The Error of Basal Area Effected by Method of Estimation

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Contents

1. INTRODUCTION

When we estimate the basal area (at breast height) of trees, if we put the true value as A, on the other hand, the estimator as A' which is actually measured, consequently $A \neq A'$ is general. If we put $A'-A=y$, "y" shows the error which comes out from estimating A. Generally, it is not considered that the error "y" is composed of a single source, but considered that the error is given practically as an accumulation of various sources of variation. For instance, the systematic error which naturally belonged to the referred instruments and the error of rounding in getting measurement, will be active on "y". Furthermore, analyzing more precisely, the error coming out from the wrong breast height will be thought as to effect results.

Among these sources of variation, excluding such the bias to be corrected clearly as the error in reading and the error kept in caliper, which are thought to be the systematic error, we can pick up the methods of estimation applied actually as the sources which might have been comparatively active and can be controlled practically. For the above method of estimation the methods to use the caliper and the tape are there, and the methods of using the former is variable with regulation of measuring directions.

If we wish to estimate exactly the basal area, we will adopt the following methods of estimation; by caliper to estimate the diameters of various directions, or to regulate the estimating of special marking in the largest diameter and smallest diameter. Such methods of estimation is based on the experimental judgement that the mean value come out from the above methods should indicate the reliable value for the approximation of diameter in genuine circle having the same area as the basal area of object. However, actually, there are many cases to get estimator through the methods of estimation which regulate the estimation in single direction from mountain side and in direction at right angle to that. The reason why we adopt so many times the later methods in comparison with the former ones will be thought that in reference to reliability the diameter value of the later is inferior to the former but can satisfy us actually in accuracy and more efficient in time (it costs little). These circumstances show us that the difference among the methods of estimation can not be treated as homogeneous in effect for the error "y".

In fact, as long as it is the results of apparent observation, if such a consideration is tried experimentally, it will not guarantee our doings sufficiently. Because it needs the judgement based on the quantitative analysis of error, in order to consider such an observation as validity. In other words, these mean the following matters. What extent in quantity the difference among methods will show? What extent in quantity error of each method will show? Is it better for us to be permitting the difference made up actually, though we find the difference theoretically? Shall we need not be paying special attention to the difference, if the difference is small as compared with the regardless error (accidental error)?

Hereby we wish to refer to the result, which we have studied, about the problem mentioned above. We are adopting the techniques of stochastics for this study.

Of the theoretical study of these problems, our study should be developed based on the theory of distribution of population; all kinds of set of curve which might be shown actually by the cross-section at breast height, presuming the circumference of cross-section as random curve. In reference to this curve population, notwithstanding that we think it extensively as the important item of study in view of various distribution of engineering rather than the simple problem of cross-section, we have scarcely studied except that it will be studied as the problem of normal stochastic process in the Brown motion in consequence. In this paper we wish to consider the same problem in view of experiment.

We wish to express our thanks to Assis. Prof. Masaji SENDA, Mr. Genichi TAGUCHI, Mr. Hiroshi KOBAYASHI and Miss. Sumiko TANAKA for their cordial guidance or help on this work.

2. DESIGN

Generally, there are so many sources of variation which have effects on the error "y"that they happen frequently to show thousands or more numbers. The error "y" is fixed as some function for so many sources of variation x_1, x_2, ..., x_n. Namely

$$y=f(x_1,\ x_2,\ \cdots\cdots,x_n)$$

Among these various sources of variation, we have the variables of continuous or discrete values. Furthermore, there are many kinds of sources of variation which can not be free from unmeasurable subjective motives, such as, temporary mood of estimator. So many quantities of sources of variation as thousands or more are there, then it is perfectly impossible to consider each effect on the error "y" experimentally and theoretically, in pursueing all kinds of sources of variation.

But there may not be so many quantities of sources of variation, which will be our object of experiment and should be controlled or might be controlled. We have picked up the methods of estimation among various sources of variation with our expectation that we can easily control the judgement come out as th result of experiment. Consequently the structure are as follows;

(error)=(error effected by the methods of estimation)+(error effected by the other kinds of methods of estimation).

