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ANALYSIS OF COMMUTATORLESS MOTOR USING STATE TRANSITION METHOD

状態推移法による無整流子電動機の解析

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(1) Introduction

In this paper, we propose a method to analize a commutatorless motor. This method enables us to analize the characterlistics of a commutatorless motor directly in time domain. We can analize by this method, not only stationary characterlistics of a commutatorless motor, but also transient characteristics when the input voltage changes instantaneously.

(2) Fundamental Equations

A commutatorless motor is constructed with a 180°-type, three-phase thyristor inverter and a synchronous motor, as is shown by Fig. 1.

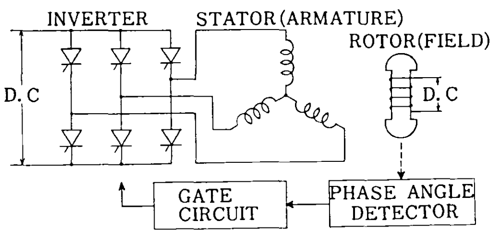

Fig. 1 Commutatorless motor

Observing on the rotor rotating with angular velocity $\dot{\theta}$ (d-q coordinate), the output voltages of the inverter (d-axis and q-axis voltage) are shown as follows respectively.

$$\left.\begin{aligned} v_d &= -V\sin(-\dot{\theta}t+30°+\delta) \\ v_q &= -V\cos(-\dot{\theta}t+30°+\delta) \end{aligned}\right\} \cdots\cdots(1)$$

where, V is the vector length of the stator voltage, and δ is the detecting angle of the rotor which corresponds to the power angle of a synchronous motor, and t is the time measured from a commutating time of the inverter.

If we denote the commutating times as $nT(n=1, 2, \cdots\cdots)$, then T equals to $\pi/3\dot{\theta}$. At this condition, the voltages v_d and v_q are given by equation (2).

$$\left.\begin{aligned} x_3(nT_+) &= x_1(nT) \\ x_4(nT_+) &= x_2(nT) \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots\cdots\cdots(2)$$

where, x_3 equals to v_d and x_4 equals to v_q. The state variables x_1 and x_2 give the initial values of x_3 and x_4 respectively. The initial conditions are given as follows.

$$\left.\begin{aligned} x_1(0) &= -V\sin(30°+\delta) \\ x_2(0) &= -V\cos(30°+\delta) \\ x_3(0) &= 0 \\ x_4(0) &= 0 \end{aligned}\right\} \cdots\cdots\cdots\cdots(3)$$

For simplisity, considering a synchronous motor without damper windings, and if the field current is constant, then the circuit equations of a synchronous motor are

$$\left.\begin{aligned} pi_f &= 0 \\ pi_d &= (-r_a i_d - L_q\dot{\theta}i_q + v_d)/L_d \\ pi_q &= (L_d\dot{\theta}i_d - r_a i_q + v_q + L_{ad}\dot{\theta}i_f)/L_q \end{aligned}\right\} (4)$$

where, i_f is the field current, i_d and i_q are the currents of d and q axis, L_d and L_q are the self-inductances of d and q axis windings, L_{ad} is the mutual inductance between d-axis and field windings, and r_a is the resistance per phase.

(3) Analysis by State Transition Method

Choosing $X=(x_1, x_2, x_3, x_4, i_f, i_d, i_q,)'$ as the state vector, the differential equation of the system except commutating time is given as follows.

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$$\dot{X}(t)=A_0X(t) \quad \cdots\cdots\cdots\cdots (5)$$

where, A_0 is the coefficient matrix defined by equations (2) and (4).

The state transition equation at commutating time is

$$X(nT_+)=BX(nT) \quad \cdots\cdots\cdots\cdots (6)$$

where, coefficient matrix B is defined by the equation (2) and the fact that the currents i_f, i_d and i_q are continuous with time. When the angular velocity $\dot{\theta}$ is constant, the time-descreet system given by equation (5) and (6) can be easily solved using state transition method.

Defining

$$H(\lambda)=\Phi(\lambda)\cdot B=\exp(A_0\lambda)\cdot B \cdots\cdots (7)$$

then, the solution of equations (5) and (6) is

$$X(nT+t)=H(t)\cdot X(nT) \quad \cdots\cdots\cdots (8)$$

If we can obtain the value of $X(nT)$, then we can compute $X(t)$ at arbitary time.

The value of $X(nT)$ at stationary condition can be obtained from equation (8) using final value theorem of z-transform.

