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Simplification of the Measurement Method of the Shear Modulus of Wood by Torsion Test

Hiroshi YOSHIHARA* and Masamitsu OHTA*

1. Introduction

To obtain the shear moduli of orthotropic material accurately, it is necessary to use the plural informations about the side planes for separating the two moduli included in the solution of differential equations. In our previous paper, strain gages were bonded at the centers of the LT-(longitudinal-tangential) and LR-(longitudinal-radial) planes for deriving the information of the LT- and LR-planes, and the shear moduli on these planes were measured accurately.[1] When the shear modulus of only one plane is needed, however, this method is inconvenient because of the useless data. Thus, the simpler method which gives the approximated values close enough to the accurate ones is required.

In this paper, we tried to simplify the measurement method of shear modulus by the torsion of a rectangular bar, and examined the validity of the simplification.

2. Theories

Figure 1 is the diagram of the torsion of a rectangular bar of which side planes are LT-(longitudinal-tangential) and LR-(longitudinal-radial) ones, respectively. When an orthotropic bar with a rectangular section is twisted around the grain axis, the shear stresses at the center of the LT- and LR-planes, τ_{LT} and τ_{LR}, are given as follows:[1]

$$\begin{cases} \tau_{LT}=\dfrac{M}{a^2 b}\cdot\dfrac{p_{LT}}{\phi} \\ \tau_{LR}=\dfrac{M}{a^2 b}\cdot\dfrac{p_{LR}}{\phi} \end{cases}, \tag{1}$$

where M is the torsional moment, a and b are the lengths of the tangential and radial directions of the specimen, respectively, and G^t_{LT} and G^t_{LR} are the mathematically-rigorous shear moduli of the LT- and LR-planes, respectively. The values of p_{LT}, p_{LR} and ϕ are given as:

$$\begin{cases} p_{LT}=-\dfrac{8}{\pi^2}\sqrt{\dfrac{G^t_{LT}}{G^t_{LR}}}\displaystyle\sum_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{(2n-1)^2}\cdot\tanh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G^t_{LT}}{G^t_{LR}}} \\ p_{LR}=1-2\left(\dfrac{2}{\pi}\right)^2\displaystyle\sum_{n=1}^{\infty}\dfrac{1}{(2n-1)^2}\left\{\cosh\dfrac{(2n-1)\pi b}{2a}\sqrt{\dfrac{G^t_{LT}}{G^t_{LR}}}\right\}^{-1} \end{cases}, \tag{2}$$

and

$$\phi=\frac{1}{3}-\frac{2a}{b}\sqrt{\frac{G^t_{LR}}{G^t_{LT}}}\left(\frac{2}{\pi}\right)^5\sum_{n=1}^{\infty}\frac{1}{(2n-1)^5}\tanh\frac{(2n-1)\pi b}{2a}\sqrt{\frac{G^t_{LT}}{G^t_{LR}}}. \tag{3}$$

The shear moduli, G^t_{LT} and G^t_{LR}, are given as follows:

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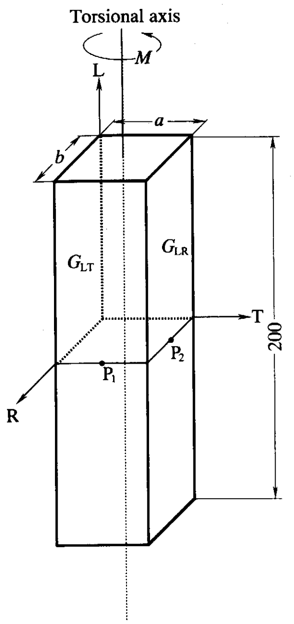

Fig. 1. Diagram of the rectangular bar subjected to the torsional force.
Unit: mm.
Notes: The values of a and b: See Table 1.
L. R, and T represent the longitudinal, radial, and tangential directions, respectively.
Strain gages were bonded at the points of P_1 and P_2.