It will be general to use caliper or tape for estimation. As a method of estimation using caliper, various kinds of regulation of measuring direction are considered. In this experiment we have chosen the following 4 methods;

M_1) to regulate the diameter measurement of a single direction from mountain side,

M_2) from mountain side, and to regulate the diameter measurement in direction at right angle to that (2 directions in total),

M_3) to measure the circumference by tape,

M_4) to regulate the diameter measurements on the largest diameter of object and in direction at right angle to that (2 directions in total).

In M_1), M_2), M_4), it is common to use caliper as a instrument. M_1) is generally thought as most rough method, on the contrary, the value estimated by the method M_4) will be accurate so much. The factor M is fixed on 4 kinds, so we get 4 levels.

In order to make the experiment more efficient, we need to consider the condition of experiment. It is generally said that the cross-section at breast height of trees is so irregular and so different that we find the shape near to the genuine circle, eggshaped figure and the other various kinds of figures. The error "y" will be effected by such a source of variation as figure F. These must be included in the conditions to consider in experiment.

For instance, if the figure of cross-section of object are near to genuine circle, by the method M_1, we can get an accurate estimator fairly, but if it is ellipse, the estimator can not be free from inaccuracy comparing with the above estimator by the measuring point of one direction actually chosen. Herewith we have tried to take the figure as one factor in order to consider the difference among methods of estimator objectively, excepting the bias of experimental value depending upon sources of figure variation. Namely, we have chosen the following 3 levels as F;

F_1) the figure thought to be comparatively near to genuine circle,

F_2) the figure thought to be comparatively near to ellipse,

F_3) the figure thought to be so-called irregular, except F_1) and F_2).

These means are based on extending the idea to omit the difference of individuals, and are adopted for the purpose of a few times of experiment, and that precisely. Because it is generally said that we had better eliminate such a bias for experiment.

We have tried to pick up the factor A, the size of basal area, as same as for figure. As a relative error, it may give an homogeneous effect, but if it has some tendency, it will be more moderate for the purpose of experiment to eliminate it as a bias. Namely, we have chosen 3 levels as follows;

A_1) around 10 cm at optional diameter of object,

A_2) around 20 cm at optional diameter of object,

A_3) around 30 cm at optional diameter of object.

In the long run, we have picked up 3 factors including F and A, as the factors of experimental condition, among the various sources of variation which might have had effects on the error y. Therefore, the formula of structure are as follows;

(error) = (error effected by the methods of estimation) + (error effected by figures) + (error effected by the sizes of basal area) + (error effected by the other kinds of sources of variation).

For the all conbinations of M and F, A which are taken as factors for experimental condition, we have executed 2 times of replication r. Because it is considered that one time of r decrease the exactness of experiment for few degree of freedom of item of error. The replication does not mean 2 times of measurements on the same cross-section, but 2 times of picking up the figure which is thought to belong to each F_i $(i=1, 2, 3)$ block. For the purpose of our experiment, it will be no efficient to measure the same cross-section repeatedly. Because we would not study the only one of basal area but in general.

As the above mentioned, we have picked up the sources of variation M, F, A as factors, 4, 3, 3 levels respectively and executed replication 2 times, then the model of structure are shown as follows,

$$y_{ijkl}=\overline{m}+M_i+F_j+A_k+I_{ij}+I_{ik}+I_{jk}+I_{ijk}+z_{ijkl}$$
$$(i=1,\ 2,\ 3,\ 4;\ j=1,\ 2,\ 3;\ k=1,\ 2,\ 3;\ l=1,\ 2)$$

In this formula, $\overline{m}$ indicates the general mean, every item of I indicates the mutual interactions among the each main effect, and z_{ijkl} indicates an accidental item.

3. SAMPLE

If we put the figure as F, and the size of area as A, each sample unit which is regulated by design is shown by $(F_jA_k)_l$. As we put $j=1, 2, 3$; $k=1, 2, 3$ and $l=1, 2$, in our experiment, the whole combination of F-A becomes 18. According to combination, 6 figures should be decided at first. In other words, 2 figures $(F_1)_1$, $(F_1)_2$ which are thought to be near to circle, 2 figures $(F_2)_1$, $(F_2)_2$ to ellipse, and 2 figures $(F_3)_1$, $(F_3)_2$ to irregular figures.

