Bull. Tokyo Univ. For., 96, 43–50 (1996)

A Stress–Strain Formula Which Can Represent the Continuous Transition from Elastic to Plastic Stress State of Wood

Hiroshi YOSHIHARA*,** and Masamitsu OHTA*

1. Introduction

In determining the stress–strain relationship of material all over the strain range, the relationship is usually separated into two phases before and after the occurrence of yielding. It is sure that this method is convenient and effective for the materials of which yield point can be identified clearly. For wood, however, it is difficult to determine the yield stress precisely, and this difficulty causes the inaccurate determination of the stress–strain relationship because the shape of the stress–strain relationship is deeply influenced by the value of the yield stress.

In our previous works, we examined the stress-strain relationship of wood in the plastic region by the strain incremental theory, and verified the validity of this theory.[1,2] In this work, we try to formulate the stress–strain relationships of wood all over the strain range with a simple hyperbolic formula, and examined the validity of this formula by uniaxial-compression tests of the specimens with various grain orientations.

2. Theories

2.1 General theory of plasticity for the stress–strain relationship

Based on the strain-incremental theory, the strain increment, $d\varepsilon$, is separated into the elastic strain increment and plastic strain increment as follows:

$$\{d\varepsilon\} = \{d\varepsilon^{e}\} + \{d\varepsilon^{p}\}\,. \tag{1}$$

Elastic strain increment in the direction oriented at ϕ with respect to the grain direction, $d\varepsilon_\phi{}^{e}$, can be represented by Hooke's law as:[3)]

$$d\varepsilon^{e}_{\phi} = \frac{d\sigma_\phi}{E_\phi}\,, \tag{2}$$

where E_ϕ is the elastic modulus of ϕ-direction (kgf/cm^2). The increments of the plastic strain components of orthotropic symmetry, $d\varepsilon^{p}_{x}$, $d\varepsilon^{p}_{y}$, and $d\gamma^{p}_{xy}$ can be represented by Prandtl–Reuss's equations as:[3)]

$$\begin{cases} d\varepsilon^{p}_{x} = \dfrac{\partial g}{\partial \sigma_x}\, d\overline{\varepsilon^{p}} \\[2ex] d\varepsilon^{p}_{y} = \dfrac{\partial g}{\partial \sigma_y}\, d\overline{\varepsilon^{p}} \\[2ex] d\gamma^{p}_{xy} = \dfrac{\partial g}{\partial \tau_{xy}}\, d\overline{\varepsilon^{p}}\,, \end{cases} \tag{3}$$

where g and $\overline{\varepsilon^{p}}$ are the plastic potential and equivalent plastic strain, respectively. Equiv-

* Department of Forest Products Science, Faculty of Agricultural Science, The University of Tokyo.
** Present address: Faculty of Science and Engineering, Shimane University.

alent stress $\bar{\sigma}$ (kgf/cm^2) is similar to the plastic potential, and is proportional to σ_ϕ as follows:

$$\bar{\sigma}=g=K_\phi\sigma_\phi, \tag{4}$$

where K_ϕ is the none-dimensional parameter of plastic anisotropy which is dependent on the grain orientation. Under the uniaxial condition, the stress components along the orthotropic axes, σ_x, σ_y, and τ_{xy} are written as:

$$\begin{cases} \sigma_x=\sigma_\phi\cos^2\phi \\ \sigma_y=\sigma_\phi\sin^2\phi \\ \tau_{xy}=\sigma_\phi\cos\phi\sin\phi. \end{cases} \tag{5}$$

By eliminating σ_ϕ, $\cos\phi$, and $\sin\phi$ from Eqs. (4) and (5), $\bar{\sigma}$ and g can be written as:

$$\bar{\sigma}=g=K_\phi\sqrt{\sigma_x^2+\sigma_y^2+2\tau_{xy}^2}. \tag{6}$$

Substituting Eqs. (5) and (6) into Eq. (3), the plastic-strain increments in the orthotropic axes can be given as follows:

$$\begin{cases} d\varepsilon_x^p=\dfrac{K_\phi^2 d\overline{\varepsilon^p}}{\bar{\sigma}}\sigma_\phi\cos^2\phi=K_\phi d\overline{\varepsilon^p}\cos^2\phi \\ d\varepsilon_y^p=\dfrac{K_\phi^2 d\overline{\varepsilon^p}}{\bar{\sigma}}\sigma_\phi\sin^2\phi=K_\phi d\overline{\varepsilon^p}\sin^2\phi \\ d\gamma_{xy}^p=\dfrac{2K_\phi^2 d\overline{\varepsilon^p}}{\bar{\sigma}}\sigma_\phi\cos\phi\sin\phi=2K_\phi d\overline{\varepsilon^p}\cos\phi\sin\phi. \end{cases} \tag{7}$$

