ON AFTER-SHOCKS.

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1.—The present note is an abstract of the paper "On After-shocks," to be published in the Report on the Mino-Owari Earthquake.

We know that any earthquake of some severity is invariably followed by a few weaker ones. Violent destructive earthquakes are followed by hundreds or thousands of minor shocks. The occurrence of after-shocks is quite natural, and it would be very strange if they did not occur. Should the latter be the case, the disturbed tract is probably being prevented from subsiding into a state of equilibrium, and may be preparing for a second strong shock. The materials utilized in this paper are those relating to the Kumamoto Earthquake of 1889, the Mino-Owari Earthquake of 1891, and the Kagoshima Earthquake of 1893.

The monthly numbers of after-shocks of the Mino-Owari Earthquakes, as recorded at several Meteorological Stations, are given at the end of the paper.

2.—Of the 3364 shocks recorded at the Gifu Meteorological Station, during two complete years between October 28th, 1891, and October, 31st, 1893, 11 were "violent," 97 were "strong," 1809 were "weak," 1,039 were "slight," and the remaining 408 were "sounds." Immediately succeeding the initial great shock, strong shocks occurred very often, but these became rarer and rarer as the days advanced. Of the 11 "violent" shocks, 10 occurred within the first four months, and the remaining 1 in September, 1892. The "strong" shocks occurred all within the first 13 months, and the "weak" shocks all within the first 20 months. Again, in the earlier epoch, namely, within the first three or four months, "weak shocks occurred most abundantly, while in the later epoch "slight" shocks occurred most abundantly, until finally we have only these latter and the "sounds." It may be supposed that points of greater unstability at the earthquake origin are being removed earlier than those of smaller unstability.

3.—The total number of after-shocks of the Kumamoto Earthquake recorded at Kumamoto up to the end of October, 1893, is 917, including two "violent" shocks which happened respectively on July 28th (initial earthquake) and August 3rd, 1889, 72 "strong" shocks, 843 "weak" and "slight" shocks and "sounds." These were all small shocks, and only the two "violent" ones reached to a distance greater than 20 *ri* from the origin (*1 ri = 2·4 miles*).

4.—In the case of these great earthquakes we have mentioned all the after-shocks have been weaker than the respective initial earthquakes. This was true also of the Yedo Earthquake in the 2nd year of Ansei (1855), and probably true of all, or nearly all, earthquakes.

5.—In the following table are compared the "frequencies" and "intensities" of after-shocks for the same three earthquakes:

"Frequency" of shocks.	I. Mino-Owari Eq.	II. Kuma-moto Eq.	III. Kago-shima Eq.	Ratio $\frac{I}{II}$	Ratio $\frac{I}{III}$
During the first 30 days	1750	341	279	5.1	6.2
During the first 2 years	3364	834		4.0	

"Activity" of shocks.	I. Mino-Owari Eq.	II. Kuma-moto Eq.	III. Kago-shima Eq.	Ratio $\frac{I}{II}$	Ratio $\frac{I}{III}$
During the first 30 days	1821	387	294	4.8	6.2
During the first 2 years	3483	907		3.8	

By the term "frequency" is meant the total number of shocks which happened during the intervals of time stated; and by the term "activity" is meant the sum of weights of these shocks taken respectively as 3, 2, and 1 for "violent," "strong," and "weak" (or "slight" or case of mere "sound") shocks.

