UDC 624.042.7: 534.51

STATISTICAL METHOD OF RESPONSE ANALYSIS FOR THE STRUCTURE MODEL SUBJECTED TO TWO-RANDOM EXCITATIONS HAVING CERTAIN TIME-LAG INTERVAL

時間差のある不規則二入力をうける構造物モデルの統計的応答解析法

by Kohei SUZUKI*
鈴木浩平

Generally machine structure system and piping system have multiple supporting and fixed points, so once an earthquake occurs these systems are subjected to more than two random inputs through these points.

Ordinarilly these multiple inputs have various different wave form. In this case it is considered the simple situation that these inputs have time-lag interval between each other.

The author has paid attention to this fact and dynamic response characteristics to earthquake motions have been investigated by analog computer.[1)]

In this paper a statistical approach of the response analysis[2)3)] is studied for the simplest one-degree-of-freedom system subjected to two gaussian random excitations having certain time-lag interval between them. Then obtained characteristics are compared with the results by analog computation.

1. Estimation of Acceleration Response

In Fig. 1 the simplest model of structure which is subjected to two random inputs is illustrated. The equation of motion for this model is given as

$$m\ddot{x}+c_1(\dot{x}-\dot{y}_1)+k_1(x-y_1)+c_2(\dot{x}-\dot{y}_2)+k_2(x-y_2)=0 \quad \cdots\cdots\cdots(1)$$

where x, y_1, y_2 are the absolute displacements, m is the mass of the model, k_1, k_2 are the stiffness, c_1, c_2 are the damping coefficient.

If the following z is introduced

$$z=x-\frac{1}{2}(y_1+y_2) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

* Dept. of Mechanical Engineering and Naval Architecture, Inst. of Industrial Science, Univ. of Tokyo.

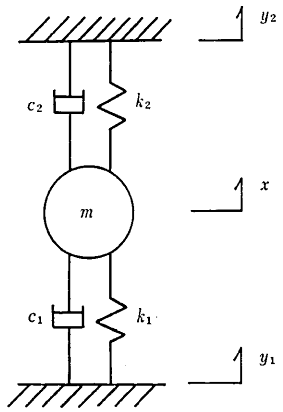

Fig. 1 Simple model of structure

eq. (1) is written as

$$\ddot{z}+h_s\omega_s(1+\alpha)\dot{z}+\frac{\omega_s^2}{2}(1+\beta)z=-\frac{1}{2}\Big\{(\ddot{y}_1+\ddot{y}_2)-h_s\omega_s(1-\alpha)(\dot{y}_1-\dot{y}_2)-\frac{\omega_s^2}{2}(1-\beta)(y_1-y_2)\Big\} \quad \cdots\cdots\cdots\cdots(3)$$

where

$$h_s=c_1/2\sqrt{mk_1}\ , \qquad \omega_s=\sqrt{\frac{k_1}{m}}$$

$$\alpha=c_2/c_1\ , \qquad \beta=k_2/k_1$$

Eq. (3) can be written using Laplace transform operator

$$\left\{\frac{s^2+h_s\omega_s(1+\alpha)s+\frac{\omega_s^2}{2}(1+\beta)}{s^2}\right\}Z(s)=-\frac{1}{2}\left\{\frac{s^2-h_s\omega_s(1-\alpha)-\frac{\omega_s^2}{2}(1-\beta)}{s^2}\cdot Y_1(s)+\frac{s^2+h_s\omega_s(1+\alpha)s+\frac{\omega_s^2}{2}(1-\beta)}{s^2}\cdot Y_2(s)\right\} \quad \cdots\cdots\cdots\cdots(4)$$

where

$$Z(s)=\mathcal{L}(z),\ Y_1(s)=\mathcal{L}(y_1) \text{ and } Y_2(s)=\mathcal{L}(y_2)$$

