

Essays on mixed oligopoly with applications to
environmental problems

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August, 2009

Acknowledgment

The chapters in this dissertation are based on various works, the details of which are given below:

- Chapter 2 is based on the paper “Partial privatization and unidirectional transboundary pollution” by Kazuhiko Kato.
- Chapter 3 is based on the paper “Can allowing to trade permits enhance welfare in mixed oligopoly?” by Kazuhiko Kato.
- Chapter 4 is based on the paper “Emission quota versus emission tax in mixed duopoly” by Kazuhiko Kato.
- Chapter 5 is based on the paper “Price competition in a mixed duopoly” by Akira Ogawa and Kazuhiko Kato.
- Chapter 6 is based on the paper “Mixed oligopoly, privatization, subsidization, and the order of firms’ moves: several types of objectives” by Kazuhiko Kato and Yoshihiro Tomaru.
- Chapter 7 is based on the paper “Robustness of ‘Endogenous timing in a mixed duopoly: Price competition’” by Kazuhiko Kato and Akira Ogawa.

First and foremost, I would like to thank Toshihiro Matsumura and Dan Sasaki for his constructive comments and advice, without which I would not have been able to write this dissertation. Further, I am grateful to Akihiko Matsui for his helpful support and

encouragement, in addition to his invaluable advice. I also thank Akira Ogawa and Yoshihiro Tomaru for permitting me to use our joint works in this dissertation. Finally, I thank Kazuharu Kiyono, Daisuke Shimizu, Susumu Cato, and Minoru Kitahara for their advice, suggestions, and support.

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Chapter 1

Overview

1.1 Introduction

There are currently a significant number of public firms across the world even though privatization has been widely and frequently observed worldwide since the 1980s.¹ Many of public firms usually compete with private firms in the same market. These markets are called *mixed oligopoly* and can be observed in several industries such as banking, broadcasting, education, energy (gas and electricity), health care, life insurance, steel, telecommunication, and transportation (railroad and airline). From the viewpoint of not only economic reforms in a practical manner but also economic theory, to analyze the effect of privatization is one of the major issues.

One of the objectives of this dissertation is to extend the previous models in mixed oligopoly theory and to examine the properties with regard to the roll of public firms including privatization. Another is to apply the mixed oligopoly theory to the environmental problems and to provide its properties. The motivation behind the latter is due to the fact that pollution generated by production has been often harming the environment

¹For details of concrete examples of privatization, see Vickers and Yarrow (1988) for U.K. and Roland (2008) for Europe, Latin America, Africa, and South Asia.

significantly in some of the aforementioned industries, and therefore not only private firms but also public firms can be regarded as important players in the environmental problems. For analyzing the environmental problems in the framework of mixed oligopoly theory, to examine the effect of privatization is also a major issue. In the subsequent, we confirm reasons for privatization.

There are several reasons for privatization.² One is that private firms can earn positive profits and improve competitiveness because their technologies are welldeveloped in relevant industries. Another reason is that an X inefficiency problem exists. From the above perspectives, when the technologies associated with relevant industries are welldeveloped, privatization of a public firm would be desirable with regard to social welfare: the government could cut subsidies and obtain tax revenue, in addition to the removal of the inefficiency.

As a basic theoretical work of a mixed oligopoly, Defraja and Delbono (1989) is often generally cited.³ Defraja and Delbono (1989) shows that there is a possibility that privatization enhances welfare in a simultaneous quantity setting game with a homogeneous product, even without positive aspects such as those mentioned above. Since the publication of Defraja and Delbono (1989), extensions of the paper have been widely analyzed. For example, Defraja and Delbono (1989) fixed the timing of the decision of each firm. The following studies relaxed the fixed timing game and considered the endogenous

²Before considering the reasons of the privatization, we may have to confirm the reasons for the establishment of a public firm. For the various reasons of the establishment of a public firm, see Anderson, de Palma, and Thisse (1997). Bös (1991) introduces various reasons of the privatization in detail.

³For pioneering works on mixed oligopoly, see Merrill and Schneider (1966) and Harris and Wien (1980). These papers show that a public firm can be regarded as an instrument of regulation in order to enhance welfare or achieve the first-best allocation.

timing game (observable delay game established by Hamilton and Slutsky (1990)): Pal (1998), Matsumura (2003), Bárcena-Ruiz (2007), Lu (2006), and Matsumura and Ogawa (2007). Other extensions have also been considered: for location choice, Cremer, Marchand, and Thisse (1991), Matsumura and Matsushima (2003), Matsumura, Ohkawa, and Shimizu (2005), and Matsushima and Matsumura (2003, 2006); for free entry, Matsumura and Kanda (2005) and Ino and Matsumura (2004).⁴

Previous studies indicate that the following three properties may be key factors in theoretical analyses of a mixed oligopoly. First, there is a possibility that the public firm makes the production allocation in the economy inefficient.⁵ Suppose that there exists a mixed duopoly with a quantity setting competition in a homogenous goods market. Since the objective of the public firm is to maximize social welfare, the public firm produces more than the private firm when both firms have a symmetric and strictly convex cost function.⁶ In this case, the marginal cost of the public firm is higher than that of the private firm in the equilibrium. In terms of the cost minimization for a certain fixed output level, the marginal costs between both firms need to be identical. When we compare mixed and pure duopolies, we find that the degree of inefficient production allocation and total output are larger in the former than in the latter. Therefore, privatization in a mixed oligopoly may enhance welfare when the number of private firms is large because underproduction by oligopoly is mitigated (Defraja and Delbono 1989).

Second, market-based regulations such as output taxes or subsidies do not directly

⁴Bös (1991), Defraja and Delbono (1990), and Nett (1993) feature a comprehensive surveys.

⁵Lahiri and Ono (1988) emphasizes that inefficiency of production allocation may occur in a pure oligopoly.

⁶When we consider the case that the marginal cost of each firm is constant and is the same for the two firms, inefficient production allocation does not occur.

affect the behavior of the public firm because the tax payment or subsidy receipt by the public firm is balanced out as the public firm also considers the tax revenue or subsidy payment. Market-based regulations only affect the behavior of the private firm. If there is one market distortion, market-based regulation could lead to social optimal allocation (White 1996). However, if there is more than one distortion in the market, direct regulation such as command and control by the regulation authority could improve welfare more than market-based regulation can.

Finally, the competitor of the public firm is quite important. In a pure duopoly, the private firm chooses its output in order to satisfy that the marginal revenue is equal to the marginal cost. In a mixed duopoly with a domestic private firm, the public firm chooses its output such that the price is equal to the marginal cost, whereas in a mixed duopoly with a foreign private firm it chooses its output such that the price is less than its marginal cost. Therefore, the results of a mixed duopoly with a foreign private firm might be quite different from those of a mixed duopoly with domestic a private firm (Fjell and Pal 1996 and Pal and White 1998). Evidently, the above difference might also alter the effects of regulations: tax payment and revenue are balanced out in a mixed oligopoly with domestic private firms, while tax payment by a foreign private firm is not balanced out and continues to be included in social welfare.

The origin of these properties is the objective of a public firm: it is often assumed that the objective of a public firm is to maximize social welfare, whereas that of a private firm is to maximize its own profits.⁷ The objective of a public firm is essential to the previous

⁷The mixed ownership of a public firm by the public and private sectors can be considered. In this case, the public firm solely pursues neither profits, nor welfare. The mixed ownership of the public firm is regarded as partial privatization. For detailed explanations of a partial privatization, see Bös (1991),

results in a mixed oligopoly.

While privatization is a growing concern and has been studied through a large volume of theoretical works, rapid technological and industrial development and economic growth has led to an increased focus on environmental problems since the 1990s. The Earth Summit was held in Rio de Janeiro in 1992 for the stabilization of atmospheric concentrations of greenhouse effect gases such as CO_2 and CH_4 . The Kyoto Protocol was adopted in COP3 in 1997 and the developed countries, except the U.S., were obligated to decrease emissions of greenhouse effect gases.⁸ Recently, the G8 Hokkaido Toyako Summit, held in 2008, set a goal to reduce the emission of greenhouse gases by over 50% at least by 2050 across the world.

The source of an environmental problem is regarded as a negative externality. There are two types of negative externalities: one is to cause the nonconvexities of the production set and the other is to cause a decrease in consumer surplus.⁹ In the subsequent discussion, we deal with the latter type of negative externalities. One of the main issues of environmental economics is to determine how to internalize the negative externalities. As pointed out by Pigou (1920), using taxes that equalize marginal private cost and marginal social cost, social optimal allocation can be realized. In addition to taxes, several environmental regulations have been considered: quotas, standards, and tradable emission permits.¹⁰ Until the 1980s, most works such as Meade (1952), Buchanan (1969), and Barnett (1980) focused on the effect of the environmental regulations in a competitive market or a monopoly market.

Fershtman (1990), and Matsumura (1998).

⁸COP3 is the abbreviated form of *The 3rd Session of the Conference of the Parties to the United Nations Framework Convention on Climate Change*.

⁹For nonconvexities, see Starrett (1972), Baumol and Oates (1988), and Xepapadeas (1997).

¹⁰For the basic analyses of environmental regulations, see Baumol and Oates (1988).

Since then, considerable number of works for environmental regulations under oligopoly and monopolistic competition have emerged.¹¹

Research works on environmental problems in a mixed oligopoly have been conducted since the advent of the 21st century. In this background, it is often observed that a mixed oligopoly markets exist in developing and developed countries where concern for the protection of the environment is high or the damage caused to the environment is serious.¹² Following Marukawa (2004) basically, we consider China, for example. In the petroleum and petrochemical industry, the three largest public firms (CNPC, Sinopec, and CNOOC) have dominated its market. However, the relaxation of regulations by the entry of WTO makes the domestic and foreign private firms enter its market. In electricity industry, there existed the five largest public firms (China Huaneng Corporation, China Datang Corporation, China Huadian Corporation, China Guodian Corporation, China Power Investment Corporation) with large-scale coal fired power generations and a number of private firms with small-scale ones.¹³ Moreover, the domestic and foreign private firms using the other power generation, such as the solar photovoltaic power generation, wind force power generation, and the incineration heat of waste disposal generation enter in this industry. In addition to these industries, there exist major public firms with private firms in many industries. In particular, big private firms exist in some industries (e.g. Jianlong Steel and Jiangsu Shagang Group in steel industry, Chery Automobile and Geely

¹¹For environmental regulations and imperfect competition, and in particular, oligopolistic market, see Xepapadeas (1997) and Petrakis, Sartzatakis, and Xepapadeas (1999).

¹²For the privatization and the environment, Bárcena-Ruiz and Garzón (2006) give examples of EU. For examples from the Central and Eastern Europe, see Bluffstone and Panayotou (2000). For examples of environmental regulations in China, see Jiang (2003).

¹³In recent days, a lot of closure of small-scale coal fired power generations occur by the policy of Chinese government (eleventh five-year plan).

Automobile in automobile Industry). In many of these industries, the pollution discharged by the production harms the environment immensely. Therefore, it would be necessary to analyze the environmental problems in the mixed oligopoly.

In the following, we survey previous works on environment problems in a mixed market. Bárcena-Ruiz and Garzón (2006), Beladi and Chao (2006), and Naito and Ogawa (2009) study the domestic market and investigate the effect of the environmental policy. Bárcena-Ruiz and Garzón (2006) determines whether or not the full privatization improves welfare in the following three cases: (1) there are no environmental problems, (2) there is an environmental problem, but no environmental regulation, (3) emission tax is imposed on the firms. They show that privatization in case (1) and (3) enhances social welfare when the number of firms is large, whereas it always decreases welfare regardless of the number of firms in (2). In the comparison of the Pigouvian tax, whose level is equal to the marginal environmental damage, the second best emission tax level is less than the Pigouvian tax level. Beladi and Chao (2006) also derives the last result. They consider emission tax in a monopoly by a partially privatized public firm. An increase in the degree of partial privatization leads to one positive effect and one negative effect on environmental damage. The positive effect is the direct decrease in pollution by the reduction in production to increase profits. The negative one is the indirect increase in pollution by the decrease in the second best emission tax rate with an increase in the degree of partial privatization. According to Beladi and Chao (2006), the above facts indicate the possibility that an increase in the degree of partial privatization leads to an increase in the level of environmental damage. Naito and Ogawa (2009) compares emission tax with emission

standard by which the government sets the abatement effort level of firms. They show that emission standard is superior to emission tax in a mixed duopoly, regardless of the degree of partial privatization of the public firm.

Ohori (2006a, 2006b) studies emission tax in international trade. Ohori (2006a) considers the third country model: a home country and foreign one export to the third country. In each country, there exists one partially privatized public firm. After the government chooses an emission tax level and a degree of partial privatization, each public firm chooses the abatement and quantity simultaneously. The paper indicates that there are some positive and negative effects on welfare when the degree of partial privatization shifts, and it shows that partial privatization is desirable. Ohori (2006b) uses the two-country model: a domestic public firm and a foreign private firm compete in a domestic market. In addition to emission tax, he considers the existence of tariff. He shows that the second best emission tax level is larger than the Pigouvian tax level because the public firm competes actively with a foreign firm. Furthermore, he shows that the reduction in tariff cannot affect the level of environmental damage.

Cato (2008) determines whether or not full privatization should be done, by focusing on the degree of environmental damage.¹⁴ He showed that welfare is smaller after privatization than that before privatization, when the degree of environmental damage is over a certain level. This is because when its degree is high, the public firm reduces its emission and output; this leads to an improvement in the environment and inefficient production

¹⁴He also considers the case where the number of private firms is endogenously determined. He shows that the magnitude of the relationship of welfare before and after privatization depends on the magnitude of the relationship between the profit of the public and the difference in environmental damage before and after privatization.

allocation in a mixed oligopoly.

As identified above, the number of research works to date is still small and the situations analyzed in these papers are not very organized. There is a need for a higher number of works focusing on environmental problems in a mixed oligopoly and well-organized for advances in this field.

1.2 Organization

This dissertation is separated by two parts. One is to determine whether or not the results obtained in previous works can be applied to a new situation (Chapters 5, 6, and 7). The other is to provide the property of privatization and environmental regulations in a mixed oligopoly when environmental problems exist (Chapters 2, 3, and 4).

Chapter 2 determines whether or not a local regional government should privatize its local public firm in a mixed duopoly when it faces unidirectional transboundary pollution problem. We consider two regions in an economy, one located upstream and the other, downstream. Under the situation where both the local public firm owned by the local government of upstream and the private firm locate and compete in upstream, we analyze two cases: (1) the private firm is owned by the private investors in upstream, and (2) it is owned by those in downstream.

Comparing the two cases, we present the following results. Partial privatization is desirable for local welfare of upstream in (1), whereas it is not always desirable in (2). In both (1) and (2), it is desirable for local welfare of downstream and entire welfare in the economy when the degree of environmental damage and the fraction of transboundary

pollution remaining in upstream are low. However, when they are large, the results change for (1) and (2).

Chapter 3 compares the effects of tradable emission permits (TEP) and non-tradable emission permits (NTEP) in a mixed oligopoly, where public firms and private firms compete in a product market. If all technologies and initial endowments of emission permits are symmetric among public and private firms and if the emission constraint is exogenous and binding, we show that social welfare is greater (smaller) under TEP than under NTEP when the weight of social welfare in each public firm's objective function and the degree of convexity of the production cost function and that of the abatement cost function are small (large).

Chapter 4 compares the emission tax with emission quota in a mixed duopoly. In a mixed duopoly, Naito and Ogawa (2009) shows that direct regulation is superior to indirect regulation, regardless of the degree of partial privatization. They regard direct regulation as emission standard such that the government sets the uniform abatement effort of each firm and indirect regulation as emission tax. In this paper, we consider another indirect regulation: the emission quota such that the regulation authority sets the emission of each firm uniformly or differentially. We show that welfare is always larger under a differentiated emission quota than under emission tax. Comparing emission tax with uniform emission quota, the superiority of environmental regulations in terms of social welfare depends on the parameters of the cost functions. If we consider the same values of these parameters used in Naito and Ogawa (2009), we can show that emission tax is superior to emission quota, that is, indirect regulation is desirable.

Chapter 5 analyzes the price competition in a mixed duopoly where one public firm and one private firm producing homogeneous products have symmetric quadratic cost functions. We consider the following three fixed timing games: they choose their prices sequentially or simultaneously, we show that there exists a case wherein the equilibrium price is the highest of all timings when the private firm is a Stackelberg leader and the public firm is a Stackelberg follower.

Chapter 6 examines the robustness of the results in earlier works for output subsidy in a mixed oligopoly, termed as “irrelevance results.” We show that the irrelevance results do not depend on the fact that each private firm maximizes its own profits.

Chapter 7 deals with the observable delay game established by Hamilton and Slutsky (1990) in the context of a mixed duopoly in price competition under differentiated product markets. We generalize the Bárcena-Ruiz (2007) model with respect to the shareholding structure so as to accommodate a private firm that is partly or completely owned by foreign investors. Even though this generalization is applied, we find that the result of Bárcena-Ruiz (2007), where both firms choose the first period, is robust.

Chapter 2

Partial privatization and unidirectional transboundary pollution

2.1 Introduction

Phenomena attributed to transboundary pollution, such as acid rain and water or air pollution, have been attracting attention since the middle of the 19th century. For example, acid rain has long been recognized as a serious environmental problem in Europe. Further, since the past few decades, acid rain has become a serious problem in East Asia.¹ Phenomena that are attributed to transboundary pollution are often considered to have been caused by production. However, such phenomena can also be caused by consumption. Recent years have witnessed a shift to a consumeristic way of life and consequently, an increase in waste. Often, household, industrial, and medical waste generated by a country

¹Nagase and Silva (2007) gives the detail extent of the damage by the acid rain in China and Japan. Ichikawa and Fujita (1995) estimate the contribution of China are about one-half of the total with respect to the wet deposition of sulfate in Japan. For the other transboundary pollution, Ohara et al. (2001) indicate a threat that an increase of ozone, sulfate, and nitrate which are causative factors of urban ozone in China may greatly impact on the air quality in Japan in the future.

or region are transported downstream to other countries or regions. For example, for the past several years, waste believed to have been generated in Russia, China, and Korea has been regularly found on the shores of northern Japan where it was carried by the sea. To solve this problem, Japan and Korea held working-level talks in February 2009.

Meanwhile, global warming continues to worsen the environment through the world. There is a possibility that global warming will affect the fraction of transboundary pollution. For example, global warming may cause the westerlies to meander, which may result in extreme weather; further natural calamities such as floods, heavy rains, and hurricanes may become more frequent in the future. The meandering of the westerlies will also affect the present amount of air-borne pollutants and toxic chemicals (which cause acid rain) that are carried between countries. Heavy rains transport city waste that may be lying on the liverbed or in waste collection sites located along the river into the river. Floods then transfer this waste to downstream areas. An increase in the atmospheric temperature and surface level of the sea and a decrease in the salinity of the seas because of melting glaciers may alter flow of the oceans, and thus, affect the amount of waste that is carried from one country to another. We can thus conclude that there is a possibility that the fraction of transboundary pollution varies even if the total level of pollution remains unchanged.

In some of the countries and regions mentioned above, there still exist mixed markets where public and private firms compete. In mixed markets, the privatization of public firms is a major issue because privatization changes the objective of the public firms. This alters the market equilibrium, which could have an impact on environmental pollution. Therefore, privatization in one country affects not only its own welfare but also the welfare

of other countries that are affected by transboundary pollution originating in that country.

We have developed our model keeping these points in mind. The model considers two regions, with one located downstream of the other, and two firms, a public firm and a private firm. We examine whether privatization of the public firm in the upstream region enhances welfare in the downstream region and the total welfare of the two regions under the unidirectional transboundary pollution.

Many earlier works analyze mixed oligopoly within the framework proposed in Defraja and Delbono (1989).² In recent years, some researches have addressed the environmental problem. Bárcena-Ruiz and Garzón (2006), Beladi and Chao (2006), Ohori (2006a, 2006b), and Kato (2006) examine environmental regulation in a mixed oligopoly and analyze the effects of privatization. Cato (2008) investigates the relationship between the degree of environmental damage and privatization. However, because these works deal with the environmental problem in one region, they do not consider transboundary pollution.

For earlier works on the transboundary pollution, Nagase and Silva (2007) is closely related to our motivation. Nagase and Silva (2007) considers the situation where one region (China) is located upstream of another region (Japan) under the unidirectional transboundary pollution.³ However, their main interest is to examine an environmental policy-making game between the two, and therefore, it is different from ours with regard to focusing on the effect of privatization. In China, a large number of public firms has been privatized since the 1990s.⁴ However, there are still a lot of mixed oligopoly in several

²Bös (1991), Defraja and Delbono (1990), and Nett (1993) provide an excellent survey of a mixed market.

³Nagase and Silva (2007) considers a competitive market and allow abatement effort and an emission tax policy.

⁴For the overview of reforms of state-owned enterprise in China, see Fernández and Fernández-

industries that depends on energy from fossil fuel, especially, coal. And thus, the analysis of the transboundary pollution in the framework of the mixed oligopoly theory may lead to a new approach to the research of the transboundary pollution problem.

The remainder of the paper is organized as follows. Section 2.2 describes our model. Section 2.3 derives the equilibrium outcome under different cases of ownership of a private firm and conducts a welfare comparison. Section 2.4 compares the results obtained in the previous section. Section 2.5 concludes the main text. Detailed calculations for the equilibrium outcome in each case and proofs of the propositions are given in the Appendices.

2.2 Model

Consider an economy of two regions: regions A and B . Region A is located upstream of region B . In this economy, there is one local public firm (firm 0) owned by the local regional government of A and one private firm (firm 1) owned by private investors from either region A or region B . Both the firms are located in region A and produce a homogeneous product that harms the environment. We call this product a “dirty good.”

Firms 0 and 1 compete in quantity. The output of firm i is denoted by q_i ($i = 0, 1$). Total output is denoted by $Q = q_0 + q_1$. We assume that the cost function of firm i is given by $c_i(q_i) = cq_i^2/2$. Given the inverse demand function of the dirty good, $p = p(Q)$, and then, the profit of firm i is

$$\pi_i(q_0, q_1) = p(Q)q_i - \frac{c_i q_i^2}{2}.$$

A representative consumer exists in each region. The representative consumer in region

Stembridge (2007).

A consumes the dirty good and a clean numeraire good. The representative consumer in region B only exists.

The representative consumer in region A maximizes $U(Q) + y$ subject to $pQ + y = m$, where p denotes the price of the dirty good, y denotes the amount of the numeraire good, whose price is normalized to 1, and m denotes the income of the representative consumer. We assume that $U(Q)$ is

$$U(Q) = aQ - \frac{Q^2}{2}. \quad (2.1)$$

Therefore, we obtain the following inverse demand function, $p(Q) = a - Q$ by solving the utility maximization problem of the representative consumer in region A .

In our model, pollution is generated by either production or consumption and is harmful to the environment. Producing or consuming one unit of a dirty good generates one unit of pollution. The pollution is converted into environmental damage which reduces the consumer surplus via a lump-sum transfer. We do not consider the case where pollution is generated by both production and consumption. In our setting, pollution is generated only in region A and there is no difference between pollution through production and that through consumption. Therefore, in the subsequent instructions and analyses, we consider the case that pollution is generated by production. The total pollution in region l is denoted by E_l ($l = A, B$); the total environmental damage in region l is denoted by $D_l(E_l) = d(E_l)^2/2$.

We assume that pollution is transboundary and can affect the environment in region B . We now explain how transboundary pollution is considered in the model. Pollution is generated only in region A because both firms produce in region A ; the amount of pollution

generated is Q . We assume that region A is located upstream of region B (along a river or in the path of a periodic wind), and therefore, some of the pollution is transported to region B . The fraction of pollution that remains in region A is θ ; therefore, the fraction of pollution transported to region B is $(1 - \theta)$. Thus, the pollution levels in regions A and B are θQ and $(1 - \theta)Q$, respectively.

This paper examines two cases of ownership of the private firm: case (h), where firm 1 is owned by private investors from region A , and case (f), where it is owned by private investors from region B . Figure 2.1 shows the two cases and the amount of pollution of two regions by the unidirectional transboundary pollution.

In the model, welfare is defined as the sum of consumer surplus, producer surplus, and environmental damage.

First, we consider case (h), wherein firm 1 is owned by private investors from region A . Welfare in region A is given by

$$w_A = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} + m. \quad (2.2)$$

Welfare in region B is given by

$$w_B = -\frac{d\{(1 - \theta)Q\}^2}{2}. \quad (2.3)$$

Welfare in the economy is defined as the sum of the welfare in regions A and B . Thus,

$$W = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} - \frac{d\{(1 - \theta)Q\}^2}{2} + m. \quad (2.4)$$

Second, we consider case (f). In this case, firm 1 is owned by private investors from region B . Welfare in region A , welfare in region B , and the total welfare are respectively

given by

$$w_A = \int_0^Q p(s)ds - p(Q)q_1 - \frac{cq_0^2}{2} - \frac{d(\theta Q)^2}{2} + m, \quad (2.5)$$

$$w_B = p(Q)q_1 - \frac{cq_1^2}{2} - \frac{d\{(1-\theta)Q\}^2}{2}, \quad (2.6)$$

$$W = \int_0^Q p(s)ds - \frac{cq_0^2}{2} - \frac{cq_1^2}{2} - \frac{d(\theta Q)^2}{2} - \frac{d\{(1-\theta)Q\}^2}{2} + m. \quad (2.7)$$

We denote the welfare of region l as “local welfare l ” and the welfare in the entire economy as “total welfare.” Further, we define the local regional government of l as “local government l .”

Here, we define the objective function of each firm. The objective functions of public firm U_0 and private firm U_1 are respectively given by

$$U_0 = \alpha W + (1 - \alpha)\pi_0, \quad \alpha \in [0, 1], \quad (2.8)$$

$$U_1 = \pi_1. \quad (2.9)$$

When $\alpha = 0$, firm 0 is a pure profit-maximizer, and when $\alpha = 1$, it is a pure local welfare-maximizer.⁵ Here, α is understood as the share holding of the public sector and $1 - \alpha$ is that of the private sector.⁶ The objective of firm 1 is to maximize its own profits.