The increment of the uniaxial plastic strain $d\varepsilon_\phi^p$ is:

$$d\varepsilon_\phi^p=d\varepsilon_x^p\cos^2\phi+d\varepsilon_y^p\sin^2\phi+d\gamma_{xy}^p\cos\phi\sin\phi. \tag{8}$$

Substituting Eq. (7) into Eq. (8),

$$d\varepsilon_\phi^p=K_\phi d\overline{\varepsilon^p}. \tag{9}$$

Thus, the total strain increment is obtained by substituting Eqs. (2) and (9) into Eq. (1) as:

$$d\varepsilon_\phi=\frac{d\sigma_\phi}{E_\phi}+K_\phi d\overline{\varepsilon^p}=\frac{d\bar{\sigma}}{E_\phi K_\phi}+K_\phi d\overline{\varepsilon^p}. \tag{10}$$

In this paper, the stress-strain relationship is defined as follows:

$$\varepsilon_\phi=f(\sigma_\phi). \tag{11}$$

Substituting Eq. (11) into Eq. (10), $d\overline{\varepsilon^p}/d\bar{\sigma}$ can be written as:

$$\frac{d\overline{\varepsilon^p}}{d\bar{\sigma}}=\frac{1}{K_\phi^2}\left[\frac{d\varepsilon_\phi}{d\sigma_\phi}-\frac{1}{E_\phi}\right]. \tag{12}$$

According to the plasticity theory, the equivalent stress-equivalent plastic strain relationship is independent of the stress conditions, and hence, $d\overline{\varepsilon^p}/d\bar{\sigma}$ of Eq. (12) can be written by a single-variable function of $\bar{\sigma}$ as follows:

$$\frac{d\overline{\varepsilon^p}}{d\bar{\sigma}}=C(\bar{\sigma}). \tag{13}$$

2.2 Formulation of the stress–strain relationship of off-axis specimens

As mentioned above, it is difficult to determine the yield stress of wood precisely, and hence, it is useful to represent the continuous transition of stress–strain relationship all over the strain range with a simple formula. In this paper, we propose a hyperbolic formula representing the stress–strain relationship as follows:

$$\varepsilon_\phi = f(\sigma_\phi) = \frac{1}{E_\phi \alpha_\phi} \sinh \alpha_\phi \sigma_\phi , \tag{14}$$

where α_ϕ is the parameter dependent on the grain orientation, and has the dimension of $(\mathrm{kgf/cm^2})^{-1}$. The proposed formula has an advantage that the stress can be represented by the strain as follows:

$$\sigma_\phi = \frac{\ln [E_\phi \alpha_\phi \varepsilon_\phi + \sqrt{\{E_\phi \alpha_\phi \varepsilon_\phi\}^2 + 1}\,]}{\alpha_\phi} . \tag{15}$$

The first derivative of Eq. (14) with σ_ϕ is

$$\frac{d\varepsilon_\phi}{d\sigma_\phi} = f'(\sigma_\phi) = \frac{1}{E_\phi} \cosh \alpha_\phi \sigma_\phi . \tag{16}$$

Substituting Eq. (16) into Eq. (12),

$$\frac{d\bar{\varepsilon}^{\mathrm{p}}}{d\bar{\sigma}} = \frac{1}{E_\phi K_\phi^2} (\cosh \alpha_\phi \sigma_\phi - 1) = \frac{1}{E_\phi K_\phi^2} \left(\cosh \frac{\alpha_\phi}{K_\phi} \bar{\sigma} - 1 \right). \tag{17}$$