$$\begin{cases} G^t_{LT} = \dfrac{\tau_{LT}}{\gamma_{LT}}, \\ G^t_{LR} = \dfrac{\tau_{LR}}{\gamma_{LR}} \end{cases} \tag{4}$$

where γ_{LT} and γ_{LR} are the shear strains at the center of LT- and LR-planes, respectively. Substituting Eq. (1) into Eq. (4), G^t_{LT} and G^t_{LR} can be represented as follows:

$$\begin{cases} G^t_{LT} = \dfrac{k_{LT}}{a^2 b} \cdot \dfrac{p_{LT}}{\phi}, \\ G^t_{LR} = \dfrac{k_{LR}}{a^2 b} \cdot \dfrac{p_{LR}}{\phi} \end{cases} \tag{5}$$

where k_{LT} and k_{LR} are the initial slopes of $M-\gamma_{LT}$ and $M-\gamma_{LR}$ relationships, respectively.

In the previous paper, we measured the values of k_{LT} and k_{LR} by strain gages bonded on the centers of each plane, and obtained the shear moduli by separating each other with Eqs. (4) and (5) by the successive approximation method. When the orthotropy is weak and the value of $\sqrt{G^t_{LR}/G^t_{LT}}$ is close to 1, the solution is similar to the isotropic materials, and the values of p_{LT}, p_{LR}, and ϕ are represented only by the aspect ratio, b/a. Here, the Coefficient ϕ^i, p^i_{LT}, and p^i_{LR} are defined as those given by substituting $\sqrt{G^t_{LR}/G^t_{LT}} = 1$ into ϕ, p_{LT}, and p_{LR} of Eqs. (2) and (3). Then, they are written as follows:

$$\phi^i = \frac{1}{3} - \frac{2a}{b}\left(\frac{2}{\pi}\right)^5 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^5} \tanh\frac{(2n-1)\pi b}{2a}, \tag{6}$$

and

$$\begin{cases} p^i_{LT} = -\dfrac{8}{\pi^2} \displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}}{(2n-1)^2} \cdot \tanh\dfrac{(2n-1)\pi b}{2a} \\ p^i_{LR} = 1 - 2\left(\dfrac{2}{\pi}\right)^2 \displaystyle\sum_{n=1}^{\infty} \dfrac{1}{(2n-1)^2} \left\{\cosh\dfrac{(2n-1)\pi b}{2a}\right\}^{-1} \end{cases}. \tag{7}$$

The shear moduli approximated by using the Coefficients ϕ^i, p^i_{LT}, and p^i_{LR}, G^i_{LT} and G^i_{LR}, are calculated from the following equation:

$$\begin{cases} G^i_{LT} = \dfrac{k_{LT}}{a^2 b} \cdot \dfrac{p^i_{LT}}{\phi^i} \\ G^i_{LR} = \dfrac{k_{LR}}{a^2 b} \cdot \dfrac{p^i_{LR}}{\phi^i} \end{cases}. \tag{8}$$

When the moment (M)-torsional angle (θ) relationship is used without measuring the strain γ_{LT} or γ_{LR}, the approximated solutions are more simplified. With the initial inclination of this $M-\theta$ relationship, the shear moduli, G^r_{LT} and G^r_{LR}, are calculated as follows:

$$\begin{cases} G^r_{LT}=r_t\cdot \dfrac{1}{ab^3}\cdot\dfrac{1}{\phi^i} \\ G^r_{LR}=r_t\cdot \dfrac{1}{a^3b}\cdot\dfrac{1}{\phi^i} \end{cases}, \tag{9}$$

where r_t is the initial inclination of $M-\theta$ relationship.

3. Experiment

3.1 Specimens

Sitka spruce (*Picea sitchensis* Carr.) was used for the specimens which had been conditioned at 20°C and 65% relative humidity before the tests.

3.2 Torsion tests

Torsion tests were made with bar-shaped specimens with axial lengths (longitudinal direction) of 200 mm. The side-lengths of the rectangular section were varied as Table 1. Biaxial strain gages (gage length=2 mm, Tokyo Sokki Co., Ltd.) were bonded on the centers of the LT- and LR-planes of the specimens for the measurement of the shear strains, γ_{LT} and γ_{LR}, respectively.