From eq. (2) and eq. (4) transfer function of relative acceleration denoted by $A(s)=\mathcal{L}(\ddot{x})$ is obtained as

$$A(s)=H_m^+(s)Y_1(s)+H_m^-(s)Y_2(s) \quad \cdots\cdots(5)$$

where

$$H_m^+(s)=\frac{1}{2}\cdot\frac{2h_s\omega_s s+\omega_s^2}{s^2+h_s\omega_s(1+\alpha)s+\frac{\omega_s^2}{2}(1+\beta)} \quad \cdots\cdots\cdots\cdots(6)$$

and

$$H_m^-(s)=\frac{1}{2}\cdot\frac{2\alpha h_s\omega_s s+\beta\omega_s^2}{s^2+h_s\omega_s(1+\alpha)s+\frac{\omega_s^2}{2}(1+\beta)} \quad \cdots\cdots\cdots\cdots(6)'$$

Assuming that y_1 and y_2 have same wave from but one of which lags behind the other with interval T_l for simplicity,

$$Y_2(s)=e^{T_l(s)}Y_1(s) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

So eq. (4) can be written as

$$|A(s)|^2=\{|H_m^+(s)|^2+|H_m^-(s)|^2 +H_m'(s)e^{-T_l(s)}+H_m''(s)^{T_l s}\}\cdot|Y_1(s)|^2 \quad \cdots\cdots\cdots\cdots(8)$$

where

$$\left.\begin{aligned} H_m'(s)&=H_m^+(s)\cdot H_m^-(-s)\\ &=\frac{-\alpha(2h_s\omega_s)^2s^2+2h_s\omega_s^3(\beta-\alpha)s+\beta\omega_s^4}{s^4+\omega_s^2\{(1+\beta)-h_s^2(1+\alpha)^2\}s^2+\frac{\omega_s^4}{4}(1+\beta)^2}\\ H_m''(s)&=H_m^+(-s)\cdot H_m^-(s)\\ &=\frac{-\alpha(2h_s\omega_s)^2s^2-2h_s\omega_s^3(\beta-\alpha)s+\beta\omega_s^4}{s^4+\omega_s^2\{(1+\beta)-h_s^2(1+\alpha)^2\}s^2+\frac{\omega_s^4}{4}(1+\beta)^2} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots(9)$$

Here simplify this problem furthermore making this system symmetric by $k_1=k_2$, $c_1=c_2$ i. e. $\alpha=\beta=1$ so that $H'_m(s)=H''_m(s)\equiv H_m(s)$.

Therefore Eq. (8) is reduced to

$$|A(s)|^2 =2|H_m(s)|^2\cdot|Y_1(s)|^2\cdot\left\{1+\frac{e^{T_l s}+e^{-T_l s}}{2}\right\} \quad \cdots\cdots\cdots\cdots(10)$$

or

$$|A(j\omega)|^2 =2|H_m(j\omega)^2|\cdot|Y_1(j\omega)|^2(1+\cos T_l\omega) \quad \cdots\cdots\cdots\cdots(10)'$$

Using Tajimi's formula under the assumption that the ground motion is stationary gaussian process, the acceleration response factor λ_a can be expressed by

$$\lambda_a^2=\frac{\int_0^\infty|A(s)|^2ds}{\int_0^\infty|Y_1(s)|^2ds}=\frac{\int_0^\infty|A(j\omega)|^2d\omega}{\int_0^\infty|Y_1(j\omega)|^2d\omega} \quad \cdots\cdots\cdots\cdots(11)$$

This λ_a determines the amplification factor, that is, the ratio of acceleration response to the ground acceleration for this system.

Considering the situation that random excitation acts on this system directly from the ground. This function has been given as follows by Tajimi and Sato respectively,

$$Y_1(s)=\frac{2h_g\omega_g s+\omega_g^2}{s^2+2h_g\omega_g s+\omega_g^2}k_T \quad \cdots\cdots\cdots\cdots(12)$$

and

$$Y_1(s) =\frac{(2h_g\omega_g s+\omega_g^2)s^2}{(s^2+2h_g\omega_g s+\omega_g^2)(\phi_1 s+1)^2(\phi_2 s+1)^2}k_S \quad \cdots\cdots\cdots\cdots(12)'$$

Where h_g and ω_g are the equivalent damping ratio and predominant circular frequency of the ground respectively, $\frac{1}{\phi_1}$ and $\frac{1}{\phi_2}$ are break frequency of the filter which suppress the high and low frequency components of the ground, and k_T and k_S are constant.

