研 究 速 報

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ON A METHOD TO OBTAIN DAMPING RATIOS BY THE POWER SPECTRUM OF IMPULSE RESPONSE

衝撃応答のパワースペクトルによる減衰定数推定の一方法について

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1. Introduction

Various methods have been proposed to obtain damping ratio of machine structure such as machine tool, piping system and so on[1),2),3),4)]. The development of transfer function analyzer made it possible to solve the equivalent damping ratio even for higher modes of the natural frequency[5),6)]. However expensive equipment has to be accommodated to perform it. Akutsu and others developed a method which made use of the power spectrum of the impulse response by means of fast Fourier transform technique[7)]. Although this simplified the method of evaluation of damping ratios, it was not satisfactory to get rid of the effect of other natural frequencies when the damping ratio for a natural frequency was evaluated.

This report investigates a method to obtain the damping ratio for the natural frequencies observed in an impulse response by dissolving its power spectrum so that each peak may correspond to that of the equivalent one-degree-of-freedom system. It is shown by the numerical simulation that the methods are effective. The method is applied to the power spectrum of the impulse response of a machine tool.

2. Characteristic of Impulse Response of One-Degree-of-Freedom System

The impulse response of one-degree-of-freedom mass-spring system is given as

$$X(t)=\frac{I\omega_0^2/k}{\omega_0\sqrt{1-\zeta^2}}e^{-\zeta\omega_0 t}\sin\omega_0\sqrt{1-\zeta^2}\,t \quad (1)$$

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where I : intensity of the impulse, ω_0 : natural circular frequency, ζ : damping ratio, k : spring constant.

The power spectrum of $X(t)$ is obtained as follows

$$S(\omega)=\frac{\omega_0^4 D}{(\omega_0^2-\omega^2)^2+4\zeta^2\omega_0^2\omega^2} \quad (2)$$

where $D=(I/k)^2$

Solid line of Fig. 1 assumes a general model of the power spectrum of the impulse response for general machine structure. If each peak is simulated by that of one-degree-of-freedom system shown by broken line and the original power spectrum is described by the sum of the each system, the damping ratio can be evaluated by taking the effect of the skirt of neighbouring natural frequency into account.

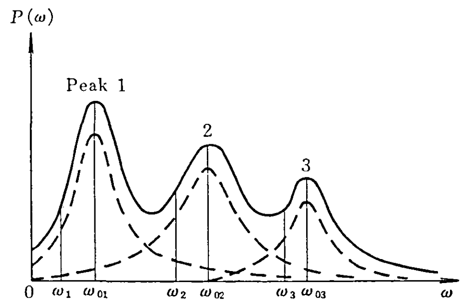

Fig. 1

The basic relation of the parameters in simulating the instrumented power spectrum by that of one-degree-of-freedom are given as follows

$$D=\frac{(\omega_1^2-\omega_0^2)^2}{\omega_0^2}\cdot\frac{P(\omega_0)}{\frac{P(\omega_0)}{P(\omega_1)}\omega_0^2-\omega_1^2} \quad (3)$$

$$\zeta=\sqrt{\frac{D}{4\cdot P(\omega_0)}} \quad (4)$$

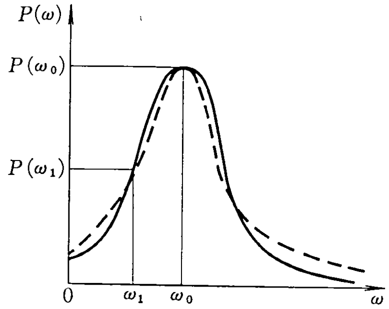

Fig. 2

Fig. 2 explains the relation of the both spectrum. This gives

$$P(\omega_1)=\frac{\omega_0{}^4D}{(\omega_0{}^2-\omega_1{}^2)^2+4\zeta^2\omega_0{}^2\omega_1{}^2} \quad (5)$$

$$P(\omega_0)=\frac{D}{4\zeta^2} \quad (6)$$

and through these (3) and (4) are obtained. (4) suggests that the damping ratio can be given by knowing the ratio of $P(\omega_1)$ to $P(\omega_0)$. The natural frequency of the damped system is obtained as

$$\omega=\omega_0\sqrt{1-2\zeta^2} \quad (7)$$

by the evaluation of ζ.

3. A Method of Damping Ratio Evaluation

The power spectrum with multi-peak is assumed to be described by the sum of equation (2), that is,

$$S_N(\omega)=\sum_{j=1}^{N}\frac{D_j\omega_{0j}{}^4}{(\omega_{0j}{}^2-\omega^2)^2+4\zeta_j{}^2\omega_{0j}{}^2\omega^2} \quad (8)$$

where j designates the system of the j th peak. The objective is to obtain D_j and ζ_j $(j=1, 2, \cdots\cdots, N)$ so that $S_N(\omega)$ adapts to the instrumented power spectrum.

