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Formulation of the Shear Stress/Shear Strain Relationship Using the Torsion Testing Data

Hiroshi YOSHIHARA*, Masamitsu OHTA** and Kazuhiro ORIGUCHI**

1. Introduction

In our previous papers, we tried to obtain the shear stress/shear strain relationship of wood by torsion of a rectangular bar, and proposed an equation deriving the shear stress/shear strain relationship from the torsional moment-shear strain diagrams.[1, 2] In that method, we used the total shear strain instead of the plastic strain, and did not consider the existence of yield stress. Based on the strain-incremental theory, however, it is inconvenient to describe the stress-strain relationship in the plastic region without considering the yield stress and plastic strain because the stress-strain relationship is usually separated into the phases before and after the occurrence of yielding.[3] When the yield stress is considered and the total strain is separated into the elastic and plastic strain components, it is rather convenient in describing the shear stress/shear strain relationship based on the plasticity theory. In addition, we found that the shear stress/shear strain relationship all over the strain range can be easily formulated by a function when the plastic strain is used. It is also convenient to give the stress-strain formula in stress analysis in various occasions.

In this paper, we tried to formulate the shear stress/shear strain relationship in the plastic region with an n-power function by separating the total strain into elastic and plastic strain components, and examined the validity of this formulation.

2. Theories

2.1 Shear stress/shear strain relationship given by the torsion test

Figure 1 shows the diagram of torsion of an orthotropic bar with a rectangular cross-section. The xyz- and XYZ-axes are defined as those of orthotropic and geometrical symmetries. Here, the x-, y-, and z-axes coincide with the tangential, radial, and longitudinal directions, respectively. The angle ϕ lying between the axes z and Z is defined as "grain angle".

When the bar is twisted around the y (Y)-axis, the shear strains at the centers of the XY- and YZ-planes, γ_{XY} and γ_{YZ}, respectively, are represented as follows:[1)]

$$\begin{cases} \gamma_{XY}=a^3bk\dfrac{G_{YZ}}{G_{XY}}p_{XY}\theta \\ \gamma_{YZ}=a^3bkp_{YZ}\theta, \end{cases} \tag{1}$$

where θ is the torsional angle, a and b are the lengths in the directions of the X- and Y-axes, respectively, and G_{XY} and G_{YZ} are the shear moduli in the XY- and YZ-planes, respectively. The values of p_{XY}, p_{YZ}, and k are represented as follows:

$$\begin{cases} p_{XY}=\dfrac{1}{a^2bk}\cdot\left[-\dfrac{8}{\pi^2}\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{(2n-1)^2}\cdot\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\right] \\ p_{YZ}=\dfrac{1}{a^2bk}\cdot\left[-\dfrac{8}{\pi^2}\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}\cdot\left\{\cosh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\right\}^{-1}\right], \end{cases} \tag{2}$$

and

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$$k=\frac{1}{3}-\frac{2a}{b}\sqrt{\frac{G_{YZ}}{G_{XY}}}\left(\frac{2}{\pi}\right)^5 \sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi b}{2a}\sqrt{\frac{G_{XY}}{G_{YZ}}}. \tag{3}$$

We suggested that the shear stresses at the center of the XY-plane, τ_{XY}, can be predicted from the torsional moment-shear strain relationship as follows:[1)]

$$\tau_{XY}=p_{XY}\left\{M+q\gamma_{XY}^2\frac{\mathrm{d}}{\mathrm{d}\gamma_{XY}}\left(\frac{M}{\gamma_{XY}}\right)\right\} \tag{4}$$

where q is derived approximately as $0.2(a^2+b^2)/ab$.[1)] Thus, the shear stress-strain relationship is predicted by Eqs. (1) and (4).

