研 究 速 報

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A STUDY ON CONFIDENCE INTERVAL IN PREDICTING ACCELERATION AMPLIFICATION FACTOR

加速度応答倍率推定時の信頼幅について

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1 Introduction

The maximum acceleration amplification factor is given as the ratio of the maximum acceleration of response of structure system to that of earthquake acceleration as input. The factor for a group of response spectra[1] with a constant damping ratio to various earthquake records scatters around a mean. The scattering occurs because of stochastic characteristics of the wave form, non-stationarity of earthquake motion and so on. The characteristic how the factor distribute were studied by making use of χ^2- and t-distribution test in previous paper.[4),7)]

On the other hand methods predicting the factor have been shown, that is, the earthquake is simulated by a stationary random vibration[2),3)] which is filtered from white noise through a one-degree-of-freedom system representing a ground model.[5)] Then the maximum is given as the value to which the probability density function of extreme reaches small enough. This makes it possible that the factor can be given by knowing the natural period of the structure system T_b, the dominant period of ground T_g and damping ratio of the system. However, the complete coincidence of the design estimation and the value actually obtained of those is hardly expected, since the dominant period of ground appears statistically in earthquakes recorded at a specific site. The natural period of constructed structures would be also different from the design value. Realization of the natural periods of the system can be considered as probabilistic event.

These necessarily require that the estimation of the acceleration amplification factor should accompany the sense of statistical characteristic as mean and variance. In this paper log-normal distributon is principally assumed for stochastic variable and the effect of the distribution to the factor is discussed.

2 Mean and Variance of the Acceleration Amplification Factor

Fig. 1 shows an example of relation between the dominant period of ground obsereved in earthquakes and frequency of the occurence at Hongo.

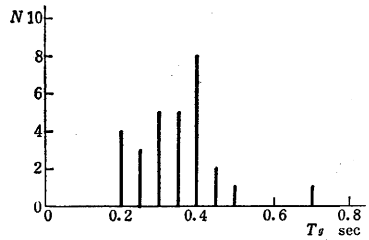

Fig. 1 The ground dominant period observed in earthquake motions and frequency of the occurences (Kanai)[6)]

This suggests that the dominant period of ground implies a probabilistic event. The mean μ and the variance σ^2 are computed as

$$\mu=0.35\,\text{sec},\quad \sigma^2=0.0109\,\text{sec}^2,\quad \sigma=0.105\,\text{sec}$$

if μ is transfered to 1.0 sec, the relative relation of μ and σ is presereved as $\sigma=0.3$ sec.

As the simplest assumption normal distribution

$$p(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} \qquad (1)$$

is applicable instead of the actual distribution. As the variance become larger, foot of the distribution merges into negative region. However, period as probabilistic variable should exist in positive. Then log-normal distribution, which is obtained by substituting $x=\log y$ to (1) and is defined only in positive,

$$p(y)=\frac{1}{\sigma\sqrt{2\pi}}\frac{1}{y}\exp\left\{-\frac{(\log y-\mu)^2}{2\sigma^2}\right\} \qquad (2)$$

is principally used. The frequency of the occurence of dominant period observed in micro-tremor also suggests approximate log-normal distribution.

The computation to obtain the mean and the variance of the amplification factor is carried out for these three cases:

1) only for the ground dominant period the probabilistic distribution is assumed,
2) only for the natural period of structural system the probabilistic distribution is assumed,
3) for both periods the probabilistic distribution is assumed under the condition that both

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variables are independent.

The computation procedure is shown by taking the case 1) as an example.

$A(T_b/T_g)$: The amplification factor given by a function of T_b/T_g through a statistical computation (Fig. 2).

$p(T_g)$: The probability density function for estimation of T_g.

Then the mean $E[A]$ and the variance $\sigma_A{}^2$ of the amplification factor are derived by

$$\left.\begin{aligned} E[A] &= \int_{T_{g1}}^{T_{g2}} A(T_b/T_g)p(T_g)dT_g \\ \sigma_A{}^2 &= \int_{T_{g1}}^{T_{g2}} \{A(T_b/T_g)\}^2 p(T_g)dT_g \\ &\quad -\{E[A]\}^2 \end{aligned}\right\} \quad (3)$$

where T_{g1} and T_{g2} show region of the integral.

Computations are performed by assuming that nominal dominant period of ground T_g is equal to 1.0 sec. Fig. 3 illustrates a relation between standard deviation σ of $p(T_g)$ and the mean or $3\sigma_A$ confidence intervals of the factor for case 1). Fig. 4 and 5 show the same relation for case 2).