Using this relation λ_a can be expressed as follows by eq. (10)′

$$\lambda_a^2(T_l) =\frac{\int_0^\infty|A(j\omega)|^2d\omega+\int_0^\infty|A(j\omega)|^2\cos T_l\omega d\omega}{\int_0^\infty|Y_1(j\omega)|^2d\omega} \quad \cdots\cdots\cdots\cdots(13)$$

When other relations exist between two inputs with time-lag T_l, $\lambda_a^2(T_l)$ can be derived in similar way.

For simple examples, when relations

$$\ddot{y}_2(t)=C\ddot{y}_1(t+T_l) \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(14)$$

$$\ddot{y}_2(t)=f(T_l)\cdot\ddot{y}_1(t+T_l) \quad \cdots\cdots\cdots\cdots\cdots(15)$$

exist,

$$|A(j\omega)|^2=|H_m(j\omega)|^2\cdot|Y_1(j\omega)|^2\cdot\{(1+e^{-2CT_l})+e^{-CT_l}\cos T_l\omega\} \quad \cdots\cdots(16)$$

and

$$|A(j\omega)|^2=|H_m(j\omega)|^2|Y_1(j\omega)|^2\cdot|1+f(T_l)e^{j\omega T_l}|^2 \quad \cdots\cdots(17)$$

are given respectively. If numerator terms is written as

$$R(T_l)=\int_0^\infty|A(j\omega)|^2\cos T_l\omega d\omega \equiv\int_0^\infty g(\omega)\cos T_l\omega d\omega \quad \cdots\cdots(18)$$

from the Wiener-Khitchine equation $R(T_l)$ represents autocorrelation function regarding $g(\omega)$ as power spectrum function.

Now let me take eq. (12) as ground characteristics $\lambda_a{}^2(T_l)$ can be evaluated from eq. (13) with aid of residue integral and Fourier integral, that is,

$$\int_0^\infty|Y_1(j\omega)|^2d\omega=\left(h_g\omega_g+\frac{\omega_g}{4h_g}\right)k_T \quad \cdots(19)$$

and

$$R(T_l)=\frac{\pi}{2}\left\{\frac{\xi_{s1}}{\eta_{s1}}e^{-\eta_{s1}|T_l|}+\frac{\xi_{s2}}{\eta_{s2}}e^{-\eta_{s2}|T_l|}+\frac{\xi_{g1}}{\eta_{g1}}e^{-\eta_{g1}|T_l|}+\frac{\xi_{g2}}{\eta_{g2}}e^{-\eta_{g2}|T_l|}\right\} \quad \cdots\cdots(20)$$

where

$$g(\omega)=\frac{(2h_s\omega_s)^2\omega^2+\omega_s^2}{(\omega^2-\omega_s^2)^2+(2h_s\omega_s)^2\omega^2}\times\frac{(2h_g\omega_g)^2\omega^2+\omega_g^2}{(\omega^2-\omega_g^2)^2+(2h_g\omega_g)\omega^2}$$
$$=\frac{\xi_{s1}}{\omega^2+\eta_{s1}^2}+\frac{\xi_{s2}}{\omega^2+\eta_{s2}^2}+\frac{\xi_{g1}}{\omega^2+\eta_{g1}^2}+\frac{\xi_{g2}}{\omega^2+\eta_{g2}^2}$$

Using above mentioned method, characteristics of the acceleration response factor as for time lag interval between two inputs are obtained.

2. Estimation of Relative Displacement Response

Next consider characteristics of the displacement as for T_l. It might be important for the structure system as crane and boiler with long natural period to estimate the effect of relative displacement as for each input-end.

Equations of motion in form of relative displacement for the aformentioned symmetrical one-degree-of-freedom system are shown as

$$\left.\begin{aligned}&\ddot{u}+2h_sw_s\dot{u}+w_s^2u\\&\quad=-\ddot{y}_1+h_s\omega_s(\dot{y}_2-\dot{y}_1)+\frac{\omega_s^2}{2}(y_2-y_1)\\&\ddot{v}+2h_s\omega_s\dot{v}+\omega_s^2v\\&\quad=-\ddot{y}_2-h_s\omega_s(\dot{y}_2-\dot{y}_1)-\frac{\omega_s^2}{2}(y_2-y_1)\end{aligned}\right\}(21)$$

