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Relationship between the Strength and Grain Orientation of Wood —Examination and modification of the Hankinson's Formula—

Hiroshi YOSHIHARA*, Shintaro AMANO** and Masamitsu OHTA*

1. Introduction

The relationships between the uniaxial strengths and the grain orientations are often discussed for the analysis of the strength properties of wood in relation to its orthotropy. Many formulas representing the relationship have been proposed[1]. These formulas contain the strength parameters in the direction of orthotropic axes; the normal strengths parallel and perpendicular to the grain, and the shear strength of orthotropic symmetry. Nevertheless, it is difficult to measure the shear strength of orthotropic symmetry properly[2], and hence, the strength formulas containing shear strength is inconvenient for the prediction of the strength corresponding to the grain orientation.

The Hankinson's formula, which does not contain the shear strength, is one of the most famous formula because of its simplicity, and is often used for many occasions. The Hankinson's formula, however, contains only two normal strength-components, and it is doubtful whether this formula can give the relationship all over the grain angle range properly.

In this paper, we examined the validity of the Hankinson's formula with the results of uniaxial-compression tests, and modified to the one which can express the relationship between the compressive strength and the grain angles more properly.

2. Theories

Hankinson proposed an equation for representing the relationship between the uniaxial strength σ_θ and the grain angle θ[3]. His formula is written as follows:

$$\sigma_\theta = \frac{XY}{X\sin^2\theta + Y\cos^2\theta}, \tag{1}$$

where X and Y are the strengths parallel and perpendicular to the grain, respectively. The Hankinson's formula is inaccurate because it has only two strength parameters. Later, Kollmann modified the Hankinson's formula by introducing the third parameter, n, for expressing the relationship more accurately as follows[4]:

$$\sigma_\theta = \frac{XY}{X\sin^n\theta + Y\cos^n\theta}, \tag{2}$$

where the value of n is 1.5–2.0 for the tensile strength and 2.5–3.0 for the compressive strength respectively. Generally, the value of n is calculated by the method of least square using several strength data corresponding to the various grain orientations. However, this method is inconvenient because much testing data is needed for the derivation of n.

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Easier representation can be given by another modification of the Hankinson's formula. Here, we introduce the third parameter X_q which is the strength in the direction inclined at 45 degrees with respect to the grain. When the Hankinson's formula agrees with X_q for $\theta=45°$, the following relationship can be obtained:

$$X_q=\frac{2XY}{X+Y}. \tag{3}$$

The influence of Y on σ_θ is small in the grain angle range of $0°\leq\theta\leq45°$. On the contrary, X has a small influence on σ_θ in the range of $45°\leq\theta\leq90°$. When the strength component of which influence is small is replaced by X_q, the relationship between σ_θ and θ would be more accurate. From Eqs. (1) and (3), hence, the strength Y is eliminated when $0°\leq\theta\leq45°$, whereas X is eliminated when $45°\leq\theta\leq90°$, and the following equation is obtained:

$$\sigma_\theta=\begin{cases}\dfrac{XX_q}{(2X-X_q)\sin^2\theta+X_q\cos^2\theta} & (0°\leq\theta\leq45°)\\[2ex] \dfrac{YX_q}{(2Y-X_q)\cos^2\theta+X_q\sin^2\theta} & (45°\leq\theta\leq90°)\end{cases}, \tag{4}$$

3. Experiment

3.1 Materials

Materials used for the specimens were Sitka spruce (*Picea sitchensis* Carr.), Akamatsu (*Pinus densiflora* Sieb. and Zucc.), Western red ceder (*Thuja plicata* D. Don), Agathis (*Agathis* sp.), Buna (*Fagus crenata* Bl.), and Katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.). Specimens were conditioned at 20°C and 65% RH before and during the tests.

3.2 Compression tests

Test specimens were cut with the dimensions of 20 mm×20 mm×40 mm. These specimens had the angles of 0 to 90 degrees at intervals of 15 degrees between the grain directions and long axes in the LR (longitudinal-radial) planes. Compression load was applied by a universal testing machine (Shimadzu AUTOGRAPH IS-5000) along the long axis of the specimen with the crosshead speed of 1 mm/min. From the stress-strain diagrams, the maximum stresses corresponding to the grain orientations were obtained.