Finally, we consider the following timing of the game. Before the game begins, the

⁵Earlier studies model the following two objectives of (full nationalized) public firm: maximization of social welfare in its region (that is, the region where it exists) and maximization of the sum of consumer surplus and producer surplus in its region. Beladi and Chao (2006) and Ohori (2006b) model the latter. In particular, they consider consumption externality. In this paper, however, we model the former, even though the objective of the public firm in Ohori (2006b) is convincing when we consider consumption externality. This is the reason why there exist two distortions in terms of social welfare maximization: both public and private firms do not maximize social welfare. In this case, distinguishing the effects on social welfare is more difficult than when the public firm is the social welfare maximizer. Furthermore, we consider that characteristic differences between consumption externality and production externality exist only in sources of pollution. If we reconsider Beladi and Chao (2006) and Ohori (2006b) using the setting of the former, some results might change.

⁶For a rationalization of this objective function, see Bös (1991) and Matsumura (1998).

public firm is perfectly owned by local government A , that is, $\alpha = 1$. When the game starts, local government A chooses the level of α , and then, the two firms choose their quantity simultaneously.

2.3 Equilibrium outcomes and welfare comparison

In this section, we derive the equilibrium outcome and compare three types of welfare before and after privatization in cases (h) and (f) . First, we examine case (h) .

2.3.1 Case (h)

We first consider the case where firm 1 is owned by private investors from region A .

Local welfare A , local welfare B , and total welfare are respectively defined as (2.2), (2.3), and (2.4).

In the second stage, each firm maximizes its objective by choosing its quantity. The first order condition of the maximization problem of firms 0 and 1 are respectively given by

$$\frac{\partial U_0}{\partial q_0} = a - (2 - \alpha + c + d\alpha\theta^2)q_0 - (1 + d\alpha\theta^2)q_1 = 0, \quad (2.10)$$

$$\frac{\partial U_1}{\partial q_1} = a - q_0 - (2 + c)q_1 = 0. \quad (2.11)$$

Solving the above first order conditions, we obtain

$$q_0^h = \frac{a(1+c-d\alpha\theta^2)}{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2}, \quad (2.12)$$

$$q_1^h = \frac{a(1+c-\alpha+d\alpha\theta^2)}{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2}, \quad (2.13)$$

$$w_A^h = \frac{2a^2(1+c)\{(1+c)(4+c-2d\theta^2)-(5+2c-4d\theta^2-2cd\theta^2)\alpha\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2} + \frac{a^2\alpha^2\{3+c-d\theta^2(3+2cd\theta^2)\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}, \quad (2.14)$$

$$w_B^h = -\frac{a^2d(2+2c-\alpha)^2(1-\theta)^2}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2}, \quad (2.15)$$

$$W^h = \frac{a^2\{2(1+c)^2(4+c-2d)-2(1+c)(5+2c-2d)\alpha-2d\alpha^2\theta^2(2+cd\theta^2)\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2} + \frac{a^2\{(3+c-d)\alpha^2+2d(2+2c-\alpha)^2\theta+4d(1+c)(-2-2c+3\alpha+c\alpha)\theta^2\}}{2\{(1+c)(3+c)-(2+c)\alpha+(1+c)d\alpha\theta^2\}^2} + m, \quad (2.16)$$

where the superscript h denotes the equilibrium outcome in case (h). In the subsequent section, this superscript is also used to represent the equilibrium outcome in the second stage. To restrict our attention to the case of the interior solution, we assume that $1+c \geq d$. We also assume that $c \geq 1$ in order to simplify the subsequent analyses.

Here, we examine the comparative statics for the equilibrium output of each firm with respect to α . We find that

$$\frac{\partial q_0^h}{\partial \alpha} < 0, \quad \frac{\partial q_1^h}{\partial \alpha} > 0, \quad \text{and} \quad \frac{\partial Q^h}{\partial \alpha} < 0, \quad \text{if and only if} \quad d\theta^2 > \frac{1}{2}. \quad (2.17)$$

In terms of local welfare A , there are two distortions in the region. One is caused by underproduction with regard to the duopolistic market and the other is caused by excess production with regard to environmental damage. A high level of d and θ imply that a large fraction of pollution remains in region A and environmental damage is large. In this case, the latter distortion dominates the former one, and therefore, the local public firm

decreases its output when it gives greater weightage to local welfare A .

In the first stage, local government A chooses α in order to maximize local welfare.⁷

Solving for α , we obtain

$$\alpha^h = \frac{(1+c)^2}{1+3c+c^2}. \quad (2.18)$$

The result shows that partial privatization is desirable for local welfare A . We also find that α^h does not depend on the fraction of transboundary pollution and the degree of environmental damage. Rather, these results depend on the functional forms of demand, cost, and environmental damage.⁸

Does partial privatization of the local public firm enhance local welfare in the other region and the total welfare? (welfare comparison)

We examine whether the optimal privatization for local welfare A enhances local welfare B and the total welfare. Comparing local welfare B and total welfare at $\alpha = 1$ and $\alpha = \alpha^h$, we obtain the following proposition.

Proposition 2.1. *When $\theta = 1$ or $d\theta^2 = 1/2$, $w_B|_{\alpha=1} = w_B|_{\alpha=\alpha^h}$. Consider the case where $\theta \neq 1$ and $d\theta^2 \neq 1/2$. Then,*

$$w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h} > 0 \quad \text{if} \quad d > \frac{1}{2} \text{ and } \theta \in \left(\sqrt{\frac{1}{2d}}, 1 \right),$$

$$w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h} < 0 \quad \text{if} \quad \begin{cases} d > \frac{1}{2} \text{ and } \theta \in \left[0, \sqrt{\frac{1}{2d}} \right), \\ d < \frac{1}{2} \text{ and } \theta \in [0, 1). \end{cases}$$

⁷The second order condition of the maximization problem is satisfied. See Appendix 2.A.

⁸The amount of total output and output level of each firm affect the decision with respect to α^h . Specifically, the total output level affects both the marginal utility of the representative consumer and the marginal environmental damage. The larger is the total output, the larger are the marginal utility and marginal environmental damage. On the other hand, the output level of each firm affects its marginal production cost: the difference between the marginal production costs of firms is maximized at $\alpha = 1$ and minimized at $\alpha = 0$. As local government chooses α taking into account both total output level and production inefficiency, partial privatization would not always be chosen given other functional forms.

Proof. See Appendix 2.B. □

Figure 2.2 illustrates Proposition 2.1 for each value of the fraction of transboundary pollution and the degree of environmental damage.

The intuition behind Proposition 2.1 is as follows. When $\theta = 1$, no fraction of the pollution caused in region A is transported to region B , and therefore, α does not affect local welfare B . When $\theta \neq 1$, some portion of the pollution generated in region A is transported to region B . Local welfare B is based on environmental damage. We know that the environmental damage function is a function of the total output and that the total output decreases (increases) with an increase in α when $\theta > (\leq) 1/\sqrt{2d}$. Suppose the case where d and θ are small (large). When the local public firm is not privatized, that is, $\alpha = 1$, it produces more (less) and the total output is larger (smaller) than when $\alpha = \alpha^h$. The larger (smaller) the total output is, the larger (smaller) the total emission is. Therefore, $\alpha = \alpha^h$ ($\alpha = 1$) is more desirable than $\alpha = 1$ ($\alpha = \alpha^h$) for local welfare B .

Next, we investigate the total welfare. We compare total welfare at $\alpha = 1$ and $\alpha = \alpha^h$. Calculating $W|_{\alpha=1} - W_{\alpha=\alpha^h}$, we obtain the following proposition.

Proposition 2.2.

$$\begin{aligned}
 W|_{\alpha=1} - W|_{\alpha=\alpha^h} > 0 & \quad \text{if} \quad d > \frac{1}{2} \text{ and } \theta \in \left(\sqrt{\frac{1}{2d}}, \bar{\theta} \right], \\
 W|_{\alpha=1} - W|_{\alpha=\alpha^h} < 0 & \quad \text{if} \quad \begin{cases} d > \frac{1}{2} \text{ and } \theta \in \left[0, \sqrt{\frac{1}{2d}} \right), \\ d > \frac{1}{2} \text{ and } \theta \in [\bar{\theta}, 1], \\ d < \frac{1}{2} \text{ and } \theta \in [0, 1], \end{cases}
 \end{aligned}$$

where $\bar{\theta}$ is the solution of $W|_{\alpha=1} - W_{\alpha=\alpha^h} = 0$.

Proof. See Appendix 2.C. □

The intuition behind Proposition 2.2 is as follows. Consider the case of $\alpha = 1$. When θ and d are either sufficiently small or large, there is a large difference between regions A and B in terms of environmental damage and a large difference between firms 0 and 1 in terms of the production cost. When partial privatization occurs, the differences become small. In other words, when θ and d are small (large), the local public firm produces more (less) when $\alpha = 1$ than when $\alpha = \alpha^h$; therefore, the difference of between the marginal production costs of the two firms are large. Moreover, total environmental damage of the two regions is large because the environmental damage function in each region is strictly increasing. Therefore, partial privatization is desirable for the entire economy when θ and d are small or large. Figure 2.3 shows these results.

2.3.2 Case (f)

We consider the case where firm 1 is owned by private investors from region B .

Local welfare A , local welfare B , and total welfare are respectively defined as (2.5), (2.6), and (2.7).

In the second stage, each firm maximizes its objective by choosing its quantity. The first order condition of the maximization problem of firms 0 and 1 are respectively given by

$$\frac{\partial U_0}{\partial q_0} = a - (2 - \alpha + c + d\alpha\theta^2)q_0 - (1 - \alpha + d\alpha\theta^2)q_1 = 0, \quad (2.19)$$

$$\frac{\partial U_1}{\partial q_1} = a - q_0 - (2 + c)q_1 = 0. \quad (2.20)$$

Solving the above first order conditions, we obtain

$$q_0^f = \frac{a(1+c+\alpha-d\alpha\theta^2)}{(1+c)(3-\alpha+c+d\alpha\theta^2)}, \quad (2.21)$$

$$q_1^f = \frac{a(1+c-\alpha+d\alpha\theta^2)}{(1+c)(3-\alpha+c+d\alpha\theta^2)}, \quad (2.22)$$

$$w_A^f = \frac{a^2\{(1+c)^2(6+c-4d\theta^2) - 2(1+c)c\alpha(1-d\theta^2) - (2+3c)(1-d\theta^2)^2\alpha^2\}}{2(1+c)^2(3-\alpha+c+d\alpha\theta^2)^2}, \quad (2.23)$$

$$w_B^f = \frac{a^2\{(2+c)(1+c-\alpha)^2 - 4(1+c)^2(1-2\theta)d + (2+c)\alpha^2d\theta^4\}}{2(1+c)^2(3-\alpha+c+d\alpha\theta^2)^2} + \frac{2a^2(-2-4c-2c^2+2\alpha+3c\alpha+c^2\alpha-2\alpha^2-c\alpha^2)d\theta^2}{2(1+c)^2(3-\alpha+c+d\alpha\theta^2)^2}, \quad (2.24)$$

$$W^f = \frac{a^2\{(1+c)^2(4+c-2d-2\alpha+4d\theta-4d\theta^2+2d\alpha\theta^2) - c\alpha^2(1-d\theta^2)^2\}}{(1+c)^2(3-\alpha+c+d\alpha\theta^2)^2} + m. \quad (2.25)$$

Here, we analyze the comparative statics for the equilibrium output of each firm with respect to α . We find that

$$\frac{\partial q_0^f}{\partial \alpha} < 0, \quad \frac{\partial q_1^f}{\partial \alpha} > 0, \quad \text{and} \quad \frac{\partial Q^f}{\partial \alpha} < 0, \quad \text{if and only if } d\theta^2 > 1. \quad (2.26)$$

In the first stage, the local government chooses α in order to maximize local welfare.

Solving for α , we obtain⁹

$$\alpha^f = \begin{cases} \bar{\alpha} & \text{if } \begin{cases} 0 < d < \frac{c}{1+2c} \text{ and } \theta \in [0, 1], \\ \frac{c}{1+2c} < d \text{ and } \theta \in [0, \sqrt{\frac{c}{d(1+2c)}}], \\ \frac{3+2c}{2(1+c)} < d \text{ and } \theta \in [\sqrt{\frac{3+2c}{2d(1+c)}}, 1], \end{cases} \\ 1 & \text{if } \begin{cases} \frac{c}{1+2c} < d < 1 \text{ and } \theta \in [\sqrt{\frac{c}{d(1+2c)}}, 1], \\ 1 < d \text{ and } \theta \in [\sqrt{\frac{c}{d(1+2c)}}, \sqrt{\frac{1}{d}}], \end{cases} \\ 0 & \text{if } \begin{cases} 1 < d < \frac{3+2c}{2(1+c)} \text{ and } \theta \in [\sqrt{\frac{1}{d}}, 1], \\ \frac{3+2c}{2(1+c)} < d \text{ and } \theta \in [\sqrt{\frac{1}{d}}, \sqrt{\frac{3+2c}{2d(1+c)}}], \end{cases} \end{cases}$$

⁹In Appendix 2.D, we show that the second order condition of the maximization problem is satisfied. For the calculation of α^f , see Appendix 2.E.

where

$$\bar{\alpha} = \frac{(1+c)\{3+2c-2d\theta^2(1+c)\}}{(3+6c+2c^2)(1-d\theta^2)}. \quad (2.27)$$

From the result, we find that α^f depends on the fraction of transboundary pollution and the degree of environmental damage. Figure 2.4 illustrates α^f for each d and θ . When both d and θ are small or large (region *I* or *IV*), partial privatization ($\alpha^f = \bar{\alpha}$) is chosen. When they take a middle value, local public firm *A* is fully privatized (region *III*) or is not privatized at all (region *II*).

The intuition behind the result is as follows. First, we consider the case where d and θ are sufficiently large. In this case, environmental damage is severe in region *A*, and thus, the local public firm produces less when $\alpha = 1$ than when $\alpha = \bar{\alpha}$. Suppose a marginal decrease of α at $\alpha = 1$. A marginal increase of output of the public firm does not affect welfare because the public firm is local welfare maximizer when $\alpha = 1$. However, the marginal decrease of output of the private firm improves welfare because it reduces the environmental damage. Therefore, (partial) privatization enhances welfare.

Second, we consider the case where d and θ are sufficiently small. In this case, the degree of environmental damage is low, and thus, we regard this case as no environmental problem in a mixed duopoly to some extent. Suppose a marginal decrease of α at $\alpha = 1$. As is the same reason mentioned in the previous paragraph, a marginal decrease of output of the public firm does not affect welfare. However, a marginal increase of output of the private firm increases consumer surplus. Therefore, (partial) privatization enhances welfare.

Finally, we consider the case where d and θ take a middle value. In this case, the equilibrium output in a mixed duopoly is nearly the same as that in a pure duopoly. For

example, consider the case where $d\theta^2 = 1$. In this case, the reaction function of the local public firm does not depend on α : the reaction function of each firm is symmetric. And thus, either full privatization or no privatization can be chosen.

Does privatization of the local public firm enhance local welfare in the other region and the total welfare? (welfare comparison)

We examine whether the optimal privatization for local welfare A enhances local welfare B and the total welfare. We compare local welfare B and total welfare at $\alpha = 1$ and $\alpha = \alpha^f$.

Here, we compare local welfare B before and after privatization. Because the level of α^f depends on the values of parameters, we separate the cases for each α^f . Figure 2.4 shows the level of α^f for the values of parameters: $\bar{\alpha}$ is chosen by the local government A in regions I and IV , 0 in region III , and 1 in region II . In each region, the results of the welfare comparison before and after privatization are as follows.

Proposition 2.3.

$$\begin{aligned}
 w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} > 0 & \quad \text{if } d > \frac{3+2c}{2(1+c)} \text{ and } \theta \in \left(\sqrt{\frac{3+2c}{2d(1+c)}}, 1 \right], \\
 w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} < 0 & \quad \text{if } \begin{cases} d > \frac{c}{1+2c} \text{ and } \theta \in \left[0, \sqrt{\frac{c}{d(1+2c)}} \right), \\ d < \frac{c}{1+2c} \text{ and } \theta \in [0, 1] \end{cases} \\
 w_B|_{\alpha=1} - w_B|_{\alpha=0} > 0 & \quad \text{if } d > 1 \text{ and } \theta \in \left(\sqrt{\frac{1}{d}}, \sqrt{\frac{3+2c}{2d(1+c)}} \right].
 \end{aligned}$$

Proof. See Appendix 2.F. □

According to Proposition 2.3, when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are low, privatization of the local public firm in region A enhances local welfare B , but when they are high, privatization worsens the welfare. Figure 2.5 shows these results.

The intuition behind Proposition 2.3 is as follows. Local welfare B is based on the sum of firm 1's profit and environmental damage. We know that the environmental damage function is a function of the total output and that the total output decreases with an increase in α when $\theta > 1/\sqrt{d}$. We also see that firm 1's profit increases with an increase in α when $\theta > 1/\sqrt{d}$ because the price of the dirty good increases with a decrease in the total output, and the output of firm 1 increases as a result of the strategic substitution effect. Thus, we find that local welfare B increases with an increase in α when $\theta > 1/\sqrt{d}$. When $\theta < 1/\sqrt{d}$, the results are opposite, that is, local welfare B decreases with an increase in α .

Lastly, we compare total welfare between $\alpha = 1$ and $\alpha = \alpha^f$. As in the case of the welfare comparison for region B , we separate the cases for each α^f . The results of the welfare comparison in terms of before and after privatization are as follows for each case.

Proposition 2.4.

$$\begin{aligned}
W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} &> 0 && \text{if } d > \frac{3+2c}{2(1+c)} \text{ and } \theta \in \left(\sqrt{\frac{3+2c}{2d(1+c)}}, 1 \right], \\
W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} &< 0 && \text{if } \begin{cases} d > \frac{c}{1+2c} \text{ and } \theta \in \left[0, \sqrt{\frac{c}{d(1+2c)}} \right) \\ d < \frac{c}{1+2c} \text{ and } \theta \in [0, 1] \end{cases}, \\
W|_{\alpha=1} - W|_{\alpha=0} &> 0 && \text{if } d > 1 \text{ and } \theta \in \left(\sqrt{\frac{1}{d}}, \sqrt{\frac{3+2c}{2d(1+c)}} \right].
\end{aligned}$$

Proof. See Appendix 2.G. □

Figure 2.6 shows Proposition 2.4. According to Proposition 2.4, when the degree of environmental damage and the fraction of transboundary pollution remaining in region A are low, privatization of the local public firm in region A enhances the total welfare because local welfare A and B both increase. However, when the same are high, local welfare B

is worsened considerably, and the total welfare decreases. Thus, in terms of total welfare, privatization is not desirable.

2.4 Comparison between cases (*h*) and (*f*)

We compare the results obtained in cases (*h*) and (*f*). There are three major points.

1. Partial privatization is chosen in case (*h*), but partial privatization, full privatization, or no privatization can be chosen in case (*f*).
2. Partial privatization enhances w_A , w_B , and W in both cases when the degree of environmental damage and the fraction of transboundary pollution remaining in region *A* are low.
3. Partial privatization enhances W and reduces w_B in case (*h*), but it reduces both w_B and W in case (*f*) when the degree of environmental damage and the fraction of transboundary pollution remaining in region *A* are high.

2.5 Concluding remarks

This paper examines the effect that the privatization of a local public firm has on local welfare in two regions and on the total welfare of the two regions when the fraction of unidirectional transboundary pollution varies. We analyze this problem by considering two separate cases of ownership of a private firm.

We discuss the possible implication of our results. Consider the example of the relationship between China located upstream and Japan, downstream. Since the 21st century, several Japanese firms have entered the Chinese market. From China's point of view, to

calculate the optimal degree of privatization in terms of welfare of China is more complex in this situation than in the situation where the competitor of the public firm is a domestic private firm: the optimal degree of privatization varies for each value of the degree of environmental damage and the fraction of transboundary pollution. Particularly, when the pollutant has a moderate degree of environment damage, Chinese government should pay attention to the trend of the fraction of transboundary pollution since there is a possibility that its fraction is affected by the recent extreme weather.

This paper uses a simple framework to consider the privatization problem in the context of unidirectional transboundary pollution problem; therefore, several extensions of this analysis are possible. For example, we can consider the case that firms can abate its pollution and the government can impose firms on the environmental regulations such as emission taxes and quotas. Note that our paper assumes that producing one unit of a dirty good generates one unit of pollution. If firms decrease pollution per unit of dirty good, partial privatization of Chinese public firms could benefit welfare in not only China but also Japan, regardless of the fraction of transboundary pollution. We can also extend our model to examine not only the case where a market for dirty goods exists in both countries but also the case where generation of the pollution occurs in the country located downstream. We leave these analyses for future research.

Appendix 2.A

The first order condition of the maximization problem of local government A

Partially differentiating w_A^h with respect to α , we obtain

$$\frac{\partial w_A^h}{\partial \alpha} = \frac{a^2(1+c)\{(1+c)^2 - (1+3c+c^2)\alpha\}(1-2d\theta^2)^2}{\{(1+c)(3+c+d\alpha\theta^2) - (2+c)\alpha\}^2} = 0. \quad (2.28)$$

We can easily find that the denominator is positive. We focus on the numerator. When $d\theta^2 = 1/2$, w_A^h does not depend on α . When $d\theta^2 \neq 1/2$, we can derive the optimal degree of partial privatization level for local government A, that is, α^h .

The second order condition of the maximization problem of local government A

To determine whether α^h is the maximizing value for w_A^h , we calculate the second order condition of the maximization problem for local government A. Then, we obtain

$$\frac{\partial^2 w_A^h}{\partial \alpha^2} = -\frac{a^2(1+c)(1-2d\theta^2)^2 X_0(c, d, \theta, \alpha)}{\{(1+c)(3+c+d\alpha\theta^2) - (2+c)\alpha\}^4} \leq 0, \quad (2.29)$$

where

$$\begin{aligned} X_0(c, d, \theta, \alpha) &= (1+c)(-3+c+3c^2+c^3) + 2(2+c)(1+3c+c^2)\alpha \\ &\quad + (3-2\alpha)d\theta^2 + (9-8\alpha)cd\theta^2 + (9-8\alpha)c^2d\theta^2 \\ &\quad + (3-2\alpha)c^3d\theta^2 > 0. \end{aligned} \quad (2.30)$$

Note that a strict inequality holds when $d\theta^2 \neq 1/2$. Therefore, the second order condition is satisfied when $d\theta^2 \neq 1/2$.

Appendix 2.B

Proof of Proposition 2.1. Calculating local welfare B when $\alpha = 1$ and $\alpha = \alpha^h$, we respectively obtain

$$w_B^h|_{\alpha=1} = -\frac{a^2(1+2c)^2(1-\theta)^2d}{2\{1+3c+c^2+(1+c)d\theta^2\}^2}, \quad (2.31)$$

$$w_B^h|_{\alpha=\alpha^h} = -\frac{a^2(1+5c+2c^2)^2(1-\theta)^2d}{2\{1+7c+5c^2+c^3+(1+c)^2d\theta^2\}^2}. \quad (2.32)$$

Comparing the above, we obtain the following equation:

$$w_B^h|_{\alpha=1} - w_B^h|_{\alpha=\alpha^h} = -\frac{a^2cd(1+c)(1-\theta)^2(1-2d\theta^2)\{2+17c+37c^2+22c^3+4c^4+2(1+c)(1+4c+2c^2)d\theta^2\}}{2\{1+3c+c^2+(1+c)d\theta^2\}^2\{1+7c+5c^2+c^3+(1+c)^2d\theta^2\}^2}.$$

From the above equation, we find that $w_B|_{\alpha=1} = w_B|_{\alpha=\alpha^h}$ when $\theta = 1$. Consider the case where $\theta \neq 1$. Whether or not $w_B|_{\alpha=1} - w_B|_{\alpha=\alpha^h}$ is positive depends on the sign of $1 - 2d\theta^2$.

Thus, we can derive Proposition 2.1. \square

Appendix 2.C

Proof of Proposition 2.2. Calculating the total welfare when $\alpha = 1$ and $\alpha = \alpha^h$, we respectively obtain

$$W^h|_{\alpha=1} = \frac{a^2\{1+5c+8c^2+2c^3+(1+2c)^2(2\theta-1)d-4c^2d\theta^2-2cd^2\theta^4\}}{2(1+3c+c^2+d\theta^2+cd\theta^2)^2} + m, \quad (2.33)$$

$$W^h|_{\alpha=\alpha^h} = \frac{a^2\{(1+6c+2c^2)(1+7c+5c^2+c^3)+(1+5c+2c^2)^2(2\theta-1)d\}}{2(1+7c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2} - \frac{a^2\{4c(1+7c+5c^2+c^3)d\theta^2+2c(1+c)^2d^2\theta^4\}}{2(1+7c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2} + m. \quad (2.34)$$

The difference between them is

$$W^h|_{\alpha=1} - W^h|_{\alpha=\alpha^h} = \frac{a^2c(-1 + 2d\theta^2)X_1(c, d, \theta)}{2\{1 + 3c + c^2 + (1 + c)d\theta^2\}^2\{1 + 7c + 5c^2 + c^3 + (1 + c)^2d\theta^2\}^2},$$

where

$$\begin{aligned} X_1(c, d, \theta) = & c + 7c^2 + 5c^3 + c^4 + 2d + 19cd + 54c^2d + 59c^3d + 26c^4d + 4c^5d \\ & - 2(1 + c)(2 + 17c + 37c^2 + 22c^3 + 4c^4)d\theta \\ & + 2d(1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + d + 6cd + 11c^2d + 8c^3d + 2c^4d)\theta^2 \\ & - 4(1 + c)^2(1 + 4c + 2c^2)d^2\theta^3 + 2(1 + c)^3(1 + 2c)d^2\theta^4. \end{aligned} \quad (2.35)$$

When $d\theta^2 = 1/2$, there is no difference between them. We consider the case where $d\theta^2 \neq 1/2$. Whether or not the difference is positive depends on both the sign of $-1 + 2d\theta^2$ and that of $X_1(c, d, \theta)$. At first glance, it is not clear whether or not $X_1(c, d, \theta)$ is positive. In the subsequent analyses, we examine the property of $X_1(c, d, \theta)$.