These samples of figure have been collected in stand of Sugi in IMASUMI at the Tokyo University Forest in Chiba. From this tsand we have picked up some cross-sections at breast height which are thought to belong to F_j respectively.

To copy the figures of trees picked up, we have used the thread-solders. We have copied it on the paper as it is, as shown in Fig. 1, removing the 3 thread-solders which have been rolled closely as possible on the circumference of the cross-section of object. As the replication is given 2 times, among the picked up trees which are classified in each F_j, we have copied respectively 2 at random actually. As the result of this operation, we have got 6 figures which are shown in Fig. 2 as the samples of figure to be given in this experiment as fundamental data.

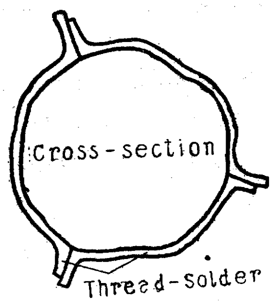

Fig. 1

After getting the samples of figure, next operation

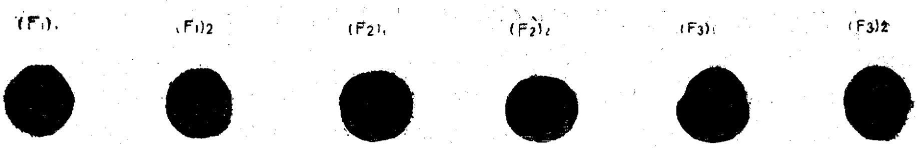

Fig. 2 Sample of figure

must be done on each A. That is, respectively for $(F_1)_1, \cdots\cdots, (F_3)_2$ shown in Fig. 2 we must make the samples of the same figure and of the different size. These are shown as 18 sample units in amount of $(F_1A_1)_1$, $(F_1A_1)_2$, $\cdots\cdots, (F_3A_3)_2$. At first, we have made the samples $(F_1A_1)_1$, $(F_1A_1)_2$, $(F_2A_1)_1$, $(F_2A_1)_2$, $(F_3A_1)_1$, $(F_3A_1)_2$ of the figure $(F_1)_1$, $(F_1)_2$, $\cdots\cdots, (F_3)_2$ and the size A_1, as the original model.

Though A_1 is regulated as a random diameter of around 10 cm in design, it is desirable to be homogeneous on size of area in order to satisfy the same condition for experiment. However, it is difficult to have the homogeneous samples on area, so we have took a photograph of samples of figure, and kept the homogeneity by the operation that we project the negative on graph through the enlargement instrument. Namely, at first we have enlarged the optional sample of figure up to around 10 cm on mean diameter, calculated the size of projected area by the graduation of graph, and then decided $(F_1A_1)_1$, $(F_1A_1)_2, \cdots\cdots, (F_3A_1)_2$ by projecting the other samples of figure so as to almost accord with the size[1].

In design, as we presume that the error y is effected by A, the tendency seems to show the loosely monotonous curve. and as the error used in actual analysis is calculated of each sample unit respectively, the problems under the same condition on area are enough able to handle in such a measure of approximation.

The samples of A_2, A_3 are made as follows. In advance we have drawn the basic line through the centre of the negatives of samples of figure and taken b as reading when we made the original model. In design, A_2, A_3 are regulated in 20 cm, 30 cm on optional diameter respectively. Consequently we calculate the values which 2 times and 3 times for each b of the original model. Secondly using the magic lantern apparatus we have decided the values which are called $2b$ respectively for the lengths of base line on the projected faces, enlarged the negative, as samples $(F_1A_2)_1$, $(F_1A_2)_2$, $(F_2A_2)_1$, $(F_2A_2)_2$, $(F_3A_2)_1$, $(F_3A_2)_2$ having A_2 size.

