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DYNAMIC RESPONSE OF FRAMEWORK STRUCTURES

骨組構造物の動的応答について

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1. Introduction

Estimating the response of a machine structure to vibratory forces is a necessary step in designing a machine to avoid damages resulting from resonance. In this paper the compliance of a structure without damping is calculated by the lumped mass method and a comparison is made between the calculated values and the experimental results.

2. Calculation

Certain machine structures can be regarded as an assembly of beams. A beam is devided into the subelement and the mass of a subelement is concentrated at the ends of it. The stiffness of the subelement are represented by the equivalent flexural, extensinal and torsional springs. Thus a framework is replaced by a spring-mass system with n degrees of freedom[1]. When the vibratory forces are applied on the system, the quation of motion of the system can be written.

$$[M]\{\ddot{x}\}+[K]\{x\}=\{f\} \qquad (1)$$

where $[K]$ is the stiffness matrix of $n\times n$,

$[M]$ is the mass matrix of $n\times n$.

$$\{f\}'=\{f_1,\ f_2,\ f_3,\ \cdots\cdots,\ f_n\}$$
$$=\{f_1\sin(pt+\alpha_1),\ f_2\sin(pt+\alpha_2),\ \cdots\cdots,\ f_n\sin(pt+\alpha_n)\} \qquad (2)$$

and

$$\{x\}'=\{x_1,\ x_2,\ x_3,\ \cdots\cdots,\ x_n\} \qquad (3)$$

where a prime $'$ means the row vector.

Forced oscillation $\{x\}$ is given by

$$\{x\}=([K]-p^2[M])^{-1}\{f\} \qquad (4)$$

Hence the compliance C_{ij} is given by

$$C_{ij}=|x_j|/|f_i|=|1/f_i\cdot([M^R]_j\cdot\{f\})| \qquad (5)$$

where $$[M^R]=([K]-p^2[M])^{-1} \qquad (6)$$

C_{ij} means the compliance at point j to the vibratory forces applied at point i and $[M^R]_j$ denotes the jth row of $[M^R]$. From equation (5) it follows that the compliance C_{ij} acts linearly on $\{f\}$. When α_i of equation (2) are different each other, C_{ij} varies with time. Fig. 1 shows a numerical calculation model idealized from the framework structure shown in Fig. 2. Table 1 gives the properties of framework elements. Each beam element is divided into subelements by solid lines. Numerical results are shown on the right side in Figs. 4 through 7. As structural damping is neglected in the numerical analysis, compliance is infinite at the resonance frequency and zero at the antiresonance frequency. Since the structure has many eigenvalues, the compliance curve is complicated in higher frequency range and then omitted in Figs. 3 through 6.

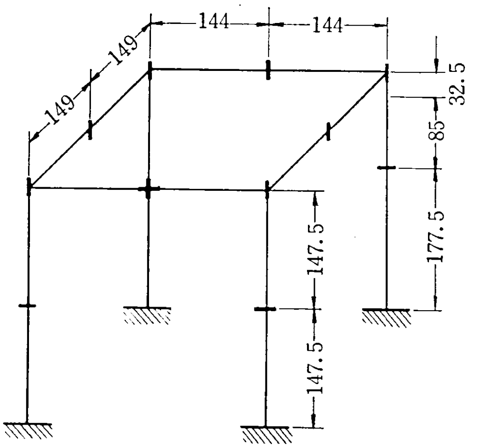

Fig. 1 Numerical calculation model

Table 1 Properties of beam element

Modulus of elasticity	2.10×10^4 kg/mm^2
Sectional area	23.0 mm^2
Moment of inertia of section (I)	10.1 mm^4
Moment of inertia of section (II)	191.6 mm^4
Polar moment of inertia of section	33.4 mm^4

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3. Experiment

The framework tested is shown in Fig. 2. The driving force is applied at the point P_1 and the amplitudes are measured at P_1 through P_{10}. Fig. 3 is a block diagram of the loading and the measurement set-up. The magnitude of driving force was measured with the strain caused at the surface of O-ring which was inserted between the driving point and an electrodynamic shaker. The amplitude of the structure was converted from alteration of capacitance of an air gap between the pickup and the structure. The magnitude of driving force was controlled to keep the amplitude nearly 0.5 mm except in the antiresonance frequency range. The driving frequency was varied stepwise and the steps were made smaller in the neighbourhood of peaks and notches. An AC-voltmeter was available for such a delicate monitoring. Typical experimental results are shown on the left in Figs. 4 through 7. It is

