研　究　速　報
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FLUCTUATIONS OF RESPONSE SPECTRA — PART 1 —

応答倍率の変動について —第1報—

—OF MULTI-DEGREES-OF-FREEDOM SYSTEM THROUGH ANALOG COMPUTATION—

—アナログ計算機による多自由度系の応答について—

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1. Introduction

The authors studied the evaluations of response spectra of multi-degrees-of-freedom system subjected to two random inputs based on the discussion in the previous paper[1]. In this study, the authors judged that it was necessary to evaluate its response spectra statistically. So they caluclated the responses to one hundred pseudo-earthquakes for each parameter by an analog computer. From these calculations, they found that the fluctuations of response or response spectra are considerably large. These fluctuations are able to be estimated through the statistical and probability theory as to be reported in next paper.

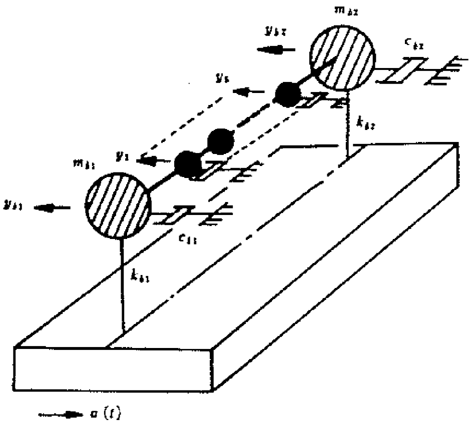

Fig. 1

2. Fundamental Equations

The analytical model in this article is shown in Fig. 1. The details and the equations of this model were described in the previous report.

The equations of the lumped mass system which is used for analog computation as follows;

$$\begin{cases} \ddot{y}_1+2\zeta\omega\dot{y}_1+2\omega^2 y_1 = -\alpha+\omega^2 y_{b1} \\ \ddot{y}_2+2\zeta\omega\dot{y}_2+\omega^2 y_1+2\omega^2 y_2+\omega^2 y_3 = -\alpha \\ \ddot{y}_3+2\zeta\omega\dot{y}_3+\omega^2 y_2+2\omega^2 y_3+\omega^2 y_4 = -\alpha \\ \ddot{y}_4+2\zeta\omega\dot{y}_4+\omega^2 y_3+2\omega^2 y_4+\omega^2 y_5 = -\alpha \\ \ddot{y}_5+2\zeta\omega\dot{y}_5+2\omega^2 y_5 = -\alpha+\omega^2 y_{b2} \end{cases} \quad (1)$$

where y_i $(i=1,\ldots,5)$ is the relative displacement of the mass i to the ground motion, $\omega^2=k/m$, $\zeta=c/2\sqrt{mk}$ and m, k, c are masses, spring constants damping coefficients respectively.

The solution of Eq. (1) can be taken in the following form,

$$y_i=\sum_{j=1}^{5} B_i^{(j)}\varphi_j(t) \quad (i=1,\ldots,5) \qquad (2)$$

Then, the equations of motion can be written in the form of generalized coordinates,

$$\ddot{\varphi}_j+2\zeta_j\nu_j\dot{\varphi}_j+\nu_j^2\varphi_j=-I_j\alpha+\omega^2 J_j y_{b1}+\omega^2 K_j y_{b1} \qquad (3)$$

where I_j, J_j, K_j are exciting coefficients to the motions of ground, building 1 and 2 respectively. Thus, absolute acceleration of the i-th mass is as follows;

$$\ddot{z}_i=\sum_{j=1}^{5} B_i^{(j)}\ddot{z}_{\varphi j} \quad (i=1,\ldots,5) \qquad (4)$$

where $\ddot{z}_{\varphi j}$ is

$$\ddot{z}_{\varphi j}=\ddot{\varphi}_j+I_j\alpha \quad (j=1,\ldots,5) \qquad (5)$$

The authors call the maximum values from Eqs. (1), (4), (6) and (7) as "direct value", "exact value", "sum of absolute values" and "root of sum of square values" respectively.

$$|\ddot{z}_i|=\sum_{j=1}^{5}|B_i^{(j)}\ddot{z}_{\varphi j}|_{\max} \qquad (6)$$

$$\ddot{z}_i|_r=[\sum_{j=1}^{5}|B_i^{(j)}\ddot{z}_{\varphi j}|_{\max}{}^2]^{1/2} \qquad (7)$$

