Bull. Tokyo Univ. For., 101, 247-252 (1999)

Shear Strength of Wood Obtained by Torsion Test

Hiroshi YOSHIHARA* and Masamitsu OHTA**

1. Introduction

There are many cases that the failure in a material is drawn by the shear forces. Thus, it is important to know the shear strength of material.

There are several examples trying to measure the shear strength of wood by torsion test. Minami¹⁾ and Suzuki and Okohira²⁾ gave the shear strengths of several wood species under the hypothesis that the specimen keeps its elastic stress condition under the torsional loading. However, the shear stress-shear strain relationship shows the elastic-plastic deformation behavior, and hence, the plastic deformation should be taken into account for obtaining the shear strength more precisely. Okusa derived the shear strength of several softwood species by the torsion tests of specimens with elliptical cross sections.³⁾ In his method, however, finite element analysis should be used, and it is impossible to evaluate the shear strength only from the torsion testing data.

In our previous paper, we suggested that the shear stress/shear strain relationship of wood can be formulated by a function.⁴˒ ⁵⁾ Based on this result, we tried to derive the real shear strengths of several wood specimens from the data of torsion tests

2. Theories

2.1 Shear stress/shear strain relationship

Figure 1 shows the diagram of torsion of an wooden bar with a rectangular cross-section. When the bar is twisted around the z-axis, the shear strain at the center of the zx-plane, γ_{zx}, is represented as follows:⁴⁾

$$\gamma_{zx}=a^3bk\,\frac{G_{yz}}{G_{zx}}\,p_{zx}\theta \tag{1}$$

where θ is the torsional angle, a and b are the lengths in the directions of the x- and y-axes, respectively, and G_{zx} and G_{yz} are the shear moduli in the zx- and yz-planes, respectively. The values of p_{zx} and k are represented as follows:

$$p_{zx}=\frac{1}{a^2bk}\cdot\left[-\frac{8}{\pi^2}\sqrt{\frac{G_{zx}}{G_{yz}}}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{(2n-1)^2}\times\tanh\frac{(2n-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}}\right], \tag{2}$$

and

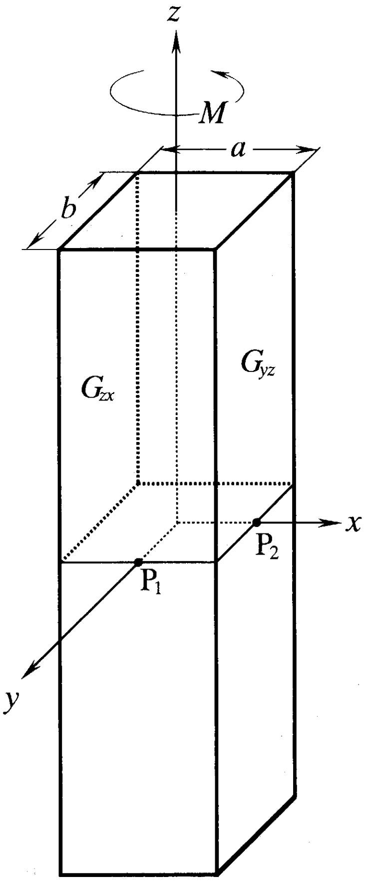

Fig. 1. Diagram of the rectangular subjected to the torsional force.

---

* Faculty of Science and Engineering, Shimane University.

** Department of Biomaterial Sciences, Graduate School of Agricultural and Life Sciences, The University of Tokyo.

$$k=\frac{1}{3}-\frac{2a}{b}\sqrt{\frac{G_{yz}}{G_{zx}}}\left(\frac{2}{\pi}\right)^{5}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{5}}\tanh\frac{(2n-1)\pi b}{2a}\sqrt{\frac{G_{zx}}{G_{yz}}}\,. \tag{3}$$