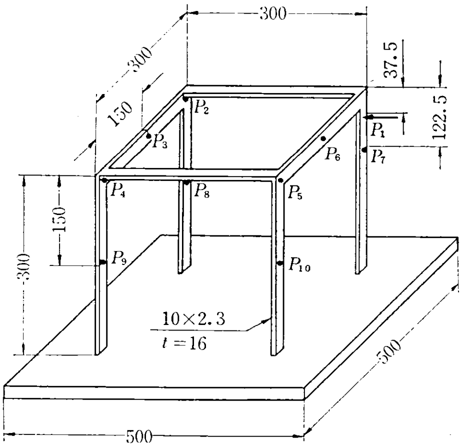


Fig. 2 Framework structure

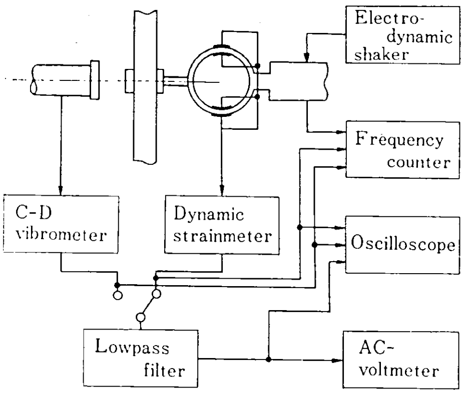


Fig. 3 Block diagram for loading and measurement

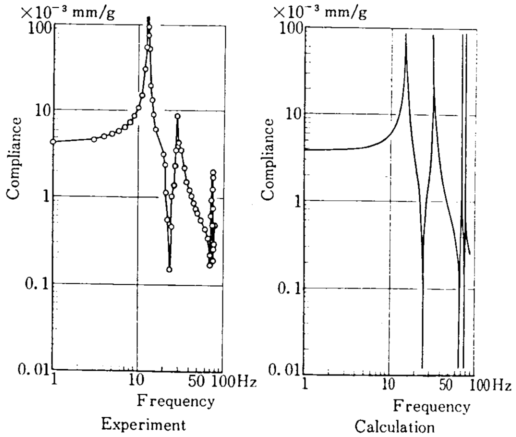


Fig. 4 Compliance at the P_1

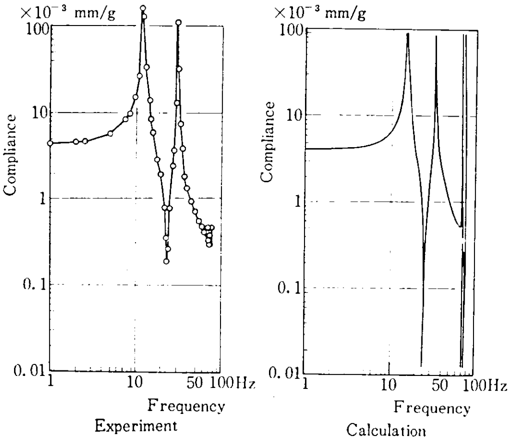


Fig. 5 Compliance at the P_2

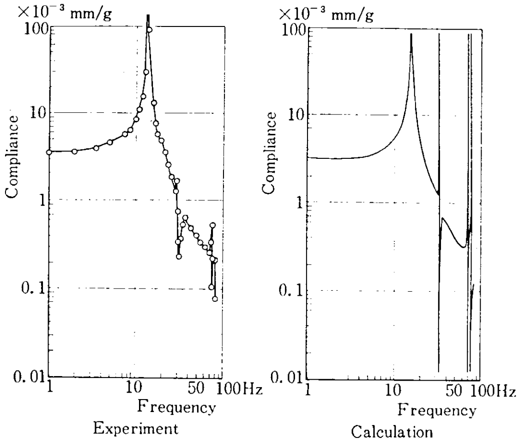
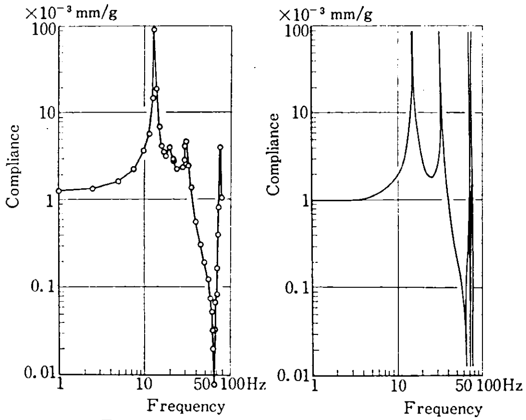


Fig. 6 Compliance at the P_3



Experiment Calculation

Fig. 7 Compliance at the P_7

noted that the value of compliance varies in wide range of 10^3-10^4 in order because of small damping of the structure used in the experiment. Each curve of the compliance shows much differnce in the higher frequency range, as there are many resonance points in a narrow frequency range and a point on the structure suddenly changes from a node of vibration to a hoop with small shifting of driving frequency. In the higher frequency range the compliance curve tends to fall and the more power is necessary to drive the structure. It must be noted that the measured value of the compliance at the peak and notch is inaccurate because of the instability of a low frequency oscillator and different response amplitude at these points.

4. Conclusion

1) The calculated curves show relatively close agreement with the experimental value. The relative error is less than 10% except in the resonance and the antiresonance frequency.

2) Frequencies of resonance and antiresonance obtained by calculation are slightly different from those which are get from experiment. It owes to the added mass effect of O-ring.

3) The small peak obviously seen at Fig. 6 of experimental curve may be caused by an inaccurate fablication of the framework structure.

4) It can be noticed that structural damping has little effect on the shape of the compliance curve in the lower driving frequency, but has much effect on the height of peak and notch in the higher frequency.

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Reference

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