First, we check the monotonicity of $X_1(c, d, \theta)$ in $\theta \in [0, 1]$. Partially differentiating $X_1(c, d, \theta)$ with respect to θ , we find that

$$\begin{aligned} \frac{\partial X_1(c, d, \theta)}{\partial \theta} = & 2d\{-(1 + c)(2 + 17c + 37c^2 + 22c^3 + 4c^4) \\ & + 2(1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + d + 6cd + 11c^2d + 8c^3d + 2c^4d)\theta \\ & - 6(1 + c)^2(1 + 4c + 2c^2)d\theta^2 + 4(1 + c)^3(1 + 2c)d\theta^3\}. \end{aligned} \quad (2.36)$$

Summing up the above terms, we find that $\partial X_1(c, d, \theta)/\partial \theta = 2dX_2(c, d, \theta)$, where

$$\begin{aligned} X_2(c, d, \theta) = & -2(1 - \theta)\{1 + 9c + 21c^2 + 25c^3 + 12c^4 + 2c^5 + 2(1 + c)^3(1 + 2c)d\theta^2\} \\ & + 2(1 - \theta)(1 + c)^2(1 + 4c + 2c^2)d\theta - c(1 + 12c + 9c^2 + 2c^3) \\ & - 4c(1 + c)^2d\theta^2. \end{aligned} \quad (2.37)$$

The above calculation shows that the second term is positive whereas the other terms are negative. As the upper bound of d is assumed to be $1 + c$, we substitute $1 + c$ into d only in the second term of the above equation. Summing up the calculation, we obtain

$$\begin{aligned} X_2(c, d, \theta) = & -2(1 - \theta)\{1 - \theta + (9 - 7\theta)c + (21 - 17\theta)c^2 + (25 - 19\theta)c^3 \\ & + 2(6 - 5\theta)c^4 + 2(1 - \theta)c^5 + 2(1 + c)^3(1 + 2c)d\theta^2\} \\ & - c(1 + 12c + 9c^2 + 2c^3) - 4c(1 + c)^2d\theta^2 < 0. \end{aligned} \quad (2.38)$$

Therefore, we find that $\partial X_1(c, d, \theta)/\partial\theta < 0$.

Second, we check the sign of $X_1(c, d, 0)$ and $X_1(c, d, 1)$. When $\theta = 0$, we find

$$X_1(c, d, 0) = c + 7c^2 + 5c^3 + c^4 + 2d + 19cd + 54c^2d + 59c^3d + 26c^4d + 4c^5d > 0. \quad (2.39)$$

When $\theta = 1$, we find

$$X_1(c, d, 1) = -c(-1 + 2d)(1 + 7c + 5c^2 + c^3 + d + 2cd + c^2d). \quad (2.40)$$

When $d > 1/2$, this term is negative. As mentioned previously, $X_1(c, d, \theta)$ decreases with respect to θ , and therefore, there exists a unique $\bar{\theta} \in [0, 1]$ at which $X(c, d, \bar{\theta})$ is equal to 0. When $d < 1/2$, this term is positive. Then, $X_1(c, d, \theta)$ is always positive in $\theta \in [0, 1]$.

Finally, we examine the magnitude of the relation between $\sqrt{1/(2d)}$ and $\bar{\theta}$. Substituting $\sqrt{1/(2d)}$ into θ in $X_1(c, d, \theta)$, we find

$$X_1(c, d, \sqrt{\frac{1}{2d}}) = d(1 + c)(3 + c)(1 + 2c)(1 + 5c + 2c^2) \left(1 - \sqrt{\frac{1}{2d}}\right)^2 \geq 0, \quad (2.41)$$

where a strict inequality holds when $d \neq 1/2$. As $X_1(c, d, \theta)$ is a decreasing function with respect to θ and $X_1(c, d, \bar{\theta}) = 0$, we find that $\sqrt{1/(2d)} \leq \bar{\theta}$, where a strict inequality holds when $d \neq 1/2$.

On the basis of the above analyses, we can draw Figure 2.3 and derive Proposition 2.2. □

Appendix 2.D

The first order condition of the maximization problem of local government A

Partially differentiating w_A^f with respect to α , we obtain

$$\begin{aligned}\frac{\partial w_A^f}{\partial \alpha} &= \frac{2a^2(1-d\theta^2)\{(1+c)(3+2c-2(1+c)d\theta^2) - (3+6c+2c^2)(1-d\theta^2)\alpha\}}{(1+c)^2(3+c-\alpha+d\alpha\theta^2)^3} \\ &= 0.\end{aligned}\tag{2.42}$$

When $d\theta^2 = 1$, w_A^f does not depend on α . When $d\theta^2 \neq 1$, we derive $\bar{\alpha}$ by solving the above equation with respect to α . Note that because both the sign and value of $\bar{\alpha}$ vary with the value of the parameters of c , d , and θ , it is necessary to examine $\bar{\alpha}$ in detail. For further details regarding α^f , see Appendix 2.E.

The second order condition of the maximization problem of local government A

To determine whether $\bar{\alpha}$ is the maximization value for w_A^f , we calculate the second order condition of the maximization problem of local government A. Then, we obtain

$$\frac{\partial^2 w_A^f}{\partial \alpha^2} = -\frac{4a^2(1+c)(1-d\theta^2)^2 Y_0(c, d, \theta, \alpha)}{(1+c)^2(3+c-\alpha+d\alpha\theta^2)^4} \leq 0,\tag{2.43}$$

where

$$\begin{aligned}Y_0(c, d, \theta, \alpha) &= c(3+3c+c^2) + (3+6c+2c^2)\alpha \\ &\quad + 3(1-\alpha)d\theta^2 + 6(1-\alpha)cd\theta^2 + (3-2\alpha)c^2d\theta^2 > 0.\end{aligned}\tag{2.44}$$

Note that a strict inequality holds when $d\theta^2 \neq 1$. Therefore, the second order condition is satisfied when $d\theta^2 \neq 1$.

Appendix 2.E

Derivation of α^f

Consider the case where $d\theta^2 \neq 1$. There is a possibility that $\bar{\alpha}$ is negative or that $\bar{\alpha}$ is greater than 1. In the subsequent analyses, we ascertain the sign and value of $\bar{\alpha}$ for each value of parameter.

First, we derive the condition where $\bar{\alpha}$ is positive. In order to obtain a positive $\bar{\alpha}$, the following conditions have to be satisfied:

$$d\theta^2 < (>) \frac{3 + 2c}{2(1 + c)} \text{ and } d\theta^2 < (>) 1. \quad (2.45)$$

As $\sqrt{(3 + 2c)/\{2d(1 + c)\}} > \sqrt{1/d}$, we obtain

$$\bar{\alpha} > 0 \text{ if } \begin{cases} d\theta^2 < 1, \\ d\theta^2 > \frac{3+2c}{2(1+c)}. \end{cases} \quad (2.46)$$

Next, we examine whether or not α^f is less than 1. Calculating $1 - \bar{\alpha}$, we obtain

$$1 - \bar{\alpha} = \frac{c - (1 + 2c)d\theta^2}{(3 + 6c + 2c^2)(1 - d\theta^2)}. \quad (2.47)$$

When the above equation is positive, $\bar{\alpha}$ is less than 1. Thus, the conditions where $\bar{\alpha}$ is less than 1 are given by

$$d\theta^2 < (>) 1 \text{ and } d\theta^2 < (>) \frac{c}{(1 + 2c)}. \quad (2.48)$$

As $1 > c/(1 + 2c)$, we obtain

$$\bar{\alpha} < 1 \text{ if } \begin{cases} d\theta^2 < \frac{c}{1+2c}, \\ d\theta^2 > 1. \end{cases} \quad (2.49)$$

Summing up the above conditions while taking into account the fact that θ must be in $[0, 1]$, we can draw Figure 2.4 and derive α^f .

Appendix 2.F

Proof of Proposition 2.3. Calculating local welfare B for each value of α^f , we obtain

$$w_B^f|_{\alpha=0} = \frac{a^2(2+c-4d+8d\theta-4d\theta^2)}{2(3+c)^2}, \quad (2.50)$$

$$w_B^f|_{\alpha=1} = \frac{a^2\{c^2(2+c)-4(1+c)^2(1-2\theta)d-2(2+2c+c^2)d\theta^2+(2+c)d^2\theta^4\}}{2(1+c)^2(2+c+d\theta^2)^2}, \quad (2.51)$$

$$w_B^f|_{\alpha=\bar{\alpha}} = \frac{a^2\{c^2(2+c)^3-(3+6c+2c^2)^2(1-2\theta)d\}}{2\{3+8c+5c^2+c^3+(1+c)^2d\theta^2\}^2} + \frac{a^2\{-(9+28c+32c^2+14c^3+2c^4)d\theta^2+(1+c)^2(2+c)d^2\theta^4\}}{2\{3+8c+5c^2+c^3+(1+c)^2d\theta^2\}^2}. \quad (2.52)$$

According to Figure 2.4, local government A does not privatize firm 0 in region II . In this case, local welfare B is unchanged. In region III , it is necessary to compare $w_B|_{\alpha=1}$ with $w_B|_{\alpha=0}$. The result is shown by

$$w_B|_{\alpha=1} - w_B|_{\alpha=0} = -\frac{2a^2(1-d\theta^2)Y_1(c, d, \theta)}{(1+c)^2(3+c)^2(2+c+d\theta^2)^2}, \quad (2.53)$$

where

$$Y_1(c, d, \theta) = (2+c)(1+3c+c^2) + (2+c)^2\theta^2 + (1+c)^2(1-\theta)^2(5+2c+d^2\theta^2) > 0. \quad (2.54)$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=0}$ depends on the sign of $1-d\theta^2$.

In regions I and IV , it is necessary to compare $w_B|_{\alpha=1}$ with $w_B|_{\alpha=\bar{\alpha}}$. The result is

shown by

$$w_B|_{\alpha=1} - w_B|_{\alpha=\bar{\alpha}} = -\frac{a^2\{c - (1 + 2c)d\theta^2\}Y_2(c, d, \theta)}{2(1 + c)^2(2 + c + d\theta^2)^2\{3 + 8c + 5c^2 + c^3 + (1 + c)^2d\theta^2\}^2}, \quad (2.55)$$

where

$$\begin{aligned} Y_2(c, d, \theta) &= c(2 + c)(7 + 16c + 10c^2 + 2c^3) + d(2 + c)(1 + 2c)(5 + 6c + 2c^2)\theta^2 \\ &\quad + d(1 + c)^2(12 + 31c + 20c^2 + 4c^3)(1 - \theta)^2 \\ &\quad + (1 + c)^2d^2\theta^2\{(5 + 10c + 4c^2)(1 - \theta)^2 + 2(2 + c)\theta^2\} > 0. \end{aligned} \quad (2.56)$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=\bar{\alpha}}$ depends on the sign of $c - (1 + 2c)d\theta^2$.

Figure 2.5 and Proposition 2.3 sum up the above analyses. □

Appendix 2.G

Proof of Proposition 2.4. Calculating total welfare for each value of α^f , we obtain

$$W_{\alpha=0} = \frac{a^2\{4 + c - 2d + 4d(1 - \theta)\theta\}}{(3 + c)^2} + m, \quad (2.57)$$

$$W_{\alpha=1} = \frac{a^2\{2 + 4c + 4c^2 + c^3 - 2d(1 + c)^2(1 - 2\theta) - 2(1 + c + c^2)d\theta^2 - cd^2\theta^4\}}{(1 + c)^2(2 + c + d\theta^2)^2} + m, \quad (2.58)$$

$$\begin{aligned} W_{\alpha=\bar{\alpha}} &= \frac{a^2\{(3 + c)(3 + 12c + 18c^2 + 10c^3 + 2c^4) - (1 - 2\theta)(3 + 6c + 2c^2)^2d\}}{2(3 + 8c + 5c^2 + c^3 + d\theta^2 + 2cd\theta^2 + c^2d\theta^2)^2} \\ &\quad - \frac{2a^2\{6 + 21c + 26c^2 + 12c^3 + 2c^4 + c(1 + c)^2d^2\theta^4\}}{2(3 + 8c + 5c^2 + c^3 + d\theta^2 + 2cd\theta^2 + c^2d\theta^2)^2} + m. \end{aligned} \quad (2.59)$$

According to Figure 2.6, total welfare as well as local welfare B is unchanged in region *II*.

In region *III*, it is necessary to compare $W|_{\alpha=1}$ with $W|_{\alpha=0}$. The result is given by

$$W|_{\alpha=1} - W|_{\alpha=0} = -\frac{2a^2(1 - d\theta^2)Y_3(c, d, \theta)}{(1 + c)^2(3 + c)^2(2 + c + d\theta^2)^2}, \quad (2.60)$$

where

$$\begin{aligned}
Y_3(c, d, \theta) &= -1 + 2c + c^2 + d(1+c)^2(5+2c)(1-\theta)^2 + d(3+3c+3c^2+c^3)\theta^2 \\
&\quad + (1+c)^2\{(1-\theta)^2 + \theta^2\}d^2\theta^2 > 0.
\end{aligned} \tag{2.61}$$

Therefore, whether $W|_{\alpha=1}$ or not is larger than $W|_{\alpha=0}$ depends on the sign of $1 - d\theta^2$.

In regions I and IV , it is necessary to compare $W|_{\alpha=1}$ with $W|_{\alpha=\bar{\alpha}}$. The result is shown by

$$W|_{\alpha=1} - W|_{\alpha=\bar{\alpha}} = -\frac{a^2\{c - (1+2c)d\theta^2\}Y_4(c, d, \theta)}{2(1+c)^2(2+c+d\theta^2)^2(3+8c+5c^2+c^3+d\theta^2+2cd\theta^2+c^2d\theta^2)^2}, \tag{2.62}$$

where

$$\begin{aligned}
Y_4(c, d, \theta) &= 17c + 47c^2 + 41c^3 + 15c^4 + 2c^5 + 3(1+c)^2d^2\theta^4 \\
&\quad + (1+c)^2d(1-\theta)^2\{12 + 31c + 20c^2 + 4c^3 + (5 + 10c + 4c^2)d\theta^2\} \\
&\quad + (7 + 24c + 25c^2 + 12c^3 + 2c^4)d\theta^2 > 0.
\end{aligned} \tag{2.63}$$

Therefore, whether or not $w_B|_{\alpha=1}$ is larger than $w_B|_{\alpha=\bar{\alpha}}$ depends on the sign of $c - (1 + 2c)d\theta^2$.

Figure 2.6 and Proposition 2.4 sum up the above analyses. □

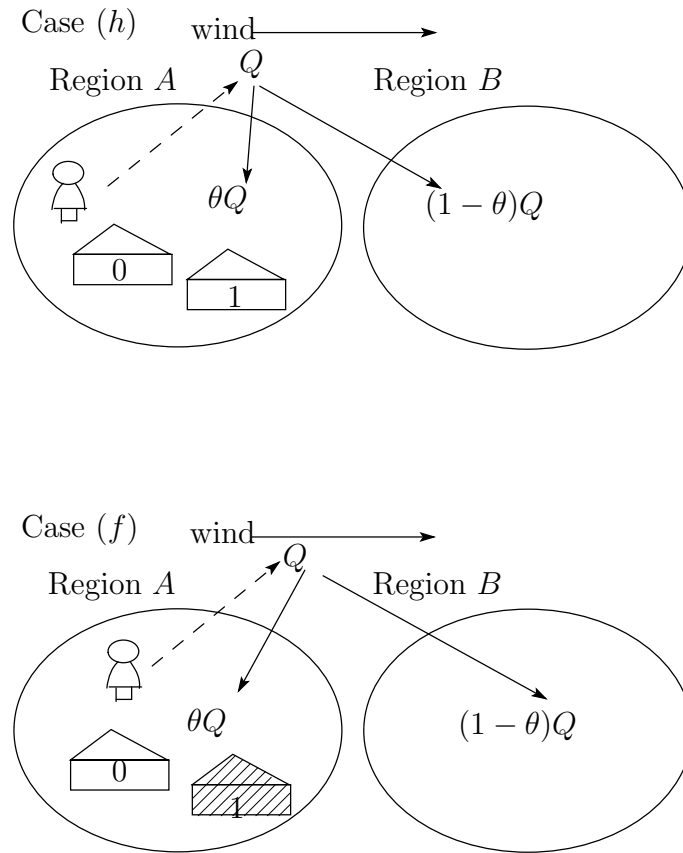


Figure 2.1: Two cases of ownership are considered in this paper. Case (h): firm 1 is owned by private investors from region A; Case (f): firm 1 is owned by private investors from region B (shaded object).

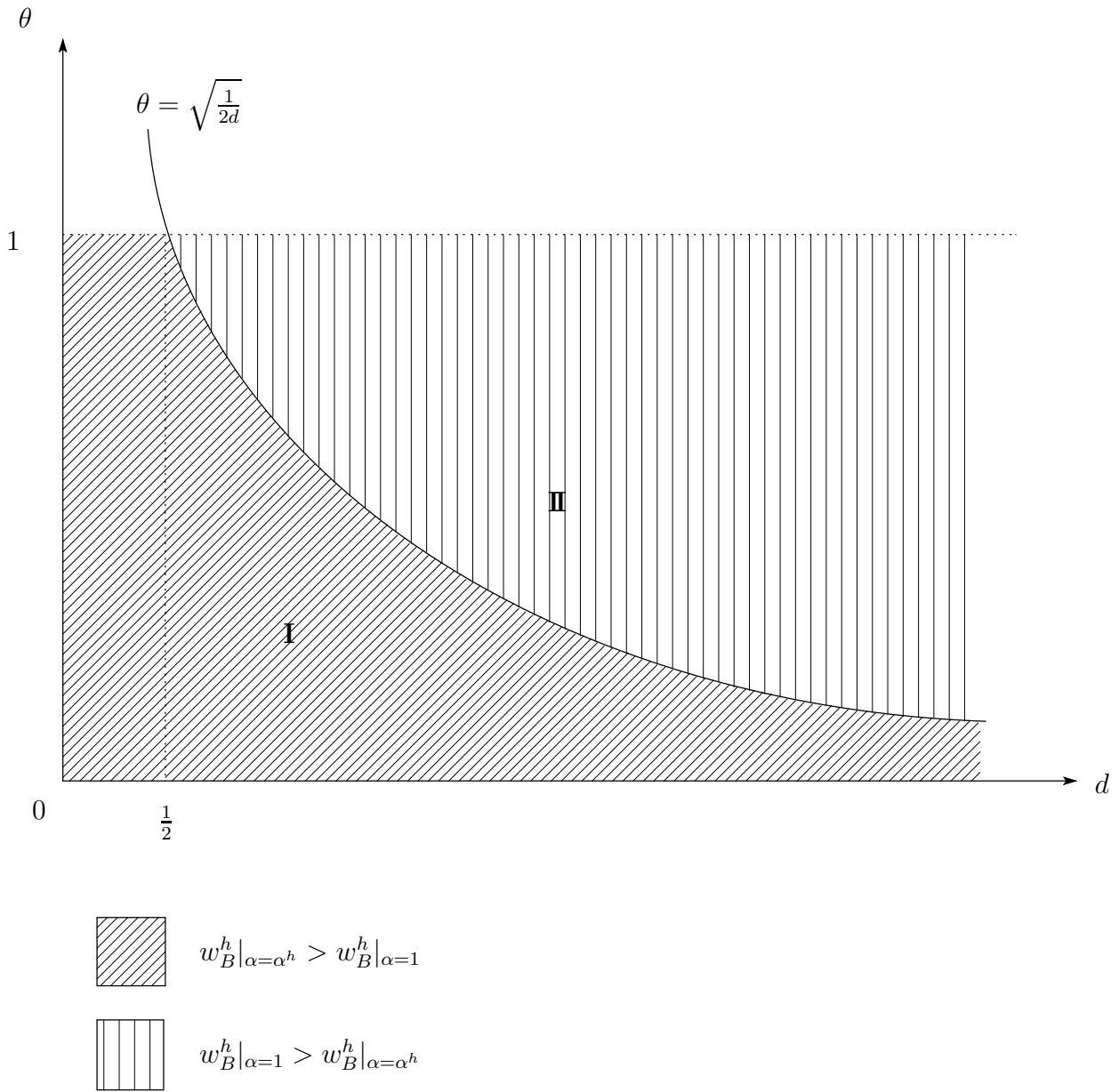


Figure 2.2: Comparisons between $w_B^h|_{\alpha=1}$ and $w_B^h|_{\alpha=\alpha^h}$ for each value of parameter of d and θ .

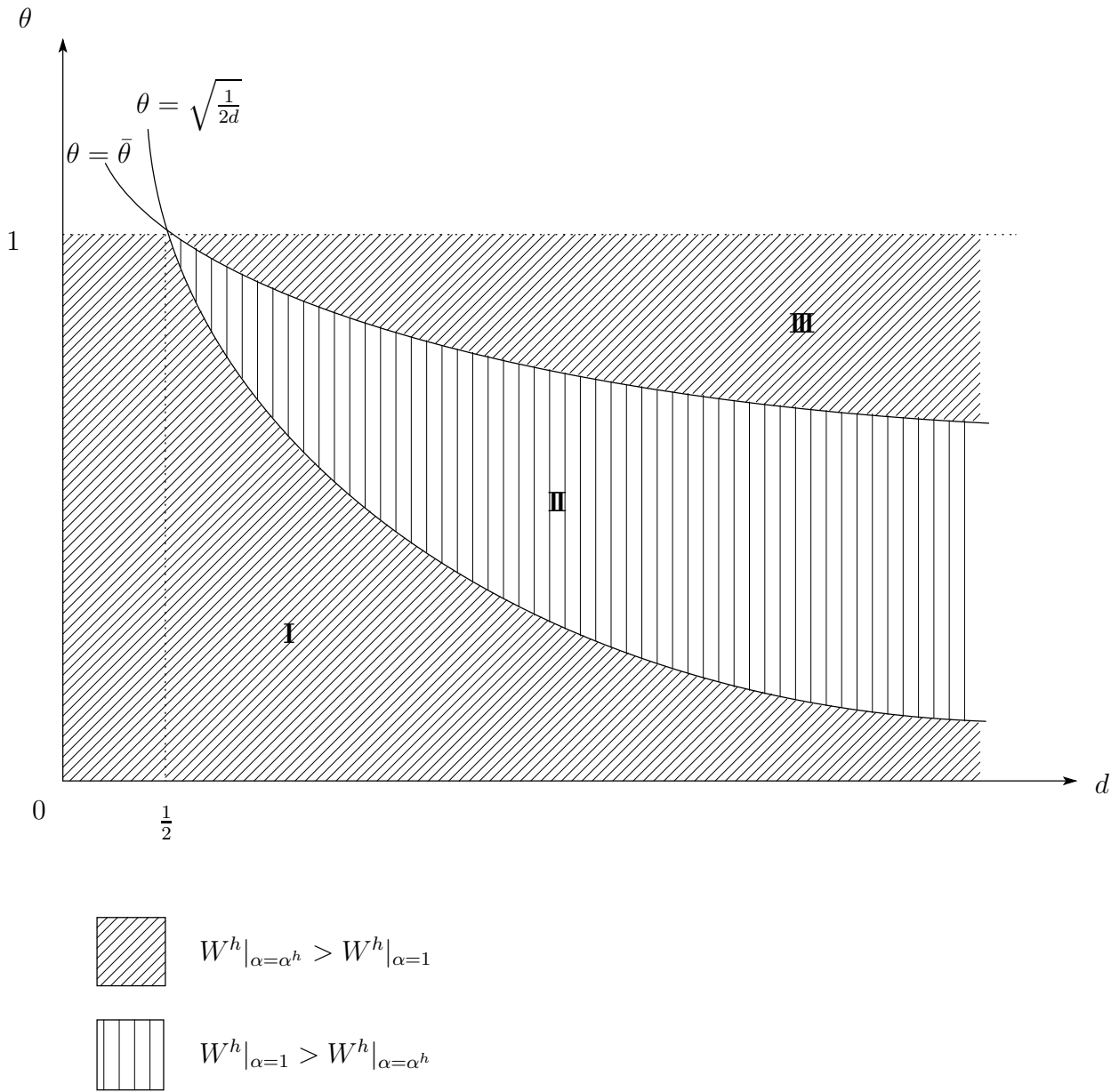


Figure 2.3: Comparison between $W^h|_{\alpha=1}$ and $W^h|_{\alpha=\alpha^h}$ for each value of parameter of d and θ .

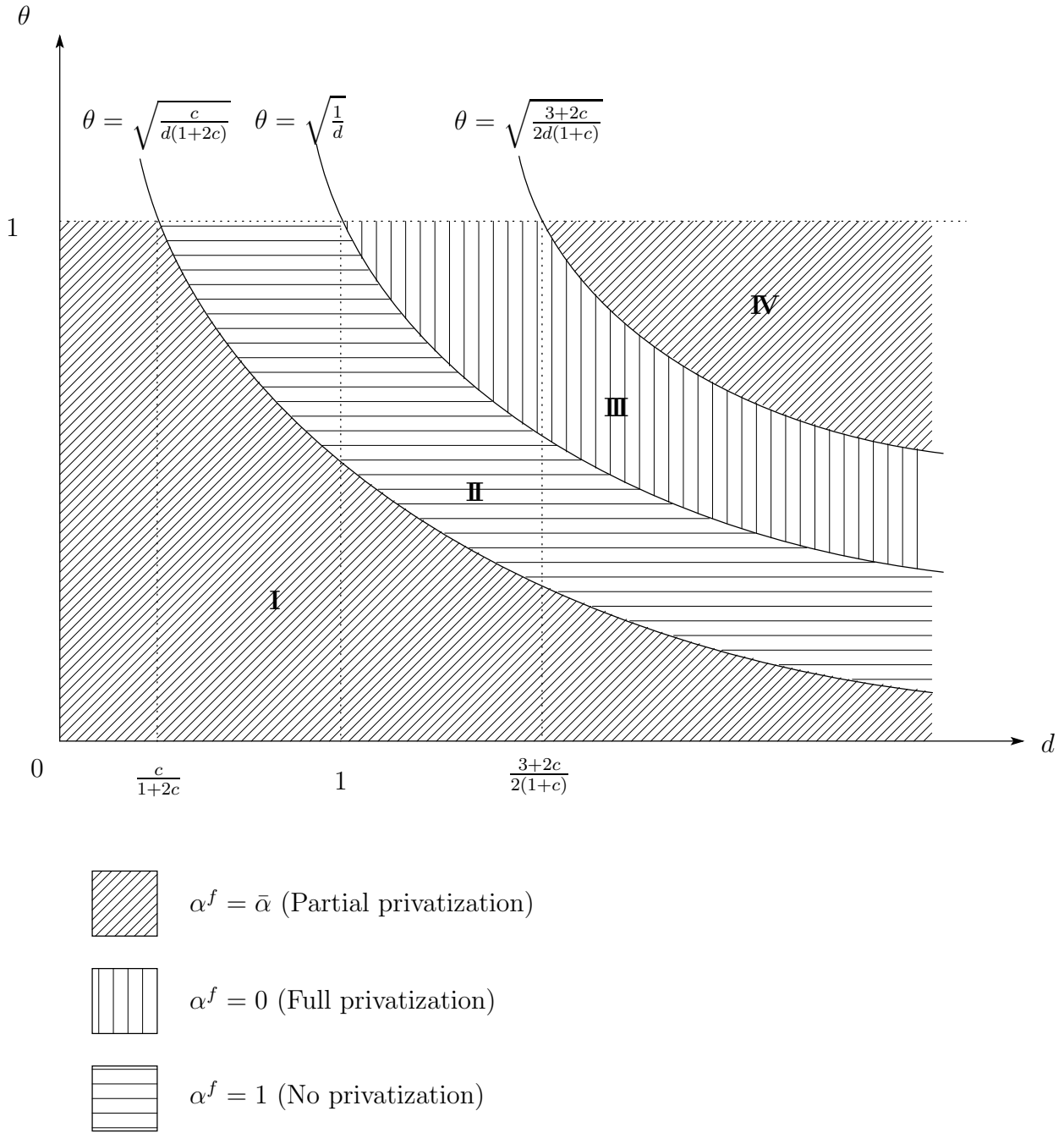


Figure 2.4: α^f for each value of parameter of d and θ .

