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A NEW FORMULATION OF NEUTRON EMISSION PROBABILITY

中性子放出確率の新公式化

by Fumio, Hara

原　文　雄*

1. Introduction

The stochastical formulation of neutron transport in the nuclear system requires the probability of neutron emission from fissile nucleus. This probability plays the important role in this stochastical formulation. But some authors have only defined the process of neutron emission as Poissonian one, or the others have not given the explicit form of this probability in their papers.

The author calculates the neutron emission probability for ^{233}U, ^{235}U and ^{239}Pu on the basis of the simple model of fission process and gives the semi-quantitative interpretations to this model from the physical points. The results show that this model is able to represent the probabilistic neutron emission process of ^{233}U, ^{235}U and ^{239}Pu.

2. A Model of Neutron Emission Process

Acording to Weinberg & Wigner[1], the neutron emission process of ^{233}U, ^{235}U and ^{239}Pu is described schematically as Fig. 1, Processes 1], 2] and 3] occur consecutively. In Fig. 1, the reactions such as α-, β- and γ-decays and delayed neutron emission are not contained for the simplicity in considerations.

Process 1] means that one neutron n collides a fissile nucleus A and they forms a compound nucleus B^* which is energetically excited.

Process 2] represents that the compound neuleus

Process 1]　$A + n \rightarrow B^*$

Process 2]　$B^* \rightarrow L^* + H^*$

Process 3]　$\begin{cases} L^* \rightarrow l \times n + L \\ H^* \rightarrow h \times n + H \end{cases}$

Fig. 1 A simple model of fission process.

B^* disintegrates into two fragments L^* and H^* which are also energetically excited.

Process 3] indicates that the excited light- and heavy-fragments emit l and h neutrons respectively.

We assume that the excited light- and heavy-fragments L^*, H^* have the following properties respectively : L^* has $\nu_{L\max}$ neutrons whose emission is considered to be probable and $\langle\nu_L\rangle$ neutrons which are average number of neutrons emitted from L^*. Similarily, H^* has $\nu_{H\max}$ and $\langle\nu_H\rangle$ neutrons respectively.

Thus we can define the probability of success for one neutron to be emitted from L^* and H^* as

$$p_L \equiv \frac{\langle\nu_L\rangle}{\nu_{L\max}}, \quad (1) \qquad p_H \equiv \frac{\langle\nu_H\rangle}{\nu_{H\max}}. \quad (2)$$

Then the probability for L^* to emit ν_L neutrons is written in the binomial form

$$P(\nu_L) = \frac{\nu_{L\max}!}{\nu_L!(\nu_{L\max}-\nu_L)!} p_L{}^{\nu_L}(1-p_L)^{\nu_{L\max}-\nu_L}. \quad (3)$$

For the heavy fragment, the probability to emit ν_H neutrons is

$$P(\nu_H) = \frac{\nu_{H\max}!}{\nu_H!(\nu_{H\max}-\nu_H)!} p_H{}^{\nu_H}(1-p_H)^{\nu_{H\max}-\nu_H}. \quad (4)$$

The event that ν neutrons are emitted from a fission reaction consists of more elementary events such that for $\nu_L = 0, 1, 2, \ldots, \nu$, L^* emits ν_L neutrons and H^* emits $\nu - \nu_L$ neutrons. Thus the probability emitting ν neutrons from a fission can be expressed by the convolution of Eq. (3) and Eq. (4). Then the neutron emission probability $P(\nu)$ has the form

$$\begin{aligned} P(\nu) = {} & \nu_{L\max}!\,\nu_{H\max}! \left[\frac{\langle\nu_H\rangle}{\nu_{H\max}}\right]^{\nu} \left[1-\frac{\langle\nu_H\rangle}{\nu_{H\max}}\right]^{-\nu} \\ & \times \left[1-\frac{\langle\nu_L\rangle}{\nu_{L\max}}\right]^{\nu_{L\max}} \left[1-\frac{\langle\nu_H\rangle}{\nu_{H\max}}\right]^{\nu_{H\max}} \\ & \times \sum_{\nu_L=0}^{\nu} \frac{\left[\frac{\langle\nu_L\rangle}{\nu_{L\max}}\right]^{\nu_L} \left[\frac{\langle\nu_H\rangle}{\nu_{H\max}}\right]^{-\nu_L}}{\nu_L!(\nu-\nu_L)!(\nu_{L\max}-\nu_L)!} * \\ & * \frac{\left[1-\frac{\langle\nu_L\rangle}{\nu_{L\max}}\right]^{-\nu_L} \left[1-\frac{\langle\nu_H\rangle}{\nu_{H\max}}\right]^{\nu_L}}{(\nu_{H\max}+\nu_L-\nu)!}. \end{aligned} \quad (5)$$

* 東京大学生産技術研究所第 2 部　柴田研究室

Fot the special case when $\nu_{L\max}=\nu_{H\max}=\nu_{\max}$ and $\langle\nu_L\rangle=\langle\nu_H\rangle=\langle\nu\rangle$, Eq. (5) can be written in more simple form.

