研 究 速 報
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ON THE CRITICAL SIZE OF DROP DETACHMENT DURING THE PROCESS OF DROPWISE CONDENSATION (II)

滴状凝縮過程における離脱液滴径について (II)

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1. Profile of Drops Placed on Smooth Inclined Surfaces

Figure 1 represents a typical profile of a drop placed on an inclined plate. It is observed that the drop adhering to the surface resisting to the external forces exhibits different angles of contact around its periphery. In Fig. 1 the angle denoted θ_a is called advancing contact angle, while θ_r is called receding contact angle. It should, however, be noticed here that these two contact angles are the ones measured within the plane of symmetry, which is to be parallel to the direction of the resultant external force. However, since the drop is three-dimensional, contact angles measured within the other planes must take different values.

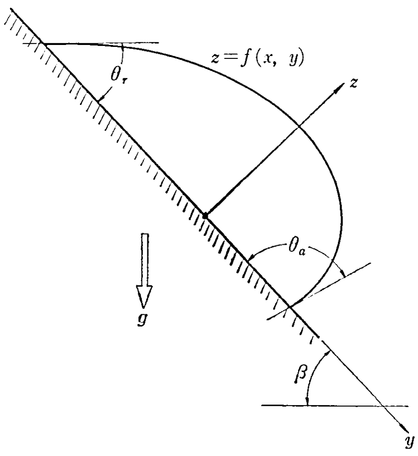

Fig. 1

Let us first assume that the inclined plate in Fig. 1 is ideally smooth and the inclination is β, and that the earth's gravity is the only external force. If the profile of the liquid-vapor interface

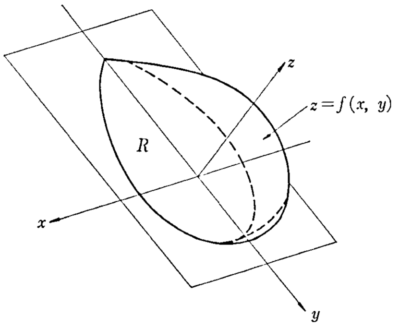

Fig. 2

is expressed as $z=f(x, y)$ (Fig. 2), then the three interfacial energies and the gravitational potential energy are expressed as follows:

$$E_{lv}=\iint_R \sigma_{lv}\sqrt{1+f_x^2+f_y^2}\,dxdy \qquad (1)$$

$$E_{ls}=\iint_R \sigma_{ls}dxdy \qquad (3)$$

$$E_{vs}=-\iint_R \sigma_{vs}dxdy+C \qquad (3)$$

$$U=-\iint_R \rho g\sin\beta\cdot yf(x, y)dxdy \qquad (4)$$

where $f_x=\partial f/\partial x$ and $f_y=\partial f/\partial y$ and R designates the region of the plate covered by the drop. Thus, the total energy of this system becomes

$$E=\iint_R \{\sigma_{lv}\sqrt{1+f_x^2+f_y^2}+(\sigma_{ls}-\sigma_{vs}) -\rho g\sin\beta\cdot yf(x, y)\}\,dxdy \qquad (5)$$

On the other hand, the mass of the liquid drop, which is to be invariable, is

$$M=\iint_R \rho f(x, y)dxdy \qquad (6)$$

Quite similar to the previous report, the profile of the drop attached to the inclined plate is so determined that the total energy E may take the minimum value under the constraint $M=$*constant*. Or, introducing the Lagrange multiplier

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λ, it becomes the variational problem of finding a function $f(x, y)$ which satisfies

$$\delta H \equiv \delta(E+\lambda M)=0 \tag{7}$$

Here the boundary condition which holds along the periphery of the drop on the plate is the natural boundary condition.

As is well known the solution to this variational problem can be found by solving Euler equation

$$\frac{\partial}{\partial x}\left(\frac{\partial F}{\partial f_x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial F}{\partial f_y}\right)-\frac{\partial F}{\partial f}=0 \tag{8}$$

$$\begin{aligned} F(x, y, f, f_x, f_y) &\equiv \sigma_{lv}\sqrt{1+f_x^2+f_y^2}+(\sigma_{ls}-\sigma_{vs}) \\ &\quad -\rho g \sin\beta\cdot y f \end{aligned} \tag{9}$$

And the natural boundary condition becomes

$$\begin{vmatrix} F-f_x\left(\frac{\partial F}{\partial f_x}\right) & -f_x\left(\frac{\partial F}{\partial f_y}\right) & 0 \\ f_y\left(\frac{\partial F}{\partial f_x}\right) & F-f_y\left(\frac{\partial F}{\partial f_y}\right) & 0 \\ \frac{\partial F}{\partial f_x} & \frac{\partial F}{\partial f_y} & 1 \end{vmatrix}=0 \tag{10}$$

Substituting F from Eq. (9) into Eq. (10) we can obtain the following relation:

$$\frac{1}{\sqrt{1+f_x^2+f_y^2}}=\frac{\sigma_{vs}-\sigma_{ls}}{\sigma_{lv}} \quad \text{on } z=0 \tag{11}$$

It can be shown easily that the right-hand side of Eq. (11) is the cosine of the contact angle θ.

Here we have reached very important conclusions. First, the angle of contact of a liquid drop placed on a smooth inclined plate is everywhere equal to θ. As before, this angle is independent of the gravity and the magnitude is quite the same as that of a drop placed on a horizontal plate. There is no reason at all to assume that the so-called advancing and receding contact angles are different. Secondly, once the above statement is admitted, the drop on the *smooth* inclined plate cannot resist at all the external forces since the adhesive force due to surface tension acting along the periphery cancels out.