In the elastic strain range, the shear stress (τ_{zx})-shear strain (γ_{zx}) relationship can be represented as follows:

$$\tau_{zx}=G_{zx}\gamma_{zx}\,. \tag{4}$$

The shear stress-shear strain relationship in the plastic strain range is determined by the following procedure. When the shear strain in the plastic region is separated into the elastic component γ_{zx}^{e} and plastic component γ_{zx}^{p}, the torsional moment (M)-plastic strain relationship is represented by the following n-power function.[5]

$$\gamma_{zx}^{\mathrm{p}}=\alpha\left(\frac{M-M_{\mathrm{y}}}{M_{\mathrm{y}}}\right)^{n}, \tag{5}$$

where M_{y} is the torsional moment at the occurrence of yielding, α and n are the material parameters. On the contrary, the shear stress is represented by the torsional moment and the plastic shear strain as follows:

$$\tau_{zx}=p_{zx}\left\{M+q_{\mathrm{p}}\gamma_{zx}^{\mathrm{p}\,2}\frac{\mathrm{d}}{\mathrm{d}\gamma_{zx}^{\mathrm{p}}}\left(\frac{M-M_{\mathrm{y}}}{\gamma_{zx}^{\mathrm{p}}}\right)\right\}, \tag{6}$$

where q_{p} is a constant value of $0.2ab/(a^2+b^2)$.[1] Eliminating M from Eqs. (5) and (6), the plastic shear strain can be given as follows:

$$\gamma_{zx}^{\mathrm{p}}=\frac{\alpha}{\left(1+q_{\mathrm{p}}\dfrac{1-n}{n}\right)^{n}}\cdot\left(\frac{\tau_{zx}-S_{zx}}{S_{zx}}\right)^{n}, \tag{7}$$

where S_{zx} is the shear yield stress given by the following equation:

$$S_{zx}=p_{zx}M_{\mathrm{y}}\,. \tag{8}$$

From Eqs.(4) and (7), the total strain in the plastic region can be given as:

$$\gamma_{zx}=\gamma_{zx}^{\mathrm{e}}+\gamma_{zx}^{\mathrm{p}}=\frac{\tau_{zx}}{G_{zx}}+\frac{\alpha}{\left(1+q_{\mathrm{p}}\dfrac{1-n}{n}\right)^{n}}\cdot\left(\frac{\tau_{zx}-S_{zx}}{S_{zx}}\right)^{n}, \tag{9}$$

By Eqs. (4) and (9), the shear stress-shear strain relationship can be formulated all over the strain range. The shear strength, F_{zx}^{ep}, is obtained by substituting the strain value at the occurrence of failure, γ_{zx}^{f}, into Eq. (9) as follows:

$$\gamma_{zx}^{\mathrm{f}}=\frac{F_{zx}^{\mathrm{ep}}}{G_{zx}}+\frac{\alpha}{\left(1+q_{\mathrm{p}}\dfrac{1-n}{n}\right)^{n}}\cdot\left(\frac{F_{zx}^{\mathrm{ep}}-S_{zx}}{S_{zx}}\right)^{n}. \tag{10}$$

2.2 Shear strength obtained by the conventional method

As mentioned above, the shear strength was conventionally derived under the hypothesis that the specimen keeps its elastic stress condition under the torsional loading. According to this hypothesis, the second term in the braces of Eq. (6) is ignored, and the shear strength, F_{zx}^{qe}, is given as follows:

$$F_{zx}^{\mathrm{qe}}=p_{zx}M_{\mathrm{f}}\,, \tag{11}$$

where M_{f} is the torsional moment at the occurrence of failure.

3. Experiment

3.1 Specimen

Sitka spruce (*Picea sitchensis* Carr.), agathis (*Agathis* sp.), katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) and buna (Japanese beach, *Fagus crenata* Bl.) were used in this experiment. Specimens were conditioned at 20℃ and 65% relative humidity before and during the tests.

