The system has five traffic signals as shown in Fig. 3

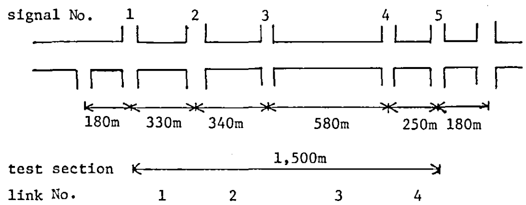


Fig. 3 Experimental signal system.

and is equipped with a mini computer. Although each stop line has two detectors, one close to the stop line and the other about 150 meters upstream, only the upstream detectors were used for the tests since the saturation flow rates are fairly consistent.

4.1 Delay vs. Cycle Time Relationship

Fig. 4 shows delay of each link as well as of the system as a whole at various cycle times. Offsets were automatically optimized through the on-line feedback control technique[2] during the course of delay observations. Delay values and traffic volumes were typewritten every fifteen minutes. Many delay data for the specified traffic volume ranges were obtained and only the averages are shown in Fig. 4.

4.2 On-line Cycle Time Optimization

Differential coefficient of delay with respect to cycle time was calculated every 15 minutes and cycle time was varied by two seconds also every 15 minutes when the coefficient value was over a certain threshold.

Fig. 5 shows the process of cycle time optimi-

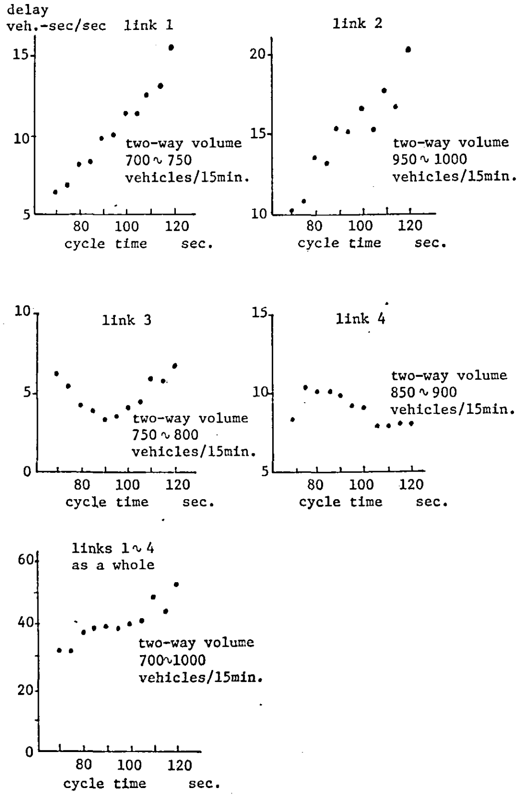


Fig. 4 Delay vs. cycle time.

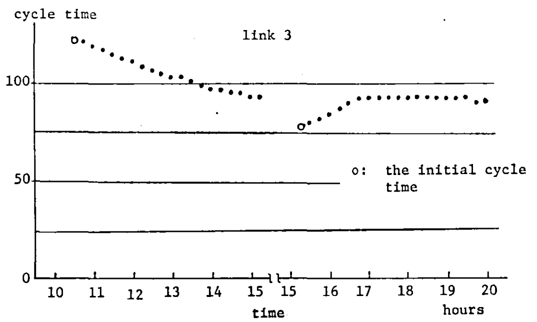


Fig. 5 Cycle time optimization process of link 3.

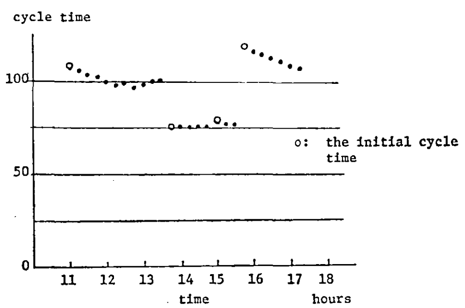


Fig. 6 Cycle time optimization process of four links as a whole.

zation when it was applied only to the link 3. The threshold was ±0.05 veh. sec. per sec. The figure shows that the feedback optimization settled cycle time at around 90 seconds which is about the optimum length as seen in Fig. 4.

Fig. 6 shows the similar process when the technique was applied to the four links as a whole. The threshold was 0.1 veh. sec. per sec. When optimization started from the initial value of 110 seconds, cycle time kept decreasing untill it reached about 95 seconds and then stayed around that value. Although it is not quite clear in Fig. 4 if there is a minimum point at about 95 seconds or not there must have been a local optimum point on that particular day of the test or otherwise, the delay curve must have been very flat about 95 seconds cycle length. With the initial value of 75 seconds, cycle time did not change because the lower limit was set at 75 seconds. When the initial value was 80 seconds, cycle time decreased and soon reached the lower limit.

5. Concluding Remarks

The method of on-line feedback optimization of cycle time presented here can be considered to work as is theoretically expected. Difficulty to avoid sitting down in local optimals and to reach the real optimum is a general problem for this type of optimization. It should be necessary, therefore, to give manually the practically allowable range of cycle time within which automatic optimization should be made.

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References

1) F. V. Webster and B. M. Cobbe, Traffic Signals, Road reserch Technical Paper No. 56, Road Research Laboratory, 1966.
2) M. Koshi, On-Line Feedback Control of Offsets for Area Control Traffic, The Fifth International Symposium on the Theory of Traffic Flow and Transportation, June, 1971.