Now (8) is partially differentiated by D_j and ζ_j $(j, \cdots\cdots, N)$, then

$$\frac{\partial S_N(\omega)}{\partial D_j}=\frac{\omega_{0j}{}^4}{(\omega_{0j}{}^2-\omega^2)^2+4\zeta_j{}^2\omega_{0j}{}^2\omega^2} \quad (9)$$

$$\frac{\partial S_N(\omega)}{\partial \zeta_j}=\frac{-8D_j\zeta_j\omega_{0j}{}^6\omega^2}{\{(\omega_{0j}{}^2-\omega^2)^2+4\zeta_j{}^2\omega_{0j}{}^2\omega^2\}^2} \quad (10)$$

are given.

The natural circular frequency ω_{0j} corresponding to the extreme power spectrum, the power spectrum for ω_{0j}, any appropriate frequency ω neighbouring ω_{0j} and the power spectrum for ω make it possible to obtain the first approximation of D_j and ζ_j by (3) and (4). The increment of $S_N(\omega)$ for that of D_j and ζ_j is

$$\Delta S_N(\omega)=\sum_{j=1}^{N}\left[\frac{\omega_{0j}{}^4}{(\omega_{0j}{}^2-\omega^2)^2+4\zeta^2{}_j\omega_{0j}{}^2\omega^2}\Delta D_j - \frac{8D_j\zeta_j\omega_{0j}{}^6\omega^2}{\{(\omega_{0j}{}^2-\omega^2)^2+4\zeta_j{}^2\omega_{0j}{}^2\omega^2\}^2}\Delta\zeta_j\right] \quad (11)$$

Substituting the first approximation D_{j1} and ζ_{j1} into (8), the first approximation of the power spectrum $S_{N1}(\omega)$ is derived. Then the increment of the power spectrum $\Delta S_N(\omega)$ is given as

$$\Delta S_N(\omega)=P(\omega)-S_{N1}(\omega) \quad (12)$$

where $P(\omega)$ is the originally instrumented power spectrum. Using (12) to (11), the equation including $2N$ unknown ΔD_j and $\Delta\zeta_j$ can be solved by putting $2N$ circular frequencies. The second approximation of D and ζ is given as

$$D_{j2}=D_{j1}+\Delta D_j \qquad \zeta_{j2}=\zeta_{j1}+\Delta\zeta_j \quad (13)$$

where suffix 1 and 2 means the order of approximation. Repeat of the procedure enables us to obtain D and ζ with better precision.

4. Numeaical Simulation

A numerical simulation is carried out for 5 peaks power spectrum. The power spectrum is composed by putting $N=5$ into (8). D_j and ζ_j computed are given in Tab. 1 and are compared with those set in the system. The spectrum at two frequencies ω_0 and $\omega_0-1.0$ for each peak is adopted to perform the simulation. In spite that only two points are used for a peak, Tab. 1 shows very good coincidence between the experimental evaluation and the true value.

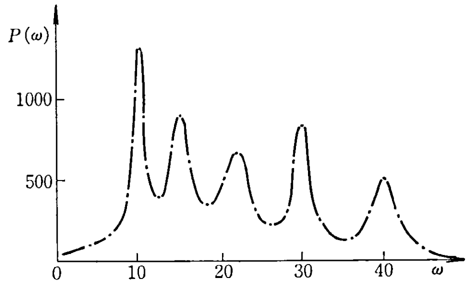

Fig. 3

Tab. 1

	D_1	D_2	D_3	D_4	D_5	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
Computed results	30. 16	30. 05	22. 42	5. 06	4. 96	0. 08	0. 10	0. 10	0. 04	0. 05
Value for the simulation model	30. 0	30. 0	25. 0	5. 5	5. 5	0. 08	0. 10	0. 10	0. 04	0. 05

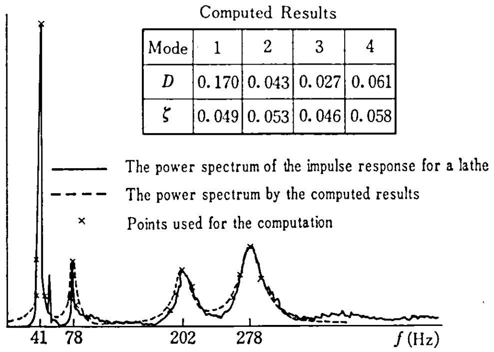

Fig. 4

5. Application to the Impulse Response of a Machine Tool

The method is applied for evaluation of the damping ratio of a machine tool. The solid line in Fig. 4 shows the power spectrum of an impulse response for a lathe. The cross marks in the figure show the points adopted for the evaluation. Three trial estimates are made by combining them with the peak. These results are averaged in order to obtain better estimate. The broken line shows the average of the estimate and fits well the original spectrum.

6. Conclusions and Ackowledgement

A method to estimate the damping ratios making use of the power spectrum of the impulse response is developed. The procedure to obtain the damping ratios is very simplified. The method has the merit that (1) the damping ratios for the natural frequencies observed in the impulse response can be estimated taking the effect of neighbouring natural frequencies, (2) as number of data necessary for the computation 2 times as that of peak is enough, (3) the procedure of the inverse Fourier transform and observation of the damped wave form for each natural frequency can be omitted.

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