For each case of A_3, the operation is as well as the above. It is considered that the differences among the area obtained are becomming larger in comparison with that of the original model. Neverthless the reason why we have adopted these operation is that it is almost impossible to do the same operation as for the original model. As mentioned above, such a measurement of approximation will be enough for our experiment, too.

By the above operation, 18 sample units n total of $(F_jA_k)_l$ are sketched. Secondly, we have measured these areas of units by planimeter and carried out the order of measurement at random, in this case. The measurements hereby obtained are presumed as the true value m of this experiment.

1) Results of these operation for reduction of the same area are shown in table 1 (I). We may think values in it satisfactory.

Finally, the sample units $(F_jA_k)_l$ sketched should be changed to make the sample units as an object actually to measure. For the above purpose, we have copied the sample units which have become the object of measuring by planimeter on the card board using a carbon paper, and then we made the model disks. In design, for the method of estimation 4 cases are regulated, consequently 5 measurements, direction from mountain side, direction at right angle to mountain side, circumference, the largest diameter, and direction at right angle to the largest diameter, are necessary for getting each estimator. In the cases of 4 measurements except circumference, we have adopted metal caliper using for construction. This is the way to make the bias of instrument in minimum. For circumference, we have adopted metal tape.

4. ANALYSIS

We have made the model disks as an object of measurement. For the purpose of experiment, 5 measurements in total by caliper and tape should be respectively done for the model disks. As the results of these measurements, we become to have estimators about M_i. Each estimator x_{ijkl}[1] which we have obtained is shown in table 1 (Ⅱ).

In addition to this, the results of measurement by planimeter for each sample unit are shown in table 1 (Ⅰ). The difference between the value of (Ⅱ) and (Ⅰ) is to be y.

If we put $100\,[(x_{ijkl}-m_{jkl})/m_{jkl}]=y_{ijkl}$, y_{ijkl} will indicates the ratio of error for each case.

At first, when we analyze each value of y_{ijkl} on the ground table, we have had the results shown in table 5. This analysis is aimed to test on the bias which kept in each method of estimation.

Table 1.

Sample unit	(Ⅰ) $m_{(F_jA_k)_l}$ Measurement of planimeter	(Ⅱ) $x_{(M_iF_jA_k)_l}$			
		M_1	M_2	M_3	M_4
$(F_1A_1)_1$	87.4	87.9	88.8	88.2	89.8
$(F_1A_1)_2$	86.6	83.9	87.6	87.7	87.8
$(F_1A_2)_1$	261.2	258.9	263.7	264.0	269.0
$(F_1A_2)_2$	262.3	252.1	265.9	265.9	264.7
$(F_1A_3)_1$	775.8	776.9	759.8	783.1	795.6
$(F_1A_3)_2$	779.8	742.7	785.8	783.1	779.2
$(F_2A_1)_1$	87.5	91.9	87.8	88.2	89.2
$(F_2A_1)_2$	87.6	97.1	86.3	88.2	88.8
$(F_2A_2)_1$	263.5	276.9	265.9	266.8	261.3
$(F_2A_2)_2$	261.3	288.2	257.1	264.9	267.0
$(F_2A_3)_1$	787.8	825.1	790.8	794.2	785.6
$(F_2A_3)_2$	788.8	874.7	778.2	800.6	798.8
$(F_3A_1)_1$	87.0	90.2	88.7	89.3	88.4
$(F_3A_1)_2$	87.0	83.9	89.9	88.8	90.9
$(F_3A_2)_1$	256.1	262.9	261.4	262.2	260.4
$(F_3A_2)_2$	260.7	248.7	267.9	265.9	271.4
$(F_3A_3)_1$	765.5	781.9	775.0	781.5	773.2
$(F_3A_3)_2$	780.8	743.7	798.8	792.6	807.3

By those results, we have become significant only for main effect M, with $\alpha=0.05$, and the variations on the other factors have been estimated to be very small. In consequence, it will be valid for us not to consider the bias which kept in each method of estimation as homogeneous, but the variations among figures and so on seem to be not necessary to have our special consideration.

Having investigated which is the significant one about M, the following has

1) From this, $x_{(M_iF_jA_k)_l}$ will be abbreviated to x_{ijkl}, and so on.

been acknowledged. Let the difference on freely 2 be D, the units of D equal 36.