Now the total (land and sea) areas of disturbance of the Mino-Owari, Kumamoto, and Kagoshima Earthquakes were respectively about 54,000, 6,500, and 5,000 square *ri*. The magnitudes of the earthquakes, if they are supposed to be proportional to their areas, are as to 11 : 1.3 : 1. The above tables show the "frequency" (or "activity") of the first earthquake to be 4 or 5 times greater than that of the second, and 6 times greater than that of the third. If we compare the two earthquakes of Kumamoto and Kagoshima, we find that the ratio of the "frequency" (or "activity") of their after-shocks as represented by the total numbers during the first 30 days is 1.3, which is exactly equal to the ratio of their areas of disturbance. The proportionality of the frequency and area is not verified between these two earthquakes and the Mino-Owari earthquake, the frequency (or activity) for the latter being distinctly smaller in proportion to its area. It is probably because the town of Gifu where the after-shocks have been recorded is not in a district which may be regarded as the proper or principal centre in the epifocal zone. Anyhow, it seems certain that the frequency (or activity) of after-shocks increases with the magnitude of the initial earthquake.

6.—Two specimen curves showing the relation of the frequency of after-shocks with time are given at the end of the paper. These curves will be seen to be very like rectangular hyperbolas.

7.—One conclusion is that the "frequency" or "activity" of after-shocks may be regarded as a measure of the magnitude of the initial earthquake that caused them. Thus in the case of the Kumamoto earthquake, the first great shock of July 28th, 1889, was followed by a second great shock five days later, on August 3rd. The second shock was thought by some people to be nearly as great as the first. But according to the above criterion, I should think that the first was double or treble as great as the second, their consequent activities of after-shocks being approximately as 90:30.

8.—To deduce theoretically the time relation of the frequency (or activity) of after-shocks, I shall assume, first, that the "activity" at any moment is proportional to the magnitude of the disturbance in the geotectonic condition then existing at the origin of the earthquake (this supposition seems probable); and, secondly, that the reduction of this latter quantity depends on the corresponding activity of after-shocks, that is, we suppose that the after-shocks remove so many points of instability or weakness at the focus. We must also suppose that each of the after-shocks has its own series of secondary after-shocks depending on its magnitude.

Let y = the activity of after-shocks at any time x;

m = the magnitude of disturbance at the focus at the same time, expressed in any arbitrary measure; and k, k^1 some constants.

The first supposition gives

$$y = k.m \; ; \qquad (a)$$

and the second gives

$$-dm = k.^1 \, y . \, dx - k^{-1} y \, dx \qquad (b)$$

These two equations give

$$-\frac{dy}{k} = y dx \; (k^1 - \tfrac{1}{k}),$$

or, on integrating,

$$\log y = -x \; (kk^1 - 1) - \log. \, c.$$

or

$$y = \frac{1}{c} e^{-x \, (kk^1 - 1)} = a . \, b^{-x}, \qquad (c)$$

where c, a and b are constants.

9.—The equation (c) as above obtained seems to be a likely one, when considered from the analogy of other physical phenomena. In applying it to practical cases, however, this is found to be entirely different from the truth, the form of the after-shock curves being, indeed, like a conic section. We shall therefore try in another way, and assume

$$y = k\,(h + x)^{-1} + k^{1}\,(h + x)^{-2} + \ldots$$

where y denotes as before the activity of after-shocks at any time x, and h, k_1 k^1...are constants. For a first approximation, we shall take

$$y = k\,(h+x)^{-1}$$

or

$$y\,(h+x) = k, \qquad (d)$$

which represents a rectangular hyperbola, and imply that the activity varies nearly inversely with time.

MINO-OWARI EARTHQUAKE.

10.—We shall here apply equation (d) to the after shocks of the Mino-Owari Earthquake. Taking the numbers expressing the "activities" for the successive twenty-two days between October 29th and November 19th, 1891, and applying the method of least squares, I obtain, for the Gifu Observation,

$$y = \frac{519.1}{x + 1.535} \qquad (e)$$

in which the middle of October 29th is the origin of time and x is expressed in days.

Similarly, by taking the numbers expressing monthly activities at Gifu for the successive 18 months, from November, 1891, till April, 1893, and applying the method of least squares, I find,

$$y = \frac{16.91}{x + 0.397} \ldots\ldots\ldots\ldots\ldots\ldots (f)$$

in which the time origin is the middle of November, 1891, and x is expressed in months, y being the total "activity" for the corresponding month divided by 30, or 31, as the case may be.