3. Experiments by Analog Computer

The authors produced 100 samples of stationary pseudo-earthquakes and those of nonstationary ones. The latters were obtained by multiplying a shaping function

$$e^{-\alpha t}-e^{-\beta t} \qquad (8)$$

to the former pseudo-earthquakes. The dynamic responses of the system to these 100 inputs were simulated by an analog computer. From these responses, they obtained the statistics of the response spectra of such multi-degrees-of-freedom system. Through these experiments, the authors used the parameters $T_g=1.0$ s, $\zeta_g=0.3$, $\Psi_1=0.053$ Hz and $\Psi_2=19.9$ Hz[2] to produce pseudo-

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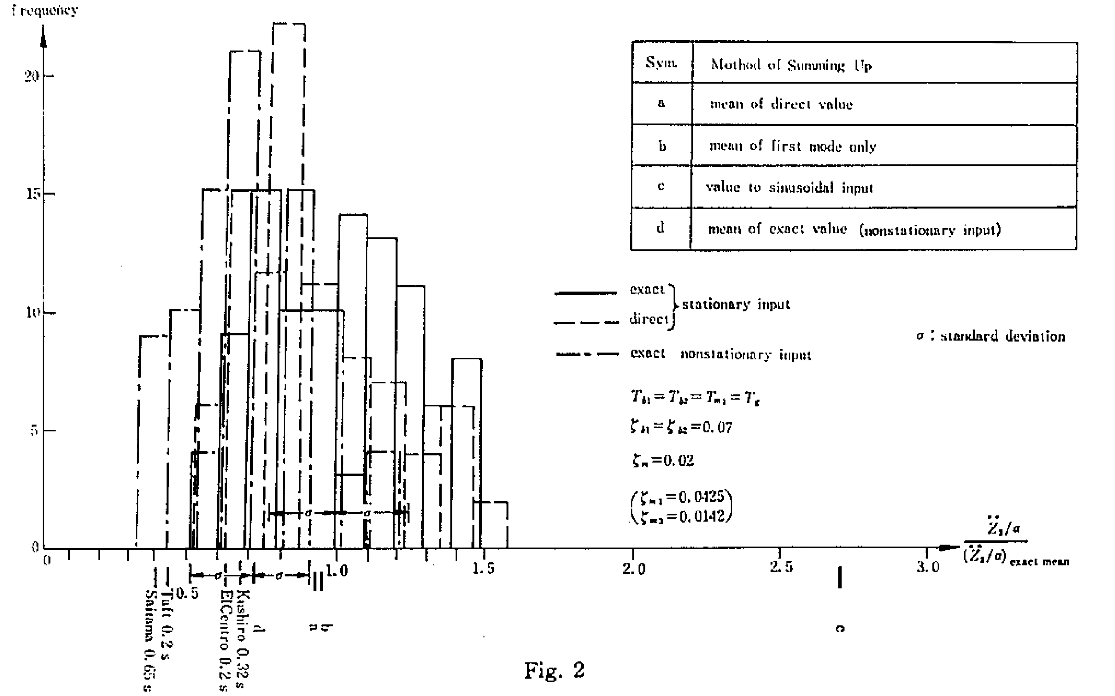

Sym.	Method of Summing Up
a	mean of direct value
b	mean of first mode only
c	value to sinusoidal input
d	mean of exact value (nonstationary input)

Fig. 2

earthquakes from a low-frequency noise oscillator. And to give the nonstationarity to them, $\alpha=0.125$ rad/s, $\beta=0.250$ rad/s in Eq. (8) were used (these values for α and β were after reference(3)).

4. Results

The discussion in this preliminary report is limited to the values of the absolute acceleration of the mass 3 at the center of the beam.

Fig. 2 shows the hystograms of response spectra to the 100 stationary pseudo-earthquakes and those of nonstationary ones, and real records; El Centro NS (May. 18, 1940), Taft NS (July. 21, 1952), Kushiro NS (Dec. 24, 1961), Saitama EW (Feb. 14, 1956).