3.2 Torsion tests

Figure 2 shows the shape of the torsion test specimens. To avoid the stress concentrations at the grip parts, specimens were cut to the dog-bone shapes, and the LR- or LT-surfaces were wider than the others to induce the shear failure on the wider surface. We defined the specimen with the wider surfaces on the LR-planes as "LR-type", and the other as "LT-type". For the LR-type, the *x*, *y*, and *z*-axes coincided with the radial, tangential, and longitudinal directions, respectively, whereas these axes coincided with the tangential, radial, and longitudinal directions, respectively, for the LT-type. These specimens were twisted around the longitudinal direction (*z*-axis), and the torsional moment-torsional angle relationships were measured. The shear moduli in the LR- and LT-planes, G_{LR} and G_{LT}, were obtained from the following equation:

$$\begin{cases} G_{\mathrm{LR}}=\left(\dfrac{M}{\theta}\right)_{\mathrm{LR}}\cdot\left[a^3b\left\{\dfrac{1}{3}-\dfrac{2a}{b}\sqrt{\dfrac{G_{\mathrm{LR}}}{G_{\mathrm{LT}}}}\left(\dfrac{2}{\pi}\right)^5\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^5}\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{\mathrm{LT}}}{G_{\mathrm{LR}}}}\right\}\right]^{-1}, \\ G_{\mathrm{LT}}=\left(\dfrac{M}{\theta}\right)_{\mathrm{LT}}\cdot\left[a^3b\left\{\dfrac{1}{3}-\dfrac{2a}{b}\sqrt{\dfrac{G_{\mathrm{LT}}}{G_{\mathrm{LR}}}}\left(\dfrac{2}{\pi}\right)^5\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^5}\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{\mathrm{LR}}}{G_{\mathrm{LT}}}}\right\}\right]^{-1} \end{cases} \tag{12}$$

where *a* and *b* are the width and thickness of the specimens, respectively, and $(M/\theta)_{\mathrm{LR}}$ and $(M/\theta)_{\mathrm{LT}}$ are the inclinations of torsional moment-torsional angle relationships of the LR- and LT-type specimens, respectively, in the elastic strain range. The shear moduli were obtained as the result of convergence by the successive approximation method.

The torsional angle θ was converted to the shear strain γ_{zx} by Eq. (1), and the torsional moment-shear strain relationship was obtained. Then, the torsional moment-plastic shear

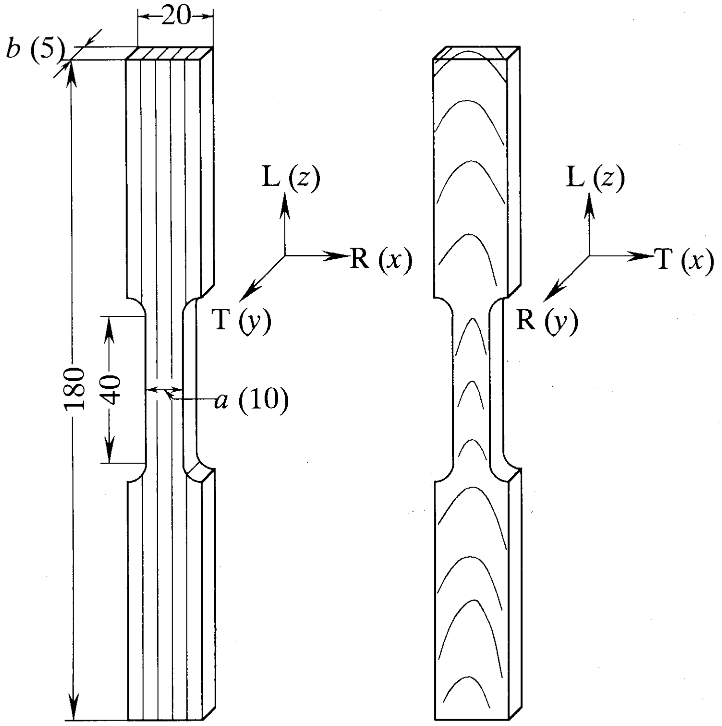

Fig. 2. Specimens used for the torsion tests (unit: mm).
Notes: Left: LR-type, Right: LT-type.

strain relationship was regressed to Eq. (5), the torsional moment at the occurrence of yielding, M_y, and the parameters α and n were given. By substituting these values into Eq. (9), the shear stress/shear strain relationship in the plastic strain region was obtained. As Eq. (10), the shear strength F^{ep}_{zx} was determined. On the other hand, the shear strength according to the conventional method, F^{qe}_{zx}, was derived by Eq. (11)

4. Results and Discussion

Table 1 shows the material parameters giving the shear stress-shear strain relationship; shear modulus, shear yield stress, and α and n determining the torsional moment-shear plastic strain relationship. From these values, the shear stress-shear strain relationship of each species was derived.