When Eq. (17) satisfies the plasticity theory, α_ϕ should be proportional to K_ϕ, and the value of $E_\phi K_\phi^2$ should be a constant value which is independent of the grain orientation. Although the elastic modulus E_ϕ and the plastic-anisotropic parameter K_ϕ are theoretically independent with each other, Eq. (17) is dependent on the grain orientation because of $E_\phi K_\phi^2$. Thus, our proposal does not satisfy the plasticity theory strictly. However, when the stress level is low and the value of $\alpha_\phi \bar{\sigma} / K_\phi$ is small, Eq. (17) can be approximated as

$$\frac{d\bar{\varepsilon}^{\mathrm{p}}}{d\bar{\sigma}} \approx \frac{\alpha_\phi^2}{2 E_\phi K_\phi^4} \bar{\sigma}^2 . \tag{18}$$

When Eq. (18) has no dependence on the grain orientation, α_ϕ can be written as

$$\alpha_\phi = \beta \sqrt{E_\phi} K_\phi^2 . \tag{19}$$

where β is a constant value of which dimension is $(\mathrm{kg/cm^2})^{-3/2}$.

2.3 Prediction of the parameters corresponding to the grain orientation

The value of E_ϕ is predicted from the rotation theory of elastic tensor as follows:

$$E_\phi = \frac{1}{\dfrac{\cos^4 \phi}{E_x} + \dfrac{\sin^4 \phi}{E_y} + \left(\dfrac{4}{E_{45}} - \dfrac{1}{E_x} - \dfrac{1}{E_y} \right) \cos^2 \phi \sin^2 \phi} , \tag{20}$$

where E_x, E_y, and E_{45} are the elastic moduli of the directions along the grain, transversely to the grain, and inclined at 45 degrees to the grain, respectively.[4]

The plastically-anisotropic parameter in the direction inclined at ϕ, K_ϕ, can be obtained from the following procedure. The plastic potential g and equivalent stress can be derived by the Hill-type quadratic function as:[5]

$$g^2 = K_x^2 \sigma_x^2 + K_y^2 \sigma_y^2 + K_s^2 \tau_{xy}^2 - K_{xy}^2 \sigma_x \sigma_y , \tag{21}$$

where K_x and K_y are the plastically-anisotropic parameters in the directions along and transversely to the grain, respectively, K_s is the parameter for the shearing in the plane of anisotropic symmetry, and K_{xy} is the parameter for the interaction term $\sigma_x\sigma_y$. Substituting Eqs. (4) and (5) into Eq. (21), K_ϕ can be represented as follows:

$$K_\phi=\sqrt{K_x^2\cos^4\phi+K_y^2\sin^4\phi+(K_s^2-K_{xy}^2)\cos^2\phi\sin^2\phi}\,. \quad (22)$$

The value of K_ϕ for $\phi=45$ degrees is defined as K_{45}. Substituting the value of K_{45} into Eq. (22), K_s and K_{xy} are eliminated, and K_ϕ can be written as:

$$K_\phi=\sqrt{K_x^2\cos^4\phi+K_y^2\sin^4\phi+(4K_{45}^2-K_x^2-K_y^2)\cos^2\phi\sin^2\phi}\,. \quad (23)$$

On the other hand, the values of K_x, K_y, and K_{45} can be derived by substituting $\beta=1$ (kg/cm^2)$^{-3/2}$ into Eq. (19) as follows:

$$\begin{cases} K_x=\sqrt{\dfrac{\alpha_x}{\beta\sqrt{E_x}}}=\dfrac{\sqrt{\alpha_x}}{\sqrt[4]{E_x}} \\ K_y=\sqrt{\dfrac{\alpha_y}{\beta\sqrt{E_y}}}=\dfrac{\sqrt{\alpha_y}}{\sqrt[4]{E_y}} \\ K_{45}=\sqrt{\dfrac{\alpha_{45}}{\beta\sqrt{E_{45}}}}=\dfrac{\sqrt{\alpha_{45}}}{\sqrt[4]{E_{45}}} \end{cases} \quad (24)$$

From Eqs. (23). and (24), the value of K_ϕ corresponding to the grain inclination ϕ can be predicted.

3. Experiment

Compression specimens of agathis (*Agathis* sp.) and katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) were cut with the dimensions of 20 mm×20 mm×40 mm. These specimens had the angles (ϕ) of 0 to 90 degrees at intervals of 15 degrees between the grain directions and long axes in the LR (longitudinal-radial) planes. Specimens were conditioned at 20°C and 65% relative humidity before and during the tests.

Strain gages (Kyowa KFC-5-C1-11, gage length=5 mm) were bonded on the both LR-plane of the specimen. Specimens were compressed by a universal testing machine (Shimadzu AUTOGRAPH IS-5000) along the long axis with the crosshead speed of 1 mm/min, and the stress–strain relationship corresponding to each grain angle was obtained. Five compression tests were made for one test.