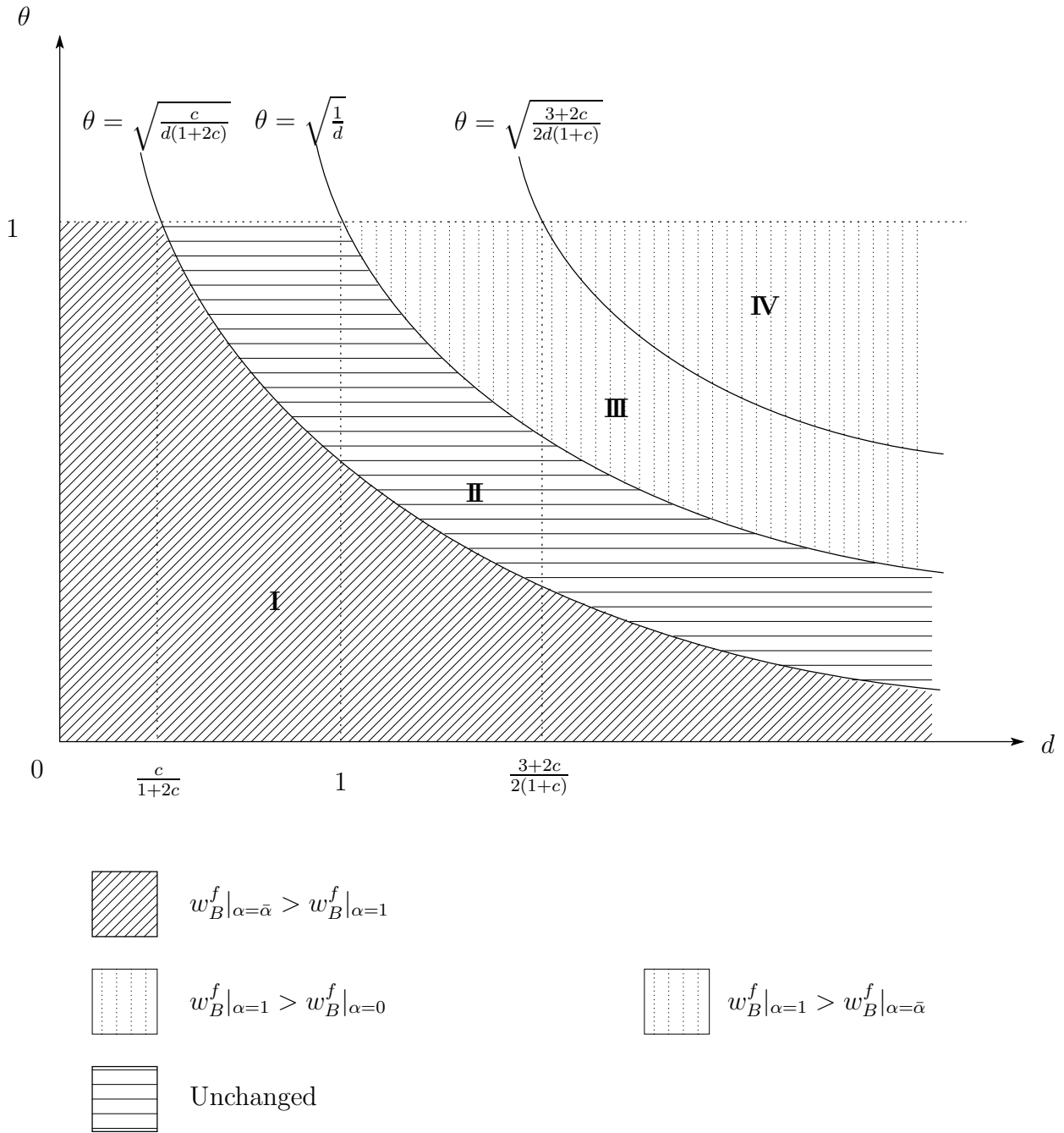


Figure 2.5: Comparisons between $w_B^f|_{\alpha=1}$, $w_B^f|_{\alpha=\alpha^h}$, and $w_B^f|_{\alpha=0}$ for each value of parameter of d and θ .

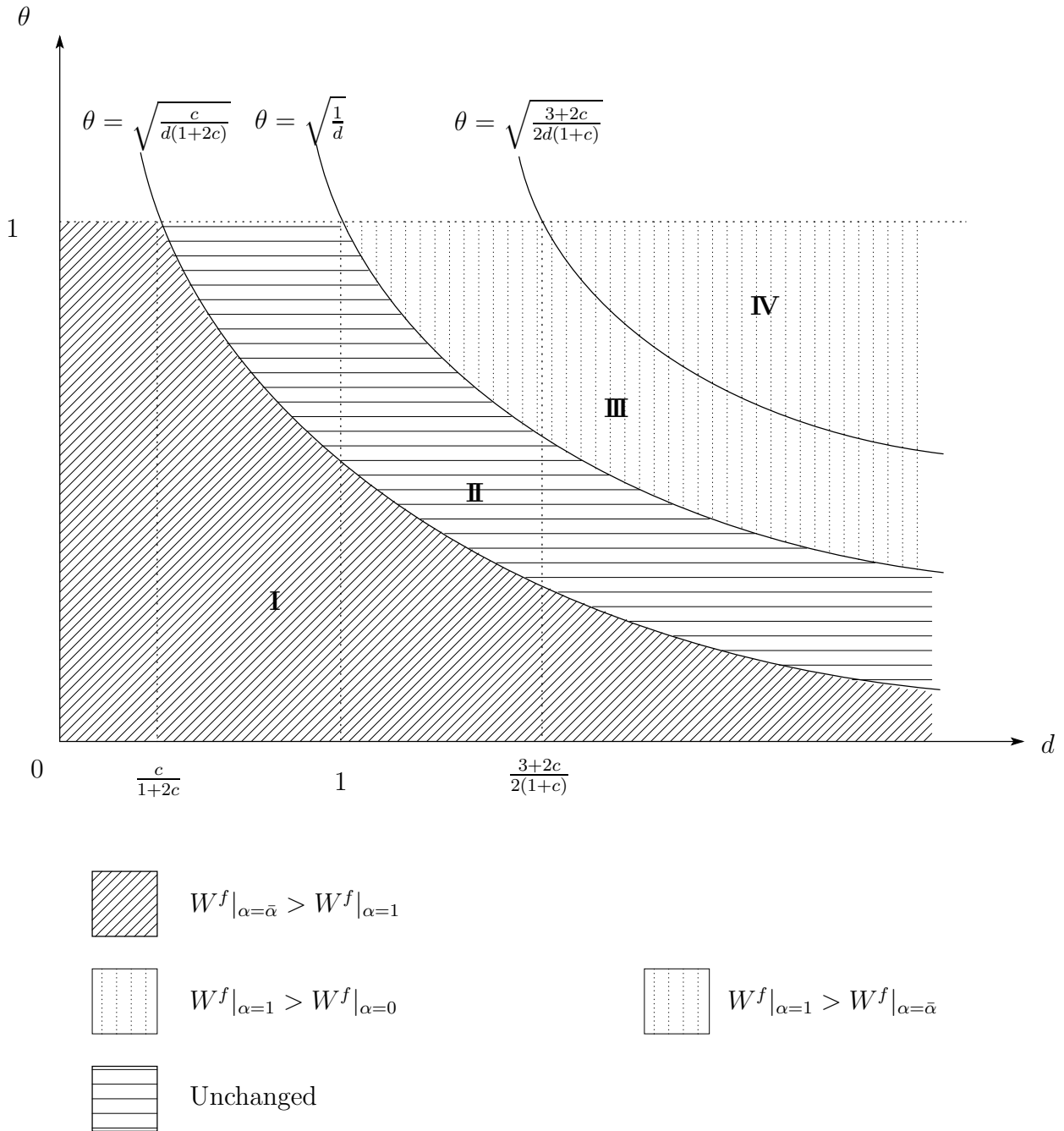


Figure 2.6: Comparisons between $W^f|_{\alpha=1}$, $W^f|_{\alpha=\alpha^h}$, and $W^f|_{\alpha=0}$ for each value of parameter of d and θ .

Chapter 3

Can allowing to trade permits enhance welfare in mixed oligopoly?¹

3.1 Introduction

In 1997, COP3 adopted the Kyoto Protocol which contained tradable emission permits (TEP) as a method of controlling global warming and went into force on the 16th of February in 2005.² Prior to this, the United States brought TEP into operation (*e.g.* see Hahn (1989)).³ TEP has been implemented experimentally in Britain since 2002 and the European Union introduced it on January 1, 2005. Many countries may follow suit to adopt TEP as one of the main environmental regulations in the future. This paper examines the effects of TEP in a mixed market, where public firms and private firms compete.⁴

Examples of mixed markets are numerous in those countries that have ratified the Ky-

¹This chapter is based on Kato (2006) in *Journal of Economics*.

²The 3rd Session of the Conference of the Parties to the United Nations Framework Convention on Climate Change.

³This case dealt not with the global warming problem but with other detrimental externality problems, such as water pollution and air pollution.

⁴A public firm is a firm that is wholly run by the public owner, or jointly owned by private and public owners.

oto Protocol. Norway's Statoil run by a public owner, and France's Renault jointly run by private owners and a public owner, compete with private firms in the same market. Furthermore, although some countries have not participated in the Kyoto Protocol yet, there is a possibility that they may use TEP for the self-restriction on their emissions or participate in the Kyoto Protocol. In China, for instance, where a large amount of greenhouse gases is discharged, mixed markets are fairly common, such as in energy industries and energy-intensive industries. If China takes its environmental problems in the future more seriously and views TEP as a successful international experiment, it may adopt TEP or participate in the Kyoto Protocol. These cases illustrate why it is worthwhile to examine the effects of TEP in a mixed market.

There are two representative cases showing how economists model the objectives of public firms. The first case is that a public firm's objective is pure social welfare maximization (De Fraja and Delbono (1989); in recent years, White (1996), Fjell and Pal (1996), Mujumdar and Pal (1998), Pal (1998)). The second case is an extension of the first: the public firm is only a partial social welfare-maximizer (Bös (1991); in recent years, Matsumura (1998), Bárcena-Ruiz and Garzón (2006), Matsumura and Kanda (2005)). In this paper we use the framework of the second. This framework is more general than the first since the second includes the first as a special case.

This paper employs a model composed of \bar{N} firms: n_0 public firms and n_1 private firms. Each public firm maximizes the weighted sum of social welfare and its own profit and each private firm maximizes its own profit. They produce output and discharge emissions. The government has a responsibility to regulate emissions, whose level is binding and

exogenously given. In order to control emissions, the government can only choose whether to allow the firms to trade emission permits or not.

We suppose that all technologies and initial endowments of emission permits are symmetric among public firms and private firms. If the emission constraint is binding, (i) under TEP, public (*resp.* private) firms are buyers (*resp.* sellers) of emission permits, (ii) when the degree of convexity of each firm's production cost function and that of each firm's abatement cost function are small and the weight of social welfare in each public firm's objective function is small (*resp.* large), social welfare is greater (*resp.* smaller) under TEP than under non-tradable emission permits (NTEP), which does not allow firms to trade emission permits, (iii) the larger the degree of convexity of each firm's production cost function or that of each firm's abatement cost function is, the narrower the range of parameters in which TEP is superior to NTEP in terms of social welfare is. If the degree of convexity of either the production cost or abatement cost functions is sufficiently large, TEP is inferior to NTEP in terms of social welfare regardless of how social welfare is weighted in public firms' objective functions.

Intuition behind result (i) is as follows. When each public firm is a pure profit-maximizer, the social welfare level under TEP is the same as that under NTEP. When each public firm takes social welfare into account in addition to its own profit, TEP no longer entails the same welfare level as NTEP. Each public firm produces more output than each private firm under both regulations, because the output is underproduced in the oligopoly market. However, the more output each public firm produces, the more emissions it discharges. Each public firm has the responsibility of making more abatement effort un-

der TEP than under NTEP. Both the marginal abatement cost and the shadow price of the emission constraint of each public firm are larger than those of each private firm under NTEP. Therefore, under TEP, public firms buy the emission permits from private firms.

Intuition behind the results (ii) and (iii) is as follows. Compared with NTEP, TEP has two positive effects and one negative effect on social welfare. One positive effect is that the total abatement cost is socially minimized at given emission levels under TEP. The other positive effect is that the total output is larger under TEP than under NTEP. Under NTEP the abatement cost of each public firm is larger than that of each private firm. Under TEP, however, they are equalized. Thereby each public firm has an incentive to produce more under TEP than under NTEP. The negative effect is that TEP causes the inefficient reallocation of production among public firms and private firms; under NTEP each public firm produces more than each private firm, and under TEP the difference in the output among them is enlarged. Thus, whether TEP is superior to NTEP or not in terms of social welfare depends on the relative size of the positive and negative effects. To examine the superiority of TEP or NTEP, we investigate the relationship among the demand and the cost structures.

Our investigation is based on some earlier works on TEP and mixed markets. Studies on the effects of TEP in markets that consist only of private firms include Malueg (1990) and Sartzetakis (1997, 2004).⁵ They compare TEP with NTEP and show that the superiority of TEP or NTEP in terms of social welfare is determined by the difference in the

⁵With respect to the case where firms have a market power in the emission permits market, see Hahn (1984). As for the case where firms have a power in both the product market and emission permits market, see von der Fehr (1993).

technologies or the initial endowments of emission permits between regulated firms.⁶ Without these differences, TEP is equivalent to NTEP in terms of social welfare. In a mixed market, however, even without such differences, the social welfare levels between these two regulations differ. This is caused by the difference in objectives between the public firm and the private firm. Usually, TEP is considered to work when there are differences with respect to the abatement technologies among firms. Even if those differences do not exist, however, firms have an incentive to trade emission permits and such trading brings benefits to this industry when the cost function is non-linear. When the output levels among the public and private firms are different, the emissions among them are also different. Therefore, the marginal abatement costs among them are different. This asymmetry is caused by the difference in the objectives among them even if the technologies among them are the same. Focusing on the efficiency of the abatement cost at given emission level, we find that TEP creates benefits because it causes the equalization of the marginal abatement costs among firms.

This paper is organized as follows. The next section describes our model. Section 3.3 derives the characteristics of the equilibrium under NTEP and section 3.4 derives those under TEP. Section 3.5 considers the comparison of NTEP and TEP and highlights the main results of this paper. Section 3.6 concludes the main text.

⁶Hung and Sartzetakis (1998) show that TEP is inferior to NTEP in terms of social welfare if the government can learn the technologies of firms and decide the initial endowments of emission permits.

3.2 Model

Consider a representative industry consisting of \bar{N} firms, n_0 public firms and n_1 private firms, all of which have the same technologies and the same initial endowments of emission permits, but different objective functions. We assume that $\bar{N} = n_0 + n_1$, $n_0, n_1 \geq 1$. They produce a homogeneous good and simultaneously compete with quantity in the product market. The inverse demand function is given by $p = A - Q$, where p is the price of the good, Q is the total output and $Q = \sum_{i=1}^{n_0} q_{0i} + \sum_{j=1}^{n_1} q_{1j}$, where q_{0i} denotes the output of public firm $0i$ ($i = 1, \dots, n_0$) and q_{1j} denotes that of private firm $1j$ ($j = 1, \dots, n_1$). We assume that A is positive and sufficiently large. The production cost function is given by $cq_l^2/2$ ($l = 01, \dots, 0n_0, 11, \dots, 1n_1$), where $c > 0$. Emissions are discharged through production. Each firm can reduce emissions either by increasing abatement efforts or by reducing output. An emission function of firm l is $E(q_l, a_l) = \rho q_l - a_l$, where $\rho > 0$ and a_l is the abatement effort of firm l . The abatement cost function is given by $ka_l^2/2$, where $k > 0$.⁷ We assume that this industry is given the total amount of emission permits \bar{E} and each firm is equally endowed with \bar{E}/\bar{N} exogenously, and this is strictly smaller than the unregulated level.^{8,9} The emission constraints are always binding for them, and thus, they

⁷We assume that the emission function and the cost function are separable with respect to the output and the abatement effort. As the example of CO_2 emissions shows, a firm can reduce emissions, even more than it discharges by its own production, by acquiring emission permits by planting trees.

⁸A possible reason for dividing \bar{E} equally for each firm is due to the same technologies of the emission and abatement among firms.

⁹In the equilibrium under no regulation, the abatement effort of each firm whose objective is to maximize its own profit is zero.

have a responsibility to abate emissions. The profit of firm l is given by¹⁰

$$\pi_l = (A - Q) q_l - \frac{c}{2} q_l^2 - \frac{k}{2} a_l^2. \quad (3.1)$$

Social welfare is the sum of the consumer's surplus, producer's surplus, and environmental damage. It is given by

$$SW = \int_0^Q (A - s) ds - \sum \frac{c}{2} q_l^2 - \sum \frac{k}{2} a_l^2 - D(\bar{E}). \quad (3.2)$$

The damage function is $D(E)$ with $D'(E) > 0$. E represents the aggregate emission level of this economy. As mentioned before, the emission constraint \bar{E} is binding in this industry. The environmental damage is a fixed value, $D(\bar{E})$.

Next we define the objective function of each firm. The objective function of public firm U_{0i} and that of private firm U_{1j} are given by

$$U_{0i} = \theta SW + (1 - \theta) \pi_{0i}, \quad \theta \in [0, 1], \quad (3.3)$$

$$U_{1j} = \pi_{1j}. \quad (3.4)$$

When $\theta = 0$, public firm $0i$ is a pure profit-maximizer, and it is a pure social welfare-maximizer when $\theta = 1$. θ is understood as the share holding of the public sector and $1 - \theta$ is that of the private sector.¹¹

The government must regulate emissions to protect the environment because of international agreements or the results of consultation with various interest groups. There are

¹⁰Under TEP, the profit of firm l is π_l plus the revenue or expenditure of trading emission permits. To simplify the explanation of each firm's objective function, we call π_l the profit of firm l .

¹¹For a rationalization of this objective function, see Bös (1991). θ can be considered to be dependent on the share holdings of the government and the number of the executives who come from government agencies because their magnitude could influence the public firm's objective.

two regulations that the government can choose. One is non-tradable emission permits (NTEP), under which firms cannot trade emission permits. The other is tradable emission permits (TEP), under which firms can trade emission permits. The government imposes one of these regulations on firms at the outset. \bar{E} is the same under two regulations.

Under TEP we assume that all firms are price takers in the emission permits market. Because of this setting, some readers may think that it is natural for the price of an emission permit, p^e , to be exogenous since all players are price takers in the emission permits market. In this paper, however, we consider p^e to be endogenous in order to clarify the effects of the public firm's objective on p^e .¹² Suppose that there are infinitely many markets which are identical to this industry in an economy. In this case, each product market is a mixed oligopoly. On the other hand, in the emission permits market, there are many groupings which consist of n_0 public firms and n_1 private firms from all industries. In other words, we consider a representative market with n_0 representative public firms and n_1 representative private firms. In addition, we consider the following situation; the government decides the degree of privatization or nationalization of public firms across nation by, for example, a 5-year plan. If we regard five years as one period, our model is consistent. Thus, we assume that the shift of the proportion of public firms or private firms occurs not only in a single market but also in the rest of all industries.

As to why we set up our model with no market power in the emission permits market, we aim to the effect of the market power in the product market. If there is market power in an emission permits market in addition to market power in the product market, it is

¹²Suppose p^e is exogenous. Then, the welfare comparison of TEP and NTEP is complex because the total emission of the industry becomes endogenous under TEP. Therefore, we have to classify the cases by the magnitude of p^e and the effect of the environmental damage. We leave this case to future research.

difficult to identify what effects cause the difference in social welfare between TEP and NTEP and to what extent each effect influences on the difference.

The timing of decision makings of the government and firms is as follows. First, the government chooses one of the environmental regulations, NTEP or TEP. Then, all firms simultaneously compete with quantity, deciding q_l and a_l . In the following, we confine our analyses to symmetric equilibria where all public firms choose the same output and abatement effort level, and all private firms do the same.

3.3 Non-tradable emission permits

In this section we derive the equilibrium of NTEP. NTEP prohibits each firm from trading emission permits. Each firm can discharge emissions as long as it obeys its own emission constraint. Maximization problems of public firm $0i$ and private firm $1j$ are given by

$$\max_{q_{0i}, a_{0i}} U_{0i}^{nt} = \max_{q_{0i}, a_{0i}} \{\theta SW + (1 - \theta)\pi_{0i}\} \quad \text{s.t.} \quad \frac{\bar{E}}{N} = \rho q_{0i} - a_{0i}, \quad (3.5)$$

$$\max_{q_{1j}, a_{1j}} U_{1j}^{nt} = \max_{q_{1j}, a_{1j}} \{\pi_{1j}\} \quad \text{s.t.} \quad \frac{\bar{E}}{N} = \rho q_{1j} - a_{1j}. \quad (3.6)$$

U_l^{nt} represents the objective function of firm l under NTEP. Let λ_l be the shadow price of the emission constraint of firm l . Solving the maximization problem of each firm, we derive the following equilibrium outcomes under NTEP.

$$\begin{aligned} q_{0i}^{nt} &= \frac{c + \hat{k} + 1}{X} M, & q_{1j}^{nt} &= \frac{c + \hat{k} + 1 - \theta}{X} M, \\ a_{0i}^{nt} &= \frac{c + \hat{k} + 1}{X} \rho M - \frac{\bar{E}}{N}, & a_{1j}^{nt} &= \frac{c + \hat{k} + 1 - \theta}{X} \rho M - \frac{\bar{E}}{N}, \\ \lambda_{0i}^{nt} &= \frac{c + \hat{k} + 1}{X} k \rho M - \frac{k \bar{E}}{N}, & \lambda_{1j}^{nt} &= \frac{c + \hat{k} + 1 - \theta}{X} k \rho M - \frac{k \bar{E}}{N}, \end{aligned}$$

where $\hat{k} = k\rho^2$, $M = A + k\rho\bar{E}/\bar{N}$, $X = (c + \hat{k} + 1 + n_1)(c + \hat{k} + 1 + \bar{N} - n_1 - \theta) - (\bar{N} - n_1)n_1$.

Then, social welfare under NTEP is

$$SW^{nt} = \frac{n_1(c + \hat{k} + 2 + n_1)\theta^2 - 2(c + \hat{k} + 1)\{\bar{N}(c + \hat{k} + 1 + n_1) + n_1\}\theta}{2X^2}M^2 + \frac{\bar{N}(c + \hat{k} + 1)^2(c + \hat{k} + 2 + \bar{N})}{2X^2}M^2 - \frac{k\bar{E}^2}{2\bar{N}} - D(\bar{E}). \quad (3.7)$$

From the results, we can easily check $q_{0i}^{nt} > q_{1j}^{nt}$ and $a_{0i}^{nt} > a_{1j}^{nt}$ and $\rho q_{0i} - a_{0i}^{nt} = \rho q_{1j} - a_{1j}^{nt} = \bar{E}/\bar{N}$. The equilibrium output of public firm $0i$ is larger than that of private firm $1j$ because public firm $0i$'s objective includes consumer's surplus partially. To satisfy the emission constraint for each firm, the equilibrium abatement effort of public firm $0i$ is larger than that of private firm $1j$. From the above, we find that the shadow price of the emission constraint of public firm $0i$ is larger than that of private firm $1j$.

3.4 Tradable emission permits

Public firm $0i$'s and private firm $1j$'s maximization problems are given by

$$\max_{q_{0i}, a_{0i}} U_{0i}^t = \max_{q_{0i}, a_{0i}} \left\{ \theta SW + (1 - \theta)\pi_{0i} + p^e \left(\frac{\bar{E}}{\bar{N}} - \rho q_{0i} + a_{0i} \right) \right\}, \quad (3.8)$$

$$\max_{q_{1j}, a_{1j}} U_{1j}^t = \max_{q_{1j}, a_{1j}} \left\{ \pi_{1j} + p^e \left(\frac{\bar{E}}{\bar{N}} - \rho q_{1j} + a_{1j} \right) \right\}. \quad (3.9)$$

U_l^t represents the objective function of firm l under TEP. As they are price takers in the emission permit market, they trade emission permits, taking the permit price p^e as given.

