研　究　速　報

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ON VIBRATION ANALYSIS OF PIPING SYSTEM (MULTI-DEGREES-OF-FREEDOM SYSTEM) SUBJECTED TO MULTI-RANDOM-INPUTS—PART 1

不規則な多入力に対する配管系（多自由度系）の振動解析について—第１報—

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1. Introduction

For earthquake-resistant piping-structures of nuclear power plants, it is necessary to be designed dynamically[1]. In this regard, the authors have to develope the fundamental theory on multi-input problems of a beam. The authors analized the dynamic behavior of the pipings subjected to two different random inputs at the both ends, that is, the problem of two random inputs to multi-degrees-of-freedom system.

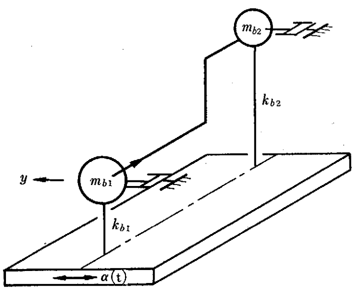

Fig. 1

2. Fundamental Equations

The analytical model[2] in this article is shown in Fig. 1. The vibrational model for buildings is shear-type one and that for pipings is extended simple beam or lumped mass system. The following assumption were made; the simple beam is Bernoulli-Euler type, and the total mass of the piping system is small enough to the mass of buildings.

The equations of motion for the buildings 1 and 2 are

$$\ddot{y}_{bi}(t)+2\zeta_{bi}\,\omega_{bi}\,\dot{y}_{bi}(t)+\omega_{bi}^2\,y_{bi}(t)=-\alpha(t) \quad (i=1,2) \qquad (1)$$

where $y_{bi}(t)$ $(i=1,2)$ is the relative displacement to the ground motion, $\omega_{bi}\,(i=1, 2)$ is the natural frequency, $\zeta_{bi}\,(i=1,2)$ is the damping ratio of the building i, and $\alpha(t)$ is the ground acceleration.

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The equation of motion for the pipings is

$$\frac{\partial^2 y(\xi, t)}{\partial t^2}+D_\xi\, y(\xi, t)=-\alpha(t) \qquad (2)$$

where differential operator $D_\xi=a^2\frac{\partial^4}{\partial\xi^4}$ for bending, or $D_\xi=-b^2\frac{\partial^2}{\partial\xi^2}$ for torsional vibration.

Boundary conditions are

$$\begin{cases} \dfrac{\partial y(0, t)}{\partial\xi}=\dfrac{\partial y(L, t)}{\partial\xi}=0 \\ y(0, t)=y_{b_1}(t), \quad y(L, t)=y_{b_2}(t) \end{cases} \qquad (3)$$

Here, $y(\xi, t)$ is the relative displacement of pipings to the ground motion, and L is a total length of the piping system.

To solve eq. (2) under conditions (3), the authors put

$$y(\xi, t)=v(\xi, t)+w(\xi, t) \qquad (4)$$

where $v(\xi, t)$ is the statical displacement and $w(\xi, t)$ is the dynamical displacement of pipings. Furthermore, $v(\xi, t)$ can be written by the relative displacements of the buildings,

$$v(\xi, t)=g_1(\xi)\,y_{b_1}(t)+g_2(\xi)\,y_{b_2}(t) \qquad (5)$$

Here $g_1(\xi)$ and $g_2(\xi)$ are the statical deformation functions given with the unit displacements of buildings 1 and 2 respectively. Then, eq. (2) is modified to

$$\frac{\partial^2 w(\xi, t)}{\partial t^2}+D_\xi\, w(\xi, t) =-\{\alpha(t)+g_1(\xi)\,\ddot{y}_{b_1}+g_2(\xi)\,\ddot{y}_{b_2}\} \qquad (6)$$

using the relation of the statical equilibrium $D_\xi\, v(\xi, t)=0$. We can solve eq. (6) for relative piping movement by means of modal analysis method, that is,

$$w(\xi, t)=\sum_{j=1}^{\infty}\varphi_j(t)\,X_j(\xi) \qquad (7)$$

Then, we can write the equations of motion in the form of generalized coordinates,