Each stress-strain curve was regressed to Eq. (14) by the method of least squares, and parameters E_ϕ and α_ϕ were obtained. Substituting the regressed value of E_ϕ and $\beta=1$ (kgf/cm^2)$^{-3/2}$ into Eq. (19), the values of K_ϕ corresponding to the grain orientations were calculated.

4. Results and Discussion

Table 1 shows the values of the elastic moduli, E_x, E_y, and E_{45}, and the plastic-

Table 1. Material parameters corresponding to the orthotropic symmetry

Species	E_x	E_y	E_{45}	K_x	K_y	K_{45}
Agathis	15.1	1.22	2.44	2.56	11.9	8.14
Katsura	12.9	1.44	2.64	3.32	10.1	8.03

Unit: E_x, E_y, and $E_{45}=\times10^4$ kgf/cm^2, K_x, K_y, and $K_{45}=\times10^{-3}$.
Note: Suffixes x and y represent the longitudinal and radial directions, respectively.

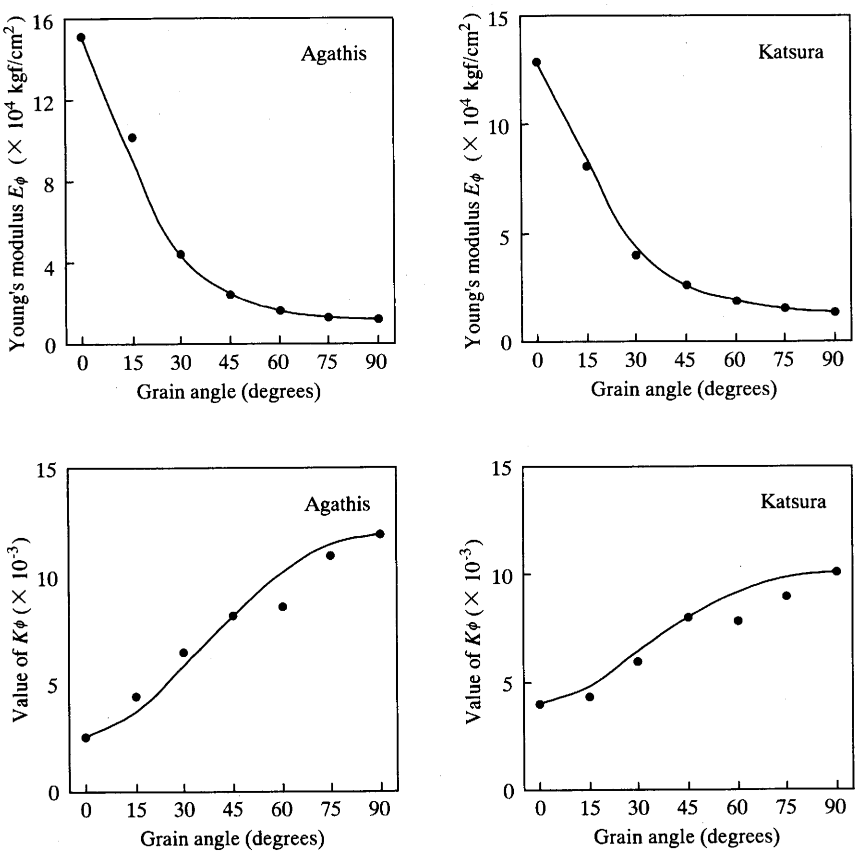

Fig. 1. Depencence of the Young's moduli E_ϕ and the plastic-anisotropic parameters K_ϕ on the grain orientations.

anisotropic parameters, K_x, K_y, and K_{45}. Substituting the elastic moduli and the plastically-anisotropic parameters into Eqs. (19) and (23), the prediction of E_ϕ and K_ϕ corresponding to the grain orientation was obtained, respectively. Figure 1 shows the comparisons of the predictions and the experimental data of E_ϕ and K_ϕ. The prediction gives rather good approximation for both species.

Substituting the predicted values of E_ϕ and K_ϕ into Eq. (14), the stress-strain curves corresponding to the grain orientations were predicted. Figure 2 shows the comparisons of experimental and predicted curves. Some predicted curves did not completely agree with the experimental data. Nevertheless, we think that the predicted formula gives good representations for the experimental curves, and the influence of the approximation on the shapes of the curves made in Eq. (18) seemed to be small. Thus, we think that the stress–strain relationship of wood all over the strain range can be represent properly by the hyperbolic formula proposed here.

