研 究 速 報
UDC 620.172.254

# FINITE ELEMENT ANALYSIS OF LOAD CELL RESPONSE IN HIGH SPEED TENSILE TESTING

高速引張試験における荷重計応答の有限要素法解析

Yoshiaki YAMADA* and Yoshihiko NAGAI*
山 田 嘉 昭・永 井 吉 彦

## Summary

Load cell response in high speed tensile testing is analysed by the finite element method as an elastic and visco-elastic wave propagation problem. Types of load measuring system studied are the conventional load cell-chuck assembly and the proposed one which is similar to that used in the split Hopkinson bar compression test. Various combinations of chuck mass and stiffness ratio of load cell to specimen have been examined. It is found that the proposed tensile load measuring system which is characterized by the small mass and the improved stiffness ratio follows faithfully the behavior of the specimen under dynamic loadings. The analysis concerns mainly with one dimensional wave, but a preliminary result for two-dimensional plane problem is given in the last section.

## 1. Introduction

Stress wave as well as vibration disturb the load cell output in high speed tensile testing. Such noises prevail when the testing speed increases or the duration time of testing from the instant of loading to specimen rupture decreases. Consequently, beyond a certain limit of loading speed, it becomes practically impossible to obtain the true material response to the applied load. To overcome these difficulties, we designed and built a new load measuring system. It consists of a long stiff load measuring transmit bar with a grip of small mass attached to it. To verify the usefulness of the proposed system, its characteristics are analyzed and compared with those of the conventional load cell-chuck assembly.

* Dept. of Applied Physics and Applied Mechanics, Inst. of Industrial Science, Univ. of Tokyo.

## 2. Method of Solution

The outline of solution procedure which is described in the preceding paper[1] and succeeded by this article is as follows.

The equation of motion of the whole system is written as

$$[M]\{\Delta\ddot{\delta}\}+[K]\{\Delta\delta\}=\{\Delta F\} \qquad (1)$$

where $[M]$ and $[K]$ denote the mass and stiffness matrices of the system respectively. Chuck mass is concentrated and contained in $[M]$. Vector $\{\Delta F\}$ corresponds to external force increment, if exists, at the nodes including both ends of the system. Assuming the linear variation of acceleration within the time interval $\Delta t$, we can replace the differential equation (1) by the following linear algebraic equation

$$\left([K]+\frac{6}{\Delta t^2}[M]\right)\{\Delta\delta\}$$
$$=\{\Delta F\}+[M]\left\{\frac{6}{\Delta t}\dot{\delta}+3\ddot{\delta}\right\} \qquad (2)$$

The material damping relevant to visco-elastic materials can be incorporated into the equation of motion (1) by adding the apparent force vector $\{F_a\}$, thus yielding

$$[M]\{\Delta\ddot{\delta}\}+[K]\{\Delta\delta\}=\{\Delta F\}+\{F_a\} \quad (3)$$

$\{F_a\}$ consists of contribution $\{F_a\}^e$ from constituent elements in mechanical model which is expressed, for example in the case of simple Maxwell material, as

$$\{F_a\}^e=\frac{\Delta t/T_r}{1+\frac{1}{2}\Delta t/T_r}A\sigma\begin{Bmatrix}-1\\1\end{Bmatrix}$$

where $T_r$ denotes the relaxation time, $\sigma$ and $A$ are the current stress and the sectional area.

## 3. Numerical Results and Characteristics of Load Measuring Systems

Figs. 1 and 2 show the models used in the numerical analysis of conventional and proposed measuring systems respectively. The stress in the specimen is represented by that of the central element distinguished by solid circle. The measuring point of stress in the load cell or transmit bar is indicated by open circle. It must be noted that the effect of reflection of wave in the load cell of the stress measuring system is not detrimental in the case of the proposed system, since the specimen breaks before the reflected wave comes back to the measuring gage station in high speed testing.

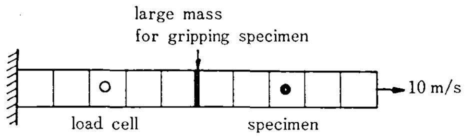


Fig. 1 Conventional system.

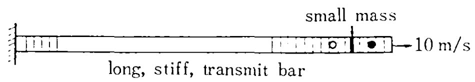


Fig. 2 Proposed system.