From equation (8), we get

$$X(nT)=H^n(T)\cdot X(0) \cdots\cdots\cdots\cdots (9)$$

substituting equation (9) to the definition of z-transform, then

$$X(z)=\sum_{n=0}^{\infty}X(nT)z^{-n}$$
$$=[U-z^{-1}H(T)^{-1}]X(0) \cdots\cdots (10)$$

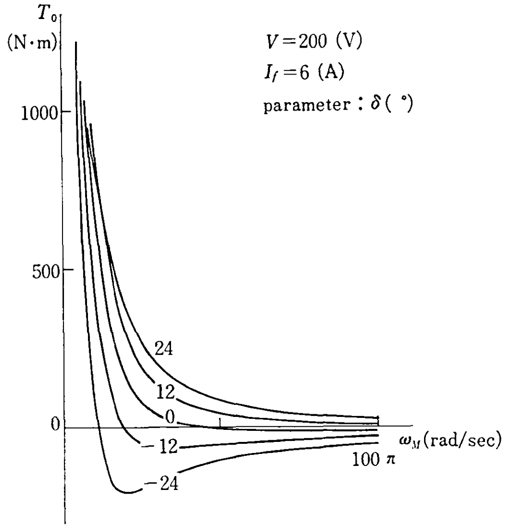

Fig. 2　Torque-speed curves of motor tested

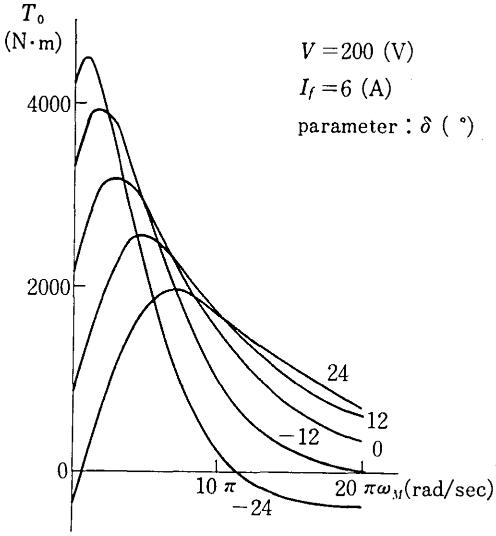

Fig. 3　Low speed part

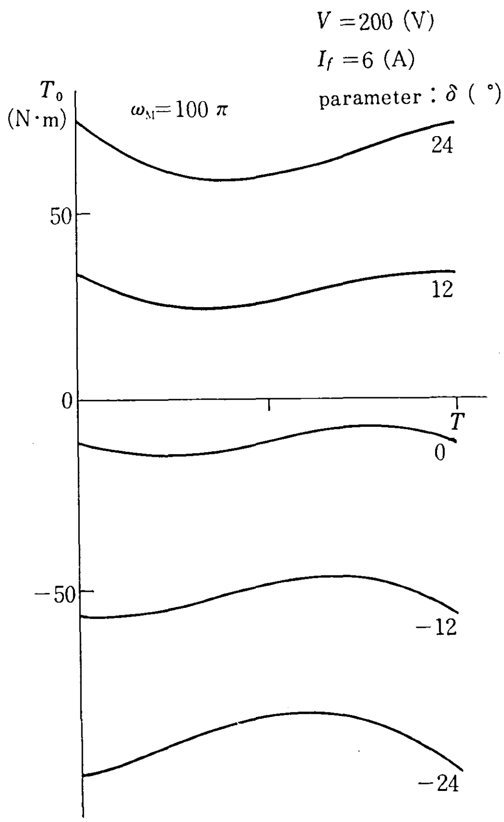

Fig. 4　Torque wave forms

where U is a unit matix. Using the final value theorem of z-transform,

$$\lim_{n\to\infty} X(nT)=\lim_{z\to 1}(1-z^{-1})X(z)$$
$$=[U-H(T)^{-1}]X(0) \quad \cdots(11)$$

By the above analysis, we can compute $\lim_{n\to\infty} X(nT)$, if the initial conditions $X(0)$ are given as equation (3). Then currents i_d and i_q at arbitary time can be computed from equation (8). Finally, we can compute output Torque T_0 and input current I_0 of the inverter. They are,

$$T_0=-(L_d-L_q)i_d i_q-L_{ad} i_f i_q \quad \cdots\cdots(12)$$
$$I_0=(v_d i_d+v_q i_q)/V \quad \cdots\cdots\cdots\cdots\cdots\cdots(13)$$

(4) Numerical Example

The characteristics of the motor tested are shown in Fig. 2∼Fig. 4. The motor constants are shown by Table 1.

In Fig. 2, average torque vs speed curves of the commutatorless motor are shown.

In Fig. 3, torque speed curves in low speed part are shown, and in Fig. 4, wave forms of instantaneous torque at angular velocity $\omega_M=100\pi$(rad/sec) are shown.

Table 1　Motor constants

10 kW, 3 Phase
2 Poles
200 V (line to line)
$r_a=0.1\Omega$
$L_d=0.00478$ H
$L_q=0.00287$ H
$L_{ad}=0.187$ H

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12 月号正誤表

ページ	段	行	種　別	正	誤
表紙 2			表紙説明	(本文 p. 18 参照)	脱落
2	右	14	本　文	1964 年新潟地震	1960 年新潟地震
3	左	下 6	〃	〃	〃
〃	〃		写真説明	〃	〃
5	右	下 14	文　献	1968 年十勝沖地震	1986 年十勝沖地震
表紙 3			筆者紹介	岡田恒男	岡田恒夫