研 究 速 報

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FLUCTUATIONS OF RESPONSE SPECTRA —PART 2—
応答倍率の変動について —第2報—
—THEORETICAL EVALUATION ON THE EFFECT OF THE EARTHQUAKE DURATION—
—地震継続時間の影響についての理論的考察—

by Nobuyuki SHIMIZU*
清 水 信 行

1. Introduction

To design highly reliable earthquake-resistant structures, we should have the knowledge on the degree of fluctuations of response spectra. These characters were studied experimentally by an analog computer simulation in a previous paper[1].

Slepian discussed on "Fluctuations of Random Noise Power" in his paper. The author introduced Rice's approximate equation[2] in his procedure. And the author could evaluate the effect of time duration T on the statistical characters of response spectra of single-degree-of-freedom system.

2. Fundamental Equations

The author assumed that the input- and output-motions of the vibration system are gaussian processes. To estimate the fluctuations of response spectra, he introduces the truncated waves of time duration T from stationary pseudo-earthquake[1]. And he assumes the fluctuations of maxima to be the ones of average power of the waves of the time duration T, that is, this means that the fluctuations of the square value of response spectrum λ^2 equal to the fluctuations of the ratio of output average power to input one of finite time duration T of the vibration system:

$$\lambda=\frac{\text{maxima of output wave}}{\text{maxima of input wave}}=\frac{\sqrt{P_0}}{\sqrt{P_i}} \quad (1)$$

where,

$$P_i=\int_0^T I_i^2(t)dt/T,\ P_0=\int_0^T I_0^2(t)dt/T \quad (2)$$

$I_i(t)$, $I_0(t)$ are input- and output-wave of the vibration system respectively, (subscripts i and o are used correspond to the input and output waves respectivly). From Rice's approximate formula, the distribution of the value χ corresponds to the root value of the average power of the waves is as follows:

$$f(\chi)d\chi=\begin{cases}\dfrac{2}{\Gamma(n+1)}\chi^{2n+1}e^{-\chi^2}d\chi\ ; & \chi\geqq 0\\ 0 & ;\ \chi<0\end{cases} \quad (3)$$

where,

$$\chi=\left(\frac{mT}{\sigma^2}\right)^{1/2}P^{1/2},\quad P=\int_0^T I^2(t)dt/T \quad (4)$$

and m, σ^2, n are mean, variance, degree of freedom of generalized χ^2 type distribution. (*cf.* reference 2)).

The joint probability distribution of χ_i and χ_0 is obtained as follows:

$$\frac{4}{\Gamma(n_i+1)\Gamma(n_0+1)}\chi_i^{2n_i+1}\chi_0^{2n_0+1}\times\exp(\chi_i^2+\chi_0^2)d\chi_0 d\chi_i \quad (5)$$

from eqs. (3), (4) and (1) under assumption that χ_i and χ_0 are independent.

From eq. (5), probability distribution of λ_* $(=\chi_0/\chi_i)$ corresponds to the response spectrum λ of eq. (1) is

$$f(\lambda_*)d\lambda_*=\begin{cases}\dfrac{2\Gamma(n_i+n_0+2)}{\Gamma(n_i+1)\Gamma(n_0+1)}\dfrac{\lambda_*^{2n_0+1}}{(1+\lambda_*^2)^{n_i+n_0+2}}d\lambda_* & ;\ \lambda_*\geqq 0\\ 0 & ;\ \lambda_*<0\end{cases} \quad (6)$$

where, the relation of λ and λ_* is

$$\lambda=K\lambda_*,\quad K=\left(\frac{\sigma_0^2}{m_0}\right)^{1/2}\left(\frac{\sigma_i^2}{m_i}\right)^{-1/2} \quad (7)$$

and m_i, σ_i^2, n_i; m_0, σ_0^2, n_0 are the same as in eq. (4).

3. Statistical Properties of Response Spectra

Statistical properties (i. e. moments) of response spectra are obtained from eq. (6). Mean value $\bar{\lambda}$, standard deviation σ_λ and relative dispersion d_λ are as follows;

$$\bar{\lambda}=K\cdot\alpha_1,\quad \sigma_\lambda=K\cdot\sqrt{\alpha_2-\alpha_1^2} \quad (8)$$
$$d_\lambda=\sqrt{\alpha_2-\alpha_1^2}/\alpha_1$$

where α_1 and α_2 are the first and second moments of λ_*, and these are obtained from eq. (6)

* Graduate Student, Plant Engg. Lab., Dept. of Mechanical Engineering and Naval Architecture, Inst. of Industrial Science, Univ. of Tokyo.