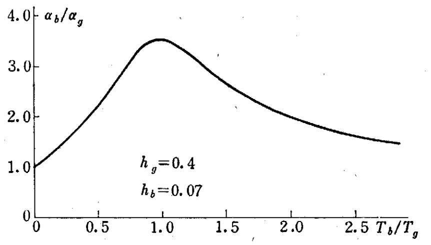

Fig. 2 Spectrum of the acceleration amplification factor by a statistical computation

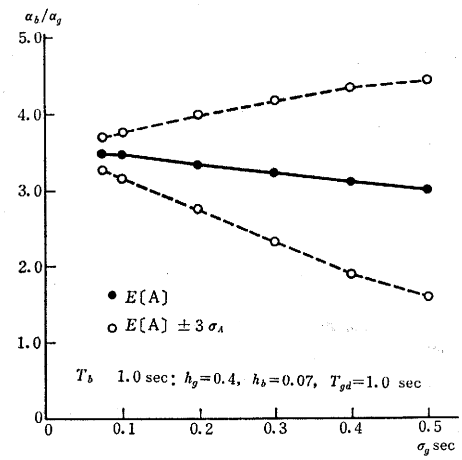

Fig. 3 Mean and confidence interval of the acceleration amplification factor (Probability density function with log-nomal distribution is assumed only for estimation of the ground dominant period)

The former is for $p(T_b)$ of log-normal distribution and the latter for that of normal distribution.

These figures show that as σ becomes large the mean of the factor decreases and the confidence interval increases. It is also known that the confidence interval by normal distribution is wider than that by log-normal for same σ.

Case 3) is supposed to be closest to actual situation, since another distribution of probabity density function is assumed in addition to that for either T_g or T_b. Fig. 6 shows some results.

Let us examine the mean and the variance of the factor for the case shown in Fig. 1. 0.35 sec, the mean, is taken as its normal ground do-

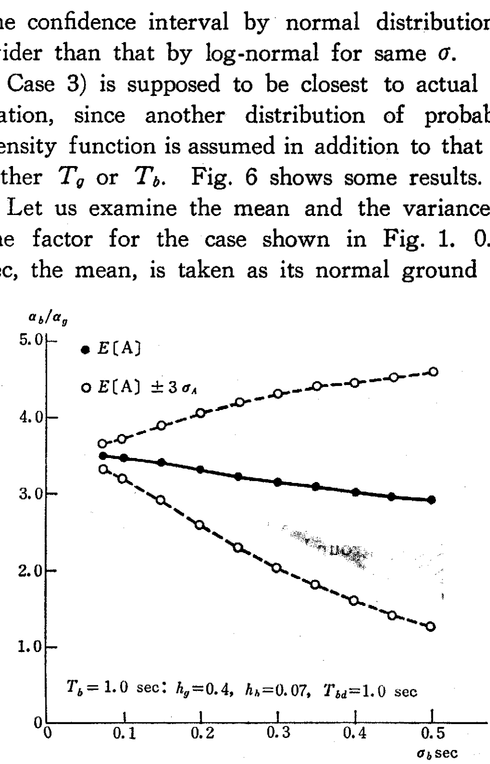

Fig. 4 Mean and confidence interval of the acceleration amplification factor (Probability density function log-normal distribution is assumed only for estimation of the natural period of structure)

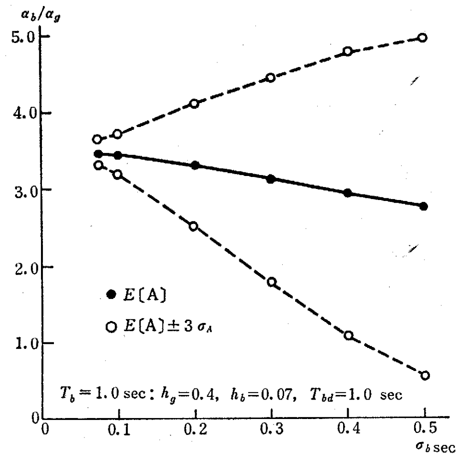

Fig. 5 Mean and confidence interval of the acceleration amplification factor (Probability density function with normal distribution is assumed only for estimation of the natural period of structure)

