研究速報

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A STUDY ON THE RESPONSE SPECTRUM OF AN APPENDAGE STRUCTURE SYSTEM
構築物・機器系の地震応答

——The Case of The Main System with Elasto-Plasticity——
——構築物の弾塑性特性の機器系にたいする影響——

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1. Introduction

The response spectrum of an appendage system that the main structure system shows elasto-plastic behaviour is analyzed. The spectrum for earthqake motion was studied in the previous work.[1] In this report the general properties of the response spectum is discussed as the response to an artificial earthquake with the aid of a stochastic approach. The result is compared with that for earthquake motion. The equivalent linearization method is applied for the deal of the non-linear characteristic. Whether elasto-plastic behaviour is allowed for the main system should be examined for the respective actual case. It is considered that the elasto-plastic characteristic simulates the move of the natural period to longer one, and the suppression of the response amplitude of the system.

Both tendencies are explained by the analysis.

2. Basic Equations for the System

The model of the system is shown as two-

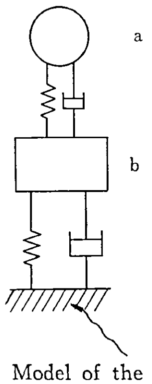

Fig. 1 Model of the system

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degree-of-freedom system in Fig. 1. The nonlinearity of the elasto-plasticity can be expressed by the system with solid friction characteristic between spring and mass. The equations of motion of this system are

$$\begin{cases} m_a\ddot{x}_a=-\mu_a(\dot{x}_a-\dot{x}_b)-k_a(x_a-x_b)-m_a\alpha(t) & (1) \\ m_b\ddot{x}_b=\mu_a(\dot{x}_a-\dot{x}_b)-\mu_b\dot{x}_b+k_a(x_a-x_b) & \\ \qquad -f_b-m_b\alpha(t) & (2) \\ f_b=k_by_b: \ \dot{x}_b=\dot{y}_b \qquad f_b<m_b|F| & (3) \\ f_b=m_bF(\dot{x}_b-\dot{y}_b)/|\dot{x}_b-\dot{y}_b|: \ f_b\geqq m_b|F| & (4) \end{cases}$$

Where m_b and m_a : mass of the main building system and the appendage system, μ_b and μ_a: damping coefficient of the main and the appendage system, k_b: spring constant of the main system in the region of linear characteristic, k_a: spring constant of the appendage system, x_b and x_a: relative displacement of the respective mass to the ground, y_b: relative displacement of the upper point of the spring for the main system to the ground, $\alpha(t)$: ground acceleration, F: acceleration corresponding to yield force, f: force by spring to the main mass.

(1) and (2) can be represented by using Laplace operator as follows,

$$\begin{cases} s^2X_a(s)=-(2\omega_ah_as+\omega_a^2)(X_a(s)-X_b(s))-R(s) & (5) \\ s^2X_b(s)=\gamma(2\omega_ah_as+\omega_a^2)(X_a(s)-X_b(s)) & \\ \qquad -2\omega_bh_bs\,X_b(s)-\bar{f}_b(s)/m_b-R(s) & (6) \end{cases}$$

where ω_b, h_b and ω_a h_a are natural circular frequency and damping ratio of the main and the appendage system respectively, γ is mass ratio m_a/m_b, $X_b(s)$, $X_a(s)$, $\bar{f}_b(s)$ and $R(s)$ are Laplace transform of x_b, x_a, f_b and $\alpha(t)$. If $U_b(s)$ and $U_a(s)$ are obtained by Laplace transform of $\dot{x}_b$ and

$\dot{x}_a$, (5) and (6) are rewritten as

$$\begin{cases} -(2\omega_a h_a s+\omega_a^2)U_b(s)+(s^2+2\omega_b h_b s+\omega_b^2)U_a(s) \\ \qquad =-sR(s) \qquad (7) \\ \{s^2+(2\omega_b h_b+\gamma\cdot 2\omega_a h_a)s+\gamma\omega_a^2\}U_b(s) \\ \qquad -\gamma(2\omega_a h_a s+\omega_a^2)U_a(s) \\ \qquad =-s\bar{f}_b(s)/m_b-sR(s) \qquad (8) \end{cases}$$