Equation (4) is given by using the total strain component γ_{XY}, and hence, it is inconvenient to use this equation for plastic analysis.[4)] We separate the total strain component in the plastic region into elastic strain component, γ_{XY}^{e}, and plastic component, γ_{XY}^{p} as follows:[5)]

$$\begin{cases}\gamma_{XY}=\gamma_{XY}^{e} & (\tau_{XY}\leq S_{XY})\\ \gamma_{XY}=\gamma_{XY}^{e}+\gamma_{XY}^{p} & (\tau_{XY}>S_{XY}),\end{cases} \tag{5}$$

where S_{XY} is the shear yield stress on the XY-plane. Similar to Eq. (4), the shear stress/shear strain relationship in the plastic strain region is written as follows:

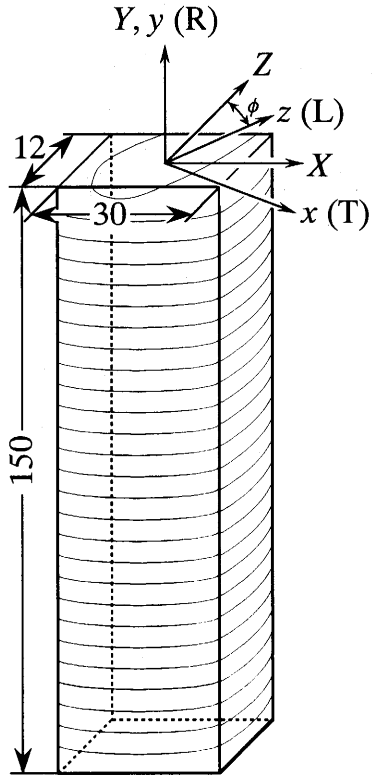

Fig. 1. Diagram of torsion-testing specimen (unit: mm).

$$\tau_{XY}=p_{XY}\left\{M+q_p\gamma_{XY}^{p2}\frac{\mathrm{d}}{\mathrm{d}\gamma_{XY}^{p}}\left(\frac{M-M_y}{\gamma_{XY}^{p}}\right)\right\}\quad(\tau_{XY}>S_{XY}), \tag{6}$$

where M_y is the torsional moment at the occurrence of yielding, and q_p is a material parameter. With this equation, we tried to formulate the shear stress/shear strain relationship by the following procedure.

The M-γ_{XY}^{p} relationship is approximated by Ludwik's n-power function as follows:[5)]

$$\gamma_{XY}^{p}=\alpha\left(\frac{M-M_y}{M_y}\right)^n, \tag{7}$$

where α is a parameter determining the volume of the plastic strain. The second term in the braces of Eq. (6) is represented by eliminating M from Eq. (7) as follows:

$$\frac{\mathrm{d}}{\mathrm{d}\gamma_{XY}^{p}}\left(\frac{M-M_y}{\gamma_{XY}^{p}}\right)=\frac{M_y}{\alpha^2}\cdot\frac{1-n}{n}\cdot\left(\frac{\gamma_{XY}^{p}}{\alpha}\right)^{1/n-2}. \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (6), the torsional moment M is eliminated, and the shear stress/shear strain relationship in the plastic strain range is given as follows:

$$\tau_{XY}=p_{XY}M_y\left\{1+\left(1+q_p\frac{1-n}{n}\right)\cdot\left(\frac{\gamma_{XY}^{p}}{\alpha}\right)^{1/n}\right\}. \tag{9}$$

When $\gamma_{XY}^{p}=0$, τ_{XY} is equal to $p_{XY}M_y$ which coincides with the shear yield stress S_{XY}. Hence, $p_{XY}M_y$ is eliminated from Eq. (9), and the shear plastic strain is represented as:

$$\gamma_{XY}^{p}=\frac{\alpha}{\left(1+q_{p}\dfrac{1-n}{n}\right)^{n}}\cdot\left(\frac{\tau_{XY}-S_{XY}}{S_{XY}}\right)^{n}. \tag{10}$$

Thus, the shear stress/shear strain relationship is represented in all over the strain range as follows:

$$\begin{cases}\gamma_{XY}=\gamma_{XY}^{e}=\dfrac{\tau_{XY}}{G_{XY}} & (\tau_{XY}\leq S_{XY})\\ \gamma_{XY}=\gamma_{XY}^{e}+\gamma_{XY}^{p}=\dfrac{\tau_{XY}}{G_{XY}}+\dfrac{\alpha}{\left(1+q_{p}\dfrac{1-n}{n}\right)^{n}}\cdot\left(\dfrac{\tau_{XY}-S_{XY}}{S_{XY}}\right)^{n} & (\tau_{XY}>S_{XY}).\end{cases} \tag{11}$$