The market clearing condition is

$$\bar{E} = \sum (\rho q_l - a_l). \quad (3.10)$$

We find that the equilibrium outcomes under TEP are

$$\begin{aligned} q_{0i}^t &= \frac{\bar{N}(c+1)}{Y}M, & q_{1j}^t &= \frac{\bar{N}(c+1-\theta)}{Y}M, \\ a_{0i}^t = a_{1j}^t &= \frac{\bar{N}(c+1)-n_1\theta}{Y}\rho M - \frac{\bar{E}}{\bar{N}}, & p^e &= \frac{\bar{N}(c+1)-n_1\theta}{Y}k\rho M - \frac{k\bar{E}}{\bar{N}}, \end{aligned}$$

where $Y = \hat{k}\{\bar{N}(c+1) - n_1\theta\} + \bar{N}\{(c+1 + \bar{N})(c+1) - (c+1 + n_1)\theta\}$. Then, social welfare under TEP is

$$SW^t = \frac{\{n_1\hat{k} + \bar{N}(c+2 + n_1)\}n_1\theta^2 - 2(c+1)\{\bar{N}(c+1 + n_1) + n_1(\hat{k} + 1)\}\bar{N}\theta}{2Y^2} \bar{N}M^2 + \frac{(c+1)^2(c + \hat{k} + 2 + \bar{N})}{2Y^2} \bar{N}^3M^2 - \frac{k\bar{E}^2}{2\bar{N}} - D(\bar{E}). \quad (3.11)$$

The equilibrium output of public firm $0i$ is larger than that of private firm $1j$ for the same reasons as in section 3.3. With respect to the equilibrium abatement effort, we can find the difference between NTEP and TEP. By trading emission permits, firms can abate emissions among them. Trading emission permits continues until each firm's marginal abatement effort level is equalized. In this paper, as the technologies of all firms are the same, therefore, the abatement effort level is the same among them. Note that the price of the emission permit is equal to the marginal abatement effort of each firm and it is not influenced by the initial allocation of each firm's emission permits.¹³

3.5 Comparison of NTEP and TEP

In this section we examine how the differences between the two environmental regulations influence the equilibrium outcomes and the social welfare levels. First, we find the following proposition.

Proposition 3.1. *(i) When $\theta = 0$, the NTEP equilibrium outcomes are the same level as the TEP ones. (ii) When $1 \geq \theta > 0$, public firms are buyers and private firms sellers of emission permits. (iii) The following relationships summarize the comparison of NTEP to*

¹³See Appendix 3.A.

TEP equilibrium outcomes:

$$\begin{aligned}
1. \quad & q_{0i}^t \geq q_{0i}^{nt}, & q_{1j}^t & \leq q_{1j}^{nt}, \\
2. \quad & a_{0i}^t \leq a_{0i}^{nt}, & a_{1j}^t & \geq a_{1j}^{nt}, \\
3. \quad & Q^t \geq Q^{nt}, & \sum a_i^t & \geq \sum a_i^{nt},
\end{aligned}$$

where strict inequalities hold for all $\theta \in (0, 1]$.

Proof. (i) Setting $\theta = 0$ in equations in sections 3.3 and 3.4 yields $q_i^t = q_i^{nt}$, $a_i^t = a_i^{nt}$. (ii)

From their respective equilibrium values we find $\lambda_{0i}^{nt} \geq p^e \geq \lambda_{1j}^{nt}$ and thus, each private firm sells permits to public firms since the permit price exceeds the marginal abatement cost of each private firm. (iii) A simple comparison of the equilibrium values of the choice variables in section 3.3 to those in section 3.4 yields the results in Proposition 3.1. \square

When $\theta = 0$, firms do not have incentives to trade emission permits under TEP since $\lambda_{0i}^{nt} = p^e = \lambda_{1j}^{nt}$. Therefore, the equilibrium outcomes under NTEP are the same level as those under TEP. When $\theta \in (0, 1]$, public firms (*resp.* private firms) are buyers (*resp.* sellers) of emission permits since $\lambda_{0i}^{nt} > p^e > \lambda_{1j}^{nt}$.

An intuition behind Proposition 3.1 (iii) is as follows. The abatement effort of public firm $0i$ (*resp.* private firm $1j$) is smaller (*resp.* larger) under TEP than under NTEP because of trading emission permits. Since the emission constraint under TEP is more relaxed than under NTEP, public firm $0i$ can increase its own output more under TEP than under NTEP. On the contrary, the output of private firm $1j$ decreases more under TEP than under NTEP. The increase in public firms' output is more than the decrease in private firms' output. As the total output is larger under TEP than NTEP, more emissions are discharged under TEP than NTEP. To satisfy the total emission constraint, therefore, the total abatement effort is greater under TEP than under NTEP.

Second, we examine how the shift in the proportion among private firms and public firms affects the equilibrium outcomes under NTEP and under TEP.

Proposition 3.2. *If n_1 increases with \bar{N} fixed,*

$$\begin{array}{ll} \text{NTEP: } & q_{0i}^{nt}, q_{1j}^{nt}, a_{0i}^{nt}, a_{1j}^{nt}, \lambda_{0i}^{nt}, \text{ and } \lambda_{1j}^{nt} \text{ increase,} & Q^{nt} \text{ and } \sum a_i^{nt} \text{ decrease.} \\ \text{TEP: } & q_{0i}^t, \text{ and } q_{1j}^t \text{ increase,} & Q^t, \sum a_i^t, a_{0i}^t, a_{1j}^t, \text{ and } p^e \text{ decrease.} \end{array}$$

Proof. Simple differentiation of the equilibrium values of the choice variables in sections 3.3 and 3.4 yields the results in Proposition 3.2. □

Under both regulations, when the number of private firms increases with the total number of firms fixed, each public firm's output increases and each private firm's output increases. This is because each private firm has an incentive to produce less output than each public firm and the strategic substitution effect works. By changing a public firm into a private firm, its firm's output decreases. A decrease in its output is larger than an increase in other firms' output, and then the total output decreases. Under NTEP, an increase in the output of each firm leads to an increase of the emission. Therefore, the abatement effort of each firm increases to satisfy the emission constraint and its shadow price of emission constraint increases. Under TEP, however, the abatement effort and the price of the emission permit decrease because the total output, and therefore total emission decreases more than before.

Third, we examine how an increase in the weight of social welfare in public firms' objectives affects the equilibrium outcomes under NTEP and under TEP.

Proposition 3.3. *If θ increases,*

$$\begin{array}{ll} \text{NTEP:} & q_{0i}^{nt}, a_{0i}^{nt}, Q^{nt}, \sum a_l^{nt}, \text{ and } \lambda_{0i}^{nt} \text{ increase,} & q_{1j}^{nt}, a_{1j}^{nt}, \text{ and } \lambda_{1j}^{nt} \text{ decrease,} \\ \text{TEP:} & q_{0i}^t, a_{0i}^t, a_{1j}^t, Q^t, \sum a_l^t, \text{ and } p^e \text{ increase,} & q_{1j}^t \text{ decreases.} \end{array}$$

Proof. Simple differentiation of the equilibrium values of the choice variables in sections 3.3 and 3.4 yields the results in Proposition 3.3. \square

As an increase in θ means that each public firm gives more weight to social welfare, it increases its output more than before. This makes each private firm's output decrease because of a strategic substitution effect. This is the same under NTEP and TEP. With respect to the abatement efforts, however, a difference between NTEP and TEP arises. Under NTEP, as each firm must obey each emission constraint, public firm $0i$ makes more abatement effort and private firm $1j$ makes less abatement effort than before. And then, shadow price of emission constraint for public firm $0i$ increases and that for private firm $1j$ decreases. On the contrary, under TEP, marginal abatement costs are equal among firms because they are price takers in the emission permit market. The larger the total output is, the larger the total emission is. The price of the emission permit increases to satisfy the total emission constraint \bar{E} . Thus an increase in θ causes an increase in their abatement efforts and the price of an emission permit.

Fourth, we compare social welfare under NTEP with that under TEP. It is difficult to calculate the condition analytically. Therefore, we confine the analysis in the case $c \geq 1$ and we compare the two social welfare levels.¹⁴ We find the following lemma:

¹⁴See more detail for the case of $c < 1$ in Appendix 3.B. $c \geq 1$ does not imply any special meaning. The reason for using the case of $c \geq 1$ is to enable us to compare social welfare under NTEP and that under TEP.

Lemma 3.1. Define $\bar{\theta} = \frac{\beta - \sqrt{\beta^2 + \alpha\gamma}}{\alpha}$. This is the only value of $\theta \in (0, 1]$ at which the social welfare under TEP equals that under NTEP.

Proof. See Appendices B and C where the values of α, β and γ are defined. □

From Lemma 3.1, we can establish the following proposition.

Proposition 3.4.

1. When $0 \leq \hat{k} \leq \phi(c, \bar{N})$, $SW^t \geq SW^{nt}$ if $0 \leq \theta \leq \bar{\theta}$,
 $SW^t < SW^{nt}$ if $\bar{\theta} < \theta \leq 1$.
2. When $\hat{k} > \phi(c, \bar{N})$, $SW^t \leq SW^{nt}$ for $\theta \in [0, 1]$,

where $\phi(c, \bar{N}) = [-\{2c^2 + (\bar{N} - 1)c - 3\} + \sqrt{9c^2 + \{\bar{N}(\bar{N}c - 2c - 2) + 18\}c + 9}]/2c$ and $\partial\phi(c, \bar{N})/\partial c < 0$.

An intuition behind Proposition 3.4 is as follows. TEP has two positive effects and one negative effect in comparison to NTEP. One of the positive effects is to increase the total output, which correspondingly increases consumer's surplus. The other positive effect is to minimize the social abatement cost given the emission level fixed. The negative effect is the inefficient reallocation of production. Thus, the superiority of TEP or NTEP in terms of social welfare depends on the relative size of the positive effects and the negative effect.

First, we consider the case where \hat{k} is small and c is nearly equal to 1. By trading emission permits, for a small θ , the increase in consumer's surplus is larger than the decrease in the producer's surplus because the output each public firm produces is slightly greater than the output each private firm produces. Therefore, the inefficient reallocation of production is small. In this case, the positive effects overcome the negative effect. However, for a large θ , the negative effect exceeds the positive effects. The output each

public firm produces is much greater than the output each private firm produces and the inefficient reallocation of production is high enough to outweigh the positive effects.

Second, we consider the case where either the degree of convexity of the production cost function or that of the abatement cost function is large. In this case, the inefficient reallocation of production is high, and then the range of the weight that TEP is superior to NTEP in terms of social welfare is narrow or even vanishes.

Note that from Proposition 3.4, when $\theta = 1$, that is, public firms are pure social welfare maximizers, social welfare is always smaller under TEP than under NTEP in any value of $c (\geq 1)$ and \hat{k} . In this case, we find that public firm $0i$'s incentive to produce more output than private firm $1j$ creates the enormous loss to social welfare by trading emission permits.

In fact, $c \geq 1$ is a sufficient condition for the social welfare function under each regulation to be strictly concave in θ . When $c \geq 1$, if the government can choose the value of θ which maximizes social welfare before the quantity competition under each regulation, we can calculate the optimal θ for each regulation, which we call θ^{*nt} for NTEP and θ^{*t} for TEP respectively and find that $\theta^{*nt} > \theta^{*t}$, where $\theta^{*nt} = (c + \hat{k} + 1)^2 / \{(c + \hat{k} + 1)^2 + (c + \hat{k})(\bar{N} - n_1)\}$ and $\theta^{*t} = \bar{N}(c + 1)^2 / \{\bar{N}(c + 1)^2 + c(\hat{k} + \bar{N})(\bar{N} - n_1)\}$. From the results, we can see that the enlargement of the inefficient reallocation of production by trading emission permits significantly does harm on social welfare.

Figure 3.1, Figure 3.2, and Table 3.1 illustrate the result of the simulation of $\bar{\theta}$ in c and $\hat{k} \in [0, 2]$ and $n_0 = n_1 = 1$.¹⁵ Figure 3.1 illustrates the relationships among c , \hat{k} , and $\bar{\theta}$

¹⁵In Table 3.1, we illustrate $\bar{\theta}$ for all ranges to enhance comprehension of Proposition 3.4 well. In Figure 3.1 and Figure 3.2, we confine the value of $\bar{\theta}$ to be $[0, 1]$.

from overall and Figure 3.2 represents them in contour. In Figure 2, in the range that c and $\hat{k} \in [0, 2]$, the number on each contour line shows the value of $\bar{\theta}$. The clear space to the left hand side of the contour line of “1” represents the pairs where social welfare is always greater under TEP than under NTEP for any θ . The clear space to the right hand side of the contour line of “0” represents the pairs where social welfare is always smaller under TEP than under NTEP for any θ . By looking at them, particularly from Figure 3.2, we can easily find that social welfare under TEP is always greater than that under NTEP when the degree of convexity of the production cost function and that of the abatement cost function is small enough. The above result is reversed, however, when they are large.

Finally, we analyze how the shift in the proportion among private firms and public firms affects $\bar{\theta}$.

Proposition 3.5. $\frac{\partial \bar{\theta}}{\partial n_1} < 0$.

Proof. Simple differentiation of the equilibrium values of choice variables yields the results in Proposition 3.5.¹⁶ □

When the number of private firms increases with the total number of firms fixed, an inefficient reallocation of production increases more than before. Therefore, the threshold $\bar{\theta}$ decreases with an increase in the number of private firms n_1 .

TEP can achieve the total cost minimization of the abatement cost at a given emission level, so it seems to be superior to NTEP in terms of social welfare. When there is a distortion in the product market, however, TEP may cause social welfare level to be lower than NTEP. This result has already been found in earlier works.¹⁷ The difference with this

¹⁶In Proposition 3.5, we only deal with the case where $\bar{\theta} \in (0, 1]$. In fact, if $\bar{\theta} < 0$, we find $\partial \bar{\theta} / \partial n_1 > 0$.

¹⁷See Malueg (1990) and Sartzetakis (2004).

paper is that attention is paid to the degree of difference in the objective functions among public firms and private firms. The demand structure and the cost structure determine whether TEP is superior to NTEP in terms of social welfare.

3.6 Concluding remarks

This paper examines the effects of a government's choice of environmental regulations in a mixed market. So far, either environmental regulation problems without a mixed market or the mixed market without environmental problems have been analyzed in earlier works. There are few papers that examine the environmental problem in a mixed market.¹⁸ We show under which conditions the government should choose TEP and NTEP when the situation for public and private firms is symmetric, with the exception of the differences in their objectives. TEP leads to the equalization of the marginal abatement costs between the firms and to an overall increase in the total output, but it also causes increased inefficiency in the reallocation of production among public and private firms. The magnitude of the weight that public firms put on social welfare and the demand and cost structures determine whether positive effects outweigh the negative effects or not.

We discuss the implication of the results in the paper. If mixed markets where public firms' objectives are to maximize social welfare are widely spread, it is desirable that the government does not allow firms to trade emission permits. For example, China has imposed the environmental regulation based on non-tradable emission permits instead of adopting tradable emission permits.¹⁹ It may be said that China has made correct choices

¹⁸Bárcena-Ruiz and Garzón (2006) examine the interaction with environmental tax and privatization in a mixed oligopoly.

¹⁹In China there is an emission permits system in which firms have to apply to the environmental

for an environmental regulation. However, if public firms are partially privatized and their objectives are not only social welfare maximization but also profit maximization, then there is a possibility that tradable emission permits is better than non-tradable emission permits type regulation in terms of social welfare. In this case, if firms consider the environment more than they used to because of public opinion, energy saving, or cost reducing and they invest in decreasing emission per output ρ and the coefficient of the abatement cost function k , the government may as well examine the introduction of tradable emission permits. The government needs to pay attention to these factors when it decides whether allowing firms to trade emission permits or not.

To simplify the analysis and to see the effect of public firms' objectives on the price of the emission permit, we assume that the price of the emission permit is endogenous and the total emission is exogenous. To focus on the market power of the product market, we also assume that under TEP firms behave as price takers in the emission permits market. We do not consider the cases where firms can buy or sell emission permits from other countries and where firms have market power in the emission permits market. In addition, we confine our analysis to quantity competition and do not consider price competition.²⁰ We wish to investigate these cases in future research.

protection administrative organization for the emission permit and follow all rules stated in the emission permit, which include the total emission restriction. In this system firms cannot trade the permits between firms.

²⁰Dastidar (1995, 1997) analyzes the case in which private firms compete in prices in a homogeneous product market with a convex cost function and shows the existence of multiple equilibria. There are few studies which analyze the same situation in a mixed oligopoly. In addition, there are also few papers which analyze price competition of the differentiated goods in a mixed oligopoly.

Appendix 3.A

The calculation of the price of the emission permit p^e The first order conditions of maximization of the objectives of each firm are as follows. Note that p^e is given in the calculation.

$$\frac{\partial U_{0i}}{\partial q_{0i}} = A - (1 - \theta + c)q_{0i} - \sum_{i=1}^{n_0} q_{0i} - \sum_{j=1}^{n_1} q_{1j} - \rho p^e = 0, \quad \text{for } i = 1, \dots, n_0, \quad (3.12)$$

$$\frac{\partial U_{0i}}{\partial a_{0i}} = -ka_{0i} + p^e = 0, \quad \text{for } i = 1, \dots, n_0, \quad (3.13)$$

$$\frac{\partial U_{1j}}{\partial q_{1j}} = A - \sum_{i=1}^{n_0} q_{0i} - \sum_{j=1}^{n_1} q_{1j} - (1 + c)q_{1j} - \rho p^e = 0, \quad \text{for } j = 1, \dots, n_1, \quad (3.14)$$

$$\frac{\partial U_{1j}}{\partial a_{1j}} = -ka_{1j} + p^e = 0, \quad \text{for } j = 1, \dots, n_1. \quad (3.15)$$

First, from solving the above $2(n_0 + n_1)$ equations, we obtain $q_{0i} = (c + 1)(A - \rho p^e)/Y'$, $q_{1j} = (c + 1 - \theta)(A - \rho p^e)/Y'$, and $a_{0i} = a_{1j} = p^e/k$ where $Y' = (c + 1 + \bar{N})(c + 1) - (c + 1 + n_1)\theta$. Next, we substitute them for q_{0i} , q_{1j} , a_{0i} , and a_{1j} in the market clearing condition (3.10), and solve for p^e , then we find the following value of p^e :

$$p^e = \frac{\rho\{\bar{N}(c + 1) - n_1\theta\}A - \{(c + 1 + \bar{N})(c + 1) - (c + 1 + n_1)\theta\}\bar{E}}{Y}k. \quad (3.16)$$

Note that the initial allocation of the emission permits among firms \bar{E}/\bar{N} does not appear in (3.10) and (3.12) - (3.15).

Then by adding

$$\frac{\{\bar{N}(c + 1) - n_1\theta\} - \{\bar{N}(c + 1) - n_1\theta\}k^2\rho^2\bar{E}}{Y} \frac{\bar{E}}{\bar{N}} (= 0) \quad (3.17)$$

to p^e and summing up, we can obtain the equilibrium value of p^e in section 3.4. Finally, we substitute it for each variable, and then we obtain the equilibrium outcome under TEP.

The reason for transforming (3.16) into p^e in section 3.4 is to compare the value of p^e with the values of λ_{0i}^{nt} and λ_{1j}^{nt} more easily.

Appendix 3.B

Welfare comparison with no limitations in the values of c and \hat{k} We define the values of α, β , and γ as follows: $\alpha = 2\hat{k}n_1 + c\hat{k}n_1 + \hat{k}^2n_1 + \hat{k}n_1^2 + 3\bar{N} + 4c\bar{N} + c^2\bar{N} + 2\hat{k}\bar{N} + c\hat{k}\bar{N} + 4n_1\bar{N} + 2cn_1\bar{N} + \hat{k}n_1\bar{N} + n_1^2\bar{N} > 0$, $\beta = \hat{k}n_1 + c\hat{k}n_1 + \hat{k}^2n_1 + 3\bar{N} + 6c\bar{N} + 3c^2\bar{N} + 4\hat{k}\bar{N} + 4c\hat{k}\bar{N} + \hat{k}^2\bar{N} + 2n_1\bar{N} + 2cn_1\bar{N} + 2\hat{k}n_1\bar{N} + 2\bar{N}^2 + 2c\bar{N}^2 + \hat{k}\bar{N}^2 + n_1\bar{N}^2 > 0$, $\gamma = \bar{N}(1 + c + \hat{k} + \bar{N})(c^3 + 2c^2\hat{k} + c\hat{k}^2 + \bar{N}c^2 - c^2 + \bar{N}c\hat{k} - c\hat{k} - 5c - 3\hat{k} - 3 - \bar{N})$, $\partial\bar{\theta}/\partial c < 0$, $\partial\bar{\theta}/\partial\hat{k} < 0$, $\beta/\alpha > 1$ and $\beta^2 + \alpha\gamma > 0$.

We conduct our analysis with no limitations on the parameters, c and \hat{k} . The conditions of a threshold θ , called $\hat{\theta} \in [0, 1]$, are $\alpha - 2\beta \leq \gamma \leq 0$ because $\hat{\theta} \geq 0$ implies $\gamma \leq 0$ and $\hat{\theta} \leq 1$ implies $\gamma \geq \alpha - 2\beta$. With respect to the relationship among the parameters' values and the magnitude relation of social welfare level under NTEP and TEP, we obtain the following conditions.

1. $SW^t \leq SW^{nt}$ for $\theta \in [0, 1]$ if and only if $\gamma > 0$,
2. $SW^t \geq SW^{nt}$ for $\theta \in [0, 1]$ if and only if $\gamma < \alpha - 2\beta$,
3. There exists $\hat{\theta} \in [0, 1]$ such that,

$$\begin{cases} SW^t \geq SW^{nt} & \text{for } \theta \in [0, \hat{\theta}], \\ SW^t < SW^{nt} & \text{for } \theta \in (\hat{\theta}, 1] \text{ if and only if } \alpha - 2\beta \leq \gamma \leq 0, \end{cases}$$

where a strict inequality holds for all $\theta \in (0, 1]$ except for $\hat{\theta}$. Section 3.5 examines the case $c \geq 1$. If $c \geq 1$, we find that $\gamma > \alpha - 2\beta$. Therefore, if $c \geq 1$, the case where TEP is superior to NTEP in terms of social welfare for $\theta \in (0, 1]$ does not exist. This appendix

considers the case, $c < 1$. In particular, we pick up the case where c is nearly equal to zero. In such a case, we find $\gamma < \alpha - 2\beta$. In this case the negative effect vanishes because $c = 0$ means both firms have the same constant marginal production cost. Therefore, when both c and \hat{k} are sufficiently small, TEP is always superior to NTEP in terms of social welfare regardless of θ .

Appendix 3.C

Proof of Lemma 3.1. The difference in social welfare levels between NTEP and TEP is

$$SW^t - SW^{nt} = \frac{\hat{k}(\bar{N} - n_1)n_1\theta^2 M^2}{2X^2Y^2}(\alpha\theta^2 - 2\beta\theta - \gamma).$$

θ that cause the social welfare under NTEP to be at the same level as that under TEP are

$$\theta = 0, \quad \frac{\beta + \sqrt{\beta^2 + \alpha\gamma}}{\alpha}, \quad \frac{\beta - \sqrt{\beta^2 + \alpha\gamma}}{\alpha}.$$

The second solution is over 1 because $\beta/\alpha > 1$. Thus only the third solution has the possibility to be in $(0, 1]$. □

$\mathbf{c} \backslash \hat{k}$	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0	1.25	1.24	1.24	1.23	1.23	1.23	1.23	1.23	1.23	1.23	1.23
0.2	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.03	1.01	1.0	0.98
0.4	1.10	1.06	1.02	0.99	0.96	0.92	0.89	0.86	0.83	0.80	0.77
0.6	1.01	0.96	0.91	0.87	0.83	0.78	0.74	0.70	0.66	0.62	0.58
0.8	0.91	0.85	0.80	0.74	0.69	0.64	0.59	0.54	0.49	0.44	0.40
1.0	0.80	0.74	0.68	0.62	0.56	0.50	0.44	0.38	0.33	0.27	0.22
1.2	0.68	0.62	0.55	0.48	0.42	0.35	0.29	0.23	0.16	0.10	0.04
1.4	0.56	0.49	0.42	0.34	0.27	0.20	0.13	0.07	0.00	-0.07	-0.13
1.6	0.44	0.36	0.28	0.20	0.13	0.05	-0.02	-0.09	-0.17	-0.24	-0.30
1.8	0.30	0.22	0.14	0.06	-0.02	-0.10	-0.18	-0.26	-0.33	-0.41	-0.48
2.0	0.17	0.08	-0.01	-0.09	-0.17	-0.26	-0.33	-0.42	-0.50	-0.58	-0.66

Table 3.1: The relationship among c , \hat{k} , and $\bar{\theta}$:

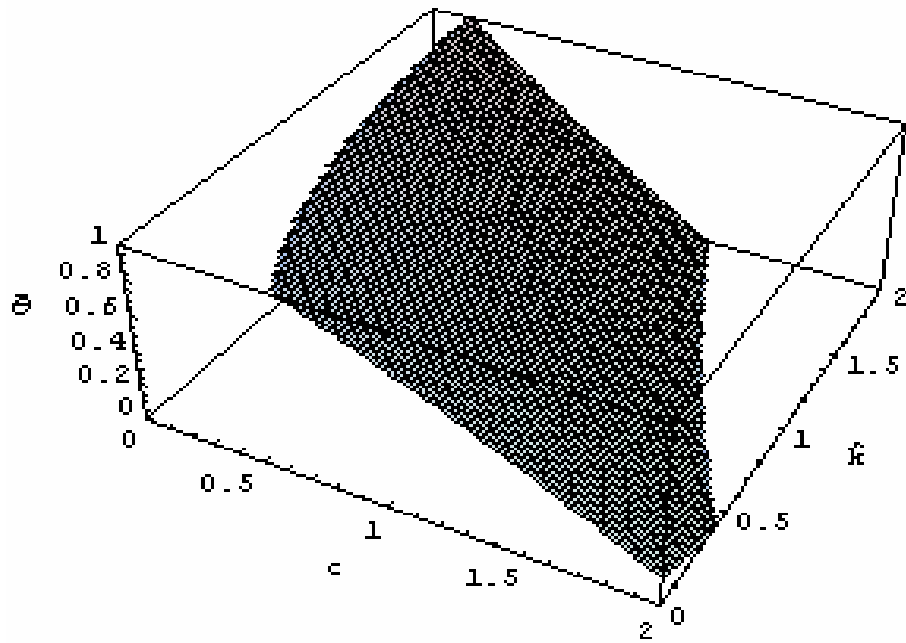


Figure 3.1: The relationship among c , \hat{k} , and $\bar{\theta}$: Overall shot

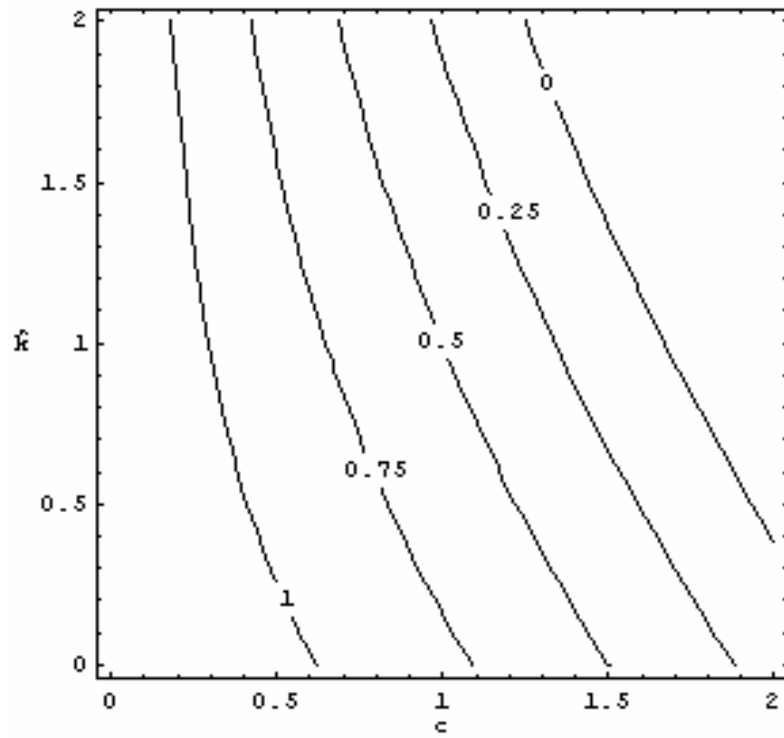


Figure 3.2: The relationship among c , \hat{k} , and $\bar{\theta}$: Topographical plot

Chapter 4

Emission quota versus emission tax in mixed duopoly

4.1 Introduction

Confronted with the environmental problem, the government often uses taxes, standards, and quotas to restrict emission that are regarded as an un-paid factor in Meade (1952). As Pigou (1920) points out, by using taxes that equalize marginal social cost and marginal private cost, social optimal allocation can be realized. Until the 1980s, many works, for example, Barnett (1980) and Baumol and Oates (1988), investigated the effects of environmental regulations in a competitive market and a monopoly. Since then, these have been analyzed in an imperfect competitive market.¹

The 21st century saw the works on environmental regulations in a mixed oligopoly. Bárcena-Ruiz and Garzón (2006) consider emission tax in the domestic market while Ohori (2006a, 2006b) and Belaldi and Chao (2006) consider the same in the international market. Kato (2006) investigates tradable emission permits and Naito and Ogawa (2009) compare

¹See Xepapadeas (1997) and Petrakis, Sartzetakis, and Xepapadeas (1999).

emission standard with emission tax. The emergence of these works is linked to the fact that the concerns with regard to the environmental problem have been growing in both the developed and developing countries, where mixed oligopoly is often seen.²

This paper investigates welfare comparison under emission taxes and quotas in a mixed duopoly.³ Further, we consider two cases under each environmental regulation: differentiated and uniform regulation level. The following situation is considered. Both firms produce the homogeneous product and have the same production and abatement technology. Emissions are produced by production. When the game starts, the government chooses the level of regulation. Following this, each firm chooses its output and abatement effort level simultaneously. We show that welfare is the largest under differentiated emission quota and that the superiority of emission tax over uniform emission quota depends on the parameters of the cost functions.