where

$$u=x-y_1\ ,\qquad v=x-y_2\ .$$

These equations are Laplace-transformed as for displacement such that

$$\left.\begin{aligned}U(s)&=\frac{1}{2}f_s(s)\{Y_1(s)+Y_2(s)\}-Y_1(s)\\V(s)&=\frac{1}{2}f_s(s)\{Y_1(s)+Y_2(s)\}-Y_2(s)\end{aligned}\right\}(22)$$

$$U(s)=\mathcal{L}(u)\ ,\qquad V(s)=\mathcal{L}(v)\ ,$$
$$Y_1(s)=\mathcal{L}(y_1)\ ,\qquad Y_2(s)=\mathcal{L}(y_2)$$

and

$$f_s(s)=\frac{2h_s\omega_ss+\omega_s^2}{s^2+2h_s\omega_ss+\omega_s^2}$$

Consider the simplest case that these two inputs have same wave form and there exists only time-lag interval between them.

Then as for $U(s)$

$$U(s)=\left\{-1+\frac{1}{2}f_s(s)\cdot(1+e^{T_ls})\right\}Y_1(s)=H_{su}(s)Y_1(s) \quad \cdots\cdots(23)$$

Hence

$$|H_{su}(s)|^2=1+\frac{1}{4}|f_s(s)(1+e^{T_ls})|^2-\frac{1}{2}\{f_s(1+e^{T_ls})+\overline{f_s(s)\cdot(1+e^{T_ls})}\} \quad (24)$$

With the assumption that the natural period of this system is taken within the range of about 0.1 s～2.0 s, the third term of the right-hand can be simplified as

$$-\frac{\omega_s^2s^2+\omega_s^4}{s^4+2\omega_s^2s^2+\omega_s^4}\left(1+\frac{e^{T_ls}+e^{-T_ls}}{2}\right)$$

In order to suppress the divergency of the integral, the filter characteristic function

$$H_f(s)=\frac{s^2}{(\psi_1s+1)^2(\psi_2s+1)^2}$$

is introduced.

Finally characteristics of the relative displacement can be estimated by

$$\lambda_u^2(T_l)=\frac{\int_0^\infty |H_{su}'(s)|^2 ds}{\int_0^\infty |Y_1(s)|^2 ds}$$

$$=\frac{\int_0^\infty |H_{su}(j\omega)|^2 |H_f(j\omega)|^2 d\omega}{\int_0^\infty |Y_1(j\omega)| d\omega} \qquad \cdots\cdots\cdots\cdots(25)$$

Concerning $v=x-y_2$, $\lambda_u^2(T_l)$ is obtained similarly.

3. Examples of the Computation and Comparison with the Results by Analog Computation

Fig. 2 and Fig. 3 show examples of the results by this statistical computation. In Fig. 2 the results of statistical analysis and those by response analysis to earthquake motion by using analog computer are compared.

Flow chart of this analog computation is illustrated in Fig. 4. In this procedure, required earthquake velocity and displacement are obtained from acceleration with aid of the proposed approximate integration by analog computer.[4] And time-lag interval T_l is generated by a time-delay device. Solid-line curve in Fig. 2 shows the response factor of this simplified model for El Centro by taking T_l as the abscissa. On the other hand, dotted-line curve is the normalized $\lambda_a(T_l)$ from eq. (11). Two lines depict pretty good concidence. These figures show wavy shape which has maximum values at $T_l=\left(n-\frac{1}{2}\right)T_s$ where $T_s=\frac{2\pi}{\omega_s}$. If damping h_s increases whole curve diminishes as shown in Fig. 3. For any case the response at $T_l=0$ takes the maximum.

In Fig. 5 an example of the relative displacement characteristics calculated from eq. (25) is illustrated.