2.2 Shear stress/shear strain relationship given by the equivalent stress/equivalent plastic strain relationship

When the stress-strain relationship in the plastic region satisfies the strain-incremental theory, the shear stress/shear strain relationship can be obtained from the equivalent stress ($\bar{\sigma}$)/equivalent plastic strain ($\overline{\varepsilon^{p}}$) relationship. Here, the equivalent stress is derived by the following Hill-type yield criterion:

$$\bar{\sigma}=S_{XY}\sqrt{\frac{\tau_{xy}^{2}}{S_{xy}^{2}}+\frac{\tau_{yz}^{2}}{S_{yz}^{2}}}, \tag{12}$$

where S_{xy} and S_{yz} are the shear yield stresses in the xy- and yz-planes which coincide with the XY-plane in the condition of $\phi=0°$ and $\phi=90°$, respectively. Then, the $\bar{\sigma}-\overline{\varepsilon^{p}}$ relationship is represented by Ludwik's power function as follows:

$$\overline{\varepsilon^{p}}=K\left(\frac{\bar{\sigma}-S_{xy}}{S_{xy}}\right)^{m}, \tag{13}$$

where K and m are the material constants. In the pure shear stress condition, the shear stress τ_{XY} is derived by substituting $\tau_{xy}=\tau_{XY}\cos\phi$ and $\tau_{yz}=\tau_{XY}\sin\phi$ as the following equation:

$$\tau_{XY}=\frac{\bar{\sigma}}{S_{xy}\sqrt{\dfrac{\cos^{2}\phi}{S_{xy}^{2}}+\dfrac{\sin^{2}\phi}{S_{yz}^{2}}}}=\frac{S_{XY}}{S_{xy}}\bar{\sigma}. \tag{14}$$

From the plasticity theory, the plastic shear strain increment $d\gamma_{XY}^{p}$ is given by the Prandtl-Reuss function as follows:

$$d\gamma_{XY}^{p}=\frac{d\bar{\sigma}}{d\tau_{XY}}d\overline{\varepsilon^{p}}=\frac{S_{xy}}{S_{XY}}d\overline{\varepsilon^{p}}. \tag{15}$$

Integrating Eq. (15), γ_{XY}^{p} is obtained as follows:

$$\gamma_{XY}^{p}=\frac{S_{xy}}{S_{XY}}\overline{\varepsilon^{p}}. \tag{16}$$

From Eqs. (13), (14) and (16), the shear stress/shear plastic strain relationship can be formulated by the power function. The shear stress/shear strain relationship can be expressed in all over the strain range as follows:

$$\begin{cases}\gamma_{XY}=\dfrac{\tau_{XY}}{G_{XY}} & (\tau_{XY}\leq S_{XY})\\ \gamma_{XY}=\dfrac{\tau_{XY}}{G_{XY}}+\dfrac{S_{xy}K}{S_{XY}}\left(\dfrac{\tau_{XY}-S_{XY}}{S_{XY}}\right)^{m} & (\tau_{XY}>S_{XY}).\end{cases} \tag{17}$$

3. Experiment

3.1 Specimens

Sitka spruce (*Picea sitchensis* Carr.) and Konara (Japanese oak, *Quercus serrata* Murray) were used in this experiment. Specimens were conditioned at 20℃ and 65% relative humidity before and during the tests.

The dimensions of specimen is shown in Fig. 1. Specimens were cut with the grain angle ϕ varying at intervals of 15 degrees from 0 to 45 degrees. By changing the grain angle, the stiffnesses on the side planes of the specimens were given in various way.[5]

3.2 Torsion tests

Biaxial strain gages (FCA-2-11, Tokyo Sokki, Co., Ltd.) were bonded on the surface centers of the XY- and YZ-planes for the measurement of the shear strains, γ_{XY} and γ_{YZ}, respectively, and the specimen was twisted around the y (Y)-axis which coincided with the radial direction. From the measured relationships between the torsional moment and shear strains, M-γ_{XY} and M-γ_{YZ}, the shear moduli of the XY- and YZ-planes, G_{XY} and G_{YZ}, respectively, were obtained by the following equations:

$$\begin{cases} G_{XY}=\dfrac{\kappa_{XY}}{a^2bk}\left[-2\left(\dfrac{2}{\pi}\right)^2\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{(2n-1)^2}\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\right] \\ G_{YZ}=\dfrac{\kappa_{YZ}}{a^2bk}\left[1-2\left(\dfrac{2}{\pi}\right)^2\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}\left\{\cosh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G_{XY}}{G_{YZ}}}\right\}^{-1}\right], \end{cases} \tag{18}$$

where κ_{XY} and κ_{YZ} are the inclination of the M-γ_{XY} and M-γ_{YZ} relationships in the elastic range, respectively.