The values of y for some particular values of x calculated from these two equations are tabulated below:—

x	Date.	Daily Activity (actual).	Same, calculated by eq. (e.)
0	1891, 10, 29	333	338
1	30	193	205
2	31	129	147
10	11,8	43	45
20	18	18	24
40	12,8	18	13
100	1892, 2, 6	4	5

x	Date.	Monthly Activity divided by the number of days of month (Actual).	Same (calculated by eq. f).
0	1891, 11	37.	43.
1	12	14.	12.
2	1892, 1	6.	7.
20	1893, 7	0.6	0.8

The two equations (e) and (f) will thus be seen to give satisfactory results. They are, of course, purely empirical, and we have no right to extend their application for values of x greater than those concerned in deducing them. By way of a trial, y's for great values of x have been calculated as shown below:—

From (e), we obtain,—

(1).— If $y=1$, $x=520$ days, or 1.4 years.
$\frac{1}{7}$ 3,600 days, or 10 years.
$\frac{1}{15}$ 7,800 days, or 21 years.
$\frac{1}{30}$ 15,600 days, or 43 years.

From (f), we obtain,—

(2).— If $y=1$, $x=16.5$ months, or 1.4 years.
$\frac{1}{7}$ 140 months, or 12 years.
$\frac{1}{15}$ 250 months, or 21 years.
$\frac{1}{30}$ 510 months, or 41 years.

The coincidence between the two results (1) and (2) is of course a matter of accidence. The meaning of these results is that the seismic "activity" (or frequency) after the elapsing of 1.4, 10, 20, and 40 years will be respectively such that one "weak" shock occurs every day, every week, every half-month, and every month. These numbers may probably be too great, and by quartering them we may venture to conclude that not until the lapse of at least some 10 years will the after-shocks or the residual effect of the great Mino-Owari Earthquake completely cease or be reduced to such a state of quiescence as having one weak shock every month. The thing is only a gross approximation, and must be taken as suggestive of the order of the quantities discussed. However, we can have an idea about the period when the minor earthquakes consequent to a big one should cease to happen.

KUMAMOTO EARTHQUAKE.

11.—I have also made similar calculations with respect to the after shocks of the Kumamoto Earthquake. I shall simply state the result, which is that 7 or 8 years will elapse before the shaken district about Kumamoto is sufficiently reduced to quiescence so as to have one "weak" shock per month. At the present date, when I am writing this note, namely, December, 1893, about 56 months after the first shock of July 28th, 1889, we have at Kumamoto on average two or three weak shocks per month.

12.—I shall not here enter into the discussion of problems which suggest themselves in connection with after-shocks (such as questions of distribution of seismic energy), but shall here briefly indicate how to estimate the probable total number of after-shocks of a great earthquake. Taking equation (*d*), we can readily demonstrate that the total number of shocks during the interval of time from $x=o$ to $x=n$ is *approximately* equal to the expression

$$k \log_{10}\left(\frac{h+n}{h}\right) \times \log_e 10 \qquad (h)$$

or, better by the expression

$$y_0 + k \log_{10}\left(\frac{h+n}{h+1}\right) \times \log_e 10 \qquad (g)$$

The probable total number of after-shocks of the Mino-Owari and the Kumamoto Earthquakes estimated by this means are respectively about 4500 and 1100.

13.—About periodicity in the frequency or activity of after-shocks.

What has been said thus far relates to the ultimate mean state of the frequency or activity of after-shocks. When examined closely, the after-shock curves will be seen to present a series of many maxima and minima, as are well shown in Pl. II.

We can readily conceive how the "activity" of after-shocks should be periodic. In deducing the equation of the "activity" as done in §8, we have assumed certain hypotheses, which concerned themselves simply with the state of distrubance at the origin. We must now suppose further that there is a kind of external action on the system, such as the weight of the superincumbent masses, which, acting continuously, finally produces fracture or faulting in the system, and thus gives rise to a maximum of the "activity" of shocks. The stress thus for a time relieved would gradually again be brought to a limit, when a second disturbance takes place, corresponding to a maximum of "activity," and so on.