The work most related to this paper is Naito and Ogawa (2009). They examine the effects of emission tax and emission standard in a mixed duopoly. In the emission standard considered in their paper, the government sets a uniform abatement effort for each firm.⁴ They show that welfare under emission standard is larger than that under emission tax regardless of the degree of partial privatization.

This paper is organized as follows. The next section describes our model. Sections 4.3 and 4.4 derive the equilibrium outcome under an emission tax and a emission quota.

²See Bárcena-Ruiz and Garzón (2006) and Jiang (2003).

³Surely, the comparison of emission quota and tax in a pure oligopoly has been done. In certain situations, the two environmental regulations have been shown to be equivalent in terms of welfare. With regard to the equivalence of emission quota and tax in a competitive market, monopoly, and pure oligopoly, see Kiyono and Okuno-Fujiwara (2003) and Kato and Kiyono (2003).

⁴This type of emission standard is called design standard. For properties of the design standard, see Besanko (1987)

Section 4.5 compares these environmental regulations. Section 4.6 concludes the main text.

The appendix provides the proof of the proposition.

4.2 Model

We follow the model used in Bárcena-Ruiz and Garzón (2006) and Naito and Ogawa (2009). Consider an industry of two firms - one public (firm 0) whose objective is to maximize social welfare and one private (firm 1) whose objective is to maximize its own profits. They produce a homogeneous good. The inverse demand function of the good is given by $p = \alpha - Q$, where $Q = q_0 + q_1$ denotes the total output; q_i ($i = 0, 1$), the output of firm i ; p , the price of the good; and $\alpha > 0$. Both firms have symmetric production cost functions given by $c_i^p(q_i) = cq_i^2/2$.

Pollution e_i is generated by production. Producing one unit of output generates one unit of pollution. Firms can reduce their pollution by reducing their output or by investing abatement effort a_i . Emission of firm i can be represented as $e_i = \max\{q_i - a_i, 0\}$. The abatement cost function of firm i is $c_i^a(a_i) = ka_i^2/2$. The profit of firm i is given by

$$\pi_i(q_0, q_1, a_i) = (\alpha - q_0 - q_1)q_i - \frac{cq_i^2}{2} - \frac{ka_i^2}{2}. \quad (4.1)$$

Welfare is the sum of the consumer's surplus, producer's surplus, and environmental damage. It is given by

$$W(q_0, q_1, a_0, a_1) = \int_0^Q (\alpha - s)ds - \sum_{i=0}^1 \frac{cq_i^2}{2} - \sum_{i=0}^1 \frac{ka_i^2}{2} - \frac{(e_0 + e_1)^2}{2}, \quad (4.2)$$

where the last term of W represents environmental damage.

The decision-makings sequence of the government and firms is as follows. First, the government chooses the levels of regulation. Then, both firms simultaneously choose their

output q_i and abatement effort a_i . We analyze this game structure under differentiated emission tax, uniform emission tax, differentiated emission quota, and uniform emission quota.

4.3 Emission tax

Suppose the situation where the government imposes on an emission tax. We consider two cases: differentiated emission tax and uniform emission tax. Since the objective of the public firm is to maximize welfare, whether the emission tax is differentiated or uniform does not affect the behavior of the public firm directly. Therefore, there is no difference between differentiated emission tax and uniform emission tax. In the subsequent section, we analyze the equilibrium under uniform emission tax t .

The maximization problem of each firm is given by

$$\max_{q_0, a_0} W(q_0, q_1, a_0, a_1), \quad (4.3)$$

$$\max_{q_1, a_1} \pi_1(q_0, q_1, a_1) - te_1. \quad (4.4)$$

We note that the the tax revenue of the government is balanced by the tax payment of the private firm, and therefore, the term related to emission tax does not appear in (4.3). The first order conditions of the above maximization problem are as follows:

$$\begin{aligned} \frac{\partial W}{\partial q_0} &= \alpha - (2 + c)q_0 - 2q_1 + a_0 + a_1 = 0, \\ \frac{\partial W}{\partial a_0} &= q_0 + q_1 - (1 + k)a_0 - a_1 = 0, \\ \frac{\partial \pi_1}{\partial q_1} &= \alpha - q_0 - (2 + c)q_1 - t = 0, \\ \frac{\partial \pi_1}{\partial a_1} &= -ka_1 + t = 0. \end{aligned}$$

Given the above behavior of each firm, the government maximizes welfare with regard to t . We can obtain the following equilibrium outcome:

$$\begin{aligned}
t^T &= \frac{k\{2(k+1)c^3 + (k^2 + 9k + 6)c^2 + (2k^2 + 8k + 3)c + 2k + 1\}\alpha}{\Delta^T}, \\
q_0^T &= \frac{(k+2)\{c^2 + (k+3)c + 1\}\{(k+1)c + 2k + 1\}\alpha}{\Delta^T}, \\
q_1^T &= \frac{c(k+2)\{(k+1)c^2 + (k^2 + 5k + 3)c + 2k^2 + 4k + 1\}\alpha}{\Delta^T}, \\
a_0^T &= \frac{\{2(k+1)c^3 + 2(k^2 + 6k + 4)c^2 + (4k^2 + 14k + 7)c + 2k + 1\}\alpha}{\Delta^T}, \\
a_1^T &= \frac{\{2(k+1)c^3 + (k^2 + 9k + 6)c^2 + (2k^2 + 8k + 3)c + 2k + 1\}\alpha}{\Delta^T}, \\
e_0^T &= \frac{\{k(k+1)c^3 + k(k^2 + 6k + 4)c^2 + (2k^3 + 8k^2 + 6k + 1)c + 2k^2 + 3k + 1\}\alpha}{\Delta^T}, \\
e_1^T &= \frac{\{k(k+1)c^3 + k(k^2 + 6k + 4)c^2 + (2k^3 + 6k^2 + k - 1)c - (2k + 1)\}\alpha}{\Delta^T}, \\
Q^T &= \frac{(k+2)\{2(k+1)c^3 + (2k^2 + 11k + 7)c^2 + (4k^2 + 12k + 5)c + (2k + 1)\}\alpha}{\Delta^T}, \\
A^T &= \frac{\{4(k+1)c^3 + (3k^2 + 21k + 14)c^2 + 2(3k^2 + 11k + 5)c + 2(2k + 1)\}\alpha}{\Delta^T}, \\
E^T &= \frac{k\{2(k+1)c^3 + 2(k^2 + 6k + 4)c^2 + (4k^2 + 14k + 7)c + (2k + 1)\}\alpha}{\Delta^T}, \\
W^T &= \frac{(k+2)\{2(k+1)c^3 + 2(k^2 + 6k + 4)c^2 + (4k^2 + 13k + 5)c + 2k + 1\}\alpha^2}{2\Delta^T},
\end{aligned}$$

where $\Delta^T = (k+1)(k+2)c^4 + (k^3 + 12k^2 + 24k + 12)c^3 + (6k^3 + 39k^2 + 57k + 22)c^2 + (8k^3 + 36k^2 + 41k + 12)c + 2(k+1)(2k+1) > 0$. A^T and E^T denote total abatement effort and total emission, respectively. Superscripts T , DQ , and UQ denote outcome under emission tax, differentiated emission quota, and uniform emission quota, respectively. In order to restrict our attention to the interior solution, we assume $c \geq 1$ and $k \geq 1$ throughout the paper.

4.4 Emission quota

In this section, we derive the equilibrium outcome for two types of emission quota: differentiated emission quota and uniform emission quota. First, we derive for the differentiated emission quota.

4.4.1 Differentiated emission quota

Maximization problems of firm 0 and firm 1 are given by

$$\max_{q_0, a_0} W(q_0, q_1, a_0, a_1) \quad \text{s.t. } \bar{e}_0 = e_0, \quad (4.5)$$

$$\max_{q_1, a_1} \pi_1(q_0, q_1, a_1) \quad \text{s.t. } \bar{e}_1 = e_1, \quad (4.6)$$

where \bar{e}_i is the emission quota imposed on firm i .

Using the method of Lagrange undetermined multiplier and calculating the first order condition of the Lagrangian function of each firm, we find that

$$\begin{aligned} \frac{\partial LW^D}{\partial q_0} &= \alpha - (2+c)q_0 - 2q_1 + a_0 + a_1 - \lambda_0^D = 0, \\ \frac{\partial LW^D}{\partial a_0} &= q_0 + q_1 - (1+k)a_0 - a_1 + \lambda_0^D = 0, \\ \frac{\partial LW^D}{\partial \lambda_0^D} &= \bar{e}_0 - q_0 + a_0 = 0, \\ \frac{\partial L\pi_1^D}{\partial q_1} &= \alpha - q_0 - (2+c)q_1 - \lambda_1^D = 0, \\ \frac{\partial L\pi_1^D}{\partial a_1} &= -ka_1 + \lambda_1^D = 0, \\ \frac{\partial L\pi_1^D}{\partial \lambda_1^D} &= \bar{e}_1 - q_1 + a_1 = 0, \end{aligned}$$

where $LW^D = W + \lambda_0^D \{\bar{e}_0 - (q_0 - a_0)\}$, $L\pi_1^D = \pi_1 + \lambda_1^D \{\bar{e}_1 - (q_1 - a_1)\}$, and λ_i^D denotes the shadow price of the emission constraint of firm i .

Given the behavior of each firm, the government maximizes welfare with regard to \bar{e}_0 and \bar{e}_1 . In the equilibrium,

$$\begin{aligned}
q_0^{DQ} &= \frac{(k+2)(c+k+1)\{c^2+(3+k)c+1\}\alpha}{\Delta^{DQ}}, \\
q_1^{DQ} &= \frac{c(k+2)\{c^2+(2k+3)c+k^2+3k+1\}\alpha}{\Delta^{DQ}}, \\
a_0^{DQ} &= \frac{\{2c^3+4(k+2)c^2+(2k^2+9k+7)c+k+1\}\alpha}{\Delta^{DQ}}, \\
a_1^{DQ} &= \frac{\{2c^3+3(k+2)c^2+(k^2+5k+3)c+k+1\}\alpha}{\Delta^{DQ}}, \\
\bar{e}_0^{DQ} = e_0^{DQ} &= \frac{\{kc^3+2k(k+2)c^2+(k^3+4k^2+3k+1)c+(k+1)^2\}\alpha}{\Delta^{DQ}}, \\
\bar{e}_1^{DQ} = e_1^{DQ} &= \frac{\{kc^3+2k(k+2)c^2+(k^3+4k^2+2k-1)c-(k+1)\}\alpha}{\Delta^{DQ}}, \\
Q^{DQ} &= \frac{(k+2)\{2c^3+(4k+7)c^2+(k+1)(2k+5)c+k+1\}\alpha}{\Delta^{DQ}}, \\
A^{DQ} &= \frac{\{4c^3+7(k+2)c^2+(3k^2+14k+10)c+2(k+1)\}\alpha}{\Delta^{DQ}}, \\
E^{DQ} &= \frac{k\{2c^3+4(k+2)c^2+(2k^2+8k+5)c+k+1\}\alpha}{\Delta^{DQ}}, \\
W^{DQ} &= \frac{(k+2)\{2c^3+4(k+2)c^2+(2k^2+8k+5)c+k+1\}\alpha^2}{2\Delta^{DQ}},
\end{aligned}$$

where $\Delta^{DQ} = (k+2)c^4 + 2(k^2+6k+6)c^3 + (k+1)(k+2)(k+11)c^2 + (k+1)(4k^2+17k+12)c + 2(k+1)^2 > 0$.

4.4.2 Uniform emission quota

Maximization problems of firm 0 and firm 1 are given by

$$\max_{q_0, a_0} W(q_0, q_1, a_0, a_1) \quad \text{s.t. } \bar{e} = e_0, \quad (4.7)$$

$$\max_{q_1, a_1} \pi_1(q_0, q_1, a_1) \quad \text{s.t. } \bar{e} = e_1. \quad (4.8)$$

As well as the differentiated emission quota, using the method of Lagrange undetermined multiplier and calculating the first order condition of the Lagrangian function of

each firm, we find that

$$\begin{aligned}
\frac{\partial LW^U}{\partial q_0} &= \alpha - (2+c)q_0 - 2q_1 + a_0 + a_1 - \lambda_0^U = 0, \\
\frac{\partial LW^U}{\partial a_0} &= q_0 + q_1 - (1+k)a_0 - a_1 + \lambda_0^U = 0, \\
\frac{\partial LW^U}{\partial \lambda_0^U} &= \bar{e} - q_0 + a_0 = 0, \\
\frac{\partial L\pi_1^U}{\partial q_1} &= \alpha - q_0 - (2+c)q_1 - \lambda_1^U = 0, \\
\frac{\partial L\pi_1^U}{\partial a_1} &= -ka_1 + \lambda_1^U = 0, \\
\frac{\partial L\pi_1^U}{\partial \lambda_1^U} &= \bar{e} - q_1 + a_1 = 0,
\end{aligned}$$

where $LW^U = W + \lambda_0^U \{\bar{e} - (q_0 - a_0)\}$, $L\pi_1^U = \pi_1 + \lambda_1^U \{\bar{e} - (q_1 - a_1)\}$, and λ_i^U denotes the shadow price of the emission constraint of firm i .

Given the behavior of each firm, the government maximizes welfare with regard to \bar{e} .

The equilibrium outcome is as follows.

$$\begin{aligned}
q_0^{UQ} &= \frac{2(k+2)(1+c+k)\{c^2 + (2k+3)c + k^2 + 3k + 1\}\alpha}{\Delta^{UQ}}, \\
q_1^{UQ} &= \frac{2(k+2)(c+k)\{c^2 + (2k+3)c + k^2 + 3k + 1\}\alpha}{\Delta^{UQ}}, \\
a_0^{UQ} &= \frac{\{4c^3 + 4(3k+4)c^2 + (12k^2 + 35k + 16)c + 4k^3 + 19k^2 + 17k + 4\}\alpha}{\Delta^{UQ}}, \\
a_1^{UQ} &= \frac{\{4c^3 + 2(5k+6)c^2 + (8k^2 + 21k + 4)c + (2k^2 + 9k + 3)k\}\alpha}{\Delta^{UQ}}, \\
\bar{e}^{UQ} = e_i^{UQ} &= \frac{k\{2c^3 + 2(3k+4)c^2 + (6k^2 + 16k + 5)c + 2k^3 + 8k^2 + 5k + 1\}\alpha}{\Delta^{UQ}}, \\
Q^{UQ} &= \frac{2(k+2)(2k+2c+1)\{c^2 + (2k+3)c + k^2 + 3k + 1\}\alpha}{\Delta^{UQ}}, \\
A^{UQ} &= \frac{2\{4c^3 + (11k+14)c^2 + 2(5k^2 + 14k + 5)c + 3k^3 + 14k^2 + 10k + 2\}\alpha}{\Delta^{UQ}}, \\
E^{UQ} &= \frac{2k\{2c^3 + 2(3k+4)c^2 + (6k^2 + 16k + 5)c + 2k^3 + 8k^2 + 5k + 1\}\alpha}{\Delta^{UQ}}, \\
W^{UQ} &= \frac{(k+2)\{2c^3 + 2(3k+4)c^2 + (6k^2 + 16k + 5)c + 2k^3 + 8k^2 + 5k + 1\}\alpha^2}{\Delta^{UQ}},
\end{aligned}$$

where $\Delta^{UQ} = 2(k+2)c^4 + 2(3k^2 + 14k + 12)c^3 + 2(3k^3 + 26k^2 + 47k + 22)c^2 + (2k^4 + 36k^3 + 111k^2 + 100k + 24)c + (k+1)(8k^3 + 33k^2 + 22k + 4) > 0$.

4.5 Comparison among emission quotas and emission tax

Using the results in the previous section, we compare the equilibrium outcome under three environmental regulations in a mixed duopoly. First, we obtain the following relationships of the three equilibrium outcomes.

Proposition 4.1.

$$\begin{aligned}
q_1^T &< q_1^{DQ} < q_1^{UQ} < q_0^{UQ} < q_0^{DQ} < q_0^T, \\
a_1^{UQ} &< a_1^T < a_1^{DQ} < a_0^T < a_0^{DQ} < a_0^{UQ}, \\
e_1^T &< e_1^{DQ} < e_1^{UQ} = e_0^{UQ} < e_0^{DQ} < e_0^T, \\
Q^{UQ} &< Q^{DQ} < Q^T, \\
A^T &< A^{UQ} < A^{DQ}, \\
E^{UQ} &< E^{DQ} < E^T.
\end{aligned}$$

Proof. Simple comparison of the three equilibrium outcomes yields the results in Proposition 4.1. □

The intuition behind Proposition 4.1 is simple. Under the emission tax policy, there is no effect on the reaction function of the public firm. Therefore, the public firm produces more than the private firm largely by investing more abatement effort. With regard to the strategic substitution, the private firm produces less output and abatement effort. Taking

into consideration the inefficient production allocation, it is desirable for the private firm to produce more. Therefore, the government chooses the differentiated emission quota in order to control the production of public firm such that it is less and that of the private firm such that it is more. Under uniform emission quota, the government cannot use the above differentiation, and hence, it decreases the difference in the emission quotas.

Note that the equilibrium outcome under emission tax and emission quota is the same if we consider the privatization of the public firm, that is, a pure duopoly.⁵

Finally, we compare welfare under the three types of emission regulations. Then, we obtain the following proposition.

Proposition 4.2.

$$\begin{aligned}
 W^{DQ} > W^{UQ} > W^T & \quad \text{if } c > \bar{c} \text{ and } k > 1, \\
 W^{DQ} > W^T > W^{UQ} & \quad \text{otherwise,}
 \end{aligned}$$

where $\bar{c} = (2 + 4k - k^2 + \sqrt{8k + 16k^2 + k^4})/\{2(k - 1)\}$ and $d\bar{c}/dk < 0$.

Proof. See Appendix 4.B. □

The intuition behind Proposition 4.2 is as follows. If the government can use the differentiated quota, welfare is the largest because it can control not only the behavior of the private firm but also that of the public firm. Whether or not uniform emission quota is superior to emission tax depends on the parameters of the cost functions. Emission tax has not only positive aspects but also negative aspects when compared to uniform emission quota: from Proposition 4.1, the total output, total emission, and inefficient

⁵See Appendix 4.A.

production allocation is larger under emission tax than under uniform emission quota. If the parameters of the cost functions are small, an increase in consumer surplus matters, and thus, it is desirable to use emission tax. If the parameters of the cost functions are large, the negative effect of inefficient production allocation is large, and then, it is desirable to use emission quota.

Finally, we comment on the emission standard considered in Naito and Ogawa (2009) denoted by superscript S and compare it with the environmental regulations in this paper. They consider uniform emission standard. Even if we allow differentiated emission standard, the second best emission standard is the same as the uniform emission standard: the sum of the emission standard levels of both firms only affects the reaction function of the public firm and the government decides its level considering the cost minimization of the abatement effort.⁶

We compare welfare under these environmental regulations. Table 4.1 shows the results for some parameters of the cost functions when $\alpha = 1$. If we set $c = k = 1$, welfare under the emission standard is the largest ($W^S > W^{DQ} > W^T > W^{UQ}$). However, if these parameters are sufficiently large, for example, $c = k = 15$, welfare under emission standard is the lowest ($W^{DQ} > W^{UQ} > W^T > W^S$). This is the reason why fixed emission abatement effort is necessary even when the abatement cost is high and there is limited control over emission levels for firms. We note that a general comparison of welfare under emission standard and other environmental regulations is complex.

⁶Appendix 4.C provides the results.

4.6 Concluding remarks

We compare the environmental regulations in a mixed duopoly under differentiated and uniform emission tax and differentiated and uniform emission quota. We obtain the following results. Welfare under differentiated emission quota is the largest; further, the superiority of emission tax over uniform emission quota depends on the parameters of the cost functions. From our results and the results of Naito and Ogawa (2009), we conclude that we have to pay attention as what are the types of environmental policies the government can set in a mixed oligopoly.

We note that the results obtained in this paper are for a very restrictive situation. However, it is difficult to analyze the comparison of environmental regulations if we mitigate this situation. How to analyze these in a more general situation is left for future research.

Appendix 4.A

After privatization We derive the equilibrium outcome under uniform emission tax and emission quota after privatization of firm 0. Note that firm 0 is a profit-maximizer after privatization. The results are as follows.

$$\begin{aligned}
 t^{TP} &= \frac{k(k+2c+6)\alpha}{\Delta^p}, \\
 q_i^{TP} &= q_i^{QP} = \frac{(k+2)(k+c+3)\alpha}{\Delta^p}, \\
 a_i^{TP} &= a_i^{QP} = \frac{(k+2c+6)\alpha}{\Delta^p}, \\
 e_i^{TP} &= e_i^{QP} = \bar{e}^{QP} = \frac{k(k+c+4)\alpha}{\Delta^p}, \\
 W^{TP} &= W^{QP} = \frac{(k+2)(k+c+4)\alpha^2}{\Delta^p},
 \end{aligned}$$

where $\Delta^p = (k+2)c^2 + (k^2 + 10k + 12)c + 4k^2 + 21k + 18 > 0$. The superscript TP (QP) denotes the equilibrium outcome under emission tax (quota) in a pure duopoly. From the results, we can see emission tax and emission quota are equivalent in a pure duopoly in terms of welfare.

Appendix 4.B

Proof of Proposition 4.2. First, we compare welfare under the two emission quotas. By definition, the government can choose $\bar{e}_0 = \bar{e}_1 = \bar{e}^{UQ}$ under the differentiated emission quota. From the results, we know that $\bar{e}_0 \neq \bar{e}_1 \neq \bar{e}^{UQ}$. Therefore, $W^{DQ} > W^{UQ}$.

Second, we compare welfare under differentiated emission quota and emission tax. We get

$$W^{DQ} - W^T = \frac{c^2 k^2 (k+2)^2 \alpha^2}{2\Delta^T \Delta^{DQ}} > 0. \quad (4.9)$$

Finally, we compare welfare under uniform emission quota and emission tax. We get

$$W^{UQ} - W^T = \frac{k(k+2)^2 \Phi(c, k) \alpha^2}{2\Delta^T \Delta^{UQ}}, \quad (4.10)$$

where $\Phi(c, k) = -1 - 2c - c^2 - 3k - 4ck + c^2 k - 2k^2 + ck^2$. We examine whether or not $\Phi(c, k)$ is positive. When $k = 1$, we get $\Phi(c, 1) < 0$. Suppose $k > 1$. Then, we obtain

$$\Phi(c, k) > 0 \quad \text{if and only if} \quad c > \bar{c} \text{ or } c < \underline{c},$$

where

$$\bar{c} = \frac{2 + 4k - k^2 + \sqrt{8k + 16k^2 + k^4}}{2(k-1)},$$

$$\underline{c} = \frac{2 + 4k - k^2 - \sqrt{8k + 16k^2 + k^4}}{2(k-1)}.$$

We can easily find that $\bar{c} > 1$ and $\underline{c} < 0$. Therefore, we obtain Proposition 4.2. \square

Appendix 4.C

Emission standard

We consider the differentiated emission standard under the framework of the basic model in Naito and Ogawa (2009).