$$\alpha_1=\Gamma(n_i+1/2)\Gamma(n_0+3/2)/[\Gamma(n_i+1)\Gamma(n_0+1)] \quad (9)$$

$$\alpha_2=(n_0+1)/n_i \quad (10)$$

4. Numerical Computations and Results

The author computes $f(\lambda)$, $\bar{\lambda}$, d_λ in the case of a single-degree-of-freedom system. The power spectra of pseudo-earthquake and absolute acceleration of the system are

$$A(\omega)=\frac{4\omega_g{}^2\zeta_g{}^2\omega^2+\omega_g{}^4}{\omega^4-2(1-2\zeta^2{}_g)\omega_g{}^2\omega^2+\omega_g{}^4}\cdot k \quad (11)$$

and

$$S_{\ddot{z}}(\omega)=\frac{4\omega_b{}^2\zeta_b{}^2\omega^2+\omega_b{}^4}{\omega^4-2(1-2\zeta_b{}^2)\omega_b{}^2\omega^2+\omega_b{}^4}\cdot A(\omega) \quad (12)$$

respectively, where ω_g, ζ_g are natural frequency and damping ratio of the ground and ω_b, ζ_b are those of the system and k is intensity of gaussian white noise. Auto-covariance functions $\psi_g(\tau)$, $\psi_{\ddot{z}}(\tau)$ of eqs. (11) and (12) are obtained by means of Fourier integral. For pseudo-earthquanke, for instance, $\psi_g(\tau)$ is

$$\psi_g(\tau)=\frac{1}{2\pi}\int_0^\infty A(\omega)e^{i\omega\tau}d\omega \quad (13)$$

and this becomes

$$\psi_g(\tau)=e^{-\alpha_g|\tau|}[\gamma_g\cos\beta_g|\tau|+\delta_g\sin\beta_g|\tau|] \quad (13)'$$

after integrating, where

$$\alpha_g=\zeta_g\omega_g, \qquad \beta_g=\sqrt{1-\zeta_g{}^2}\,\omega_g$$

$$\gamma_g=(1+4\zeta_g{}^2)/8\zeta_g,\quad \delta_g=(1-4\zeta_g{}^2)/8\sqrt{1-\zeta_g{}^2}.$$

Fig. 1 shows the normalized auto-covariance function $\rho(\tau)(=\psi(\tau)/\psi(0))$ for the absolute acceleration of the pseudo-earthquake and the system.

Fig. 2 shows the probability density function $f(\lambda)$ of the response spectra. As T becomes larger, $f(\lambda)$ is peaking up around the mean value of the response spectra and the standard deviation becomes smaller.

Fig. 3 shows the curves of the relative dispersion d_λ for T_b/T_g. For $T_b/T_g=0.5\sim1.0$, the change of these values is considerably large and for $T_b/T_g\gtrsim1.2$, these values make a straight line.

5. Conclusions and Acknowlegements

The author obtained the following conclusions from the above discussions;

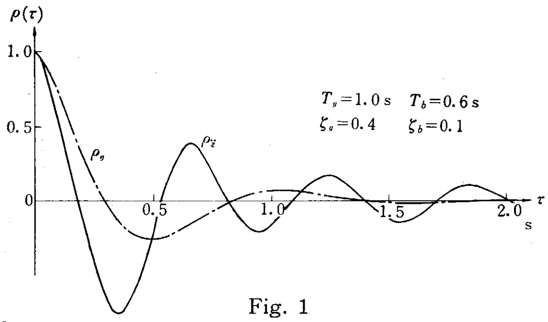

Fig. 1

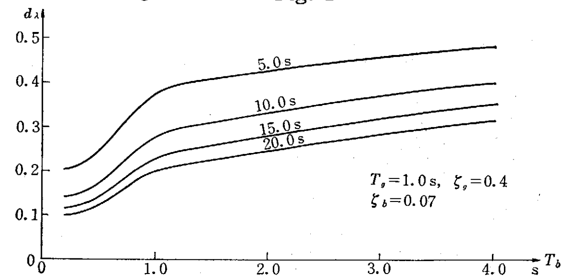

Fig. 2

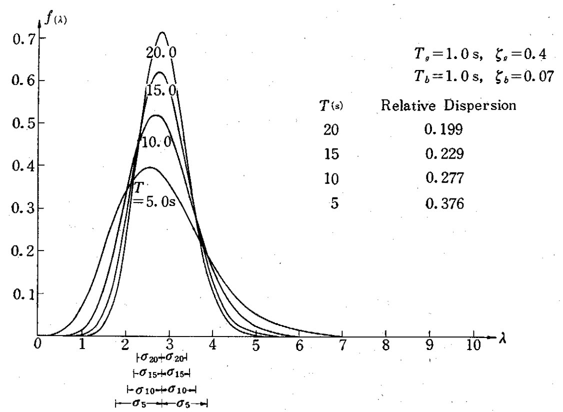

Fig. 3

(1) The duration of earthquake is the important factor for the fluctuations of response spectra.

(2) Mean value of response spectra is influenced little by the duration of earthquake, but the standard deviation is influenced.

(3) Relative dispersion is almost 0.2～0.4 for $T_b/T_g=0.2\sim0.4$ and $T/T_g=20.0\sim5.0$.

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