The fluctuations in "activity" of after-shocks for a particular earthquake may consist of several sets of periods, and these may be different for different earthquakes. The average lengths of periods for the Mino-Owari Earthquake are 4.5 days, 33 days, and 6.3 months, and those for the Kumamoto Earthquake are 4.6 days, 33 days, and 9.4 months. Besides these three distinct sets of periods, there seems to exist also a series of shorter periods, only a few hours in length, and another set of periods much longer than those above mentioned may possibly exist.

I shall here note that regular fluctuations in the activity or frequency may also be detected with respect to ordinary small earthquakes. Thus, for instance, in the seismic frequency for Tokio, the annual period, and a longer period, about four years in length, can distinctly be seen. These questions will form the subject of another paper to be shortly given.

14.—The probable estimated total numbers (n) of after-shocks of the Mino-Owari Earthquake for several places, and the distances (r) of these latter from the central portion of the Neô Valley, where the shock had been strongest, are given in the following list :—

	Gifu.	Nagoya.	Tsu.	Kioto.	Osaka.	Tokio.
n......	4,500	2,000	350	140	80	30
r......	7 *ri*	15	25	25	36	68

Of the above six places, Gifu, Nagoya, and Tsu are situated along the elongated Meizoseismal Zone, and the remaining three places Kioto, Osaka, and Tokio are situated in directions more or less perpendicular to the latter. These two groups must be discussed separately.

15.—Relation of the barometric height and the "activity." I have examined the after-shocks of the Mino-Owari Earthquake in relation to the barometric heights and also in relation to the fluctuations of the latter, but found no definite result.

Kagoshima Earthquake.

16.—The Kagoshima earthquake is as yet only a few months old. The results of the calculations made about it are that the after-shocks will probably cease after two or three years, and that the total number of after-shocks will be about 600.