In the grain angle range used here, the yield stress on the XY-plane was always smaller than that on the YZ-plane, and hence, the shear stress/shear strain relationship on the YZ-plane cannot be determined. In this study, hence, we examined the shear stress/shear strain relationship on the XY-plane. From the M-γ_{XY} diagram, the torsional moment at the occurrence of yielding, M_y, was obtained. The shear yield stress, S_{XY}, can be calculated by the following equation:

$$S_{XY}=p_{XY}M_y\,. \tag{19}$$

According to the numerical calculations conducted in a previous paper, however, the real value of shear yield stress is about 80% of that obtained from Eq. (19).[2] Here, we evaluated the shear yield stess as 80% of that derived by Eq. (19). The torsional moment (M)-plastic shear strain (γ^{p}_{XY}) relationship was regressed to Eq. (7), and the shear stress/shear strain relationship was formulated to Eq. (11).

3.3 Compression tests

To determine the equivalent stress/equivalent plastic strain relationship, uniaxial compression tests were made. The dimensions of the short column specimens were 40 (L)×20 (R)×20 (T) mm. Uniaxial strain gages (gage length=2 mm, FTA-2-11, Tokyo Sokki Co., Ltd.) were bonded on the centers of the LR-planes, and a load was applied along the long axis of each specimen at the crosshead speed of 1 mm/min. From the obtained stress (σ)-plastic strain (ε^{p}) relationships, the equivalent stress ($\bar{\sigma}$) and equivalent plastic strain ($\overline{\varepsilon^{\mathrm{p}}}$) were calculated by the following equations:

$$\begin{cases} \bar{\sigma}=S_{xy}\dfrac{\sigma}{Y} \\ \overline{\varepsilon^{\mathrm{p}}}=Y\dfrac{\varepsilon^{\mathrm{p}}}{S_{xy}}, \end{cases} \tag{20}$$

where Y is the compressive yield stress. The equivalent stress/equivalent plastic strain relationships were approximated into Eq. (13), and the parameters K and m were obtained. The shear stress/shear strain relationship was derived by substituting the values of K and m into Eq. (17). This shear stress/shear strain relationship was compared to that obtained by the torsion testing data.

Table 1. Shear moduli and shear yield stresses in the XY-plane corresponding to the grain angles (unit: kgf/cm^2)

Species	Grain angle							
	Shear modulus G_{XY}				Yield stress S_{XY}			
	0°	15°	30°	45°	0°	15°	30°	45°
Spruce	770	860	960	1480	13	12	12	13
Konara	2800	3000	3300	4100	31	28	33	36

Table 2. Paramete α and n obtained by regressing the torsional moment-shear strain relationship into Ludwik's n-power function

Species	Grain angle							
	α ($\times 10^{-3}$)				n			
	0°	15°	30°	45°	0°	15°	30°	45°
Spruce	3.79	2.86	2.13	2.35	1.14	1.93	1.54	1.27
Konara	3.35	5.29	4.09	4.47	1.96	1.51	1.83	1.58

Table 3. Parameters K and m obtained by regressing the equivalent stress/equivalent plastic strain relationship into the Ludwik's power function

Species	K ($\times 10^{-2}$)	m
Spruce	0.12	1.65
Konara	4.15	2.28

4. Results and Discussion

Table 1 shows the shear moduli and the yield shear stresses on the XY-planes, G_{XY} and S_{XY}, respectively, corresponding to the grain angles, and Table 2 shows the parameters α and n obtained by the regression of M-γ_{XY}^{p} relationship to Ludwik's power function. Substituting these values into Eq. (11), the shear stress/shear strain relationships corresponding to the grain angles were obtained. Table 3 shows the values of K and m derived from the equivalent stress/equivalent plastic strain relationships. Substituting these values into Eq. (17), the shear stress/shear strain relationships were obtained. Figure 2 shows the comparisons of the shear stress/shear strain relationships obtained by Eqs. (11) and (17). In deriving the relationship by Eq. (11), the value of q_p should be determined. After several trials, we determined that q_p was equal to $0.2(a^2+b^2)/ab$ as similar to q in Eq. (4). The stress-strain behaviors of both species were different with each other. The shear stress/shear strain diagrams of spruce show less curvature, whereas those of konara show more sharp ones. We thought that the proposed formula represented these tendencies effectively, and that the shear stress/shear strain relationship can be formulated by approximating the torsional moment-shear strain relationship.