Then

$$D^2/(36\times 13.9) \geqq F^1_{36}(0.05)=4.11$$

$$\therefore \ |D| \geqq 45.4$$

In this experiment, the amount of the ratio of error for each M_i is as follows,

$$\left.\begin{matrix} M_1: & -33.7 & M_3: & 25.2 \\ M_2: & 14.8 & M_4: & 32.5 \end{matrix}\right\} \cdots\cdots\cdots\cdots\cdots (1)$$

therefore, it has been supposed that only tne ratio of error in M_1 (namely-33.7) is too small. However these do not mean that the bias of M_1 is large. We can understand that the estimations by M_2, M_3, and M_4 have the tendency to give the bias for the plus direction as average compare with the true value and on the contrary the estimation by M_1 has the tendency to give the bias for the minus direction.

After testing the difference of mean value, we have investigated about variance.

As we have done replication 2 times against each figure and each size, then we can get the ranges of each stratum M-F-A. Table 6 shows the results of analysis on the ranges. The analysis is aimed to test on homogeneity of variance. If we put range as R, it is generally acknowleged that

$$E\{(x-m)^2\}=\sigma^2, \qquad E\{R/d_N\}=\sigma$$

According to table 6, it is recognized that main effects M, F and interaction $M\times F$, with $\alpha=0.01$, have become significant. In consequence, it will be valid for us not to consider the standard deviation as homogeneous. The standard deviation is differed among F too, and we can understand that the difference is varied much by the conbination of M and F.

When we investigated which difference is significant among M, the following has been acknowledged. Let the difference on freely 2 be D, the units of D equal 18.

Then

$$D^2/(18\times 0.33) \geqq F^1_{12}(0.05)=4.75$$

$$\therefore \ D \geqq 5.3$$

In this test, the amount of each M_i is as follows,

$$\begin{matrix} M_1: & 79.9 & M_3: & 3.6 \\ M_2: & 12.8 & M_4: & 18.6 \end{matrix}$$

therefore, it has been supposed that M_1, M_2, M_3, and M_4 are mutually different. If the standard deviation is varied, under the effect of F, it will be estimated to show the different value mutually as average.

As the results of table 5 and 6, we have known that it is necessary to consider M_1 separate from M_2, M_3, and M_4 on bias, and that each M_1, M_2, M_3, M_4 can not be considered as the same on variation moreover the size is different according to combinations of method and figure. Secondly in accordance with these results, we have investigated how much possibility of the ratio of error in each case of M-F would happen.

As it is well known that $M\times F$ is highly significant by table 6, we must consider F when we estimate the size of error. It is not necessary to consider A.

By the ground table aimed to get table 6, we can make table 2 as follows when we calculate the ranges of each stratum in M-F.

Table 2. %

Figure / Method	F_1	F_2	F_3
M_1	3.87	15.60	7.17
M_2	1.23	2.00	1.03
M_3	0.40	0.30	0.50
M_4	2.03	1.70	2.57

Table 3. %

Figure / Method	F_1	F_2	F_3
M_1	3.43	13.82	6.35
M_2	1.09	1.77	0.91
M_3	0.35	0.27	0.44
M_4	1.80	1.51	2.28

The coefficient $1/d_2$, by which the number of measure (N) become 2, is 0.8862. Therefore, in multiplying each value of table 2 by this value we can get table 3. Each value of table 3 can be thought to be the estimators $\hat{\sigma}_{ij}$ for the standard deviation in each st ratum of M_iF_j.

We can consider that individual measurement belonging to M_iF_j have the same standard deviation as shown in table 3. Accordingly estimating on a single tree, the ratio of error in extent as mean value $\pm 3\ \hat{\sigma}_{ij}$ can be thought possible to happen.

Next, we have estimated how much possibility of the ratio of error would happen in each stratum of M-F as average. On bias $M \times F$ shows only the size which can be disregarded. Accordingly if we consider bias as homogeneous in spite of F, A, the ratio of error in M_iF_j can be got as shown in table 4.