Torsional moment was applied with manual torsion testing equipment. The bar with a rectangular cross section shrinks along the torsional axis by twisting. When the shrinkage is restricted, tensile stress occurs in the specimen. In this experiment, thus, one of the grips grasping the specimen was equipped to slide along the torsional axis for preventing the axial restriction. From the initial slopes of $M-\gamma_{LT}$, $M-\gamma_{LR}$, and $M-\theta$ relationships, the values of k_{LT}, k_{LR}, and r_t were obtained, respectively. Substituting k_{LT}, k_{LR} into Eq. (5), the shear moduli G^t_{LT} and G^t_{LR} were calculated by the method of successive approximation. The shear moduli G^i_{LT}, G^i_{LR}, G^r_{LT}, and G^r_{LR} were calculated by Eqs. (8) and (9). These approximated moduli were compared with the accurate moduli G^t_{LT} and G^t_{LR}.

4. Results and Discussion

Figure 2 shows the shear moduli of the LT- and LR-planes obtained by the accurate solution, G^t_{LT} and G^t_{LR}. When the aspect ratio was far from 1, the shear moduli at the wider plane (G^t_{LT} of Type G and G^t_{LR} of Type A) tended to be measured large. This phenomenon was because of the axial restriction which could not reduced entirely. We think that it is difficult to reduce the restriction effect entirely when the specimen has the narrow cross section.[2)]

Figure 3 shows the ratios of the approximated shear moduli to those obtained by the

Table 1. Lengths of the sides on rectangular sections and aspect ratios of the torsion-test specimens

Type of specimen	A	B	C	D	E	F	G
a (mm)	12	12	12	12	36	72	108
b (mm)	108	72	36	12	12	12	12
Aspect raio *b*/*a*	9	6	3	1	1/3	1/6	1/9

Notes: The characters *a* and *b* represent the lengths of tangential and radial directions, respectively.

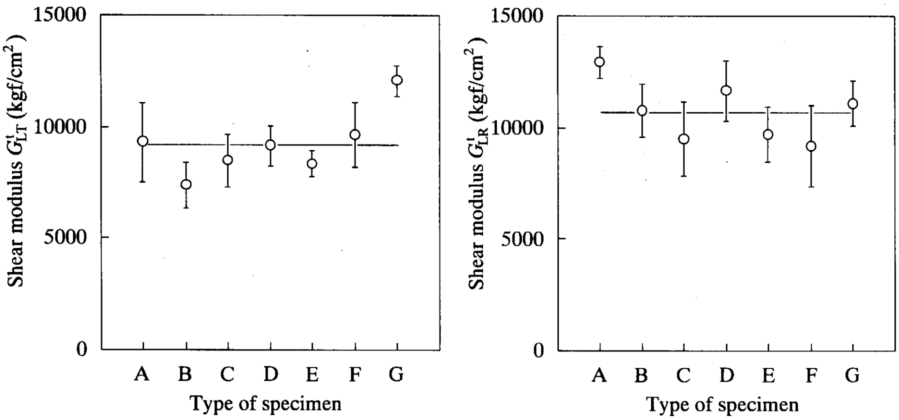

Fig. 2. Shear moduli obtained from the accurate solution, G^t_{LT} and G^t_{LR}, corresponding to the type of the specimens.
Legend: ○: Mean, I: Standard deviation, ——: Mean of all specimens.
Notes: Suffixes L, R, and T represent the longitudinal, radial, and tangential directions, respectively. A–G: See Table 1.