5. Conclusion

We tried to formulate the stress-strain relationship of wood by a hyperbolic function, and examined the validity of the proposal by the uniaxial-compression tests of specimens with various grain orientations.

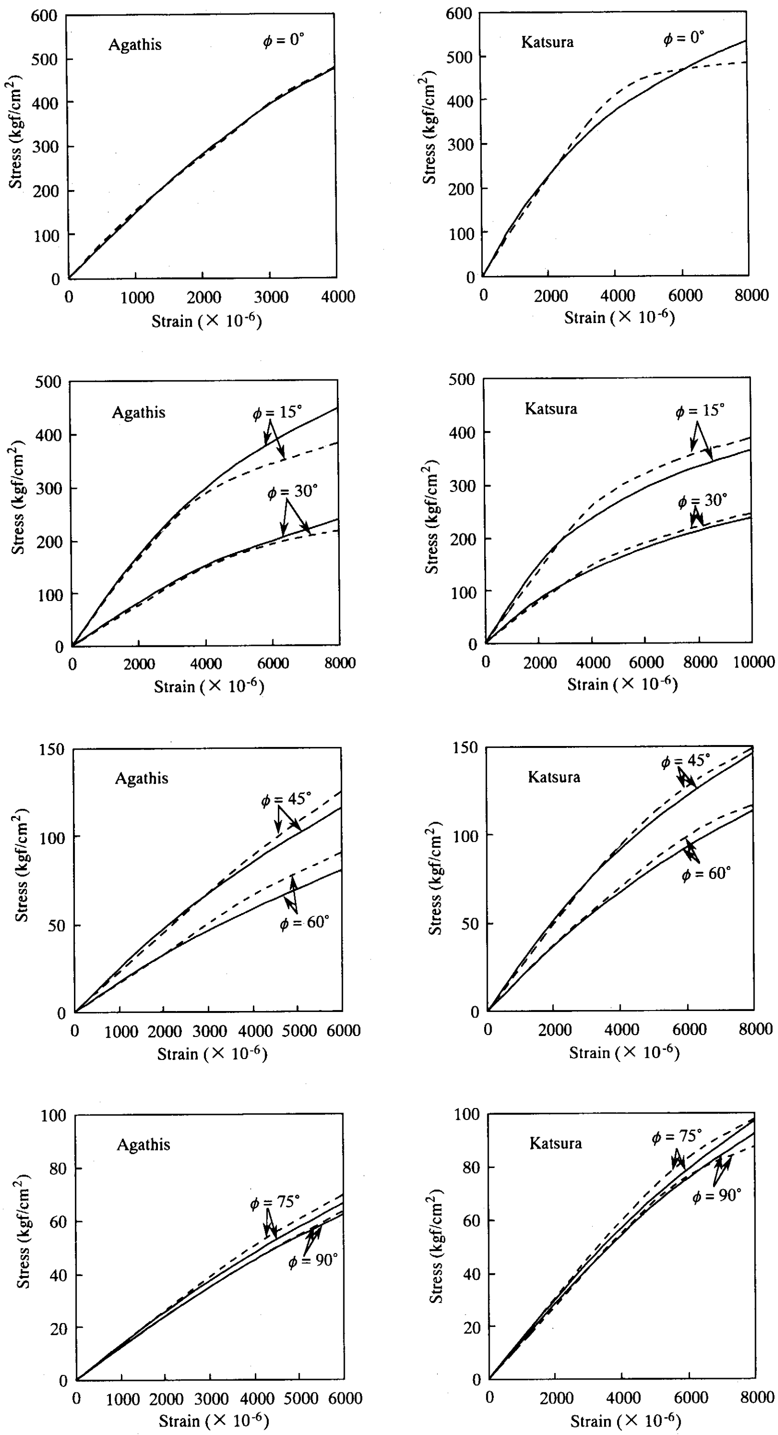

Fig. 2. Stress–strain relationships corresponding to the grain orientations.
Legend: ——, Theoretical curve; ----, Experimental curve.

Some predicted curves did not completely agree with the experimental data. On the whole, however, our proposal gave the good representations for the experimental data. Thus, we conclude that the stress-strain relationship of wood all over the strain range can be represent properly by the hyperbolic formula proposed here.