Table 2 shows the shear strengths obtained by the different procedures, F^{ep} and F^{qe}, of each species and the ratio of F^{ep}/F^{qe}. In this experiment, the values of elastic-plastic strengths were about 70% of those obtained by the conventional method although the shear stress-shear strain relationships varied by the species. There are several results of mechanical tests which show the coincidence between the testing data and theoretical analyses more precisely when the shear strength is evaluated as about 70% of that obtained by the conventional solution.[6,7]

In measuring the shear strength by torsion test, the shear stress/shear strain relationship should be taken into account. As mentioned, however, the procedure in determining the shear stress-strain relationship is quite complicated. From this viewpoint, hence, 70% reduction of the strength given by the conventional method may be effective for a rough evaluation of the real shear strength. Of course, we think that more experimental data should be collected to verify the applicability of 70% reduction for various species.

Table 1. Material parameters giving the shear stress/shear strain relationship

Species	LR-plane				LT-plane			
	G_{LR}	S_{LR}	α	n	G_{LT}	S_{LT}	α	n
Spruce	0.47	3.0	3.9	1.7	0.92	5.1	5.1	1.8
Agathis	0.92	6.2	0.8	3.7	0.66	4.7	2.3	2.0
Katsura	0.76	6.6	1.1	2.1	0.69	6.4	1.8	2.8
Buna	0.87	4.4	0.4	3.3	0.59	4.4	2.4	2.3

Unit: G_{LR} and G_{LT}: GPa, S_{LR} and S_{LT}: MPa, α: 10^{-3}

Table 2. Shear strengths predicted by the torsion tests

Species	LR-plane			LT-plane		
	F^{ep}_{LR}	F^{qe}_{LR}	F^{ep}_{LR}/F^{qe}_{LR}	F^{ep}_{LT}	F^{qe}_{LT}	F^{ep}_{LR}/F^{qe}_{LT}
Spruce	**11.3**	16.3	0.69	**15.5**	25.4	0.61
Agathis	**12.8**	20.8	0.61	**12.2**	16.4	0.74
Katsura	**17.9**	21.7	0.82	**13.7**	16.8	0.82
Buna	**18.3**	24.6	0.74	**14.7**	19.4	0.76

Unit: MPa.
Note: F^{ep} and F^{qe} were obtained from Eqs. (10) and (11), respectively.

5. Conclusion

We tried to obtain the real shear strength of wood by torsion tests. The shear strength given by the formulation was about 70% of that obtained by the method conventionally adopted

Summary

In this paper, we tried to obtain the real shear strength of wood by torsion tests with considering the plastic deformation.

Sitka spruce (*Picea sitchensis* Carr.), agathis (*Agathis* sp.), katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) and buna (Japanere beech, *Fagus crenata* Bl.) were used for the specimens. These specimens were twisted around the longitudinal axis, and the shear stress-shear strain relationships of LR- (longitudinal-radial) planes and LT- (longitudinal-tangential) planes were obtained. These relationships were formulated by n-power functions, and the shear strengths were predicted by putting the shear strain at the occurrence of failure on the formula. On the other hand, the shear strength was independently calculated by the conventional method which is based on the hypothesis that the specimen keeps its elastic stress condition under the torsional loading, and the strengths obtained by the different methods were compared with each other.

Although the shear stress-shear strain relationships varied by the species, the values of elastic-plastic strengths were about 70% of those obtained by the conventional method. There are several results of mechanical tests which show the coincidence between the testing data and theoretical analyses more precisely when the shear strength is evaluated as about 70% of that obtained by the conventional solution.