3. Results

The author calculated Eq. (5) by digital computer for some hundred cases of parameters $\nu_{L\max}$, $\nu_{H\max}$, $\langle\nu_L\rangle$ and $\langle\nu_H\rangle$. The results which are the most close representation of experimental data obtained by B.C. Diven et al[2] are shown in Table 1. And for comparison with experiments, the experimental distributions obtained by B.C. Diven et al[2] are also given in Table 1.

For the case when the energy of incident neutron is 80 KeV, the parameters $\nu_{L\max}$, $\nu_{H\max}$, $\langle\nu_L\rangle$, and $\langle\nu_H\rangle$ of ^{233}U are 4. 0, 4. 0, 2. 1 and 1. 7 respectively. Thus, $p_L=0.525$ and $p_H=0.425$ are obtained for ^{233}U. Similarily in the same incident neutron energy as above case, for ^{235}U, $\nu_{L\max}=\nu_{H\max}=4.0$, $\langle\nu_L\rangle=2.1$ and $\langle\nu_H\rangle=1.5$, so that $p_L=0.525$ and $p_H=0.375$, and for ^{239}Pu, $\nu_{L\max}=4.0$, $\nu_{H\max}=8.0$, $\langle\nu_L\rangle=1.7$ and $\langle\nu_H\rangle=1.5$, so that $p_L=0.425$ and $p_H=0.190$.

For the case when the energy of incident neutron is 1. 25 MeV and fissile nucleus is ^{235}U, $\nu_{L\max}=7.0$, $\nu_{H\max}=8.0$, $\langle\nu_L\rangle=0.5$ and $\langle\nu_H\rangle=2.5$, consequently $p_L=0.072$ and $p_H=0.313$ are obtained.

Here we should note that for the above values of the parameters, the cases of exchanged subscripts L and H give us the same results as Table 1, because of the symmetry of Eq. (5) with respect to subscripts L and H.

The probability distributions of neutron emission from ^{233}U, ^{235}U and ^{239}Pu for 80 KeV incident neutron and from ^{235}U for 1. 25 MeV are shown in Figs. 2, 3, 4 and 5 respectively. These Figs. contain also the results obtained by R.B. Leachman[3], Poisson's distributions having the same average value as the results obtained from Eq. (5) and experimental distributions obtained by B.C. Diven et al.[2]

4. Discussions

We can interpret the validity of values of parameters $\nu_{L\max}$, $\nu_{H\max}$, $\langle\nu_L\rangle$ and $\langle\nu_H\rangle$ from the fundmental properties of nucleus.

For example, in the case of ^{235}U, the compound nucleus B^* of process 2] in Fig. 1 has 144 neutrons and disintegrates into two fragments L^* and H^*. In this fission process, from the mass distribution of fission products[1], we can lead that the fission mode in which the neutron numbers of L' and H' are 50 and 82 respectively is most probable. Here it should be noted that neutron numbers 50 and 82 are said as "Magic Numbers" and the nuclei with 50 or 82 neutrons are very stable energetically. Then L^*+H^* has totally 12 $(=144-(50+82))$ superfluous neutrons. The average value of excitation energy of both fragments is about 35MeV as showing below. It would be accept-

Table 1 The probabilities of neutron emission from a fission of ^{233}U, ^{235}U and ^{239}Pu. The author's results obtained from Eq. (5) are shown in the columns named "AUTHOR" and for comparison with the experimental distributions, Diven's results are also show in the columns named "EXPERIMENT".

P(ν)	^{233}U (80 KeV)		^{235}U (80 KeV)		^{239}Pu (80 KeV)		^{235}U (1.25 MeV)	
	AUTHOR	EXPERIMENT	AUTHOR	EXPERIMENT	AUTHOR	EXPERIMENT	AUTHOR	EXPERIMENT*
P(0)	0.0058	0.010±0.008	0.0063	0.027±0.004	0.0001	−0.01±0.01	0.0026	0.02 ±0.0
P(1)	0.1608	0.150±0.024	0.1726	0.158±0.010	0.0082	0.11±0.03	0.0629	0.092±0.03
P(2)	0.3160	0.326±0.037	0.3291	0.339±0.014	0.1856	0.13±0.06	0.3415	0.308±0.06
P(3)	0.2977	0.301±0.044	0.2974	0.305±0.015	0.5990	0.56±0.08	0.4359	0.408±0.07
P(4)	0.1477	0.176±0.041	0.1357	0.133±0.013	0.1704	0.11±0.08	0.1340	0.099±0.06
P(5)	0.0526	0.042±0.028	0.0443	0.038±0.009	0.0313	0.06±0.09	0.0206	0.06 ±0.05
P(6)	0.0150	−0.010±0.017	0.0116	−0.001±0.003	0.0047	0.05±0.08	0.0024	0.01 ±0.02
P(7)	0.0036	0.006±0.009	0.0026	0.001±0.002	0.0006	0.00±0.06	0.0002	0.00 ±0.0
P(8)	0.0008	−0.002±0.002	0.0005	0.000±0.000	0.0001	−0.01±0.03	0.0000	—
⟨ν⟩	2.661	2.585±0.062	2.578	2.47±0.03	3.047	3.048±0.045	2.708	2.73
⟨σ²⟩	1.466	1.16	1.369	1.22	0.566	0.43	0.773	1.00