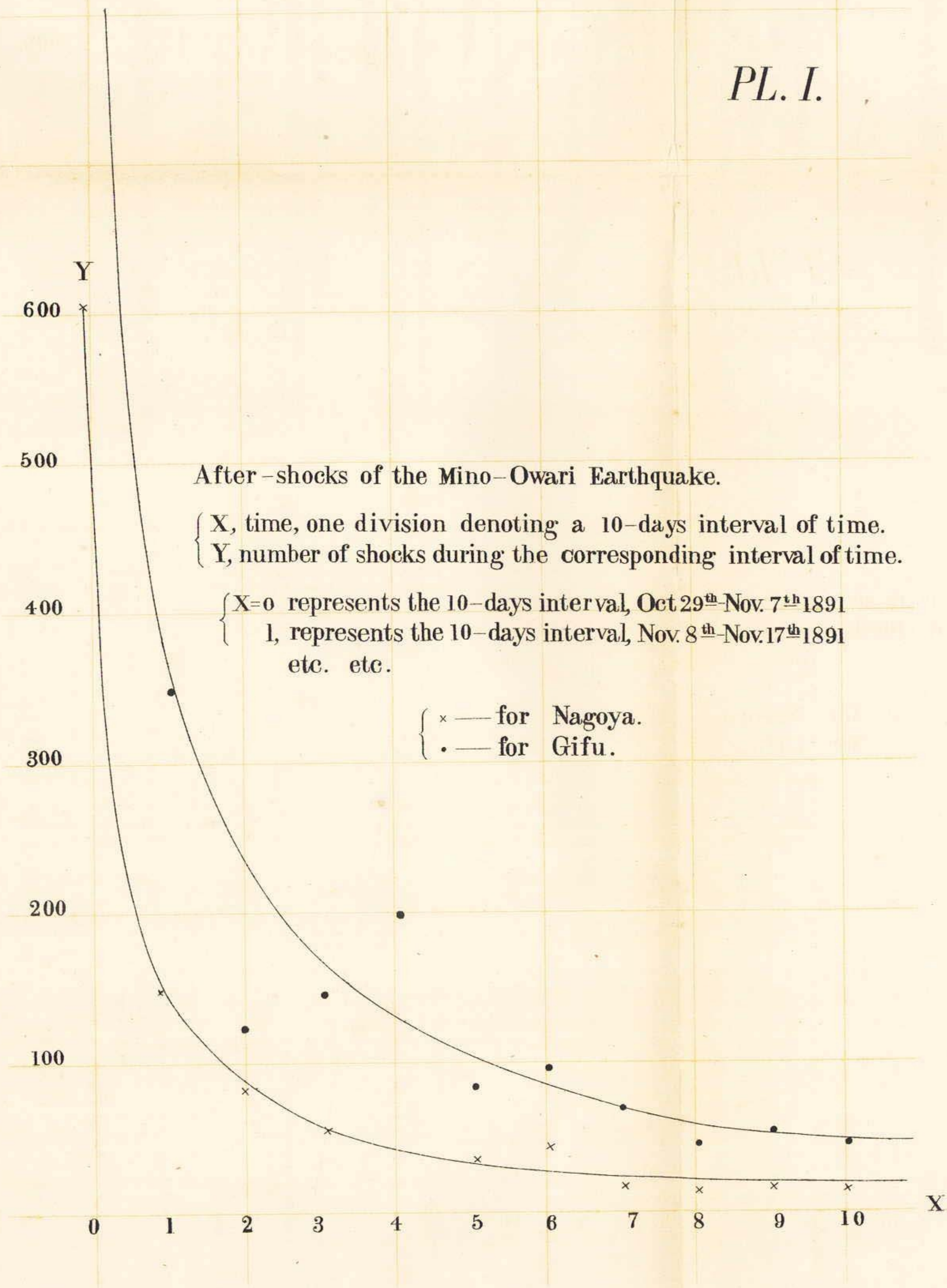
MONTHLY NUMBER OF AFTER-SHOCKS OF THE MINO-OWARI EARTHQUAKE.

(FOR THE FIRST TWO YEARS).

