研 究 速 報

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MEASUREMENT OF THERMOPHYSICAL PROPERTIES OF BIOLOGICAL SYSTEMS - PART 1

生物体における熱的物性値の測定 第1報

—Fundamental Principles—

一基本原理一

by Ichiro TANASAWA* and Tadashi KATSUDA*

棚 沢 一 郎・勝 田 直

1. Introduction

Measurement (especially, *in vivo* measurement) of the thermophysical properties of tissues or organisms of animals (especially, of man) have been attempted in response to the demand from various fields of biological science, such as medicine, physiology, sanitary science and so forth. The attempts, however, have obtained only insufficient harvest because of the difficulty in dealing with the living objects.

A little consideration will reveal that the difficulty arises from the requirements as follow which should be satisfied on the measurement:

(a) The thermophysical properties of a living tissue may differ considerably from those of a dead tissue. Therefore, *in vivo* measurement is wanted.

(b) Since most tissues are neither uniform nor homogeneous, the measurement on as small a portion as possible is required.

(c) In case when a measuring device is inevitably inserted into the living tissue, the local destruction of the tissue and the influence upon the surroundings should be minimized.

(d) In connection to the above, the time required for the measurement should be as short as possible.

In the present report, argument will be focussed chiefly on a couple of typical properties, i. e., the thermal conductivity and thermal diffusivity. The methods hitherto employed to measure the thermal conductivity and diffusivity of animal tissues may be roughly classified into three categories, in each of which the measurement is done

(1) by cutting off the tissue from the living body and using a conventional (*in vitro*) method of measurement.

(2) by inserting a temperature sensitive element (such as a thermister or a thermocouple) into the living tissue and measuring the change in the temperature or the heat flow under an appropriate condition.

(3) by bringing a material, whose thermophysical properties are known, into contact with the tissue, measuring the change in the temperature inside the material and calculating the unknown properties of the tissue.

If these three methods are compared to the four requirements stated before, it is obvious that the first one is irrelevant since the measurement on living tissues is demanded. The second and the third method are to be subject to further discussion.

The method reported by Chato[1], which is a representative example of the second method, in which small, spherical thermister bead is used, is based on a very excellent idea, but it has also quite a few defects. For instance; local destruction and inflammation of the tissue are likely to be caused by the insertion of the thermister bead; the exact position of the inserted bead is hardly recognized; especially, it is very difficult to know how near the peripheral blood vessels exist; although the calculation of the heat conduction are done

* Dept. of Mechanical Engineering and Naval Architectute, Inst. of Industrial Science, Univ. of Tokyo.

assuming semi-infinite medium, the actual measurement has to be made by inserting the thermister bead fairly near to the underneath of the body surface; the effect of the leading wire(s) may not be neglisible; etc.

As to the third method, Nukiyama[2] and Umehara[3] have published the reports. An advantage of this method over the others is that it needs no destruction at all of the tissues, while it has a disadvantage that only the properties in the vicinity of the surface are measurable. In particular, since the theoretical analysis in the above cited studies is carried out under the assumption of the contact between two semi-infinite solids, it becomes necessary in the actual measurement to create contacting surface with sufficient area. However, the body surfaces are in general of very complex shape and it is not always possible to get enough plane area. Moreover, even if it be possible, the requirement (b) is likely to be violated.

From the argument as above, it is concluded that the contact method is the most appropriate for *in vivo* measurement of the thermophysical properties of living tissues, though there remains some room for improvement. In this report, the description of the modified contact method by the authors is presented. The results of the measurement and the discussion upon the possibility of the practical use of this method is left for the next report.

2. Basic Principle of the Measurement

The contact method modified by the authors is as follows: One end of a solid cylindrical copper bar whose thermophysical properties are known in advance is brought into contact with the surface of the tissue. The change in the temperature inside the copper bar is measured and recorded from this instant on. Using this record the thermophysical properties are calculated by applying the result of the transient heat conduction analysis. Figure 1 shows the model for it.

Prior to the analysis, several assumptions for simplification are introduced. At first, it is assumed that the material I (the living tissue) is a homogeneous, isotropic and semi-infinite solid. The effects caused by the blood flow and the heat generation inside are temporarily ignored. Meanwhile, the material II (the copper bar) is assumed to be a semi-infinite solid cylinder and one-dimensional heat conduction along the axis is considered. It is also assumed that no heat is transferred across the parts of the surfaces of the both materials where they are exposed to the atmosphere.

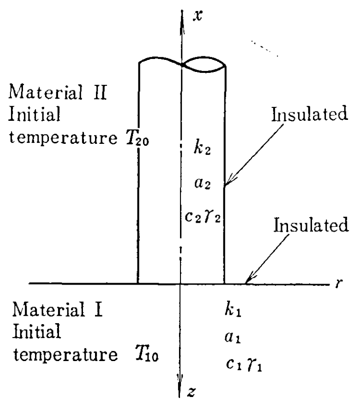

Fig. 1 Model for analysis.

In order to obtain rigorous solution to this two-body problem, a set of partial differential equations with appropriate initial and boundary conditions as shown below should be solved analytically.