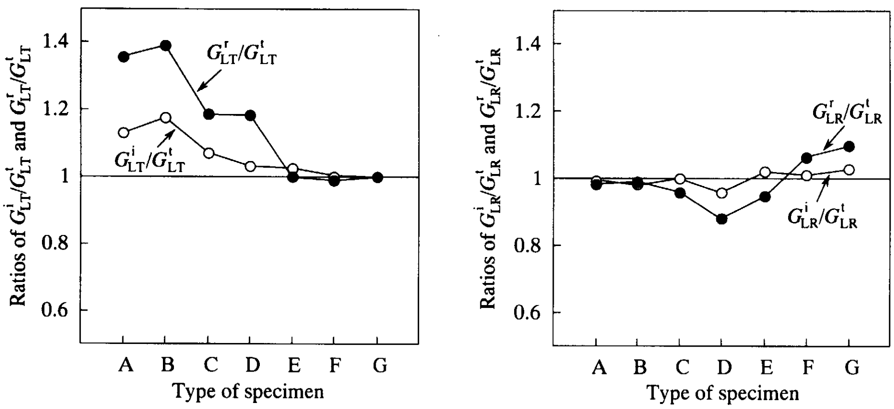

Fig. 3. Ratios of the shear moduli obtained by the approximated methods to those by the accurate solution.
Notes: G^r_{LT} and G^r_{LR} are obtained from Eq. (4), G^i_{LT} and G^i_{LR} are obtained from Eq. (8), and G^t_{LT} and G^t_{LR} are obtained from Eq. (9).
A–G: See Table 1.

accurate solution corresponding to the types of the aspect ratios of the rectangular section. When the specimen had narrow cross section, the approximated shear moduli at the wider planes (LT-planes for Types E, F, and G, and LR-planes for Types A, B, and C) were close enough to the moduli obtained from the accurate solutions, G^t_{LT} and G^t_{LR}. When the specimen has an intermediate aspect ratio (Type D), however, the shear moduli obtained from the torsional moment-torsional angle relationships, G^r_{LT} and G^r_{LR}, could not be measured precisely. On the contrary, the moduli obtained from the information of one strain gage, G^i_{LT} and G^i_{LR}, were close enough to the accurate solutions even when the aspect ratio was intermediate. Thus, we considered that the shear modulus at one plane can be measured properly by bonding a strain gage on the plane when the specimen has an intermediate aspect ratio.

Nevertheless, we cannot find out from the experimental results whether this method is always applicable for the material with the shear moduli of which values are apart from each other. Here, several values of G^{t}_{LT} and G^{t}_{LR} were given, and the approximated shear moduli, G^{i}_{LT}, G^{i}_{LR}, G^{r}_{LT}, and G^{r}_{LR}, were simulated. The given values of G^{t}_{LT} and G^{t}_{LR} varied as Table 2 because the shear modulus on the LR-plane does not usually exceed the twice of the shear modulus on the LT-plane.[3,4] The aspect ratios of the specimens were same as in Table 1.

Figure 4 shows the simulation results. Similar to the experimental results, this figure indicates that the shear modulus can

Table 2. Shear moduli G^{t}_{LT} and G^{t}_{LR} given as the material parameters

	G^{t}_{LT}	G^{t}_{LR}
Case 1	5.0	10.0
Case 2	6.0	10.0
Case 3	7.0	10.0
Case 4	8.0	10.0

Unit: $\times 10^3$ kgf/cm^2.