The above statements seem bewitching. However, we have to seek another cause which actually keeps the drop at rest on the inclined surface. The authors consider that it is the *roughness* of the surface.

2. Profile of Drops on Rough Inclined Surfaces

Let us assume that the roughness of the surface of the plate is expressed by a known function $z=h(x, y)$. Tracing the similar procedure as the preceding section, Eqs. (1) to (6) are modified to the following forms, respectively:

$$E_{lv}=\iint_R \sigma_{lv}\sqrt{1+f_x^2+f_y^2}\,dxdy \tag{12}$$

$$E_{ls}=\iint_R \sigma_{ls}\sqrt{1+h_x^2+h_y^2}\,dxdy \tag{13}$$

$$E_{vs}=-\iint_R \sigma_{vs}\sqrt{1+h_x^2+h_y^2}\,dxdy+C \tag{14}$$

$$U=-\iint_R \rho g \sin\beta\cdot y\{f(x, y)-h(x, y)\}\,dxdy \tag{15}$$

$$\begin{aligned} E=\iint_R [&\sigma_{lv}\sqrt{1+f_x^2+f_y^2} \\ &+(\sigma_{ls}-\sigma_{vs})\sqrt{1+h_x^2+h_y^2} \\ &-\rho g \sin\beta\cdot y\{f(x, y)-h(x, y)\}]\,dxdy \end{aligned} \tag{16}$$

$$M=\iint_R \rho\{f(x, y)-h(x, y)\}\,dxdy \tag{17}$$

Thus, repeating the principle of the minimum energy, the profile $f(x, y)$ of the drop adhered to the rough inclined surface $h(x, y)$ should be so determined that the total energy E expressed by Eq. (16) may take the minimum value under the constraining condition that M of Eq. (17) remains constant. Hence, Eqs. (7) and (8) remain the same except that the definition of F in Eq. (8) should be

$$\begin{aligned} F &\equiv \sigma_{lv}\sqrt{1+f_x^2+f_y^2} \\ &\quad +(\sigma_{ls}-\sigma_{vs})\sqrt{1+h_x^2+h_y^2} \\ &\quad -\rho g \sin\beta\cdot y\{f(x, y)-h(x, y)\} \end{aligned} \tag{18}$$

instead of Eq. (9).

In the present case, the boundary condition at the surface of the plate should be

$$f(x, y)=h(x, y) \quad \text{at the drop periphery}$$

and the natural boundary condition is reduced to

$$\begin{vmatrix} F-f_x\left(\dfrac{\partial F}{\partial f_x}\right) & -f_x\left(\dfrac{\partial F}{\partial f_y}\right) & -h_x \\ f_y\left(\dfrac{\partial F}{\partial f_x}\right) & F-f_y\left(\dfrac{\partial F}{\partial f_y}\right) & -h_y \\ \dfrac{\partial F}{\partial f_x} & \dfrac{\partial F}{\partial f_y} & 1 \end{vmatrix}=0 \tag{19}$$

Substitution of F from Eq. (17) into Eq. (19) yields

$$\frac{f_x h_x+f_y h_y+1}{\sqrt{1+f_x^2+f_y^2}\sqrt{1+h_x^2+h_y^2}}=\frac{\sigma_{vs}-\sigma_{ls}}{\sigma_{lv}} \tag{20}$$

The left-hand side of Eq. (20) represents the scaler product between the unit vector normal to the drop surface and the unit vector normal to the rough plate surface. And it is, of course, equal to the cosine of the angle between these unit vectors. Since the right-hand side of Eq. (20) represents the cosine of the contact angle θ, Eq. (20) signifies that the drop contacts the surface of the plate with the angle θ everywhere around the periphery.

By the above statement we may explain why the drop can stay on an inclined plate against the action of the external forces. Figure 3 illustrates the mechanism: A drop on a rough surface adjusts itself so that adhesive force due to surface tension generated between its periphery and the roughness of the plate works most effective against the external forces. Considering on the plane of symmetry (Fig. 3), both ends of the drop profile come just on the point of the roughness curve where the gradient of the roughness, $|\partial h(0, y)/\partial y|$, takes the maximum. In this case, the angles of

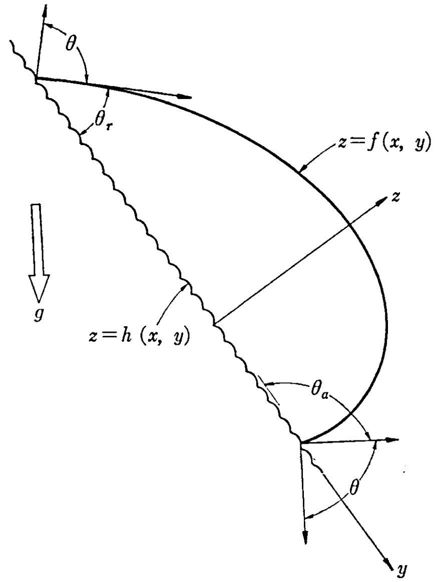

Fig. 3

contact still remain θ on the both ends. For the person who observes this phenomenon with his eyes, the roughness of the surface would be too small to be noticed, and the drop seems to be adhered to the surface, which is seemingly smooth, with different contact angle at each end. This might be the reason why the advancing and receding contact angles are observed.

Thus, if the surface roughness is, as an approximation, expressed by a (three-dimensional) periodic function as is schematically illustrated in Fig. 3, and the angle between its envelope (apparently flat surface) and the maximum gradient of the roughness is θ_s, the apparent advancing and receding angles are expressed as follows;

$$\theta_a=\theta+\theta_s\,;\ \theta_r=\theta-\theta_s \tag{21 a, b}$$

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