Table 4. %

Figure / Method	F_1	F_2	F_3
M_1	−6.32~2.58	−19.80~16.06	−10.11~6.37
M_2	−0.59~2.23	−1.48~3.12	−0.36~2.00
M_3	0.94~1.86	1.06~1.74	0.84~1.96
M_4	−0.52~4.14	−0.14~3.76	−1.14~4.76

The value hereby given will be calculated as follows. Taking the instance of M_1F_1, such is the case that on the basal area the estimation has been done in measuring a single direction from mountain side against the objective cross-section which shows the figure supposed to be circle. As the ratio of error concerning M_1 is −33.7 shown as 18 units in total[1)], the average is −33.7 /18=−1.87. If we take the reliability as 95 %, the confidence interval will become $\pm 3.18 \times 3.43/\sqrt{6} = \pm 4.45$ from $\hat{\sigma}_{11}$ is 3.43 by table 3. Then it is shown as (−6.32~2.58).

In this experiment, it has been expected that in the case of the estimation of basal area by the method M_i which has been done against a group of trees supposed to belong to the strata of F_j the ratio of error effected by the method of estimation will have the same size as shown in table 4 in average. However the value is shown as (bias $\pm t \cdot \hat{\sigma}_{ij}/\sqrt{N}$), and then it increases and decreases by the number N of component trees of objective group. Accordingly in the case of $N > 100$, the confidence interval will be highly small.

When our object of estimation is not for a single tree but for a group of trees, how much possibility of the ratio of error shall we consider to happen?

1) C.f. (1)

The actual stand are set of the cross-section at breast height which shows the various figures. Then it is naturally varied according to the component ratio of F. It must be almost impossible for us to estimate classifing the individual figure of component trees against the set of various F. Now, we have investigated that how much the ratio of error in average shall be expected if we presume the component ratios of F_1, F_2, F_3 in objective stand as homogeneous.

As $\hat{\sigma}_{ij}$ are shown by table 3, the average standard deviations $\hat{\sigma}_i$ for M_i are estimate as follows,

$$\hat{\sigma}_1=\sqrt{(3.43^2+13.82^2+6.35^2)/9}=5.20$$
$$\hat{\sigma}_2=\sqrt{(1.09^2+1.77^2+0.91^2)/9}=0.76$$
$$\hat{\sigma}_3=\sqrt{(0.35^2+0.27^2+0.44^2)/9}=0.20$$
$$\hat{\sigma}_4=\sqrt{(1.80^2+1.51^2+0.28^2)/9}=0.78$$

On the contrary, the estimators $\hat{y}_i$ of mean on M_i are supposed to be $\hat{y}_1=-1.87$, $\hat{y}_2=0.82$, $\hat{y}_3=1.40$, $\hat{y}_4=1.81$ by (1). Finally we shall get the following formulas,

$$\left.\begin{array}{ll} M_1: & -1.87\pm3\times5.22/\sqrt{N} \\ M_2: & 0.82\pm3\times0.76/\sqrt{N} \\ M_3: & 1.40\pm3\times0.20/\sqrt{N} \\ M_4: & 1.81\pm3\times0.78/\sqrt{N}\ (\text{unit } \%) \end{array}\right\} \cdots\cdots(2)$$

N indicates the number of component trees in objective stand of investigation. From those results the ratios of error happening in the estimation can be considerd to decrease according to increase of N.

In the case of judgement concerning the precision of the quality on methods of estimation, it is necessary to consider the 2 factors of both bias and variability. From the results of table 5, it is recognized that evidently we can not give either superiority or inferiority on bias. Because the differences among the methods have been understood as the difference by the plus or the minus direction. However as the results of table 6, M_1, M_2, M_3, and M_4 have been understood to be unequal in variability. Accordingly we could have got the small order of variability M_3, M_2, M_4, M_1 as the order of the quality on methods of estimation.

It is remarkable to use the method by tape of M_3. In other words, the variation of M_3 is highly small under each F. It is recognized that the estimators by M_3 have tendency to give excessive value not only on the average but also on each case. But this matter will not be so important. Because it is supposed that small variability is more important for our estimation, whatever bias it may be. Especially these will be more important when we take the single tree as object of our investigation. Therefore the estimation seems to be highly efficient, which gives the corrective coefficient 1/1.014 to the estimator obtairned by M_3.