4. Results and discussion

Table 1. Compressive strengths in the directions parallel and perpendicular to the grain, X and Y, and strengths in the direction inclined at 45 degrees with respect to the grain, X_q

Species	X	Y	X_q
Spruce	412	64	117
Akamatsu	536	46	92
Western red ceder	284	36	68
Agathis	416	66	150
Buna	452	113	183
Katsura	447	83	148

Unit: kgf/cm²

Table 1 shows the strengths X, Y, and X_q. These values were substituted into the Hankinson's formula and the modified formula represented by Eqs. (1) and (4), respectively, and the comparisons of these formulas were obtained as shown in Fig. 1. The Hankinson's formula gives a good prediction for the relationships between the compressive strengths and the grain angles except the relationship for Agathis which was located at the lower position than the testing data. On the contrary, the modified formula shows a good agreement for all data.

The low value of strength predicted by the Hankinson's formula often occurs when

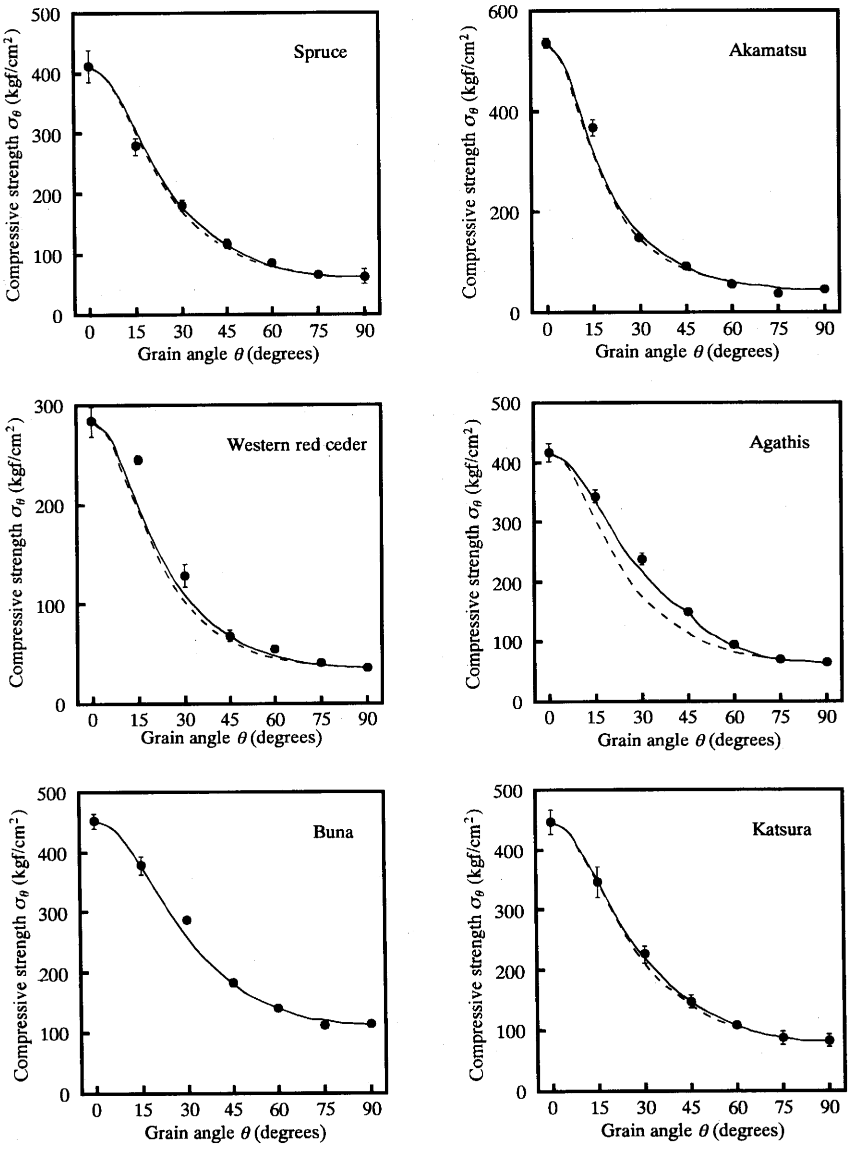

Fig. 1. Compressive strengths corresponding to the grain angles.
Legend: Solid dots: Mean, Horizontal bars: Standard deviation, Solid lines: Modification of Hankinson's formula, Dashed lines: Conventional Hankinson's formula.
Notes: Both predictions for Buna are almost overlapped.

