研 究 速 報

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ANALYSIS OF INVERTER-INDUCTION MOTOR SYSTEM

インバーター誘導電動機系の解析

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1. Introduction

An induction motor, when supplied by a static inverter, is an attractive variable speed drive for industry. Since the inverter performs a series of discontinuous switching operations, the behavior of the motor is a succession of responses to these transients. The research described in this paper is an attempt to analyze an inverter-induction motor system by state transition method, which has the following advantages.

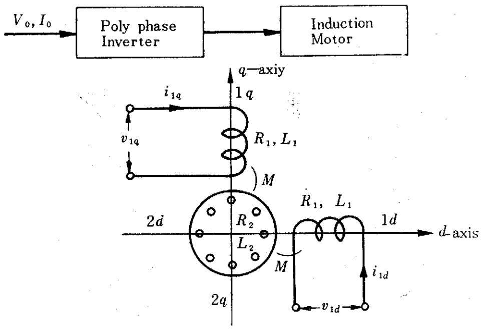
1) An inverter-induction motor system can be regarded as a type of direct current machine.

2) Computation by digital computers can be easily done.

3) Analytic solution can be obtained for constant speed operation of the motor.

2. The frame of reference rotating discontinuously

An inverter-induction motor system is shown in Fig. 1. The analysis is based on two axis theory.



$$\begin{bmatrix} v_{1d} \\ v_{1q} \\ v_{2d} \\ v_{2q} \end{bmatrix} = \begin{bmatrix} R_1+L_1P & & MP & \\ & R_1+L_1P & & MP \\ M\theta & M\dot{\theta} & R_2+L_2P & L_2\dot{\theta} \\ -M\dot{\theta} & MP & -L_2\dot{\theta} & R_2+L_2P \end{bmatrix} \begin{bmatrix} i_{1d} \\ i_{1q} \\ i_{2d} \\ i_{2q} \end{bmatrix}$$

Fig. 1 Inverter and Induction motor

The output voltage waveforms of a three-phase inverter are shown in Fig. 2. The output voltages are devided into six states with the phase difference of 60° as shown in Fig. 3. The stator field direction jumps discontinuously by an angle 60° in the same direction, each time when a driving signal of the inverter comes in.

Observed from the frame of reference rotating discontinuously as shown in Fig. 4 ($\alpha-\beta$ coordinate), the output voltage of the inverter is a constant vector. By this transformation, an impulsive "fictitious voltage" appears during each commutation.

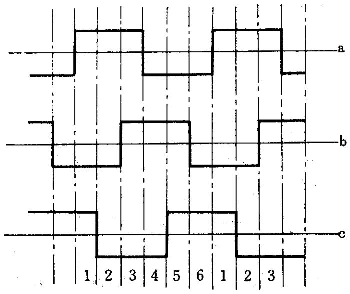


Fig. 2 Output voltage wave forms of a three-phase inverter.

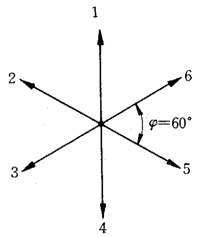


Fig. 3 Stator voltage vectors.

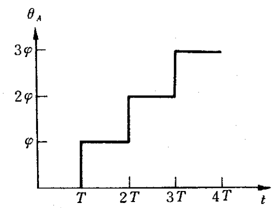


Fig. 4 Discontinuous rotation of the frame of reference.

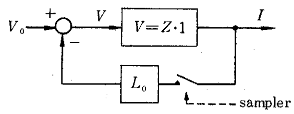


Fig. 5 Block diagram representation of a inverter-induction motor system.

The block diagram of the system with respective to this frame of reference is shown in Fig. 5. This system is called a "time descrete system" and can be analyzed with use of control theory. The phenomena that occur during each commutation are shown as follows. In the form of a state transition equation,

$I(nT_+)=BI(nT)$　　T; sanpling period

$$B=\begin{array}{c|c|c|c|c|}
1\alpha & \cos\varphi & \sin\varphi & & \\
1\beta & -\sin\varphi & \cos\varphi & & \\
2\alpha & & & \cos\varphi & \sin\varphi \\
2\beta & & & -\sin\varphi & \cos\varphi
\end{array} \qquad (1)$$

If the inverter has ideal switching characteristics, we have

$$V_0 I_0 = v_{1\alpha} i_{1\alpha} + v_{1\beta} i_{1\beta}$$

$$I_0 = \frac{v_{1\alpha}}{V_0} i_{1\alpha} + \frac{v_{1\beta}}{V_0} i_{1\beta}$$

By this relation, the inverter-induction motor system can be regarded as a type of direct current machine. Especially when $v_{1\beta}$ is chosen to be equal to zero, we have

$$I_0 = \frac{v_{1\alpha}}{V_0} i_{1\alpha}$$

In this case, the wave from of $i_{1\alpha}$ is equal to that of I_0, and the constant $v_{1\alpha}/V_0$ represents the turn ratio of the inverter transformers.