It is pointed out that this displacement character-

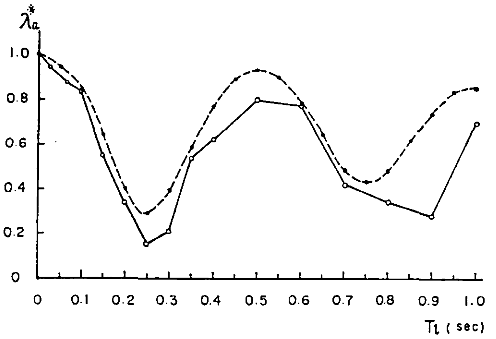

Fig. 2 Acceleration respone as abscissa T_l (T_s=0.5 sec, h_s=0.02)

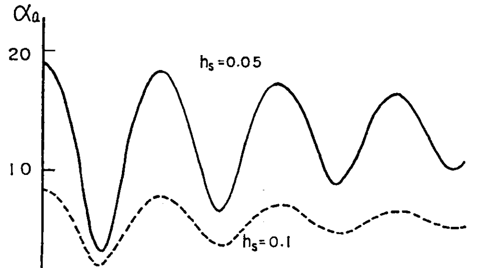

Fig. 3 Theoretical response curve by eq. (13) ($T_s=T_g$=0.5 sec, h_g=0.4)

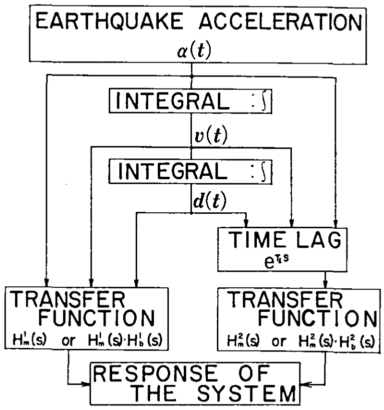

Fig. 4 Flow chart of response analysis by analog-computer

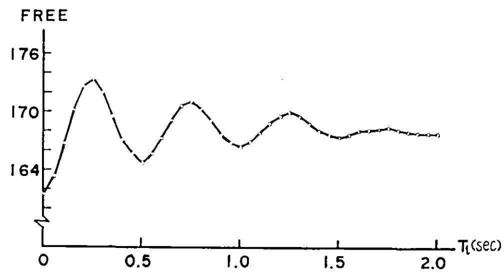

Fig. 5 Theoretical displacement response curve as abscissa T_l (T_l=0.5 sec, h_s=0.2)

istic has minimum value at $T_l=0$ and also shows wavy shape in which maximum and minimum points are taken at $T_l=\left(n-\frac{1}{2}\right)T_s$ and $T_l=(n-1)T_s$ respectively. This tendency is conspicuous in case of the response analysis to earthquake, however comparison is difficult owing to the effect and direct current component within the earthquake motions.

In any case small time-lag T_l might have a serious effect on the structure system.

This statistical method of analysis might be able to apply to more complicated system. For example let me take the machine structure system illustrated in Fig. 6, which is connected with two different building system. In this case relation

$$|A(s)|^2 = |H_m(s)|^2 |1+H_{b2}(s)e^{T_l s}|^2 |H_{b1}(s)|^2 |H_g(s)|^2 \qquad \cdots\cdots\cdots\cdots(26)$$

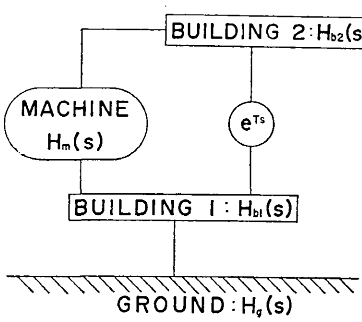

Fig. 6 Model of building-machine structure system

is obtained. However calculation becomes more sophisticated, the estimation of such a complicated system is possible in similar way.

4. Acknowledgement

In closing the author would like to express sincere thanks to Associate Professor H. Sato who always gives continuing guidance and advices. He also wishes to thank Professors A. Watari, S. Fujii and H. Shibata for their helpful advices. Several stimulating discussions with Dr. N. Shimizu are gratefully acknowledged.

(Manuscript received June 21, 1971)

References

1) H. Sato and K. Suzuki; A Study on Response Analysis of Machine Structure System Subjected to Two Seismic Motions. Bull. ERS. No. 4 (1970-12)
2) H. Sato; On the Response Spectrum of the Machine Structure System to the Strong Motion Earthquke. Bull. JSME. 9-36 (1966-11)
3) H. Tajimi; Basic Theories on Aseismic Design of Structure Rep. of the Inst. of Ind. Sci., Univ. of Tokyo. 8-4 (1953-4)
4) K. Suzuki and H. Sato; On a Method to Obtain Displacement Wave Form from Record of Earthquake Acceleration. SEISAN KENKYU 22-1. (1970-1)