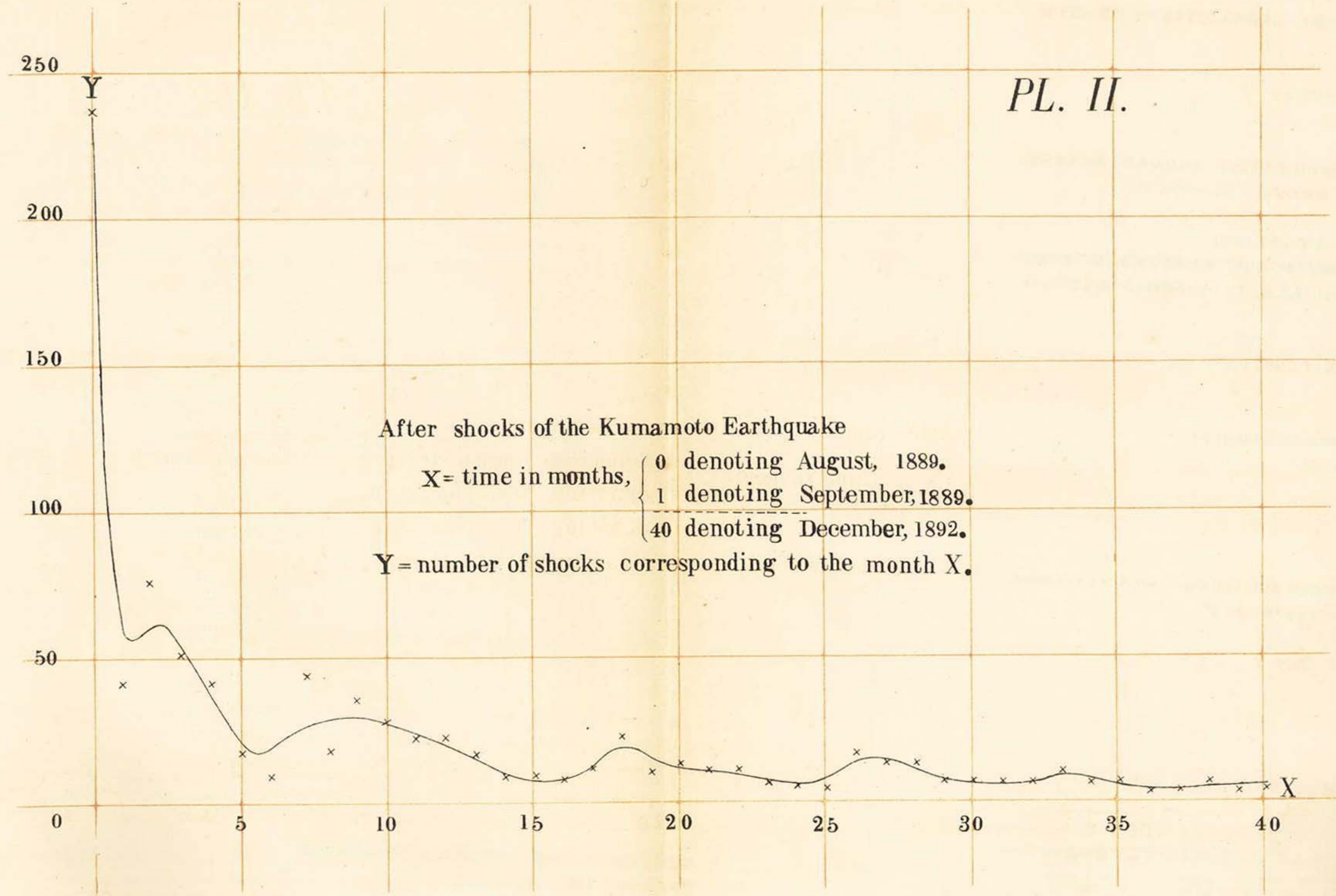
Year, Month.	Gifu.	Nagoya.	Tsu.	Osaka.	Kioto.
1891, 10	720	683	140	46	69
11	1087	415	89	7	20
12	416	112	24	4	10
1892, 1	164	43	12	2	5
2	114	29	10	2	5
3	87	86	7	1	1
4	90	11	4	0	2
5	54	11	1	0	0
6	30	12	1	0	3
7	35	4	1	0	0
8	52	15	7	0	1
9	107	13	2	1	1
10	47	7	4	0	2
11	68	11	0	0	0
12	39	10	2	2	2
1893, 1	31	5	2	0	2
2	20	4	2	0	1
3	52	9	1	1	0
4	59	10	1	0	1
5	32	14	1	2	0
6	12	9	3	1	0
7	18	11	0	0	0
8	13	11	0	1	0
9	20	8	0	0	0
10	16	6	0	1	1
Sum	3383	1549	314	71	126

December, 1893.

PL. I.


After-shocks of the Mino-Owari Earthquake.
X, time, one division denoting a 10-days interval of time.
Y, number of shocks during the corresponding interval of time.
X=0 represents the 10-days interval, Oct 29th-Nov. 7th 1891
1, represents the 10-days interval, Nov. 8th-Nov. 17th 1891
etc. etc.
× — for Nagoya.
• — for Gifu.
Y
600
500
400
300
200
100
0
1
2
3
4
5
6
7
8
9
10
X

PL. II.



250
200
150
100
50
0
Y
X
5
10
15
20
25
30
35
40
After shocks of the Kumamoto Earthquake
X = time in months, 0 denoting August, 1889.
1 denoting September, 1889.
40 denoting December, 1892.
Y = number of shocks corresponding to the month X.