5. Conclusion

We tried to formulate the shear stress/shear strain relationship of wood by approximating the torsional moment-shear strain relationship with Ludwik's n-power function, and concluded that the shear stress/shear strain relationship can be well formulated when the torsional moment-shear strain relationship is derived by the power function.

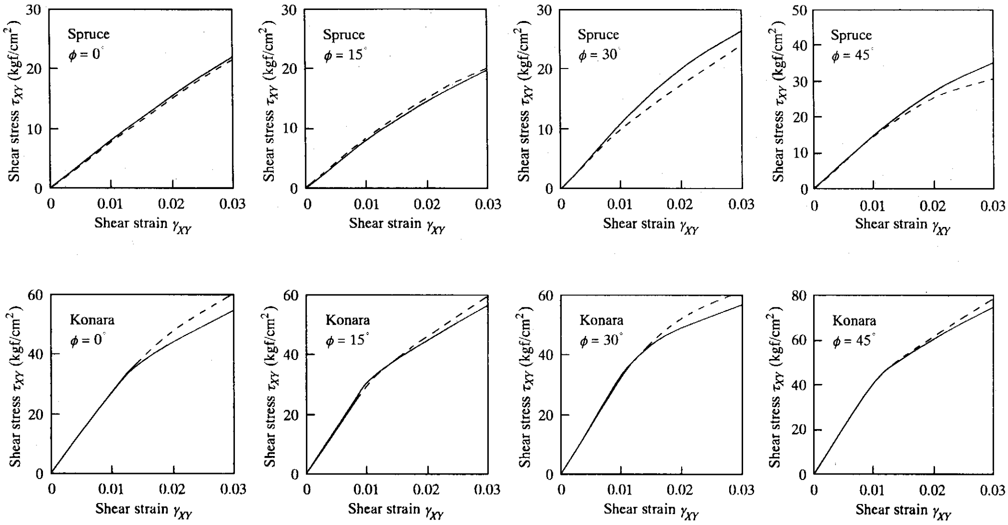

Fig. 2. Comparison of the shear stress/shear strain relationship predicted from the torsion testing data and the equivalent stress/equivalent plastic strain relationship.
Legend: Solid and dashed lines are obtained from the torsion tests and the equivalent stress/equivalent plastic strain relationships, respectively.

Summary

In this paper, we tried to formulate the shear stress/shear strain relationship of wood by approximating the torsional moment-shear strain relationship with Ludwik's n-power function.

Sitka spruce (*Picea sitchensis* Carr.) and konara (Japanese oak, *Quercus serrata* Murray) were used for the specimens. Specimens were cut so as to have various angles between the grain and the geometrical axes. These specimens were twisted around the radial axis, and the torsional moment-shear strain relationships were obtained. This relationship was approximated to Ludwik's n-power function, and this relationship was transformed into the shear stress/shear strain relationship which was formulated by a similar power function. The applicability of the shear stress/shear strain formula was examined by comparing with that predicted from the equivalent stress/equivalent plastic strain relationship.

We concluded that the shear stress/shear strain relationship can be well formulated when the torsional moment-shear strain relationship is derived by the power function.

Key words: shear stress, shear strain, torsion test, Ludwik's n-power function

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ねじり試験による木材のせん断応力–せん断ひずみ関係の定式化

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要旨

矩形棒のねじり試験から得られたねじりモーメント–せん断塑性ひずみ関係をべき乗関数に近似することによって，木材のせん断応力–せん断ひずみ関係を定式化することを試みた。