$$\ddot{\varphi}_j(t)+2\zeta_j\,\nu_j\,\dot{\varphi}_j(t)+\nu_j^2\,\varphi_j(t) =-\{\alpha_{0j}\,\alpha(t)+\beta_{0j}\,\ddot{y}_{b_1}(t)+\gamma_{0j}\,\ddot{y}_{b_2}(t)\} \quad (j=1, 2, \cdots) \qquad (8)$$

where $X_j(\xi)$ is the normalized mode-shape function of the j-th mode. The authors also assumed the damping ratio ζ_j would be small. The right hand side of eq. (8) is also written as

$$-\{\beta_{0j}\ddot{z}_{b_1}(t)+\gamma_{0j}\ddot{z}_{b_2}(t)\},$$

where $\ddot{z}_{bi}(t)$ $(i=1,2)$ is the absolute acceleration of the building i. Here α_{0j}, β_{0j} and γ_{0j} $(i=1, 2, \cdots)$ are exciting coefficients to external uniform acceleration and the effects of motions of supporting buildings.

3. Response Analysis in Statistical Approach

From eqs. (1), (4), (7) and (8), we can obtain the displacement, velocity and acceleration of pipings. The absolute acceleration $\ddot{z}(\xi, t)$ is as follows:

$$\ddot{z}(\xi,t)=\begin{cases}\sum_{j=1}^{\infty}X_j(\xi)\left[\frac{\nu_j{}^2}{\nu_j'}\int_0^t\{\beta_{0j}\ddot{z}_{b_1}(\sigma)+\gamma_{0j}\ddot{z}_{b_2}(\tau)\}\right.\\ \left.\cdot e^{-\zeta_j\nu_j(t-\tau)}\sin\{\nu_j'(t-\sigma)+2\phi_j\}d\sigma\right] \quad (9)\\ \quad ;\ 0<\xi<L\\ \ddot{z}_{b_1}(t);\ \xi=0\\ \ddot{z}_{b_2}(t);\ \xi=L\end{cases}$$

where $\nu_j'=\nu_j\sqrt{1-\zeta_j{}^2}$ and $\phi_j=\tan^{-1}\frac{\zeta_j}{\sqrt{1-\zeta_j{}^2}}$.

To discuss the statistical characteristics of the dynamical response, the authors introduced the white-noise type pseudo-earthquake. In such cases we can assume that the ground acceleration is a stationary random process and its mean is zero.

$H_{\ddot{z}}(i\omega, \xi)$ is transfer function through the system and $A(\omega)$ is the power spectrum of the ground acceleration. Then, the variance of the response of pipings $\ddot{z}(\omega, \xi)$ can be written as

$$E\{\ddot{z}^2(\xi,t)\}=\int_{-\infty}^{\infty}|H_{\ddot{z}}(i\omega,\xi)|^2A(\omega)d\omega \quad (10)$$

$H_{\ddot{z}}(s, \xi)$ is as follows;

$$H_{\ddot{z}}(s,\xi)=\sum_{j=1}^{\infty}X_j(\xi)[H_{\ddot{z}j}(s)\{\beta_{0j}H_{\ddot{z}b_1}(s)+\gamma_{0j}H_{\ddot{z}b_2}(s)\}] \quad (11)$$

Here, $H_{\ddot{z}j}(s)$ $(j=1, 2, \cdots)$, $H_{\ddot{z}b_1}(s)$ and $H_{\ddot{z}b_2}(s)$ are transfer functions of acceleration through pipings in j-th mode and both buildings. Each of then has the same form, for example,

$$H_{\ddot{z}j}(s)=\frac{2\zeta\nu_j s+\nu_j{}^2}{s^2+2\zeta_j\nu_j s+\nu_j{}^2}. \quad (12)$$

If we assume that the ground acceleration is Gaussian process, and put the amplification factor of response through the system as following;

$$\lambda(\xi)=\sqrt{\frac{E\{\ddot{z}^2(\xi,t)\}}{E\{\alpha^2(t)\}}} \quad (13)$$

the amplification factor of the maximum acceleration of pipings to that of the ground is

$$\lambda^2(\xi)=\frac{\int_{-\infty}^{\infty}|H_{\ddot{z}}(s,\xi)|^2|H_g(s)|^2}{\int_{-\infty}^{\infty}|H_g(s)|^2}*$$
$$*\frac{\left|\frac{s^2}{(\Psi_1s+1)^2}\frac{1}{(\Psi_2s+1)^2}\right|^2kd\omega}{\left|\frac{s^2}{(\Psi_1s+1)^2}\frac{1}{(\Psi_2s+1)^2}\right|^2kd\omega} \quad (14)$$

Where $H_g(s)$ is a transfer function of the ground and $\left|\frac{s^2}{(\Psi_1s+1)^2}\frac{1}{(\Psi_2s+1)^2}\sqrt{k}\right|^2$ is the power spectrum of acceleration in source.