3. Analysis by state transition method

By two-phase symmetrical component method, the circuit equation of a induction motor expressed in complex variables is

$$\dot{X}=AX \qquad X=[e_s \;\; i_{1f} \;\; i_{2f}]' \qquad (2)$$

$$A=\frac{1}{\sigma^2}\begin{array}{|c|c|c|}
\hline
1 & & \\
\hline
L_2 & -(R_1L_2+jM^2\dot{\theta}) & R_2M-jML_2\dot{\theta} \\
\hline
-M & R_1M+jML_1\dot{\theta} & -(R_2L_1-jL_1L_2\dot{\theta}) \\
\hline
\end{array}$$

$$e_s=\frac{v_{1\alpha}}{\sqrt{2}}, \qquad i_{1f}=\frac{i_{1\alpha}+ji_{1\beta}}{\sqrt{2}},$$

$$i_{2f}=\frac{i_{2\alpha}+ji_{2\beta}}{\sqrt{2}}$$

The state transition equation at each sampling point is

$$X(nT_+)=B_0X(nT) \qquad B_0=\begin{array}{|c|c|c|}
\hline
1 & & \\
\hline
 & e^{-j\varphi} & \\
\hline
 & & e^{-j\varphi} \\
\hline
\end{array} \qquad (3)$$

The solution of Eq. (2) and (3) is given as follows.

$$X(t-nT)=H(t-nT)X(nT),$$

$$H(\lambda)=\Phi(\lambda)B_0$$

$$X(nT)=\mathfrak{z}^{-1}[(U-z^{-1}H(T))^{-1}]X(0),$$

$$\Phi(\lambda)=\begin{array}{|c|c|}
\hline
1 & 0 \\
\hline
(\lambda) & \phi(\lambda) \\
\hline
\end{array} \qquad (4)$$

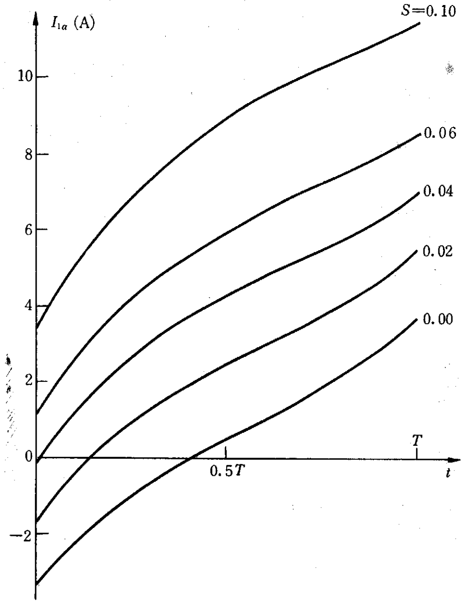


Fig. 6　Computed curves of stator current $i_{1\alpha}$ Inverter freq. 50 Hz

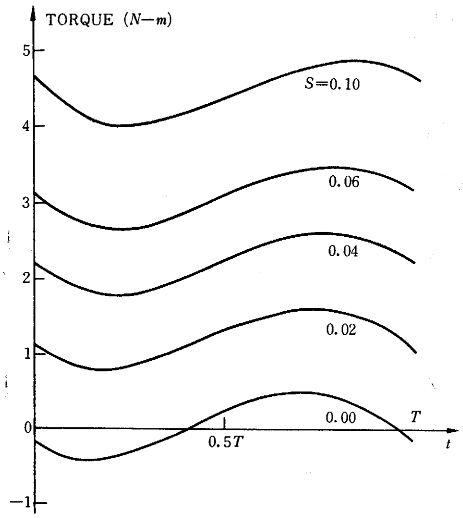


Fig. 7　Computed curves of output torque T_0 Inverter freq. 50 Hz

The output torque of the motor is given by

$$T_0=2MR_e[j\overline{i_{1f}(t)}^{*}i_{2f}(t)] \qquad (5)$$

Computed characteristics for a 0.75 kW induction motor are shown in Fig. 6 and 7. The motor constants are as follows.

$R_1=2.78\,\Omega$,　　$L_1=L_2=0.213\,H$,
$M=0.206\,H$,　　$R_2=1.71\,\Omega$

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