試験にはシトカスプルース (*Picea sitchensis* Carr.) およびコナラ (*Quercus serrata* Murray) の矩形棒を用いた。繊維方向と幾何学主軸間のなす角（繊維傾斜角）を変化させることによってせん断力がはたらく面の異方性の程度をさまざまに変化させた。これらの試験体を，半径方向を中心軸としてねじり，剛性の小さい面のねじりモーメント–せん断塑性ひずみ関係を Ludwik 型のべき乗関数に近似した。このねじりモーメント–せん断塑性ひずみ関係を，同様のべき乗関数を用いて塑性域におけるせん断応力–せん断ひずみ関係に変換した。この式の適用性を，相当応力–相当塑性ひずみ関係から予測されるせん断応力–せん断ひずみ関係と比較することにより検討した。その結果，ねじりモーメント–せん断塑性ひずみ関係をべき乗関数に近似することにより，木材のせん断応力–せん断ひずみ関係を同様のべき乗関数で表現できると結論した。

Abstract

Genetic Diversity and Structure of Hinoki (*Chamaecyparis obtusa*) in Chichibu District

Ding-Qin Tang, Hailong Shen and Yuji Ide

The genetic diversity and structure of hinoki were investigated in natural forests and plantations in Chichibu district by using 10 allozyme loci in 8 enzyme systems as marker genes. The analysis of gene composition and genetic structure, both intrastand and interstand, showed that in natural forests, 95.4% of the total genetic variation was maintained intrastand. It is shown that Chichibu natural forests possess great genetic variability. The analysis of estimated average heterozygosities revealed that plantations possess as large an intrastand genetic diversity as natural forest. But only 1.7% of genetic variation (G_{ST}) was attributable to genetic differences interstand, much less than the 4.6% in natural forests. Notably, plantations possessed less genetic variation interstand than the natural forests. The estimated average multilocus heterozygote deficit reached -0.091, which deviated significantly from the expected value under panmixia that was fitted in natural forests. This reconfirmed that some procedures during afforestation gave rise to a reduction of homozygotes and resulted in an excess of heterozygotes.

Formulation of the Shear Stress/Shear Strain Relationship Using the Torsion Testing Data

Hiroshi Yoshihara, Masamitsu Ohta and Kazuhiro Origuchi

In this paper, we tried to formulate the shear stress/shear strain relationship of wood by approximating the torsional moment-shear strain relationship with Ludwik's n-power function.

Sitka spruce (*Picea sitchensis* Carr.) and konara (Japanere oak, *Quercus serrata* Murray) were used for the specimens. Specimens were cut so as to have various angles between the grain and the geometrical axes. These specimens were twisted around the radial axis, and the torsional moment-shear strain relationships were obtained. This relationship was approximated to Ludwik's n-power function, and this relationship was transformed into the shear stress-shear strain relationship which was formulated by a similar power function.

The applicability of the shear stress/shear strain formula was examined by comparing with the relationships predicted from the equivalent stress/equivalent plastic strain relationship.

We concluded that the shear stress/shear strain relationship can be well formulated when the torsional moment/shear strain relationship is derived by the power function.

Establishment of a Callus Culture System of *Populus euphratica, Populus alba* cv. Pyramidalis and *Populus maximowiczii*×*Populus plantierensis*

Hailong SHEN, Shin WATANABE and Yuji IDE

A basic protocol of callus culture of *Populus euphratica, Populus alba* cv. Pyramidalis and *Populus maximowiczii*×*Populus plantierensis* (FS-51) was established. Callus was induced on MS medium containing BAP, NAA and 2,4-D in combination or alone. The callus could be subcultured on MS medium supplemented with 0.2 mg/*l* BAP and 1.0 mg/*l* NAA. When callus was cultured on the medium containing BAP in combination with NAA or GA_3, adventitious shoots were regenerated. The shoots were successfully rooted on 1/2 MS medium.

A Study on Landscape Formation and the Influence of Hang-Zhou West Lake in China

Yue SHEN

This research elucidates the landscape composition and landscape formation of Hong-Zhou West Lake (H-Z. W. L), and studies the landscape forming of other landscape areas based on H-Z. W. L. The main method of the research was to make out DTM according to topographical maps and perform quantitative analysis. In conclusion, the features of the landscape composition of H-Z. W. L are 1) both collected and expanded landscape, 2) three-layer structural landscape, 3) a skillful combination of man-made landscape and natural landscape. The formation methods are 1) promoting a layer-structure, 2) laying out atmosphere, 3) creating a typical landscape. In the lates projects of landscaping, the landscape composition and formation methods of H-Z. W. L were referred to and a common landscape appeared.