Key words: Shear strength, Torsion test, Formulation

References

1) MINAMI, Y.: *Mokuzai Kogyo,* **8**, 32–34 (1953).
2) SUZUKI, N. and OKOHIRA, Y.: *Bull. Mie Univ. Dept. Agr.,* **65**, 46–49 (1982).
3) OKUSA, K.: *Mokuzai Gakkaishi,* **23**, 217–227 (1977).
4) YOSHIHARA, H., Ohta, M.: *Mokuzai Gakkaishi,* **43**, 457-463 (1997).
5) YOSHIHARA, H., OHTA, M.: *Bull. Tokyo Univ. Forest,* **99**, 11–17 (1998).
6) YOSHIHARA, H., OHTA, M.: *Mokuzai Gakkaishi,* **37**, 511–516 (1991).
7) MASUDA, M., TANOURA, O.: *Bull. Kyoto Univ. Forest,* **67**, 158–166 (1995).

(Received Oct. 30, 1998)
(Accepted Mar. 17, 1999)

ねじり試験で得られた木材のせん断強度

吉原　浩*・太田正光**

（* 島根大学総合理工学部材料プロセス工学科,
** 東京大学大学院農学生命科学研究科）

要　旨

塑性変形を考慮し，矩形棒のねじり試験から，木材のせん断強度を求めた。

試験にはシトカスプルース (*Picea sitchensis* Carr.), アガチス (*Agathis* sp.), カツラ (*Cercidiphyllum japonicum* Sieb. and Zucc.) およびブナ (*Fagus crenata* Bl.) を用いた。これらの試験体を，繊維方向を中心軸としてねじり，まさ目面および板目面のせん断応力–せん断ひずみ関係を得た。このせん断応力–せん断ひずみ関係をべき乗関数を用いて定式化し，破壊発生時のひずみの値をこの式に代入することでせん断強度を求めた。一方，ねじり負荷では破壊まで弾性状態を保つという従来より行われている方法でねじりせん断強度を求め，それぞれの値を比較した。その結果，樹種によってせん断応力–せん断ひずみ関係は多様であったが，いずれの樹種においても定式化によって得られた真のせん断強度の値は従来の方法で得られた強度の約70％程度となった。

キーワード：せん断強度，ねじり試験，定式化

Shear Strength of Wood Obtained by Torsion Test

Hiroshi YOSHIHARA and Masamitsu OHTA

In this paper, we tried to obtain the real shear strength of wood by torsion tests with considering plastic deformation.

Sitka spruce (*Picea sitchensis* Carr.), agathis (*Agathis* sp.), katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) and buna (Japanese beech, *Fagus crenata* Bl.) were used as specimens. These specimens were twisted around the longitudinal axis, and the shear stress-shear strain relationships of LR- (longitudinal-radial) planes and LT- (longitudinal-tangential) planes were obtained. These relationships were formulated by n-power functions, and the shear strengths were predicted by putting the shear strain at the occurrence of failure into the formula. On the other hand, the shear strength was independently calculated by the conventional method which is based on the hypothesis that the specimen keeps its elastic stress condition under the torsional loading. The strengths obtained by the different methods were compared with each other.

Although the shear stress-shear strain relationships varied with species, the values of the elastic-plastic strengths were about 70% of those obtained by the conventional method. There are several results of mechanical tests which show the coincidence between the testing data and theoretical analyses more precisely when the shear strength is evaluated as about 70% of that obtained by conventional solution

---

Ginkgolide Production in *Ginkgo biloba* Trees and Cultured Cells

Hiroki INOUE, Shigehiro KAMODA, Tamami TERADA
and Yoshimasa SABURI

Ginkgolide content in various parts of ginkgo trees and in some cultured cells was studied.

Ginkgolides were detected in all the parts of ginkgo trees examined. Especially, roots of young trees contained much more ginkgolides than green leaves. Albumens contained ginkgolide B as the main ingredient.

Several cultured cells produced ginkgolide B, but the concentration was far less than any part of mother plants. Embryo-derived cultured cells generally produced ginkgolide B, whereas petiole- and cambium-derived calli showed poorer results. Greening of the cultured cells under illumination seemed to have no connection with ginkgolide production.