The representation as for the nonlinear characteristic is given as

$$\begin{cases} Z(s)=U_b(s)-Y_b(s) & (9) \\ Y_b(s)=s\bar{F}(s)/\omega_b^2 & (10) \\ \bar{F}(s)=Z(s) & (11) \end{cases}$$

where $Y_b(s)$ and $\bar{F}(s)$ are Laplace transform of y_b and F, $Z(s)$ is Laplace transform of input to the nonlinear characteristic and κ is the equivalent linearized gain of the nonlinearity. Taking (4) into the consideration and substituting (8) into (11), $U_b(s)$ and $U_a(s)$ are obtained as

$$\begin{cases} U_b(s)=[\{s^2+(1+\gamma)2\omega_a h_a s+(1+\gamma)\omega_a^2\}R(s) \\ \qquad +(s^2+2\omega_a h_a s+\omega_a^2)\kappa Z(s)]/\Delta \qquad (12) \\ U_a(s)=[\{s^2+(2\omega_b h_b+(1+\gamma)2\omega_a h_a)s \\ \qquad +(1+\gamma)\omega_a^2\}R(s) \\ \qquad +(2\omega_a h_a s+\omega_a^2)\kappa Z(s)]/\Delta \qquad (13) \\ \Delta=-[s^3+\{2\omega_b h_b+(1+\gamma)2\omega_a h_a\}s^2 \\ \qquad +\{2\omega_b h_b\cdot 2\omega_a h_a+(1+\gamma)\omega_a^2\}s+\omega_a^2\cdot 2\omega_b h_b] \\ \qquad\qquad (14) \end{cases}$$

In order to acquire the relation between $R(s)$ and $Z(s)$

$$U_b(s)=(1+\kappa s/\omega_b^2)Z(s) \qquad (15)$$

which is obtained from (9), (10) and (11), is substituted into (12). Then it is expressed as

$$\mathrm{B}(s)\cdot R(s)=-\mathrm{A}(s)Z(s) \qquad (16)$$

where

$$\begin{aligned} A(s)=&\kappa s^4+[\omega_b^2+\kappa\{2\omega_b h_b+(1+\gamma)2\omega_a h_a\}]s^3 \\ &+[\omega_b^2\{2\omega_b h_b+(1+\gamma)2\omega_a h_a\} \\ &+\kappa\{2\omega_b h_b\cdot 2\omega_a h_a+\omega_b^2+(1+\gamma)\omega_a^2\}]s^2 \\ &+[\omega_b^2\{2\omega_b h_b\cdot 2\omega_a h_a+(1+\gamma)\omega_a^2\} \\ &+\kappa(\omega_b^2\cdot 2\omega_a h_a+\omega_a^2\cdot 2\omega_b h_b)]s \\ &+\omega_b^2\cdot\omega_a^2(\kappa+2\omega_b h_b) \qquad (17) \end{aligned}$$

$$B(s)=\omega_b^2\{s^2+(1+\gamma)2\omega_a h_a s+(1+\gamma)\omega_a^2\} \qquad (18)$$

If $Z(s)$ is used for (12), the velocity and the displacement response of the main system are as follows

$$\begin{aligned} U_b(s)=&-(\kappa s+\omega_b^2)\{s^2+(1+\gamma)2\omega_a h_a s \\ &+(1+\gamma)\omega_a^2\}R(s)/A(s) \qquad (19) \end{aligned}$$

$$X_b(s)=U_b(s)/s \qquad (20)$$

and as for the appendage system

$$\begin{aligned} &U_a(s)-U_b(s) \\ &=-s\{\kappa\cdot 2\omega_b h_b s+\omega_b^2(\kappa+2\omega_b h_b)\}R(s)/A(s) \\ &\qquad (21) \end{aligned}$$

$$X_a(s)-X_b(s)=(U_a(s)-U_b(s))/s \qquad (22)$$

$$\begin{aligned} s^2X_a(s)+R(s)=&-(2\omega_a h_a s+\omega_a^2)\{\kappa\cdot 2\omega_b h_b s \\ &+\omega_b^2(\kappa+2\omega_b h_b)\}R(s)/A(s) \\ &\qquad (23) \end{aligned}$$

are given. These are the response of the relative velocity and displacement to the main system, and the acceleration response.

κ and F can be given as[2)]

$$\kappa=\sqrt{2/\pi}\cdot(F/\sqrt{I_z}) \qquad (24)$$

$$F=\beta\sqrt{I_g} \qquad (25)$$

where

$$I_g=(1/2\pi)\int_0^\infty|R(s)|^2d\omega,\quad I_z=(1/2\pi)\int_0^\infty|Z(s)|^2d\omega$$

$$\begin{aligned} R(s)=&[(2\omega_g h_g s+\omega_g^2)s^2/ \\ &(s^2+2\omega_g h_g s+\omega_g^2)(\psi_1 s+1)^2(\psi_2 s+1)^2]K \\ &\qquad (27) \end{aligned}$$ [1)3)4)]

then κ is obtained for a given β by using (22), (23) and (24) for (21). In (24) ω_g and h_g: the ground predominant circular frequency and the equivalent damping ratio, ψ_1 and ψ_2: the time constant of low pass and high pass filter and K: a constant representing a constant power spectrum at the base.