The government sets the abatement effort of each firm given by \bar{a}_i . In this case, the firm only chooses its output level. The maximization problem of each firm is

$$\max_{q_0} W(q_0, q_1, \bar{a}_0, \bar{a}_1), \quad (4.11)$$

$$\max_{q_1} \pi_1(q_0, q_1, \bar{a}_1). \quad (4.12)$$

The first order conditions of the above maximization problem are

$$\begin{aligned} \frac{\partial W}{\partial q_0} &= \alpha - (2 + c)q_0 - 2q_1 + \sum_{i=0}^1 \bar{a}_i = 0, \\ \frac{\partial \pi_1}{\partial q_1} &= \alpha - q_0 - (2 + c)q_1 = 0. \end{aligned}$$

The government chooses \bar{a}_0 and \bar{a}_1 to maximize welfare, given the firms' behavior. We obtain the following equilibrium outcome.

$$\begin{aligned}
q_0^S &= \frac{\{(k+2)c^3 + 2(2k+5)c^2 + 2(k+7)c + 4\}\alpha}{\Delta^S}, \\
q_1^S &= \frac{(c+1)\{(k+2)c^2 + 2(2k+3)c + 2k\}\alpha}{\Delta^S}, \\
a_i^S &= \bar{a}_i^S = \frac{(c+1)(2c^2 + 7c + 2)\alpha}{\Delta^S}, \\
e_0^S &= \frac{\{kc^3 + (4k+1)c^2 + (2k+5)c + 2\}\alpha}{\Delta^S}, \\
e_1^S &= \frac{(c+1)\{kc^2 + (4k-1)c + 2(k-1)\}\alpha}{\Delta^S}, \\
Q^S &= \frac{\{2(k+2)c^3 + 9(k+2)c^2 + 4(2k+5)c + 2(k+2)\}\alpha}{\Delta^S}, \\
A^S &= \frac{2(c+1)(2c^2 + 7c + 2)\alpha}{\Delta^S}, \\
E^S &= \frac{\{2kc^3 + 9kc^2 + 2(4k+1)c + 2k\}\alpha}{\Delta^S}, \\
W^S &= \frac{\{2(k+2)c^3 + 4(2k+5)c^2 + (7k+22)c + 2(k+2)\}\alpha^2}{2\Delta^S},
\end{aligned}$$

where $\Delta^S = (k+2)c^4 + 2(4k+7)c^3 + 4(5k+8)c^2 + 2(8k+13)c + 4(k+1) > 0$.

	W^S	W^{UQ}	W^T	W^{DQ}
$c = k = 1$	$\frac{69}{254}$	$\frac{177}{667}$	$\frac{17}{64}$	$\frac{93}{350}$
$c = k = 10$	$\frac{273}{3736}$	$\frac{57903}{792601}$	$\frac{20057}{274549}$	$\frac{28983}{396731}$
$c = k = 15$	$\frac{148189}{2783258}$	$\frac{1042967}{19586779}$	$\frac{4557377}{85586944}$	$\frac{521747}{9798334}$

Table 4.1: Welfare under emission standard, uniform emission quota, emission tax, and differentiated emission quota for several values of c and k in the case of $\alpha = 1$

Chapter 5

Price competition in a mixed duopoly¹

5.1 Introduction

This paper analyzes the price competition in a homogeneous product market under a mixed duopoly. We consider the case that cost functions are symmetric between two firms and they are strictly convex.² In our model, one private firm and one public firm exist. The former maximizes its own profits. The latter maximizes a weighted average of social welfare and its own profits.³ Since we do not understand well which firm is a first-mover, we compare three timings of price setting: (timing *S*) Both firms set those prices simultaneously. (timing *V*) First the private firm sets its price, and second the public firm does one. We call this situation "private price leadership". (timing *B*) First, the public firm sets its price, and second the private firm does one. We name this situation as "public price leadership".

¹This chapter is based on Ogawa and Kato (2006) in *Economics Bulletin*.

²We consider a quadratic cost function.

³For a rationalization of the objective, see Bös (1991). Using such a objective function, we can deal with several types of public firm.

From the seminal work of Defraja and Delbono (1989), mixed oligopoly becomes one of the major topic in the theory of industrial organization. Many studies consider quantity competition and deals with asymmetric linear cost function or quadratic cost function. However, only few analyzes about price competition. Thus this paper answers a part of left question.

Dastidar (1995) studies a price competition with homogeneous product markets under private oligopoly. They show that the equilibrium prices are multiple in a pure strategy if cost functions are symmetric between both firms.

We show that the equilibrium price under S have a range and it equals to Dastidar (1995) even though the public firm exists in the market. We also find that the equilibrium price under V is higher than the one under B and exceeds the range of the one under S under some condition.⁴

This paper has 4 sections. Section 5.2 builds the model. Section 5.3 solves the equilibrium. Section 5.4 concludes the paper.

5.2 Model

Suppose there are a homogeneous product market which consists of one private firm and one public firm. The demand function is given by $D(p) = a - p$, where a is positive and sufficiently large. The cost function is given by cq_i^2 , where c is positive and q_i is the output of firm i , $i = 0, 1$.

We introduce the following assumptions:⁵

⁴Dastidar (1995) focuses on a simultaneous price competition, and thus some of our main results are not compared with it.

⁵These assumptions are also used in Dastidar (1997).

Assumption

1. *Firms have to supply all the demand it faces.*
2. *When both firms choose the same price, they share the demand equally, that is, when they choose the same price, p , each firm supplies $\frac{1}{2}D(p)$ respectively.*

The profits of firm i is given by

$$\pi_i = \begin{cases} p_i(a - p_i) - c(a - p_i)^2 & \text{if } p_i < p_j, \\ p_i\left\{\frac{1}{2}(a - p_i)\right\} - c\left\{\frac{1}{2}(a - p_i)\right\}^2 & \text{if } p_i = p_j, \\ 0 & \text{if } p_i > p_j. \end{cases} \quad (5.1)$$

Social welfare is given by

$$\begin{aligned} SW &= \text{consumer's surplus} + \text{producer's surplus} \\ &= \frac{1}{2}(q_0 + q_1)^2 + \pi_0 + \pi_1 \end{aligned} \quad (5.2)$$

The objective function of the public firm U_0 and that of the private firm U_1 are given by

$$U_0 = \alpha SW + (1 - \alpha)\pi_0, \quad (5.3)$$

$$U_1 = \pi_1. \quad (5.4)$$

5.3 Equilibrium

We consider the three types of the price competition; simultaneous (S), private price leadership (V), and public price leadership (B). We restrict our attention to the situation where each firm chooses pure strategies.

5.3.1 Simultaneous price competition

Proposition 5.1. *In the equilibrium, both firms choose the same price p^S within $\left[a\frac{c}{c+2}, a\frac{3c}{3c+2}\right]$.*

Proof. First we consider an undercut incentive. Since the public firm counts private firm's profits, the public firm's incentive is weaker than the private firm's one. Thus we focus on the private firm. Suppose the public firm sets the price $p_0 \leq a\frac{3c}{3c+2}$. If the private firm sets the price $p_1 = p_0 - \epsilon$, then the increase of revenue is less than the increase of cost. Second we consider pullup incentive. If $U_i < 0$ by setting the same price as the opponent's, firm i can increase U_i by pulling up the price.⁶ Since the public firm counts the consumer's surplus, a price that causes negative U_0 is lower than the one that causes negative U_1 . Thus we focus on the private firm. Calculating $U_1 = 0$, we have $p_1 = a\frac{c}{c+2}$. \square

Note that this proposition is the same result and the similar intuition of Dastidar (1995) because the public firm have a weak incentive to set a different price from the one of the private firm and thus the incentive is not binding. In other words, the public firm is not beneficial or harmful for social welfare in simultaneous case.

5.3.2 Private price leadership

Proposition 5.2. *1. In the equilibrium, both firms choose*

$$p^V = \begin{cases} a\frac{c+1}{c+2} & (\text{Case 1, 2}), \\ a\frac{c(1+\alpha)+1-\alpha+\sqrt{(-\alpha^3+2c\alpha^2+3\alpha+2c\alpha-2)/(2c+2-\alpha)}}{c(1+\alpha)+2} & (\text{Case 3}), \\ a\frac{(3-\alpha)c}{2(1-\alpha)+(3-\alpha)c} & (\text{Case 4, 5}), \end{cases}$$

⁶Then firm i supplies nothing.

where

$$\begin{aligned}
\text{Case 1: } & \alpha \leq \frac{\sqrt{17}-3}{2}, c \geq \frac{2(1-\alpha)}{\alpha+1}, \\
\text{Case 2: } & \alpha > \frac{\sqrt{17}-3}{2}, c \geq \frac{\alpha^2-9\alpha+2+\sqrt{\alpha^4+14\alpha^3+37\alpha^2-4\alpha+4}}{4\alpha}, \\
\text{Case 3: } & \alpha > \frac{\sqrt{17}-3}{2}, \frac{2(\alpha-1)^2}{3\alpha-1} \leq c < \frac{\alpha^2-9\alpha+2+\sqrt{\alpha^4+14\alpha^3+37\alpha^2-4\alpha+4}}{4\alpha}, \\
\text{Case 4: } & \alpha \leq \frac{\sqrt{17}-3}{2}, c < \frac{2(1-\alpha)}{\alpha+1}, \\
\text{Case 5: } & \alpha > \frac{\sqrt{17}-3}{2}, c < \frac{2(\alpha-1)^2}{3\alpha-1}.
\end{aligned}$$

2. We note that p^V in case 1 and 2 exceeds p^S if $c < 2$.

Proof. As we mentioned at the proof of Proposition 5.1, the public firm has only weak incentive to undercut. Hence, the private firm can choose p^V from wider range than the one of p^S . In case 1 and 2, profit-maximizing price subjected to the range is inner solution, $a\frac{3c}{3c+2}$. If $c < 2$, it exceeds $a\frac{3c}{3c+2}$. \square

Note that the public firm may be harmful for social welfare in private price leadership case.

5.3.3 Public price leadership

Proposition 5.3. 1. In the equilibrium, both firms choose

$$p^B = \begin{cases} a\frac{\{1-\alpha+(1+\alpha)c\}}{2+(1+\alpha)c} & \text{if } c > \frac{2(1-\alpha)}{1+\alpha}, \\ a\frac{3c}{3c+2} & \text{if } c \leq \frac{2(1-\alpha)}{1+\alpha}. \end{cases}$$

2. When $\alpha = 1$, $p^B = a\frac{c}{c+1}$, which coincides with the price under first best.

3. $\frac{\partial p^b}{\partial \alpha} < 0$, $\frac{\partial SW^b}{\partial \alpha} > 0$.

Proof. As we mentioned at the proof of Proposition 5.1, the private firm chooses the same price as the public one if it is in $[a\frac{c}{c+2}, a\frac{3c}{3c+2}]$. The public firm knows it and maximizes U_0 subjected to the range because in the case that the public firm sets price outside the

range, the private firm chooses a different price. In such a situation, the production cost increases dramatically and thus U_1 is damaged. If $c > \frac{2(1-\alpha)}{1+\alpha}$, the maximization problem has an inner solution. If not, it has a corner solution.

The part 2 is derived by a simple comparison. About part 3, the larger a weighted average of social welfare is, the more the public firm concerned with social welfare. As the production inefficiency does not occur as long as both firms supply, the public firm can decrease the price to increase the consumer surplus.

□

Note that the public firm may be beneficial for social welfare in public price leadership case.

Now, we compare p^B with p^V and find that $p^B < p^V$. The intuition behind this result is that the public firm has an incentive to decrease the price since the public firm has an incentive to enhance social welfare.

5.4 Concluding remarks

We analyze three types of price competition with homogeneous products and symmetric quadratic cost functions under mixed duopoly. We find that the equilibrium price in private price leadership case is higher than the one in simultaneous case under some condition of cost parameter, and always exceeds the one in public price leadership case. We also find that the public firm chooses the same price as the private firm chooses regardless of the timing of the price setting.

We have the following intuition from the results: The public enterprises are often

justified by the reason that they are conscious of social welfare and enhance it. However, even if they acts for the improvement of social welfare, the existence of them may lead worse outcome because private firms would take advantage of such a behavior. Therefore, a price monitoring in mixed markets is quite important and privatization would be promoted when a highly marked-up price is sustained.

Chapter 6

Mixed oligopoly, privatization, subsidization, and the order of firms' moves: several types of objectives¹

6.1 Introduction

Recently many works have investigated mixed oligopoly. Usually they investigate the consequences of privatization of a public firm.² In such existing models, a public firm and private firms often compete in a homogeneous good market and it is often assumed that a public firm maximizes social welfare and each private firm maximizes its own profits.³

In this stream of mixed oligopoly theories, White (1996), Poyago-Theotoky (2001), Myles (2002), Fjell and Heywood (2004), and Tomaru (2006) examine the relationship between the output subsidy in mixed oligopoly and that in private oligopoly. Especially Poyago-Theotoky (2001) and Myles (2002) showed that the optimal subsidy level, all firms' profits, the output level and welfare are identical regardless of whether (a) the public firm

¹This chapter is based on Kato and Tomaru (2007) in *Economics Letters*.

²For example, see DeFraja and Delbono (1989).

³Some other objectives of a public firm are considered in some papers. They assumed that the public firm maximizes a weighted average of social welfare and its own profits, for example. See Bös (1991) and Matsumura (1998).

and all n private firms choose their output simultaneously, (b) the public firm is privatized and all $n + 1$ private firms choose their output simultaneously, or (c) the public firm acts as a Stackelberg leader and all n private firms do as Stackelberg followers. These results are obtained in the model where a welfare-maximizing public firm competes with profit-maximizing private firms.

In a real world, the private firms do not always seem to be profit maximizers, however. Kaneda and Matsui (2003) pointed it out and presented an n -firm Cournot oligopoly model in which each firm's objective is to maximize the weighted average of its own profits and another factor. In this setting, they showed that firms whose realized profits are the largest are not generally profit maximizers. Thus we will not be surprised if the private firms have other objectives which are slightly different from maximization of their own profits. Taking this into account, can we nevertheless find that the results obtained by Poyago-Theotoky (2001) and Myles (2002) hold? The purpose of this paper is to answer this question.

6.2 Model and Main results

We assume that there are n private firms and one public firm producing a homogeneous good. The output of the private firm i is denoted by q_i ($i = 1, 2, \dots, n$) and that of the public firm by q_0 . Total output is $Q = q_0 + \sum_{i=1}^n q_i$. The cost function of firm j is $C(q_j)$, $j = 0, 1, \dots, n$. The inverse demand function in the market is given by $P(Q)$. We put the following assumption.

Assumption 6.1. *The inverse demand function and cost function satisfy the following*

properties.

- (i) $P(Q)$ is twice-continuously differentiable for all $Q \geq 0$, $P(Q) > 0$, and $P'(Q) < 0$,
- (ii) $C(q_j)$ is twice-continuously differentiable, strictly convex for all $q_j > 0$, and $C(0) \geq 0$.

The profits of firm j are

$$\pi_j(Q, q_j, s) = P(Q)q_j - C(q_j) + sq_j, \quad j = 0, 1, \dots, n.$$

where s is the output subsidy. Social welfare is defined by the sum of consumer surplus plus profits less the cost of the subsidy. Thus

$$W(q_0, q_1, \dots, q_n) = \int_0^Q P(z)dz - C(q_0) - \sum_{i=1}^n C(q_i). \quad (6.1)$$

Note that social welfare does not directly depend on the subsidy.

The objective of the public firm is to maximize social welfare. The objectives of private firms are not always to maximize its own profits though they are symmetric among private firms. Each private firm maximizes the weighted average of its own profits and some other objectives. This objective function of private firm i ($i = 1, 2, \dots, n$) U_i is given by⁴

$$U_i(q_1, q_2, \dots, q_i, s, \theta) = (1 - \theta)\pi_i(Q, q_i, s) + \theta F(Q, q_i), \quad (6.2)$$

where $\theta \in [0, 1)$ is the weight which private firm i puts on the objective other than its own profits. Notice that $\theta = 0$ implies that the firm maximizes its own profits. We put the following assumptions to guarantee the unique existence of Nash equilibrium in each subgame.⁵

⁴This formulation of each private firm's objectives follows from Kaneda and Matsui (2003)

⁵See Novshek (1985) and Kolstad and Mathiesen (1987).

Assumption 6.2. *The marginal revenue $P'(Q)q_j + P(Q)$ satisfies the following properties.*

$$P''(Q)q_j + P'(Q) < 0 \text{ for all } q_j \geq 0 \text{ and for all } Q \geq 0.$$

Assumption 6.3. *The function "F" satisfies the following properties.*

$$\frac{\partial F}{\partial Q} \leq 0, \quad \frac{\partial^2 F}{\partial Q \partial q_i} + \frac{\partial^2 F}{\partial Q^2} \leq 0, \quad \text{and} \quad \frac{\partial^2 F}{\partial q_i^2} + \frac{\partial^2 F}{\partial q_i \partial Q} \leq 0.$$

As to the function F , we give examples, *Revenue*: $F(Q, q_i) = P(Q)q_i$ and *Negative cost*: $F(Q, q_i) = -C(q)$.⁶

We consider a setting where the government sets the output subsidy in the first stage. In the subsequent stages, firms compete in quantity. As to the subsequent stages, we consider the following three cases: (a) *Mixed Oligopoly*: the public firm and n private firms simultaneously choose their output; (b) *Private Oligopoly*: the public firm is privatized and all $n + 1$ private firms simultaneously choose their output; (c) *Stackelberg*: the public firm acts as a Stackelberg leader and n private firms do as Stackelberg followers.

For our analysis we define the output level q^* by

$$P((n + 1)q^*) = C'(q^*).$$

q^* is the output level such that each firm equalizes its own marginal cost to the market price.

Thus, $(q_0, q_1, \dots, q_n) = (q^*, q^*, \dots, q^*)$ is the first-best allocation. Given the assumptions on $C(\cdot)$, $P(\cdot)$, and $F(\cdot)$, it is unique for fixed n . Further define the subsidy s^* :

$$s^* = - \left[q^* P'((n + 1)q^*) + \frac{\theta}{1 - \theta} \left(\frac{\partial F((n + 1)q^*, q^*)}{\partial q_i} + \frac{\partial F((n + 1)q^*, q^*)}{\partial Q} \right) \right].$$

This is also uniquely identified. We put the following assumption.

⁶For further examples and explanations of these objectives, see Kaneda and Matsui (2003). However, we put the stronger assumption on $F(\cdot)$ than Kaneda and Matsui (2003), therefore, we give only these two examples which satisfy Assumption 6.3.

Assumption 6.4. *At subsidy s^* , each firm producing output q^* can obtain non-negative profits, that is,*

$$(1 - \theta) [P((n + 1)q^*)q^* - C(q^*) + s^*q^*] + \theta F((n + 1)q^*, q^*) \geq 0.$$

Now, we examine the three cases (a), (b), and (c) below.

(a) Mixed Oligopoly

First, we consider the mixed oligopoly. The first order condition of firm 0 is

$$P(Q) - C'(q_0) = 0, \tag{6.3}$$

and that of firm i ($i = 1, 2, \dots, n$) is

$$(1 - \theta) [P(Q) + P'(Q)q_i - C'(q_i) + s] + \theta \left[\frac{\partial F(Q, q_i)}{\partial q_i} + \frac{\partial F(Q, q_i)}{\partial Q} \right] = 0. \tag{6.4}$$

By solving the above $n + 1$ first order conditions, we obtain $q_0^m(s)$ and $q_i^m(s)$ which are the equilibrium output level of firm 0 and firm i in the second stage. In the first stage, the government sets the following subsidy level s^m :

$$s^m = - \left\{ q_i^m(s^m) P'(Q^m(s^m)) + \frac{\theta}{1 - \theta} \left[\frac{\partial F(Q^m(s^m), q_i^m(s^m))}{\partial q_i} + \frac{\partial F(Q^m(s^m), q_i^m(s^m))}{\partial Q} \right] \right\},$$

where $Q^m(s^m)$ denotes $q_0^m(s^m) + \sum_{i=1}^n q_i^m(s^m)$. And then, all firms choose their output level so as to equalize their marginal costs to price. Thus subsidy s^m ensures $q_0^m(s^m) = q_i^m(s^m) = q^*$ ($i = 1, 2, \dots, n$).

(b) Private Oligopoly

Second, we consider the private oligopoly. There are $n + 1$ firms whose objectives are symmetric among them. In the equilibrium, each firm including firm 0 chooses its output

so as to satisfy the equation (6.4). We denote $q_0^p(s)$ and $q_i^p(s)$ as the equilibrium output level of firm 0 and firm i in the second stage. When the government sets the subsidy level s^p in the private oligopoly to be equal to s^m , we find $q_0^p(s^p) = q_i^p(s^p) = q^*$ ($i = 1, \dots, n$).

(c) Stackelberg

Finally, we consider the case where the public firm acts as a Stackelberg leader and all private firms Stackelberg followers, that is, the public firm chooses its output in the second stage and all private firms do in the third stage.

Each firm i chooses its output so as to satisfy the equation (6.4). Given this optimization behavior, firm 0 chooses its output. We define $q_i^{pl}(q_0, s)$ for $i = 1, \dots, n$ as the equilibrium output level of firm i in the third stage. In the second stage, firm 0 chooses its output level so as to satisfy the following equation:⁷

$$P(Q^{pl}(s)) - C'(q_0^{pl}(s)) + \sum_{i=1}^n \left[P(Q^{pl}(s)) - C'(q_i^{pl}(q_0^{pl}(s), s)) \right] \frac{\partial q_i^{pl}(q_0^{pl}(s), s)}{\partial q_0} = 0,$$

where $q_0^{pl}(s)$ is the equilibrium output level of firm 0 and $Q^{pl}(s)$ is the sum of equilibrium output level of all firms in the second stage.

In the first stage, the government sets s^{pl} to be equal to s^m , and then both the first order conditions of firm i and that of firm 0 become the conditions that the marginal cost of each firm is equal to price. Therefore, $q_i^{pl}(q_0^{pl}(s^{pl}), s^{pl}) = q_0^{pl}(s^{pl}) = q^*$ ($i = 1, \dots, n$) holds.

Hence subsidization achieves the output level under first-best in all three cases. Moreover, the optimal subsidies are identical in these cases. The proceeding results are summa-

⁷In the third stage, the equilibrium output level of all private firms are equal to the same as $q^{pl}(q_0, s)$ because of the symmetric objective among all private firms and the uniqueness of Nash equilibrium in the subgame. We examine the sign of $\frac{\partial q_i^{pl}(q_0, s)}{\partial q_0}$ and find that it is negative.

rized in Proposition 6.1.

Proposition 6.1. *Under Assumptions 6.1-6.4, in three cases, (1) given $\theta \in [0, 1)$, the optimal subsidies are identical, (2) for all $\theta \in [0, 1)$, equilibrium output and social welfare are identical and the first-best allocation is achieved.*

Suppose the following situation: there are two private firms (firm 1 and firm 2) and one public firm (firm 0). $p(Q) = a - q_0 - q_1 - q_2$, $F(Q, q_j) = p(Q)q_j$, and $c(q_j) = kq_j^2/2$, $j = 0, 1, 2$ are assumed. Then, in the equilibrium, $s^* = s^h = \frac{(1-k\theta)a}{(3+k)(1-\theta)}$ ($h = m, p, pl$) and $W(s^*) = \frac{3a^2}{2(3+k)}$ in all three cases.

Note that the irrelevance result does not always hold if different firms have different weights, $\theta_j \neq \theta_l, j \neq l, j, l = 0, \dots, n$. Suppose that $a = 100$, $k = 2$, $\theta_0 = 1/6$, $\theta_1 = 1/2$, and $\theta_2 = 1/3$ in the above example. Then the optimal subsidies in three cases are different: $s^m \approx 6.07$, $s^p \approx 11.29$ and $s^{pl} \approx 6.27$. Furthermore, under these subsidies, social welfare in three cases are also different: $W(q_0^m(s^m), q_1^m(s^m), q_2^m(s^m)) \approx 2996.50$, $W(q_0^p(s^p), q_1^p(s^p), q_2^p(s^p)) \approx 2990.65$, and $W(q_0^{pl}(s^{pl}), q_1^{pl}(q_0^{pl}(s^{pl}), s^{pl}), q_2^{pl}(q_0^{pl}(s^{pl}), s^{pl})) \approx 2996.52$.

Now we consider the following case: different firms have different weights, $\theta_j \neq \theta_l$, and the government can impose individual subsidy level s_j on each firm.

In this case, we find that the first-best allocation can be achieved in all three cases when the government imposes each firm on the following subsidy:

$$s_j^* = - \left\{ q^* P'(Q^*) + \frac{\theta_j}{1 - \theta_j} \left[\frac{\partial F(Q^*, q^*)}{\partial q_j} + \frac{\partial F(Q^*, q^*)}{\partial Q} \right] \right\}.$$

Note that s_j^* only depends on the weight θ_j . Under these individual subsidies, the firm which has higher θ receives high (low) subsidy if $\partial F/\partial Q + \partial F/\partial q_j < (>) 0$.

6.3 Concluding remarks

A series of existing works demonstrated that there are no consequences from privatization of a public firm in a mixed oligopoly when the government uses a subsidy to ensure the first-best allocation. These works are based on the fact that all private firms behave as profit-maximizers like other works on a mixed oligopoly. In this paper, we show that the results obtained by such existing works hold even if each private firm's objective is not profit maximization.

We finally note the following 4 important assumptions to derive an irrelevance result originated from Poyago-Theotoky: (1) all private firms have symmetric objective functions among them; (2) the public firm maximizes social welfare; (3) the objectives among the public firm and the private firms are symmetric when the public firm is privatized; (4) all firms have an identical production technology.

Chapter 7

Robustness of “Endogenous timing in a mixed duopoly: Price competition”

7.1 Introduction

Mixed markets wherein the government-owned firm(s) and purely private firm(s) compete are seen world wide. We see various public firms – of which the government partly or completely owns shares and has some control over the firms – in several industries such as banking (e.g., JP Bank in Japan), insurance (e.g., JP insurance in Japan), oil (e.g., PDVSA in Venezuela), motor vehicle (e.g., Renault in France and Volkswagen in Germany), railway (e.g., SNCF in France and Amtrak in the U.S.), heavy industry (e.g., BAE systems in the U.K. and Finmeccanica in Italy), and public utility (e.g., GDF Suez in France and EDF in France). Some public firms are key players in international competition.¹ Recently, the U.S. government introduced capital injection for firms such as American Insurance Group and General Motors.² Since public firms or capital-injected firms often face political pressure,

¹For example, GDF Suez is a major supplier of natural gas to the U.K., Belgium, and Germany.