At last, the results of our test on the absolute value of y_{ijkl} are shown in table 7. This is understood as the results of (bias+variation). The error itself, which arises from our actual estimation for the basal area at breast height of a single tree, would have such a tendency. It is recognized that the effect of A would have a quadratic tendency.

Table 5.

Factorial effect	SS	df	MS
M	148.31	3	49.43*
F	45.36	2	22.68
A	5.99	2	3.00
$M\times F$	5.22	6	0.87
$M\times A$	1.98	6	0.33
$F\times A$	1.88	4	0.47
$M\times F\times A$	4.42	12	0.37
$R(MFA)$	501.23	36	13.92
T	714.39	71	

Table 6.

Factorial effect	SS	df	MS
M	400.03	3	133.34**
F	57.25	2	28.63**
A	1.39	2	0.70
$M\times F$	165.20	6	25.86**
$M\times A$	1.71	6	0.28
$F\times A$	4.30	4	1.08
E	3.94	12	0.33
T	633.82	35	

Table 7.

Factorial effect		SS	df	MS
A	lin	932.81	1	932.81**
	res	1766.65	1	1766.65**
M		1868.03	3	622.68**
F		278.69	2	139.35
$M\times F$		1325.58	6	220.93*
$M\times A$		1332.80	6	222.12*
$F\times A$		155.67	4	38.92
$M\times F\times A$		821.78	12	68.48
$R(MFA)$		2783.80	36	77.33
T		11265.81	71	

5. SUMMARY

It is supposed that the error y, which happened on our estimation of basal area at breast height, is generally effected by various sources of variation. We have picked up the methods of estimation M as a factor among various source sof variation, and have considered theeffects of M, F, and A on y, considering the figure F and the size of basal area A as the experimental condition. In this experiment we have taken the levels of M, F, and A as 4, 3, and 3, which are as follows replication has been excuted 2 times.

M_1) The method to regulate the diameter measurement of a single direction from mountain side.

M_2) The method to regulate the diameter measurements from mountain side and in direction at right angle to that.

M_3) The method to measure the circumference by tape.

M_4) The method to regulate the diameter measurements on the largest diameter of object and in direction at right angle to that.

F_1) The figure thought to be comparatively near to genuine circle.

F_2) The figure thought to be comparatively near to ellipse.

F_3) The figure thought to be so-called irregular except F_1) and F_2).

A_1) Around 10 cm at optional diameter of object.

A_2) Around 20 cm at optional diameter of object.

A_3) Around 30 cm at optional diameter of object.

If we put $100[(x_{ijkl}-m_{jkl})/m_{jkl}]=y_{ijkl}$, y_{ijkl} will indicate the ratios of error on $(M_iF_jA_k)_l$. ($i=1, 2, 3, 4$; $j=1, 2, 3$; $k=1, 2, 3$; $l=1, 2$). (c.f. table 1). Table 5 shows the results of analysis about y_{ijkl}. Table 6 shows the results of analysis concerning the ranges which calculate in each M-F-A from 2 replications. From the results of table 6, the estimates $\hat{\sigma}_{ij}$ of the standard deviation for each stratum M-F are considered as same as table 3. Consequently estimating on a single tree, the ratio of error in extent as mean value $\pm 3\hat{\sigma}_{ij}$ can be thought possible to happen. Then the errors of ratio on M_iF_j have been estimated as table 4.

When our object of estimation is not for a single tree but for a group of trees, the ratio of error naturally varies according to the component ratios of F_j on objective stand. Now, if we presume the component ratios of F_1, F_2, F_3 in objective stand as homogeneous, the ratios of error on each M_i shown as same as (2) in average are estimated.