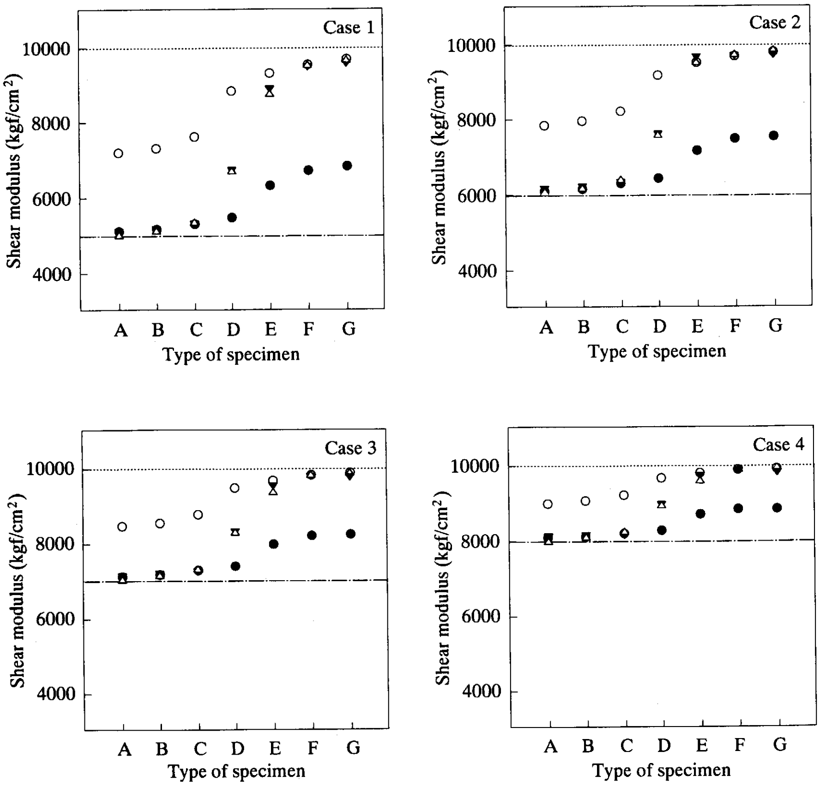

Fig. 4. Shear moduli calculated by the quasi-isotropic solutions.
Legend: -·-·-·-: Rigorous value of the shear modulus on the LT-plane, G^{t}_{LT}, ············: Rigorous value of the shear modulus on the LR-plane, G^{t}_{LR}, ● and ○: Shear moduli of LT and LR planes given from the torsional moment-shear strain relationships, G^{i}_{LT} and G^{i}_{LR}, ▼ and △: Shear moduli of LT and LR planes given from the torsional moment-torsional angle relationship, G^{r}_{LT} and G^{r}_{LR}.
Nores: A–G: See Table 1.
Case 1–4: See Table 2.

be obtained properly from the torsional moment-torsional angle relationship when the plane on which shear modulus is intended to be measured is wider than the other. As mentioned above, however, the moduli of the specimen with extremely narrow cross section would be measured inaccurately because of the axial restriction effect. On the contrary, the shear modulus can be measured more precisely by bonding a strain gage on the measured plane even when the specimen has an intermediate aspect ratio. Of course, the shear modulus cannot be measured precisely when the values of the shear moduli on the LT- and LR-planes are extremely different from each other. However, the shear modulus on the LR-plane does not usually exceed the twice of that on the LT-plane.[3,4] Thus, we think that the shear moduli on the LT- or LR-planes can be measured by bonding one strain gage on the measured plane.

5. Conclusion

We tried to simplify the measurement method of the shear modulus of wood by the torsion, and following results were obtained.

(1) When the cross section of the specimen was narrow enough (aspect ratio = 1/9 or 9 in this experiment), the shear moduli obtained from the three procedures were close enough to each other. However, the axial restraint effect was remarkable for the specimen with narrow cross-section, and the shear moduli were evaluated as large.

(2) The shear modulus can be approximated properly by bonding one strain gage on the measured plane even when the specimen has an intermediate aspect ratio of 1/3 to 3. The shear modulus on the LR-plane of wood does not usually exceed the twice of that on the LT-plane, and hence, we think that the shear moduli on the LT- or LR-planes can be measured by bonding one strain gage on the measured plane.

Summary

In a previous paper, we measured the shear moduli of wood by the torsion of a rectangular bar accurately. In this method, two strain gages were bonded on the LR- and LT-planes to measure the shear strains. Although this method is mathematically-accurate, it is not convenient because of the necessity of two torsional moment-shear strains relationships. Here, we tried to simplify the measuring method by using only one torsional moment-shear strain relationship.

In this experiment, specimens with various rectangular cross sections of which aspect ratios (*b*/*a*) varied from 1/9 to 9 were twisted around the longitudinal directions. The shear moduli were calculated by the following three different relationships; two torsional moment-shear strains relationships, one torsional moment-shear strain relationship, and the torsional moment-torsinal angle relationship. The moduli obtained by these three procedures were compared with each other. The results were summarized as follows:

(1) When the cross section of the specimen was narrow enough (aspect ratio = 1/9 or 9 in this experiment), the shear moduli obtained from the three procedures were close enough to each other. However, the axial restraint effect was remarkable for the specimen with narrow cross-section, and hence, the shear moduli were evaluated as large.

(2) The shear modulus can be approximated properly by bonding one strain gage on the measured plane even when the specimen has an intermediate aspect ratio of 1/3 to 3. The shear modulus on the LR-plane of wood does not usually exceed the twice of that on the LT-plane. Thus, we think that the shear moduli on the LT- or LR-planes can be measured effectively by bonding one strain gage on the measured plane.