The hystograms and values are obtained by the various of estimation (direct, exact, sum of absolute and root of sum of square values, *cf.* § 2). Mean value of the response spectra to nonstationary inputs

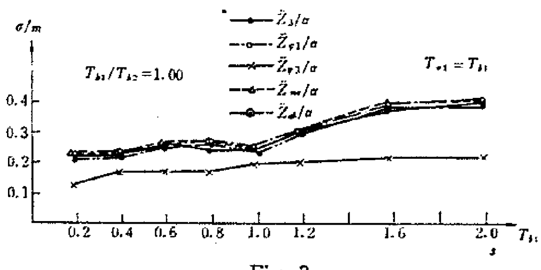

Fig. 3

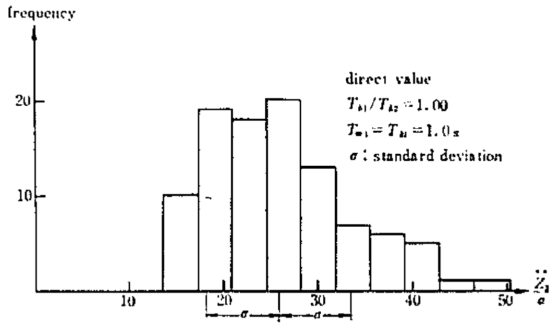

Fig. 4 (a)

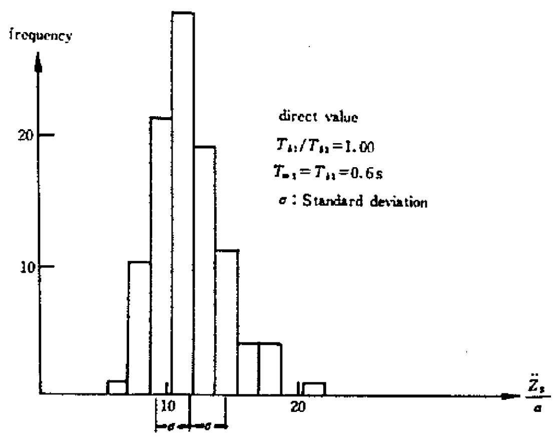

Fig. 4 (b)

is about 70% of that to stationary. This is close to the values for Kushiro and El Centro earthquakes. It would seem that the stronger nonstationarity of the inputs (*i. e.* larger value of the coefficient α and β in Eq. (8)), the less mean value of response spectra. Relative dispersion to the stationary and nonstationary inputs, however, are almost equal each other. These results seem to be considerably important character for the fluctuations of response spectra. Fig. 3 shows relative dispersion σ/m of response spectra. This relative dispersion is almost constant (0. 2～0. 3) for any case of T_{b1}/T_{b2} and of T_{b1} under fixed T_{b1}/T_{b2} so far as the authors' experiments concern,

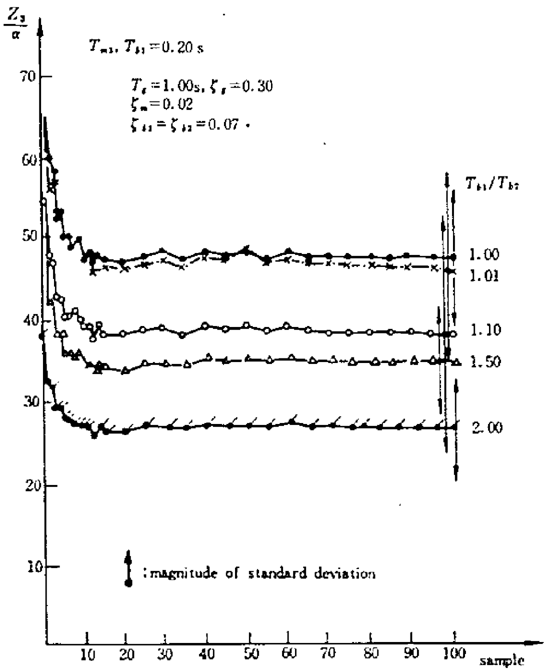

Fig. 5

where T_{b1} and T_{b2} are the natural period of buildings 1 and 2 respectively. Figs. 4 (a) and 4 (b) show the hystograms of response spectra for T_{b1} under fixed T_{b1}/T_{b2}.

5. Conclusions and Acknowledgment

From and analog computer simulation, the authors obtained the following results;

(1) Relative dispersion is considerably large, and its value takes nearly 0. 3.

(2) This means that the reliability of response calculus is so low that a single result of computation may have an error exceed 90% of the value according to the law of 3σ.

(3) The mean value might be said to be relatively stable, if the number of response computations for a particular system is exceed twenty (Fig. 5).

(4) Relative dispersion change little for the various parameters (T_m, T_{b1}, etc).

(5) It would seem that the relative dispersion stend to increase as complexity of the system.

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