In the case of judgement concerning the precision of the quality on methods of estimation, it is necessary to consider the 2 factors of both bias and variability. From the results of table 5, it is recognized that evidently we can not give either superiority or inferiority on bias. Because the differences among the methods have been understood as the difference by the plus direction and the minus one. However as the results of table 6, M_1, M_2, M_3, and M_4 have been understood not to be same of variability. Accordingly we could have got the small order of variability M_3, M_2, M_4, M_1 as the order of the quality on methods of estimation

It is remarkable to use the method by tape of M_3. In other words, the variation of M_3 is highly small. It is recognized that the estimators by M_3 have tendency to give excessive value, but this matter will not be so important. Because it is supposed that small variability is more important for our estimation, whatever bias it may be. Especially these will be more important when we take a single trees as object of our investigation.

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胸高断面積の推定方法による誤差（摘　要）

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胸高断面積を推定する際に生ずる誤差 y は，一般に，数多くの変動原因によつて影響されていると考えられる。われわれは，数多い変動原因のなかから因子として推定方法 M をとりあげ，又，実験条件として形状 F，断面積の大きさ A を考慮して，y に対する M, F, A の効果を考察した。

推定には，輪尺又は巻尺を用いるのが一般であろう。そして，輪尺を用いる推定方法としては，推定する場合の測定方向の規定がいろいろと考えられる。本実験においては，M に対して

M_1）山側からの1方向の直径測定を規定する方法。

M_2）山側から，及び，これと直角方向の直径測定（従つて，合計2方向）を規定する方法。

M_3）外周を巻尺で測定する方法。

M_4）対象断面積の最長径，及び，これと直角方向の直径測定（従つて，合計2方向）を規定する方法。

の4つの推定方法をえらんだ。

尚，F，A に対しては，次に示すように夫々3水準をとり，又，繰返しは2回行つた。

F_1）形状が比較的正円に近いと看做される断面。

F_2）形状が比較的楕円に近いと看做される断面。

F_3）F_1），F_2）以外の所謂不規則な形状と看做される断面。

A_1）任意径が大約 10 cm 程度の断面。

A_2）任意径が大約 20 cm 程度の断面。

A_3）任意径が大約 30 cm 程度の断面。

$100[(x_{ijkl}-m_{jkl})/m_{jkl}]=y_{ijkl}$（$i=1, 2, 3, 4$；$j=1, 2, 3$；$k=1, 2, 3$；$l=1, 2$）(第1表参照)とすれば，$y_{ijkl}$ は $(M_iF_jA_k)_l$ についての誤差率と考えられる。第5表は，y_{ijkl} について分析した結果を示す。繰返しの2回から，M-F-A ごとに Range をとつて分析した結果が第6表である。第6表の結果から，M_iF_j の各階層ごとの標準偏差の推定量 $\hat{\sigma}_{ij}$ は第3表程度と推定されるから，単木個々の推定を行なう場合には，M_i の平均値 $\pm 3\hat{\sigma}_{ij}$ 程度の誤差率が生起する可能性が考えられる。又，この結果から，M_iF_j の誤差率は第4表に示す如く推定された。

推定の対象が，単木個々ではなく，その集団を対象とした場合の誤差率は，当然，対象林分における F_j の構成歩合によつて変化するだろう。今，対象林分における F_1, F_2, F_3 の構成歩合が均一であると仮定すると，M_i によつて平均 (2) に示される程度の誤差率が考えられる。

われわれは，推定方法の精度上の良否を判断する場合には，“偏り”と“変動”の2つの要素についての考察を必要とする。第5表の結果から，偏りについては優劣はつけえないことをしつた。それは，方法間の有意差が正負の方向による差異として認められたからである。しかしながら，第6表の結果から，変動については M_1，M_2，M_3，M_4 ともに同一とは認めえないことをしつた。従つて，推定方法の精度上の良否の順序として，われわれは，変動の小さい順序 M_3，M_2，M_4，M_1 をえたことになる。

注目される方法は，M_3 の巻尺による方法であろう。それは，変動が非常に小さいことである。推定値は，過大な値を与える傾向があることが認められるが，このことは大した問題ではないとおもわれる。何故ならば，偏りはどのような値でも，われわれの推定には変動が小さいことの方が重要とおもわれるからである。単木を調査対象とした場合には，このことは特に重要であるだろう。