Conduction equations:

$$\frac{\partial T_1}{\partial t}=a_1\left(\frac{\partial^2 T_1}{\partial z^2}+\frac{1}{r}\frac{\partial T_1}{\partial r}+\frac{\partial^2 T_1}{\partial r^2}\right) \quad \text{(1a)}$$

$$\frac{\partial T_2}{\partial t}=a_2\left(\frac{\partial^2 T_2}{\partial z^2}+\frac{1}{r}\frac{\partial T_2}{\partial r}+\frac{\partial^2 T_2}{\partial r^2}\right) \quad \text{(1b)}$$

Initial conditions:

$$T_1=T_{10}, \quad T_2=T_{20} \quad \text{at } t=0 \quad \text{(2a, b)}$$

Boundary conditions:

$$\text{Material I}: \left[\frac{\partial T_1}{\partial z}\right]_{z=0}=0 \quad (r>R) \quad \text{(3a)}$$

$$\text{Material II}: \left[\frac{\partial T_2}{\partial r}\right]_{r=R}=0 \quad (x>0) \quad \text{(3b)}$$

$$\text{Contact surface}: k_1\left[\frac{\partial T_1}{\partial z}\right]_{z=0}=-k_2\left[\frac{\partial T_2}{\partial x}\right]_{x=0} \quad \text{(3c)}$$

$$[T_1]_{z=0}=[T_2]_{x=0} \quad (t>0) \quad \text{(3d)}$$

However, it seemed extremely difficult to solve the above equations analytically. Thus the authors sought for an approximate solution by combining solutions for a few basic heat conduction problems and proceeding from the simpler case to the more complexed problems. Since the description in detail of the procedure cannot be presented here because of the space limitation, only the final results are presented below:

$$T_2=T_{20}+\frac{1}{k_2}\sqrt{\frac{a_2}{\pi}}\frac{\Delta T}{A}\sum_{n=0}^{\infty}\left(-\frac{B}{A}\right)^n \times \mathrm{erfc}\left(\frac{nR}{2\sqrt{a_1 t}}+\frac{x}{2\sqrt{a_2 t}}\right) \quad (4)$$

$$[T_1]_{r=0}=T_{10}+\frac{\Delta T}{\sqrt{\pi a_1}}\frac{1}{\gamma_1 c_1}\sum_{n=0}^{\infty}\left(-\frac{B}{A}\right)^n \times \left\{\mathrm{erfc}\,\frac{nR+\sqrt{R^2+z^2}}{2\sqrt{a_1 t}}-\mathrm{erfc}\,\frac{z+nR}{2\sqrt{a_1 t}}\right\} \quad (5)$$

where

$$\Delta T=T_{20}-T_{10} \quad (6)$$

$$A=\frac{1}{k_1}\sqrt{\frac{a_1}{\pi}}+\frac{1}{k_2}\sqrt{\frac{a_2}{\pi}},\quad B=-\frac{1}{k_1}\sqrt{\frac{a_1}{\pi}} \quad (7a,\ b)$$

If dimensionless parameters

$$T_1^*=\frac{T_1-T_{20}}{T_{10}-T_{20}},\quad T_2^*=\frac{T_2-T_{20}}{T_{10}-T_{20}} \quad (8a,\ b)$$

$$\alpha=\frac{a_1}{a_2},\quad \beta=\frac{k_1}{k_2} \quad (9a,\ b)$$

$$x^*=\frac{x}{R},\quad z^*=\frac{z}{R};\quad t^*=\frac{a_2 t}{R^2} \quad (10a,\ b,\ c)$$

are introduced, Eqs. (4) and (5) can be rewritten as

$$T_2^*=\frac{1}{1+\frac{\sqrt{\alpha}}{\beta}}\sum_{n=0}^{\infty}\left(\frac{1}{1+\frac{\beta}{\sqrt{\alpha}}}\right)^n \times \mathrm{erfc}\left(\frac{n}{2\sqrt{\alpha t^*}}+\frac{x^*}{2\sqrt{t^*}}\right) \quad (11)$$

$$T_1^*=1-\sum_{n=0}^{\infty}\left(\frac{1}{1+\frac{\beta}{\sqrt{\alpha}}}\right)^{n+1} \times \left\{\mathrm{erfc}\left(\frac{n+z^*}{2\sqrt{\alpha t^*}}\right)-\mathrm{erfc}\,\frac{n+\sqrt{1+z^{*2}}}{2\sqrt{\alpha t^*}}\right\} \quad (12)$$

It should be noted here that in Eqs. (11) and (12) if the terms in the summation sign corresponding to $n\geq 1$ are omitted, the results are identical with the solutions for the contact between two semi-intinite media. This fact suggests that the solutions for the present problem are approximated by the solutions for semi-infinite media for a certain intial period, and then the effect caused by the fact that one of the materials is a cylinder becomes dominant.

Figure 2 shows an example of the change in

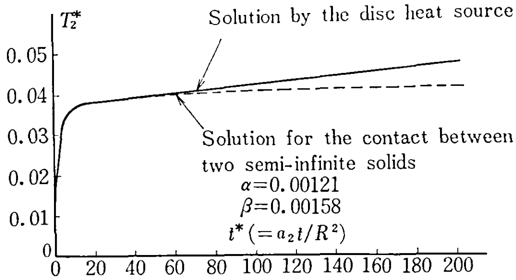

Fig. 2 An example of the temperature change inside the measuring bar ($x^*=1.0$).

T_2^* with respect to time where the thermophysical properties of water ($k_1=0.526$ kcal/mh°C, $a_1=0.000529\ m^2/h$) and of copper ($k_2=332$ kcal/mh°C, $a_2=0.435\ m^2/h$) are assumed for the materials I and II, and then $\alpha=0.00121$ and $\beta=0.00158$. It is seen that the effect of the finite contact surface does not appear up to $t^*\simeq 20$. Use is made of this fact on the measurement of the thermophysical properties of the living tissues.

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