* from Fig. 3 of Leachman's paper (3)

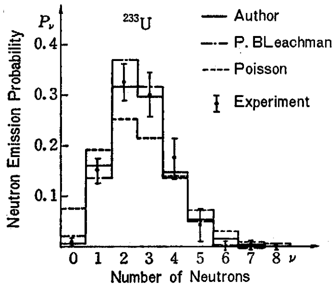

Fig. 2 Probability distribution of neutron emission from a fission of ^{233}U. (The kinetic energy of incident neutron is 80 KeV.)

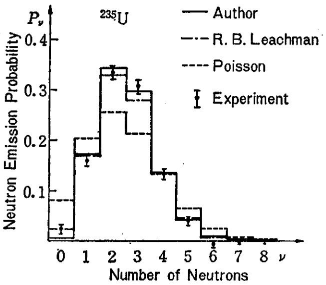

Fig. 3 Probability distribution of neutron emission from a fission of ^{235}U. (The kinetic energy of incident neutron is 80 KeV.)

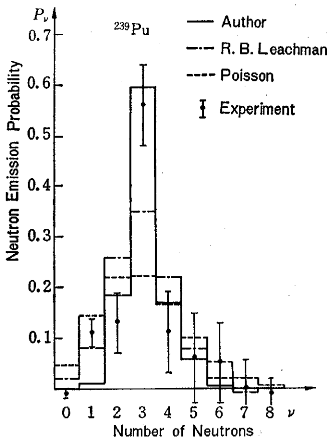

Fig. 4 Probability distribution of neutron emission from a fission of ^{239}Pu. (The kinetic energy of incident neutron is 80 KeV.)

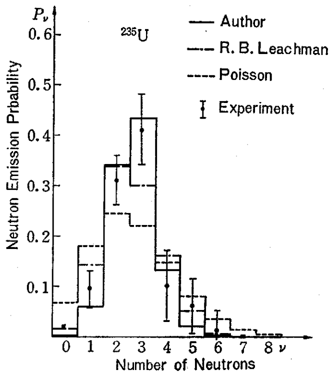

Fig. 5 Probability distribution of neutron emission from a fission of ^{235}U. (The kinetic energy of incident neutron is 1. 25 MeV.)

able to predict that the maximum value of excitation energy is about 70 (=2×35) MeV. On the other hand, there is a fact that the average binding energy of a nucleon is about 8 MeV. Now, the value obtained from dividing the maximum excitation energy by the average binding energy would become the number of neutrons which are probable to be emitted from both fragments. Then the number of neutron to be emitted is, at most, about 8 (≒70 ÷8). If we assume that the superfluous neutrons are symmetrically distributed to each fragment, we can obtain $\nu_{L\,max}=4.0$ and $\nu_{H\,max}=4.0$.

The same interpretation as for ^{235}U will be applied to the case of ^{233}U, but for ^{239}Pu, the difference between $\nu_{L\,max}$ and $\nu_{H\,max}$ is a problem unsolved in this short paper.

The energy released from a fission is about 200 MeV and the total kinetic energy of framents is about 165 MeV[1]. The difference between the released energy and total kinetic energy is about 35 (=200−165) MeV and this energy would be used to excite the both fragments. If we assume that the excitation energy of each fragment is about 17. 5 (=35÷2) MeV, then the average number of neutrons to be emitted becomes about 2 (≒17. 5 ÷8), since the binding energy is about 8 MeV per nucleon. Thus $\langle\nu_L\rangle$ and $\langle\nu_H\rangle \fallingdotseq 2$ is explained by the above simple discussions.

For the other fissile nuclei, the similar discussions will be applicable, but some modifications will be necessary to explain the different features from ^{235}U.

5. Conclusions

The author obtained the following conclusions from the above discussions:

(1) The convolution of two binomial distributions is a closer representation of experimental distribution rather than Poisson's distribution for the neutron emission from ^{233}U, ^{235}U and ^{239}Pu.

(2) The parameters $\nu_{L\,max}$, $\nu_{H\,max}$, $\langle\nu_L\rangle$ and $\langle\nu_H\rangle$ can be semi-quantitatively interpreted from the view point of fundamental properties of nucleus.

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References

1) A.M. Weinberg & E.P. Wigner; The Physical Theory of Neutron Chain Reactors (The University of Chicago Press, 1958)
2) B.C. Diven et al; Phys. Rev. **101**, 1012, (1956)
3) R.B. Leachman; Phys. Rev. **101**, 1005, (1956)