A Study on the

Composite Wood-Concrete T-Beam Bridge

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## Acknowledgement

Contents

I Introductory

Some methods of effective application of the "Composite Wood-Concrete Beams"[1] to minor road-bridges for the purpose of increasing strength and durability of the wooden beams were originally provided by Toshigoro TAKAHASHI,[2] who reported[3] the result of his experiment in 1940 and recommended the use of beams composed of ordinary Portland-cement and "Ezomatsu" (*Picea jezoensis* CARR.).[4]   Several road-bridges were built of the C. W. C. beams of this kind in Hokkaido prefecture, where they proved excellent properties both in strength and durability even 10 years after their erection.

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[1] In this article, denoted by "C. W. C. beams"   [2] Roadengineer, Bureau of Civil Engineering Works, Prefectural Government of Hokkaido   [3] 高橋敏五郎 (Toshigoro TAKAHASHI) : 「木コンクリート橋」道路, 昭15, 第2巻 12號 ("Wood-concrete Bridges", Roads, Vol. 2, No. 12), 1940
[4] A kind of timber popularly used for wooden structures, produced exclusively in Hokkaido Island

The results of some supplementary experiments of the beams of the same kind were also reported recently. Dr. Takeo FUKUDA [2] performed further theoretical analysis [3] of the C.W.C. beams and gave formulas similar to those used for the design of the reinforced concrete beams. The theory is based on the assumptions that, 1) the stresses are always less than the proportional limits and stress-strain proportionality is assured, 2) any cross-section of a beam remains plane even after bending, 3) a certain ratio of the modulus of elasticity of the concrete and that of the wood never changes in any stage of bending, and 4) there is no tensile stress in the concrete. According to the theory, following formulas are derived for the case of the beam with T-shaped section and rectangular connecting teeth, which prevent slip between the concrete and the wood, as shown in Fig. 1.

Fig. 1 Composite wood-concrete T-beam
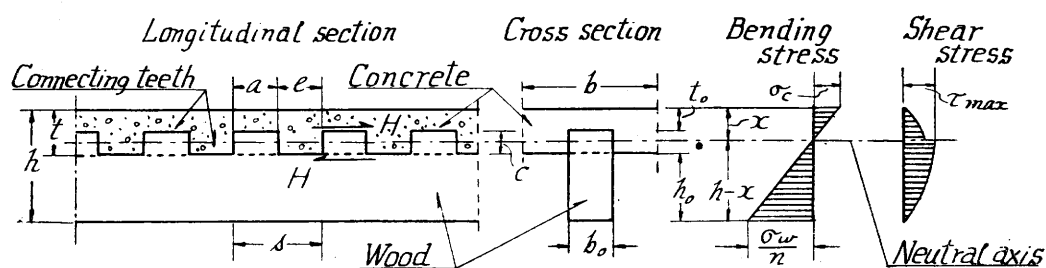
(1) Position of the neutral axis: The equation of the condition that, the total sum of the geometrical moments of the effective cross-sectional areas in regard to the neutral axis must be zero, is given in the expression,

$$A_c \left(x - \frac{t_o}{2}\right) - nA_w \left(h - x - \frac{h_o}{2}\right) = 0$$

where $A_c \fallingdotseq t_o b$ = Effective sectional area of the concrete

$A_w \fallingdotseq h_o b_o$ = Effective sectional area of the wood

h = Total depth of the beam

t_o = Effective depth of the concrete-slab

h_o = Effective depth of the wood-web

b = Width of the concrete-slab

b_o = Width of the wood-web

(1) 伊福部宗夫 (Muneo IFUKUBE):「木コンクリート橋設計に關する考察」道路, 昭23, 2號
    ("Considerations on the design of wood-concrete bridges", Roads, No. 2, Feb. 1948, pp.32~33)
(2) Prof. of Civil Engineering, 2nd Faculty of Engineering, Tokyo University
(3) 福田武雄 (Takeo FUKUDA), 木構造學 (Wooden Structures), Tokyo, 1949, pp.163~175

x = Vertical distance to the neutral axis from the upper surface of the concrete-slab