Key words: Shear modulus, Torsion test, Orthotropy, Isotropy, Strain gage

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ねじり試験による木材のせん断弾性係数測定の簡略化

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要　旨

既報では，矩形棒の2つの側面中央（板目面およびまさ目面）にひずみゲージを貼付してねじり，2つのねじりモーメントせん断ひずみ関係から数学的に厳密な方法でせん断弾性係数を算出したが，多少煩雑な計算が必要であった。そこで今回は，必要とする面（板目面およびまさ目面のいずれか一面）の中央にのみひずみゲージを貼付してねじり，1つのねじりモーメント-せん断ひずみ関係からせん断弾性係数が求められるのかどうか検討した。

試験では断面寸法比 (b/a) が 1/9 から 9 の矩形棒を繊維方向を中心軸としてねじった。せん断弾性係数は以下に示す3つの関係すなわち (i) 2つのねじりモーメント-せん断ひずみ関係，(ii) 1つのねじりモーメント-せん断ひずみ関係 (iii) ねじりモーメント-ねじり角の関係から計算し，それぞれの値を比較した。結果を以下に示すと，

(1) 断面寸法比が十分に大きければ（本実験では断面寸法比＝1/9 あるいは 9），3つの関係から得られたせん断弾性係数は互いに近接した。しかし，断面寸法比が大きくなるとねじり軸方向の拘束効果が避けられず，せん断弾性係数は大きく見積もられる傾向があった。

(2) ひずみゲージを必要とする一面にのみ貼付した場合，断面寸法比が大きくなくても（断面寸法比が 1/3 から 3 の範囲）比較的正確にせん断弾性係数が算出できた。一般的に，木材のまさ目面のせん断弾性係数は板目面のせん断弾性係数の2倍以上にはならないので，この方法でまさ目および板目のせん断弾性係数を求めることは可能であると考えられた。

キーワード: せん断弾性係数，ねじり試験，異方性，等方性，ひずみゲージ

Simplification of the Measurement Method of the Shear Modulus of Wood by Torsion Test

Hiroshi YOSHIHARA and Masamitsu OHTA

In a previous paper, we accurately measured the shear moduli of wood by the torsion of a rectangular bar. In this method, two strain gages were bonded on the LR- and LT-planes to measure the shear strains. Although this method is mathematically accurate, it is not convenient because of the necessity of two torsional moment-shear strain relationships. Here, we tried to simplify the measuring method by using only one torsional moment-shear strain relationship.

In this experiment, specimens with various rectangular cross sections, of which the aspect ratios (b/a) varied from 1/9 to 9, were twisted around the longitudinal axis. The shear moduli were calculated by the following three different relationships; two torsional moment-shear strain relationships, one torsional moment-shear strain relationship, and the torsional moment-torsional angle relationship. The moduli obtained by these three procedures were compared with each other. The results were summarized as follows:

(1) When the cross section of the specimen was narrow enough (aspect ratio = 1/9 or 9 in this experiment), the shear moduli obtained from the three procedures were close enough to each other. However, the axial restraint effect was remarkable for the specimen with narrow cross-section, and hence, the shear moduli were found to be large.

(2) The shear modulus can be approximated properly by bonding one strain gage on the measured plane even when the specimen has an intermediate aspect ratio of 1/3 to 3. The shear modulus on the LR-plane of wood does not usually exceed twice that on the LT-plane. Thus, we conclude that the shear moduli on the LT- or LR-planes can be measured effectively by bonding one strain gage on the measuring plane.

Utilization and Evaluation of Exterior Wood VII
Thermal sensation by radiant heat

Satoshi SHIDA and Mari KOIKE

The effect of radiant heat from several materials on the thermal sensation of human skin was considered through the measurement of radiant heat by heat-flow meters.

The heat energy that induces thermal sensation on human skin was evaluated to be 66 W/m^2 on average. The height generating thermal sensation (H.G.T.S.) on human skin, which is the distance between the heated material and the hand model, was the lowest in copper and was the highest in granite. The maximum heat was absorbed at a height of 3 cm by the hand model. The absorbance decreased in accordance with the increase of height. Wood had an intermediate value of radiant heat between those of metal and stone and showed an intermediate level of efficiency in inducing radiant thermal sensation.