Parameters or data in the computation, if otherwise stated, are as follows.

load cell:

number of element divisions

5 (for conventional system)

50 (for proposed system)

length of individual element

$\Delta l = 10\,\text{mm}$

Young's modulus $E = 20\,000\,\text{kg/mm}^2$

density $\rho = 0.0008\,\text{kg}\cdot\text{msec}^2/\text{mm}^4$

sectional area $A = 100\,\text{mm}^2$

specimen:

number of element division

5 (standard specimen)

length of individual element

$\Delta l = 10\,\text{mm}$

Young's modulus $E = 2\,000\,\text{kg/mm}^2$

density $\rho = 0.00008\,\text{kg}\cdot\text{msec}^2/\text{mm}^4$

sectional area $A = 100\,\text{mm}^2$

tensile testing speed

$V = 10\,\text{mm/msec}$

Specimen data given above represent the plastics which are assumed, in the examples studied, to have the same sectional area as the load cell. It must be noted, however, that the numerical results are applicable to metal (e. g. steel) specimen whose sectional area is approximately one-tenth of the load cell.

Figs. 3 and 4 compare the response of the conventional and proposed load measuring systems. It can be seen that in the conventional method, the vibration of load cell superposes on the stress wave which propagates and repeats

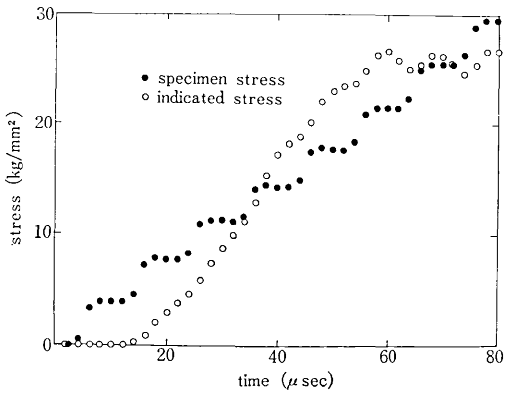


Fig. 3 Response of conventional system with relatively heavy chuck for gripping specimen.

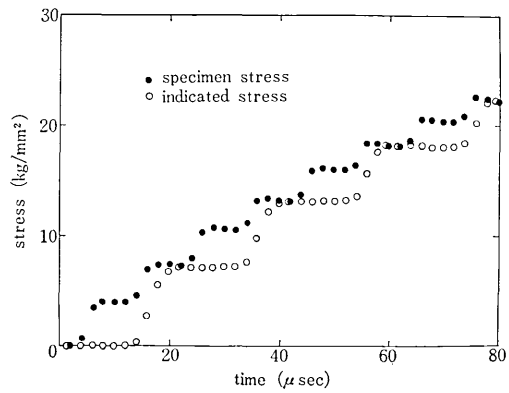


Fig. 4 Response of proposed system with chuck of small mass for gripping specimen.

reflections in specimen and load cell; thus the record of the stress being deteriorated. Moreover, the conventional system is not suitable for the stress measurement under very high speed test conditions, since the specimen breaks under the influence of the stress wave reflection which disturbs the true record of the material behavior.

Fig. 5 exemplifies the effect of the mass of chuck in the case of the conventional system. It is concluded that the mass shoud be kept as small as possible. Also important is the ratio of stiffness between the load cell and specimen under test. As shown in Fig. 6, the stress in the specimen dose not increase monotonously, when the stiffness of the specimen is increased and made, for example, equal to that of the load cell. This fact should be accounted for in the test intended to obtain the true work-hardening characteristics of materials.

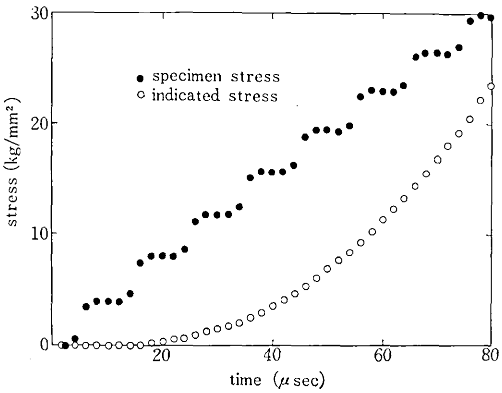


Fig. 5 Response of conventional system. The mass of chuck is ten times of the actual one.

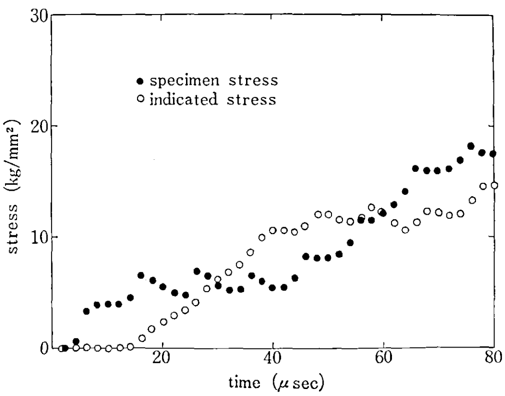


Fig. 6 Response of conventional system. Stiffness of the specimen is increased ten times and made equal to that of the load cell.