the value of X_q is extremely larger than $2XY/(X+Y)$. This drawback can be reduced by introducing other parameters. Of course, the formula with many parameters becomes difficult to use in spite of its generality. Kollmann's modification is one of the typical examples in which difficulties can be expected. It is sure that our modification has a drawback in that the relationship lacks in smoothness at $\theta=45°$. However, this modification is convenient in predicting the relationship more accurately than the Hankinson's formula by introducing only one additional parameter.

5. Conclusion

We examined the applicability of the Hankinson's formula, which gives the relationship between the uniaxial strength and the grain orientation, by the compression tests of Spruce, Akamatsu, Western red ceder, Agathis, Buna, and Katsura, and modified his formula by introducing the strength in the direction of 45° with respect to the grain.

The Hankinson's formula gave a proper prediction for the relationships between the compressive strengths and the grain angles except for Agathis which was located at the lower position than the testing data. On the contrary, the modified equation yielded a good relationship for all data.

Summary

We examined the applicability of the Hankinson's formula, which gives the relationship between the uniaxial strength and the grain orientation, by the compression tests of Sitka spruce (*Picea sitchensis* Carr.), Akamatsu (*Pinus densiflora* Sieb. and Zucc.), Western red ceder (*Thuja plicata* D. Don), Agathis (*Agathis* sp.), Buna (*Fagus crenata* Bl.), and Katsura (*Cercidiphyllum japonicum* Sieb. and Zucc.). The specimens were cut with the grain angles of 0 to 90 degrees at the interval of 15 degrees between the grain direction and long axis, and uniaxial compression tests were made. The relationships between the compressive strengths and the grain angles were represented by the Hankinson's formula and the formula modified by introducing the strength in the direction of 45 degrees with respect to the grain.

The Hankinson's formula gave a proper prediction for the relationships between the compressive strengths and the grain angles except for Agathis which was located at the lower position than the testing data. On the contrary, the modified equation yielded a good relationship for all data.

Key words: Hankinson's formula, Grain orientation, Uniaxial-compression test, Uniaxial strength

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木材の纖維傾斜角と強度の関係—Hankinson の式の検討と改良—

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要　旨

纖維傾斜角と強度の関係を与える Hankinson の式を，様々な木材の単軸圧縮試験によって検討した。また，纖維方向と 45°傾いた方向の強度を Hankinson の式に導入することによって改造した。

45°傾いた方向の強度が大きい場合，Hankinson の式では全体の纖維傾斜角と強度の関係を的確に表現することができなかった。これに対して，改造した式は全体の纖維傾斜角と強度の関係を的確に表現することができた。

キーワード: Hankinson の式，纖維傾斜角，単軸圧縮試験，圧縮強さ

Abstracts

Relationship between the Strength and Grain Orientation of Wood
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Hiroshi YOSHIHARA, Shintaro, AMANO and Masamitsu OHTA

We examined the Hankinson's formula, which gives the relationship between the uniaxial strength and the grain orientation, by the uniaxial-compression tests of the specimens with several grain orientations, and modified with taking account of the strengths for the specimens with the grain inclinations of 45 degrees.

The Hankinson's formula gave a proper prediction for the relationships between the compressive strengths and the grain angles except for Agathis which was located at the lower position than the testing data. On the contrary, the modified equation yielded a good relationship for all data.

L-Phenylalanine Ammonia-Lyase Activities in Cell Suspension Cultures of *Eucalyptus polybractea* (III)
—Effects of phytohormones on L-phenylalanine ammonia-lyase activities—

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Effects of the lack and addition of phytohormones on cell growth and PAL activity were investigated in suspension cell cultures of *Eucalyptus polybractea.*

The cells cultured in the medium lacking 2,4-D or kinetin could not grow. PAL activity was increased and depressed in the absence of 2,4-D and kinetin, respectively.

Addition of GA_3 or ABA to the control medium reduced the cell growth and induced PAL activity. Especially, GA_3 caused a remarkable increase in PAL activity.