Then the variance of the respective response can be given by using same sort of equation as (26) taking the κ. $\kappa\to\infty$ corresponds to the linear system and the equations lead by (19), (20), (21), (22) and (23) are make equal to those obtained for the linear system.

3. Discussions for the Spectrum

Fig. 2 shows the response spectrum of acceleration amplification factor in terms of ratio of the standard deviation for the appendage system, this makes it obvious that the existence of the elasto-plasticity in the main structure system causes the release of the acceleration amplification factor in

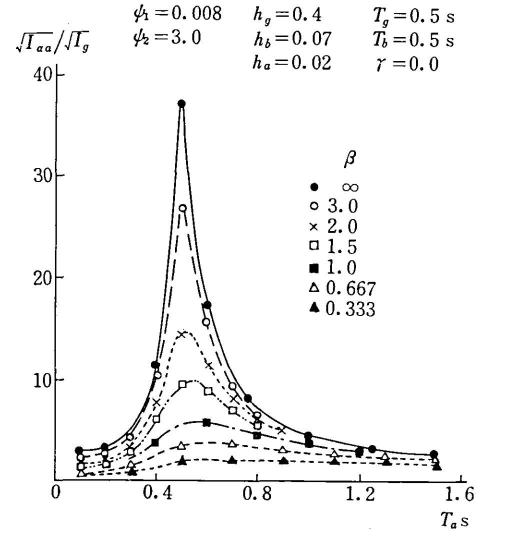

Fig. 2 The response spectrum of the acceleration amplification fator of the appendage system by the stochastic approach

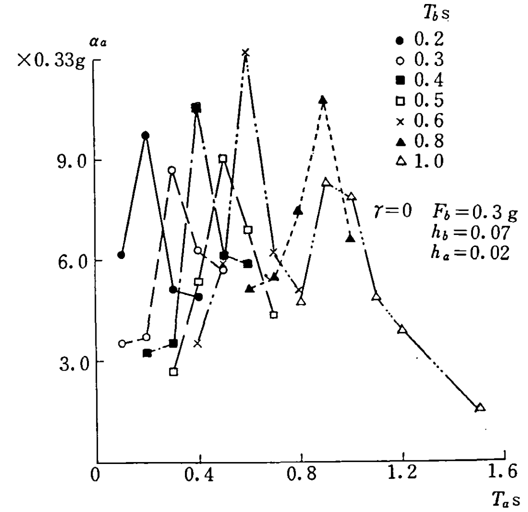

Fig. 3 Comparable spectrum with Fig. 2 for El Centro earthquake

the appendage system, which was already studied for earthquake motion. The spectrum for small β shows that the equivalent natural period of the main system moves to longer region.

Fig. 3 is the spectrum for El Centro earthquake. Looking at the spectrum for $T_b=0.5s$, the amplification factor is about 9.0 for $T_m=0.5s$. This was about 17.7 for the linear system. $F=0.3g$ in Fig. 3 corresponds to $\beta=0.910$. $\beta=1.5$ in Fig. 2 takes the factor of 9.5 at $T_m=0.5s$. This suggests that of the estimate β by the stochastic

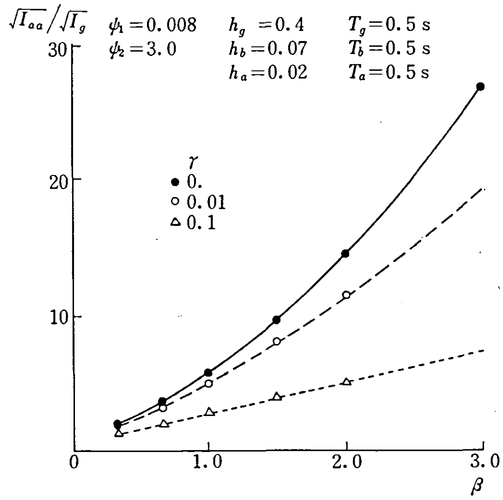

Fig. 4 A characteristic of peak in Fig. 2

approach is larger than that for earthquake motion by the factor 1.5 approximately. This was also applicable for the spectrum of single-degree-of-freedom system. Fig. 4 illustrates the relation between β and the amplification factor taking γ as the parameter.

Thus the system equation is given for the assummed structure system, in terms of the equivalent linearization gain. The effect of the elasto-plasticity to the response spectrum of the appendage system is obtained by the stochastic approach. This will help to reveal the general characteristic of the spectrum to earthquake motion.

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