4. Evaluation of Transfer Function

Developing $|H_{\ddot{z}}(s,\xi)|^2$ after substituting eq. (11), the authors obtained the following relation:

$$|H_{\ddot{z}}(s,\xi)|^2=\sum_{j=1}^{\infty}X_j{}^2(\xi)[|H_{\ddot{z}j}(s)|^2\cdot\{\beta_{0j}H_{\ddot{z}b_1}(s)+\gamma_{0j}H_{\ddot{z}b_2}(s)\}^2]$$
$$+\sum_{\substack{j=1\\(j\neq k)}}^{\infty}\sum_{k=1}^{\infty}X_j(\xi)X_k(\xi)[\{H_{\ddot{z}j}(s)H_{\ddot{z}k}(s)\}\cdot\{\beta_{0j}H_{\ddot{z}b_1}(s)+\gamma_{0j}H_{\ddot{z}b_2}(s)\}\cdot\{\beta_{0k}H_{\ddot{z}b_1}(s)+\gamma_{0k}H_{\ddot{z}b_2}(s)\}] \quad (15)$$

Tajimi already evaluated λ of the single-freedom system to a single input. In his paper[3] $|H_{\ddot{z}j}(s)H_{\ddot{z}k}(s)|$ in eq. (15) was evaluated as $Q^2{}_{pq}$, and the values of $\lambda_{pq}{}^2$ introduced by integrating $Q_{pq}{}^2$ were tabulated in Table 3 of Tajimi's paper. This value turns to zero, if the phase difference between both responses of j-th and k-th modes is equal to $\pi/2$.

If each mode considerably separates from others, the phase differences of the particular mode, which is in resonance, to adjacent modes are near to $\pi/2$ under low damping condition. So, in such conditions, the values of $\lambda_{pq}{}^2$ are usually small, as we can see the values in Tajimi's table. The scheme of "root of sum of squares" comes from neglecting such terms. The authors followed this scheme and neglected the second term of eq. (15), then

$$|H_{\ddot{z}}(s,\xi)|^2\fallingdotseq\sum_{j=1}^{\infty}X_j{}^2(\xi)|H_{\ddot{z}j}(s)|^2\cdot\{\beta_{0j}{}^2|H_{\ddot{z}b_1}(s)|^2+\gamma_{0j}{}^2|H_{\ddot{z}b_2}(s)|^2+2\beta_{0j}\gamma_{0j}[H_{\ddot{z}b_1}(s)H_{\ddot{z}b_2}(s)]\} \quad (16)$$

is introduced. In this relation again we find cross-terms $[H_{\ddot{z}b_1}(s)H_{\ddot{z}b_2}(s)]$. If the vibration characteristics of both buildings are similar, eq. (16) can be deduced to a single-input relation with the relation

$$\alpha_{0j}=\beta_{0j}+\gamma_{0j}. \qquad (17)$$

On the other hand, if their eigen-frequencies separate enough, again the results of integrating these terms can be neglected. The relation

$$|H_{\ddot{x}}(s,\xi)|^2 \doteqdot \sum_{j=1}^{\infty} X_j^2(\xi)|H_{\ddot{x}j}(s)|^2 \cdot \{\beta_{0j}^2|H_{\ddot{x}b_1}(s)|^2+\gamma_{0j}^2|H_{\ddot{x}b_2}(s)|^2\} \qquad (18)$$

shows that the scheme of "root of sum of squares" would be effective both to modal analysis and to response analysis to multi-inputs, if we assumed that the inputs to both buildings were synchronized and in-phase. Also the authors should mention that this scheme is not in safety-side always, because the relation of eq. (18) is not $\geqq$, only $\doteqdot$.

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