²The U.S. government is not the only actor in terms of the recent capital injection. Some newspapers have reported that the Taiwanese authority, for example, plans to introduce capital injection in the semiconductor industry.

these firms do not necessarily act as profit maximizers.³ Thus, analyses of mixed markets are needed to understand this situation.

Since the seminal work of De Fraja and Delbono (1989), many in-depth theoretical analyses on mixed oligopoly have been conducted.⁴ In this paper, we focus on those that treat the endogenous timing game with a mixed oligopoly. Since the government can affect the behavior of public firms, it may be able to choose the most desirable timing of these firms in terms of social welfare. In other words, endogenous timing would have a non-negligible policy implication. With respect to the timing game, several papers have been published in the framework of the observable delay game introduced by Hamilton and Slutsky (1990). Pal (1998) considers a mixed oligopoly with quantity competition in a homogenous goods market and shows that the timing in the equilibrium is two-type sequential decision making the duopoly case: private(public) leader with public(private) follower.⁵ Matsumura (2003) analyzes quantity competition between one public firm and one private firm that is owned by a foreign investor. He demonstrates that the timing in the equilibrium is sequential decision making with public leadership. Bárcena-Ruiz (2007) focuses on a mixed duopoly with price competition between a public firm and a domestic private firm under differentiated product markets. He assumes linear demand functions and linear cost functions, and he points out that both firms have incentives to be the first

³For example, Japan Post (JP) was forced to change its business plan so as to rebuild its office in central Tokyo due to political pressure. Some analysts claim that the result of this will be a reduction in JP's profits.

⁴Examples of recent research are as follows: Ishibashi and Matsumura (2006) consider R&D development. Han and Ogawa (2008) deal with international competition. Bárcena-Ruiz and Garzón (2006) focus on environmental policy.

⁵Matsumura and Ogawa (2007) consider partial privatization with the model analyzed by Pal (1998) and show that private leadership is more robust than public leadership. However, Matsumura (2003) shows a contrastive result. In other words, the shareholders are important for the timing structure in equilibrium if there is quantity competition.

mover, and therefore they choose the first period.

Here, we attempt to check the robustness of the result shown by Bárcena-Ruiz (2007) by extending his model so as to deal with various shareholding structures. This extension allows us to analyze the case where the competitor is partly or completely owned by foreign investors, and we find that the result is robust.

The remainder of this paper is organized as follows. Section 7.2 explains the model. Section 7.3 analyzes three fixed timing games as the subgames of the endogenous timing game. Section 7.4 investigates the equilibrium outcome and presents the results. Section 7.5 provides the conclusion of the paper. Appendix provides the proof of proposition and the results of the calculation.

7.2 Model

Suppose that there is an economy with a duopolistic sector where one public firm (firm 0) and one private firm (firm 1) produce differentiated products and a competitive numeraire sector. The representative consumer maximizes $U(q_0, q_1) + y$ subject to $p_0q_0 + p_1q_1 + y \leq m$, where q_i ($i = 0, 1$) denotes the amount of product i and p_i denotes its price, y denotes the amount of the numeraire good whose price is normalized to 1, and m represents the total income of the representative consumer. $U(q_0, q_1)$ is a quadratic and strictly concave function: $U(q_0, q_1) = a(q_0 + q_1) - (q_0^2 + 2bq_0q_1 + q_1^2)/2$, where a is a sufficiently large positive number and $b \in (0, 1)$.⁶ Solving the maximization problem of the representative consumer,

⁶This function is based on Singh and Vives (1984) and used in Bárcena-Ruiz (2007). A sufficiently large a guarantees that the maximized profits are strictly positive even if the rival firm sets the own price at 0.

we obtain the following demand functions:

$$q_i(p_0, p_1) = \frac{(1-b)a - p_i + bp_j}{(1-b)(1+b)}, \quad (i \neq j, i = 0, 1). \quad (7.1)$$

These demand functions (7.1) have the following properties,

$$\frac{\partial q_i(p_0, p_1)}{\partial p_i} < 0, \frac{\partial q_i(p_0, p_1)}{\partial p_j} > 0, \text{ and } \left| \frac{\partial q_i(p_0, p_1)}{\partial p_i} \right| > \left| \frac{\partial q_i(p_0, p_1)}{\partial p_j} \right|, (i \neq j, i, j = 0, 1) \quad (7.2)$$

The first inequality of (7.2) indicates that the demand function strictly satisfies the law of demand. The second inequality suggests that the products are substitutes. The third inequality means that the “own effect” is larger than the “cross effect.”

Hereafter, we focus on the duopolistic sector. The duopolists face demand function $q_i(p_0, p_1)$ and choose p_i . Let c denote the marginal cost for each firm.⁷ We assume $c > 0$. The profit of firm i is given by $\pi_i(p_0, p_1) \equiv (p_i - c)q_i(p_0, p_1)$. Note that $q_0(c, 0) > 0$ and $q_1(0, c) > 0$ under a sufficiently large a .

Firm 0 is a domestic public firm that maximizes domestic social welfare, and firm 1 is owned not only by domestic private investors but also by foreign private ones, and it maximizes its own profits. Domestic social welfare W is the sum of consumer surplus and producer surplus, and it is given by

$$W(p_0, p_1) = U(q_0(p_0, p_1), q_1(p_0, p_1)) - \sum_{i=0}^1 p_i q_i(p_0, p_1) + \pi_0(p_0, p_1) + \theta \pi_1(p_0, p_1) + m, \quad (7.3)$$

where θ represents the ratio between domestic and foreign private investors. When $\theta = 1$ ($\theta = 0$), firm 1 is owned only by domestic private investors (foreign private investors). In this case, the profit of firm 1 is included (not included) in social welfare.

⁷The reader may be interested in the case where the marginal cost is increasing. If we use the following quadratic cost function, $c(q_i) = cq_i^2/2$, ($i, j = 0, 1$), we find that there is a unique equilibrium wherein both firms choose the first period in the observable delay game.

Using (7.1) and (7.2), we can find the following properties of $W(p_0, p_1)$ and $\pi_i(p_0, p_1)$:⁸

$$\frac{\partial^2 \pi_i(p_0, p_1)}{\partial p_i^2} < 0, \quad \text{and} \quad \left| \frac{\partial^2 \pi_i(p_0, p_1)}{\partial p_i^2} \right| > \left| \frac{\partial^2 \pi_i(p_0, p_1)}{\partial p_j \partial p_i} \right|, \quad (7.4)$$

$$\frac{\partial^2 W(p_0, p_1)}{\partial p_0^2} < 0 \quad \text{and} \quad \left| \frac{\partial^2 W(p_0, p_1)}{\partial p_1^2} \right| > \left| \frac{\partial^2 W(p_0, p_1)}{\partial p_0 \partial p_1} \right|, \quad (7.5)$$

$$\frac{\partial^2 \pi_1(p_0, p_1)}{\partial p_0 \partial p_1} > 0 \quad \text{and} \quad \frac{\partial^2 W(p_0, p_1)}{\partial p_1 \partial p_0} \geq 0. \quad (7.6)$$

(7.4) and (7.5) imply that the second order condition of the maximization problem of each firm is satisfied. (7.6) implies that the actions of the firms are strategic complements unless θ is equal to 0.⁹

We now present the observable delay game. The firms independently announce the time at which they will set their prices and are committed to this choice. After the announcement, each firm then chooses the price, knowing when the other firm will set its price. Formally, the game is played as follows. In the first stage, each firm independently selects $t_i \in \{I, II\}$, where t_i indicates the time when price p_i is set. In the second stage, there are two periods (period *I* and period *II*) in which to set prices. $t_i = I$ implies that firm i sets its price in period *I* and $t_i = II$ implies that it sets its price in period *II*. In the second stage, after observing t_0 and t_1 , firm i , selecting $t_i = I$, chooses its price p_i first. If firm i selects $t_i = II$, it delays the decision until the next period. In period *II*, the price set by the firm that selects period *I* is observed, and then the firm that selects period *II* sets its price. At the end of this game, the market opens and each firm sells its output,

⁸From (7.1) and (7.2), we obtain

$$\frac{\partial^2 W}{\partial p_1 \partial p_0} = \theta \frac{\partial q_1}{\partial p_0} \geq 0, \quad \frac{\partial^2 W}{\partial p_0^2} = \frac{\partial q_0}{\partial p_0} < 0, \quad \frac{\partial^2 \pi_1}{\partial p_0 \partial p_1} = \frac{\partial q_1}{\partial p_0} > 0, \quad \text{and} \quad \frac{\partial^2 \pi_i}{\partial p_i^2} = 2 \frac{\partial q_i}{\partial p_i} < 0, \quad (i = 0, 1).$$

Note that $\partial q_1 / \partial p_0 = \partial q_0 / \partial p_1$ for the symmetry of the demand function.

⁹If $\theta = 0$, the decision of the public firm is independent from its rival's price.

which is equal to the demand, q_i .

7.3 Fixed timing game

Before presenting the equilibrium outcome in the observable delay game, we investigate each of the games of fixed timing. The fixed timing games are as follows: (S) both firms select the same period ($t_0 = t_1 = I$ or II); (L) firm 0 selects $t_0 = I$ and firm 1 selects $t_1 = II$; (F) firm 1 selects $t_1 = I$ and firm 0 selects $t_0 = II$. We use the superscript of S , L , and F in order to indicate the equilibrium outcome of each fixed timing game.

7.3.1 Game (S)

First, we consider fixed timing game (S). The first order condition of each firm's maximization problem is given by

$$\frac{\partial W(p_0, p_1)}{\partial p_0} = (p_0 - c) \frac{\partial q_0(p_0, p_1)}{\partial p_0} + \theta(p_1 - c) \frac{\partial q_1(p_0, p_1)}{\partial p_0} = 0, \quad (7.7)$$

$$\frac{\partial \pi_1(p_0, p_1)}{\partial p_1} = q_1(p_0, p_1) + (p_1 - c) \frac{\partial q_1(p_0, p_1)}{\partial p_1} = 0. \quad (7.8)$$

From equation (7.8), we can derive the reaction function of firm 1 and denote $p_1 = r_1(p_0)$. From (7.4) and (7.6), we can find $0 < r_1'(p_0) < 1$. From $q_1(0, c) > 0$ and (7.2), we can find $r_1(p_0) > c$ for all $p_0 \geq 0$.

Substituting $r_1(p_0)$ into p_1 in equation (7.7), we obtain the following Lemma 7.1.

Lemma 7.1. $c \leq p_0^S < p_1^S$ for all $\theta \in [0, 1]$, where a strict inequality holds when $\theta \in (0, 1]$.

Lemma 1 is derived by using $r_1(p_0) > c$, $|\partial q_0/\partial p_0| \geq |\theta(\partial q_1/\partial p_0)|$, $\partial q_0/\partial p_0 < 0$, and $\theta(\partial q_1/\partial p_0) \geq 0$.

7.3.2 Game (L)

Second, we consider fixed timing game (L). First, we consider period *II*. In this period, firm 1 prices $r_1(p_0)$, from equation (7.8). Given firm 1's pricing, firm 0 chooses the price to maximize its objective, $W^l(p_0) \equiv W(p_0, r_1(p_0))$, in period *I*. Solving the maximization problem, we obtain the following first order condition:

$$\frac{dW^l(p_0)}{dp_0} = (p_0 - c) \left(\frac{\partial q_0}{\partial p_0} + \frac{\partial q_0}{\partial p_1} r'_1 \right) + \theta(r_1 - c) \left(\frac{\partial q_1}{\partial p_0} + \frac{\partial q_1}{\partial p_1} r'_1 \right) - (1 - \theta)q_1 r'_1 = 0. \quad (7.9)$$

From (7.8), we can find

$$q_1 = -(p_1 - c) \frac{\partial q_1}{\partial p_1}. \quad (7.10)$$

Using (7.10), we rewrite the first order condition in (7.9), and then,

$$\frac{dW^l(p_0)}{dp_0} = (p_0 - c) \left(\frac{\partial q_0}{\partial p_0} + \frac{\partial q_0}{\partial p_1} r'_1 \right) + (r_1 - c) \left(\theta \frac{\partial q_1}{\partial p_0} + \frac{\partial q_1}{\partial p_1} r'_1 \right) = 0. \quad (7.11)$$

Solving the above equation with regard to p_0 , we obtain the equilibrium price of the public firm p_0^L , and this allows us to also obtain $p_1^L = r_1(p_0^L)$.¹⁰ Note that with regard to (7.11), we find that $\partial q_0/\partial p_0 + (\partial q_0/\partial p_1)r'_1 < 0$ and $r_1 - c > 0$, whereas $\theta(\partial q_1/\partial p_0) + (\partial q_1/\partial p_1)r'_1$ is ambiguous. p_0^L can be larger, smaller, or equal to c ; it depends on the sign of $\theta(\partial q_1/\partial p_0) + (\partial q_1/\partial p_1)r'_1$.¹¹

Note that $p_0^L < p_1^L$ for all $\theta \in [0, 1]$. The reason is as follows. From the analysis in game (S), we find $r_1(0) > c$ and $0 < r'_1 < 1$. Hence, $p_1 = r_1(p_0)$ and $p_1 = p_0$ have a unique intersection at (\bar{p}, \bar{p}) , where $\bar{p} > c$. As we know that p_0^L satisfies $p_0^L \cdot dW^l/dp_0 = 0$ and

¹⁰There is a possibility that the solution of (7.11) may be negative. In this case, $p_0^L = 0$.

¹¹From (7.11), we obtain $\text{sign}(p_0^L - c) = \text{sign}(\theta(\partial q_1/\partial p_0) + (\partial q_1/\partial p_1)r'_1)$. Using the specific function, we derive $\text{sign}(p_0^L - c) = \text{sign}(\theta - 1/2)$.

dW^l/dp_0 is a decreasing function with regard to p_0 , evaluating dW^l/dp_0 at $p_0 = \bar{p}$, we find the following relationship.

$$\left. \frac{dW^l(p_0)}{dp_0} \right|_{p_0=\bar{p}} = (\bar{p} - c) \left\{ \left(\frac{\partial q_0}{\partial p_0} + \theta \frac{\partial q_1}{\partial p_0} \right) + \left(\frac{\partial q_0}{\partial p_1} + \frac{\partial q_1}{\partial p_1} \right) r'_1 \right\} < 0. \quad (7.12)$$

Thus, we obtain $p_0^L < p_1^L$ for all $\theta \in [0, 1]$.

7.3.3 Game (F)

Finally, we consider fixed timing game (F). First, we consider period *II*. In this period, firm 0 prices $r_0(p_1)$, from equation (7.7). Using (7.5) and (7.6), we find that $r_0(p_1)$ satisfies

$$r'_0(p_1) > 0, \quad |r'_0(p_1)| < 1, \quad r_0(c) = c, \quad \text{when } \theta \in (0, 1]. \quad (7.13)$$

Note that firm 0 prices c when $\theta = 0$.

Given firm 0's pricing, firm 1 chooses the price to maximize its objective $\pi_1^f(p_1) \equiv \pi_1(r_0(p_1), p_1)$ in period *I*. The first order condition is

$$\frac{d\pi_1^f}{dp_1} = q_1 + (p_1 - c) \left(\frac{\partial q_1}{\partial p_0} r'_0 + \frac{\partial q_1}{\partial p_1} \right) = 0. \quad (7.14)$$

Solving the above equation with regard to p_1 , we obtain the equilibrium prices $p_0^F = r_0(p_1^F)$ and p_1^F when $\theta \in (0, 1]$. In this situation, we find $p_i^F > c$ because $(\partial q_1/\partial p_0)r'_0 + \partial q_1/\partial p_1 < 0$, $r_0(c) = c$, and $q_1(c, c) > 0$. We can show that $p_0^F < p_1^F$ since $p_i^F > c$, $r_0(c) = c$, and $0 < r'_0 < 1$. We note that if $\theta = 0$, the equilibrium price is $(p_0^F, p_1^F) = (c, r_1(c))$. Since $p_0^F = c < r_1(c) = p_1^F$, $p_0^F < p_1^F$. Therefore, we obtain $p_0^F < p_1^F$ regardless of θ .

7.4 Equilibrium in the observable delay game

Now, we derive the whole game by comparing the three fixed-timing games: games (S), (L), and (F).

7.4.1 Comparison of the equilibrium prices

First, we compare the equilibrium prices of each firm among games (S), (L), and (F).

Comparison between games (S) and (L)

Note that dW^l/dp_0 is a monotonically decreasing function in p_0 because $d^2W^l/dp_0^2 < 0$.

Evaluating (7.9) at p_0^S , we check whether the sign is positive or negative.

$$\left. \frac{dW^l}{dp_0} \right|_{p_0=p_0^S} = \left\{ (p_0^S - c) \frac{\partial q_0}{\partial p_1} + (r_1(p_0^S) - c) \frac{\partial q_1}{\partial p_1} \right\} r'_1 < 0, \quad (7.15)$$

because (7.2), Lemma 7.1, and $r'_1 > 0$. Thus, we obtain $p_0^L < p_0^S$. As we know that $p_1^L = r_1(p_0^L)$, $p_1^S = r_1(p_0^S)$, and $r'_1 > 0$, we obtain the following Lemma 7.2.

Lemma 7.2. $p_i^L < p_i^S$ for all $\theta \in [0, 1]$, ($i = 0, 1$).

Comparison between games (S) and (F)

When $\theta = 0$, we can easily find that $p_i^S = p_i^F$. Therefore, we consider the case where $\theta \in (0, 1]$. Note that $d\pi_1^f/dp_1$ is a monotonically decreasing function in p_1 because $d^2\pi_1^f/dp_1^2 < 0$. Evaluating $d\pi_1^f/dp_1$ at p_1^S , we find

$$\left. \frac{d\pi_1^f}{dp_1} \right|_{p_1=p_1^S} = (p_1^S - c) \frac{\partial q_1}{\partial p_0} r'_0 > 0, \quad (7.16)$$

because (7.2), Lemma 7.1, and $r'_0 > 0$. Thus, we find

$$p_1^S < p_1^F. \quad (7.17)$$

Now, we examine the magnitude relation between p_0^S and p_0^F when $\theta \in (0, 1]$.

As we know that $p_0^F = r_0(p_1^F)$, $p_0^S = r_0(p_1^S)$, and $r'_0 > 0$, we obtain

$$p_0^S < p_0^F. \quad (7.18)$$

Summing up these results, we obtain the following Lemma 7.3.

Lemma 7.3. $p_i^S \leq p_i^F$, where a strict inequality holds $\theta \in (0, 1]$, ($i = 0, 1$).

From Lemma 7.2 and Lemma 7.3, we obtain the following Proposition 7.1.

Proposition 7.1. $p_i^L < p_i^S \leq p_i^F$ for all $\theta \in [0, 1]$, ($i = 0, 1$), where a strict inequality holds $\theta \in (0, 1]$.

7.4.2 Comparison of the profits of firm 1

Here we compare the profits of firm 1 among games (S), (L), and (F).¹²

First, we compare the profit of firm 1 in game (S) and that in game (L). The equilibrium prices of the two games lie on the reaction function of firm 1. We examine how an increase of the price of firm 0 on the reaction function of firm 1 influences the profit of firm 1.

$$\begin{aligned} \frac{d\pi_1(p_0, r_1(p_0))}{dp_0} &= \left\{ (r_1 - c) \frac{\partial q_1}{\partial p_1} + q_1 \right\} r_1' + (r_1 - c) \frac{\partial q_1}{\partial p_0}, \\ &= (r_1 - c) \frac{\partial q_1}{\partial p_0} > 0. \end{aligned} \tag{7.19}$$

By $r_1 > c$ and Lemma 7.2, we find $\pi_1^L < \pi_1^S$ for all $\theta \in [0, 1]$.

Second, we compare the profits of firm 1 between games (S) and (F). We can easily find $\pi_1^S = \pi_1^F$ when $\theta = 0$. We consider the case where $\theta \in (0, 1]$. By definition, firm 1 can choose the price which is equal to p_1^S in game (F). We know that $p_1^S \neq p_1^F$ in this range of θ ; therefore, we obtain $\pi_1^S < \pi_1^F$.

To sum up these results, we obtain the following Proposition 7.2.

Proposition 7.2. $\pi_1^L < \pi_1^S \leq \pi_1^F$ for all $\theta \in [0, 1]$, where a strict inequality holds $\theta \in (0, 1]$.

¹²We provide the equilibrium price of each firm, the profit of firm 1, and social welfare in each game in Appendix 7.A.

7.4.3 Comparison of welfare

First, we compare welfare between games (L) and (S). By definition, firm 0 can choose the price which is equal to p_0^S in game (L). We know that $p_0^L \neq p_0^S$; therefore, we obtain $W^S < W^L$.

Second, we compare welfare between games (S) and (F). We can easily find $W^S = W^F$ when $\theta = 0$. When we consider the case where $\theta \in (0, 1]$, it is difficult to find the proof without using calculation. Thus, we use it to obtain $W^F < W^S$.¹³

To sum up the results, we obtain the following Proposition 7.3.

Proposition 7.3. $W^F \leq W^S < W^L$ for all $\theta \in [0, 1]$, where a strict inequality holds $\theta \in (0, 1]$

Finally, we derive the equilibrium of the observable delay game in our model. We restrict our attention to the equilibrium which is not supported by a weakly dominated strategy of either firm at least.¹⁴ From Propositions 7.2 and 7.3, we obtain the following Proposition 7.4.

Proposition 7.4. *In a subgame perfect Nash equilibrium (SPNE), both firms choose $t_0 = t_1 = I$.*

The intuition behind Proposition 7.4 is as follows. The public firm has an incentive to lower the prices, whereas the private firm has an opposite-directed incentive, i.e., to raise

¹³For the proof, see Appendix 7.B.

¹⁴When $\theta = 0$, there is another SPNE, supported by a weakly dominated strategy of the public firm and a weakly dominant strategy of the private firm, in which the public firm chooses $t_0 = II$ and the private firm chooses $t_1 = I$.

the prices. Since prices are a strategic complement, both firms desire to be the first mover. These incentives are highlighted in Bárcena-Ruiz (2007).

The aforementioned incentive of the public firm is affected by the shareholding structures. Since the public firm takes into account the profit of the domestic shareholder, the incentive becomes weak (strong) when the ratio of domestic shareholders is high (low). Therefore, Proposition 7.4 holds.

7.5 Concluding remarks

We examine the observable delay game of a mixed duopoly in price competition under differentiated product markets. We extend the model proposed by Bárcena-Ruiz (2007) with respect to the shareholding structure. We find that the result of Bárcena-Ruiz (2007) that both firms choose the first period is robust, even though we modify the model.

We note that the ratio of shareholders is significant because it may be the reduced form of global competition with respect to the corporate origin. However, not only the ratio of domestic firms but also the total number of firms may affect the result.¹⁵ This issue remains for future research.

¹⁵Lu (2006) deals with quantity competition among one public firm, n domestic private firms, and m foreign private firms in a homogeneous goods market.

Appendix 7.A

The equilibrium price of each firm, the profit of firm 1, and social welfare in each game

Game (S)

$$p_0^S = \frac{(1-b)ab\theta + (2-b\theta)c}{2-b^2\theta}, \quad p_1^S = \frac{(1-b)a + (1+b-b^2\theta)c}{2-b^2\theta},$$

$$\pi_1^S = \frac{(1-b)(a-c)^2}{(1+b)(2-b^2\theta)^2},$$

$$W^S = \frac{\{5+3b+2(1-b-3b^2-b^3)\theta - (1-b-2b^2)b^2\theta^2\}(a-c)^2}{2(1+b)(2-b^2\theta)^2} + m.$$

Game (L) ¹⁶

$$p_0^L = \frac{b(1-b)(2\theta-1)a + (4+b-2b^2-2b\theta)c}{4-b^2-2b^2\theta},$$

$$p_1^L = \frac{(1-b)(2-b^2)a + (2+2b-b^3-2b^2\theta)c}{4-b^2-2b^2\theta},$$

$$\pi_1^L = \frac{(1-b)(2-b^2)^2(a-c)^2}{(1+b)(4-b^2-2b^2\theta)^2},$$

$$W^L = \frac{\{5+3b-b^2-b^3+2(1-b-2b^2)\theta\}(a-c)^2}{2(1+b)(4-b^2-2b^2\theta)} + m.$$

Game (F)

$$p_0^F = \frac{(1-b)ab\theta + (2-b\theta-b^2\theta)c}{2(1-b^2\theta)}, \quad p_1^F = \frac{(1-b)a + (1+b-2b^2\theta)c}{2(1-b^2\theta)},$$

$$\pi_1^F = \frac{(1-b)(a-c)^2}{4(1+b)(1-b^2\theta)},$$

$$W^F = \frac{\{5+3b+2(1-b-6b^2-2b^3)\theta - (3-3b-8b^2)b^2\theta^2\}(a-c)^2}{8(1+b)(1-b^2\theta)^2} + m.$$

¹⁶Note that the equilibrium outcomes in game (L) are derived by considering the case of an interior solution. We omit them in the case of a corner solution ($p_0^L = 0$).

Appendix 7.B

Proof of the latter of Proposition 7.3. Since we cannot show the relationship without specific functions, the proof uses them.

$$W^S - W^F = -\frac{(a-c)^2(9b^2\theta - 2b^2\theta^2 - 4b^4\theta^2 + b^4\theta^3 - 4)(1-b)b^2\theta}{8(b^2\theta - 1)^2(b^2\theta - 2)^2(b+1)}. \quad (7.20)$$

Therefore, if $y(b, \theta) \equiv 9b^2\theta - 2b^2\theta^2 - 4b^4\theta^2 + b^4\theta^3 - 4 < 0$, $W^S - W^F > 0$.

$$\begin{aligned} \frac{\partial y}{\partial \theta} &= b^2 (3b^2\theta^2 - 8b^2\theta - 4\theta + 9), \\ &= b^2[(8 - 3\theta)(1 - b^2\theta) + 1 - \theta]. \end{aligned} \quad (7.21)$$

Since $b < 1$, (7.21) is larger than 0. Substituting θ with 1,

$$y(b, 1) = -(1-b)(1+b)(4-3b^2) < 0. \quad (7.22)$$

Hence, $W^S - W^F > 0$. □

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