Summary

We formulated the stress-strain relationships of wood using a hyperbolic function over the whole of the strain range, and examined the validity of the results by uniaxial-compression tests.

Agathis (*Agathis* spp.) and katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) with various grain orientations were used for the experiment. These specimens were compressed, and the stress-strain curves were regressed using the hyperbolic function by the method of least squares. The parameters corresponding to the grain orientations were obtained from the experimental data along the orthotropic axes, and the stress-strain curves corresponding to the various grain orientations were predicted.

Some predicted curves did not completely agree with the experimental data. On the whole, however, our proposal gave a good representation for the experimental data. Thus, we conclude that the stress-strain relationship of wood over the whole strain range can be represented properly by the hyperbolic formula proposed here.

Key words: stress–strain relationship; grain orientation; plastic region; hyperbolic function; uniaxial compression test.

References

1) YOSHIHARA, H. and OHTA, M.: Mokuzai Gakkaishi, **38**, 759–763, 1992.
2) YOSHIHARA, H. and OHTA, M.: Mokuzai Gakkaishi, **40**, 263–267, 1994.
3) YAMADA, Y.: Sosei-Nendansei (Plasticity and Visco-elasticity), Baifu-kan, Tokyo, 1980.
4) JENKIN, C. F.: Report on materials of construction used in aircraft and aircraft engines. H.M. Stationary Office, London, 1920.
5) HILL, R. (1948). Proc. Roy. Soc., A, **193**, 281–297.

(Received Apr. 26, 1996)
(Accepted Jul. 19, 1996)

弾性から塑性に至る変化を一義的に表現できる木材の応力-ひずみ関係式

吉原　浩*,**・太田正光*

(* 東京大学農学生命科学研究科, ** 現在, 島根大学総合理工学部)

要　旨

木材の応力-ひずみ関係を双曲線関数で定式化し，その妥当性を単軸圧縮試験から検討した。

さまざまな繊維傾斜角を持つアガチス (*Agathis* sp.) およびカツラ (*Cercidiphyllum japonicum* Sieb. and Zucc.) を試験体に用いた。これらの試験対を単軸圧縮試験し，得られた応力-ひずみ関係を最小二乗法で双曲線関数に回帰した。また，各繊維傾斜角に対応する応力-ひずみ関係のパラ

メータを，異方性対称軸に対するパラメータから予測し，全体の応力-ひずみ関係を定式化した。
いくつかの不一致は存在したとはいえ，定式化された応力-ひずみ関係は実験から得られた応力-ひずみ関係と全体的によく一致した。したがって，ここで用いた式は木材の応力-ひずみ関係をよく表現できると結論した。

キーワード：応力-ひずみ関係，双曲線関数，繊維傾斜角，単軸圧縮試験

A Stress–Strain Formula Which Can Represent the Continuous Transition from Elastic to Plastic Stress State of Wood

Hiroshi YOSHIHARA and Masamitsu OHTA

The stress-strain relationships of wood were formulated by a hyperbolic function over the whole of the strain range, and the validity of the results examined by uniaxial-compression tests.

Agathis (*Agathis* spp.) and katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.) with various grain orientations were used for the experiment. These specimens were compressed, and the stress-strain curves were regressed using the hyperbolic function by the method of least squares. The parameters corresponding to the grain orientations were predicted from the experimental data along the orthotropic axes, and the stress-strain curves corresponding to the grain orientations were predicted.

Our proposal gave a good representation for the experimentally obtained stress-strain curves. Thus, we that our proposal is useful for the prediction of the stress-strain relationships over the whole strain range.

Establishment of Advanced Tissue Culture Techniques in *Betula platyphylla* var. *japonica* and in Dipterocarpaceae species

Lu-Min VAARIO

This research focuses on woody plant tissue culture, and consists of two parts. The first part describes the importance of suberized periderm in organo-genesis of woody plants based on studies of *Betula platyphylla* var. *japonica*. The results suggest that two chemical compounds are effective in suberization of the periderm and advantitious bud differentiation. In this part an advanced plant regeneration system for root explants is established, and the possibility to transform plants by Agrobacterium-mediation is demonstrated. The second part applies the established regeneration system to Dipterocarpaceae trees. This was conducted with the *S. leprosula* and *S. johorensis* species.