## 4. Two Dimensional Wave Problem

The method of analysis can be easily extended to the two dimensional case by employing appropriate mass and stiffness matrices $[M]$, $[K]$ and apparent load vector $\{F_a\}$. Care should be taken, however, of the relative mesh sizes between spatial element division and time interval for the temporal integration. This problem has been discussed by Fujii[2)].

The rectangular element of Fig. 7 and mesh division of Fig. 8 are used and the plane elastic stress wave is analyszed. The ratio of time interval $\Delta t$ and the length $\Delta l$ $(=2a=2b)$ of the rectangular element is taken as

$$\Delta t=\Delta l/c_1 \tag{4}$$

with $$c_1=\sqrt{\frac{K+4G/3}{\rho}}=\sqrt{\frac{1-\nu}{(1-2\nu)(1+\nu)}}\sqrt{\frac{E}{\rho}} \tag{5}$$

where $c_1$ corresponds to the velocity of longitudinal elastic wave[3)].

Figs. 9 and 10 depict the effect of Poisson's ratio on wave form. In the case of Fig. 9 with $\nu=0$, the problem degenerates essentially to one dimensional problem, since the transverse reflection of wave from the side wall does not occur.

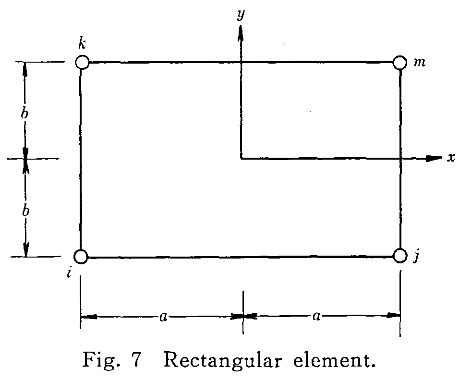


Fig. 7 Rectangular element.

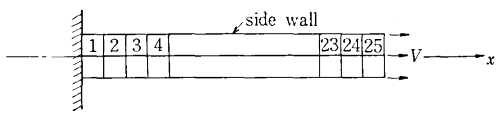


Fig. 8 Element division of plane model.

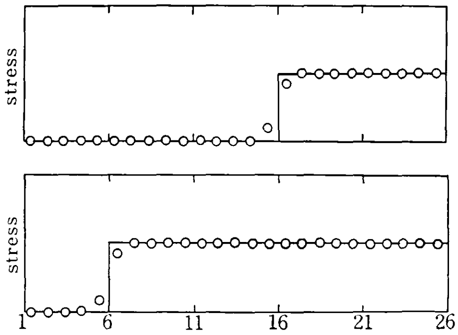


Fig. 9 Two-dimensional stress wave propagation ($\nu=0$).

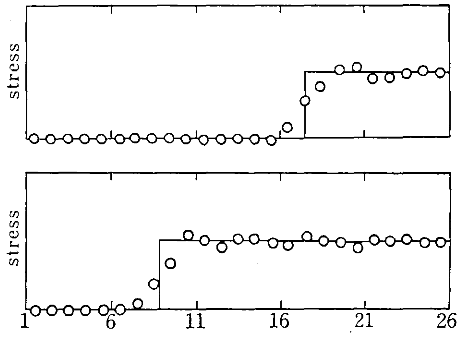


Fig. 10 Two-dimensional stress wave propagation ($\nu=0.3$).

In contrast, the transverse wave superposes on the longitudinal one, as can be seen from Fig. 10 for the case of elastic wave with $\nu=0.3$.

## 5. Conclusion

By introducing a new measuring system, it has become possible to pursue the true characteristics of response of materials to the high speed loadings. To verify the effectiveness of the proposed method, propagation of one dimensional wave in the system of load cell, specimen and chuck mass is analysed by the finite element procedure. Further, the method is extended to cover stress waves in the two dimensional continuum media.

(Manuscript received March 26, 1973)

### References

1) Y. Yamada and Y. Nagai, Analysis of One-dimensional Stress Wave by the Finite Element Method, Seisan Kenkyu, **32** - 5 (1971), pp. 186-189.
2) H. Fujii, Stability and Convergence of Finite Element Schemes for Vibration Problem in Elasticity Theory, Recent Advances in Computational Methods in Structural Mechanics and Design, Univ. of Alabama Press (1972), pp. 201-218.
3) Kolsky, H., Stress Waves in Solids, Dover Publications, Inc, (1963) p. 13.