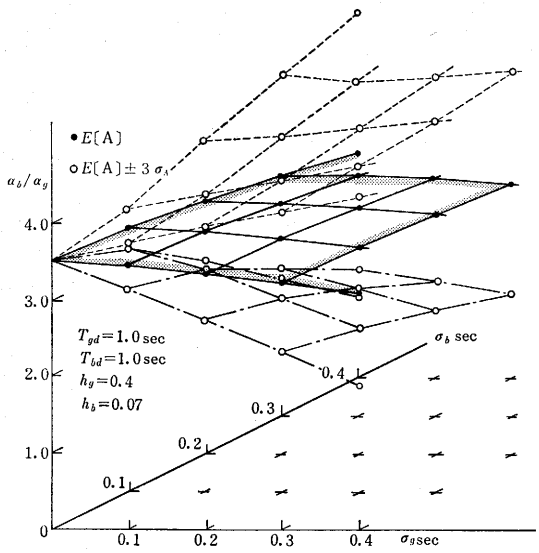

Fig. 6 Mean and confidence interval of the acceleration amplification factor (Probability density function with log-normal distribution is assumed for estimation of both the natural period of structure and the ground dominant period)

minant period. The natural period of structure system would be set as $T_b=0.35$ sec, too. This means that both period of the system are nominally equal and the condition is worst in view of the aseismic design. Provided that the amplification factor is given by Fig. 2, the mean and the variance can be computed as

$$E[A]=3.15, \quad \sigma_A^2=0.185, \quad \sigma_A=0.430$$

Theoretical value which corresponds to same standard deviation $\sigma=0.3$ is obtained as follows from Fig. 3, 4 and 5 respectively.

$$E[A]=3.23 \qquad \sigma_A=0.327$$
$$E[A]=3.17 \qquad \sigma_A=0.377$$
$$E[A]=3.12 \qquad \sigma_A=0.443$$

Thus, the mean obtained by the actual example can be considered same as theoretical one, and the variance is close to that by normal distribution, though it is only one example at present.

According to reference 4) the confidence interval $3\sigma_A$ is given as $3\sigma_A=0.45$ for damping ratio of the structure $h=0.05$. In order to obtain same amount of interval, $\sigma_g/T_{gd}=\sigma_b/T_{bd}\fallingdotseq 0.15$ can be given from the theoretical computatation show in Fig. 3 and 4. Suffix d means design value. This suggests that the confidence interval of response spectrum can be predicted by assuming a probability density function for the estmation of natural period of the system.

3 Conclusions and Acknowledgment

Thus a method to obtain the mean and the variance which are expected in predicting the acceleration amplification factor of structure to earthquakes has been shown and several conclusions are derived.

1) The mean and the variance of the factor based on normal distribution give smaller and larger results respectively than those by log-normal distribution for same variance of the distribution.
2) The mean and the variance of the factor are also given for the case that both periods of the system are taken as probabilistic variables. Generally they are given as a curved surface which is a function of the variance of the probability density function.
3) The mean and the variance of the factor based on an actual relation between the ground dominant period and numbers of the occurence at a specific site are compared with the theoretical estimation. It showed that the estimation can be a good approximation. However, the same kind of observation data should be accumlated in future to make the method generalize.
4) The variance of the probability density function which gives same amount of confidence interval as that for maxima of response spectra is discussed. This is also should be studied in future.

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References

1) G. W. Housner et al. : Spectrum Analysys of Strong Motion Earthquakes, Bull, Seism. Soc. Am., 43-2, 1953-4
2) H. Tajimi: Basic Theories on Aseismic Design of Structures (in Japaneses), Rep. IIS., Univ. of Tokyo, 8-4, 1959-3.
3) H. Sato: A Study on Aseismic Design of Machine Structure (in Japanese), Rep. IIS., Univ. of Tokyo, 15-1, 1965-11.
4) H. Sato: On the Confidence Intervals of the Acceleration Amplification Factor of Response Spectrum (in Japanese), Jour. IIS., Univ. of Tokyo, 20-6, 1968-6.
5) K. Kanai: Semi-empirical Formula for the Seismic Characteristics of the Ground, Bull. ERI, Univ. of Tokyo, 35, 1957-6.
6) Y. Otsuki and K. Kanai: Aseismic Design (in Japanese), Corona Co. Ltd., 1962.
7) H. Umemura, Y. Osawa and Y. Matsushima: A Study on Response Spectra to Earthquake Motions Observed in Japan (in Japanese), Bull. RI., Fac. of Eng., Univ. of Tokyo, 26. 1967-12.