$n = E_w/E_c$ = Ratio of the modulus of elasticity

E_w = Modulus of elasticity of the wood

E_c = Modulus of elasticity of the concrete

From which, the position of the neutral axis,

$$x \doteqdot \frac{A_c t_o + n A_w h}{2A_c + n A_w} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(1)$$

(2) Moment of inertia of the section: The effective moment of inertia I of the beam-section in regard to the neutral axis is given with the terms of x and other known values;

$$I = \frac{b}{3}\left[x^3 - (x-t_c)^3 \right] + \frac{n b_o}{3}\left[(h-x)^3 - (h-x-h_o)^3 \right] \quad \cdots\cdots\cdots\cdots\cdots\cdots(2)$$

(3) Extreme fiber stresses: If the bending moment M at any point of the beam due to the external forces is given, then the extreme fiber stress σ_c of the concrete and σ_w of the wood,

$$\sigma_c = \frac{M}{I} x, \qquad \sigma_w = n\sigma_c \frac{1-k}{k} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(3)$$

where, $k = k/h$

(4) Maximum shear stress: Denoting S the shearing force at any point of the beam due to the external forces, the maximum shear stress,

$$\tau_{max} = \frac{SG}{I b_o} \doteqdot \frac{3}{2}\frac{S}{b_o h} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

where, G = Geometrical moment, in regard to the neutral axis, of the part of the section divided by the same axis

$I/G \doteqdot \dfrac{2}{3} h$ (approximately in most cases)

(5) Stress in the connecting teeth: Assuming that the horizontal shear stress is uniformly distributed along a short distance s of the base line of one set of the teeth, and denoting H the total stress along the base surface of length s, then

$$H = \tau_{max} b_o s$$

Hence, the horizontal shear stresses in the concrete and in the wood,

$$\tau_c = \frac{H}{be}, \qquad \tau_w = \frac{H}{b_o a} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(5)$$

If these formulas are reasonably applicable, the necessary numerical values required for the adequate design of the C. W. C. T-beams can be easily obtained.

The authors attempted to investigate and examine the applicabilities of these formulas for the bridge design by means of the laboratory experiments.

II The preliminary experiments

A. Notes on the materials used for the C. W. C. test-beams

Properties of the materials, e. i. wood, cement, gravel and sand, used for the test-beams are shown in tables I ~ IV in general.

Table I   Wood

Wood species	"Sugi" (*Cryptomeria japonica* D.DON) *			
Timber	Grown in Akita Prefectu e			
Average width of year- ing	2.5~4.0 mm			
Specific gravity	Air dry 0.43~0.52 Absolute dry 0.38~0.46			
Mecha ical pro erties (at moisture content 13~15%)	Ultimate strength	Compression (// to the grain)	280~305	kg/cm²
		Tension (// to the grain)	190~300	"
		Bending	370~450	"
		Shear (// to the grain)	50~ 80	"
	Modulus of elasticity for tension		$4.7 \times 10^4 \sim 9.1 \times 10^4$ kg/cm²	

* "Sugi" is one of the most popular kind of the structural timbers all over the country except in Hokkaido

Table II   Cement

Commercial name	Chichibu Silica Ce nent	
Manufactured by	Chichibu Cement Co., Saitama Prefecture	
Specific gravity	2.91	
Ultimate strength examined by J. E. S. Standard mortar test (28th day)	Compression	Average 97~220 kg/cm² 212 "
	Bending	Average 46.8~56.2 kg/cm² 50.7 "

Table III   Gravel

Kind of gravel	Specific gravity	Maximum size	Composition	
			Grade of size	Percentage by weight
Natural gravel taken from the river bed of Arakawa, Saitama Prefecture	2.63	20 mm	5~10 mm	45 %
			10~15	45
			15~20	10
			Total	100 %

B. Composition of the concrete mix used for the experiments

To secure excellent qualities such as strength, workability, consistency and plas-

ticity of the concrete, J. E. S. standard slump-test and strengh-test were performed, while the water-cement ratio and the gravel-sand ratio were changed 40~70% and

Table IV   Sa·d

Kind of Sand	Specific gravity	Composition	
		Grade of particles	Percentage by weight
Natural sand taken from the river bed of Arakawa, Saitama Prefecture	2.65	<0.15 mm 0.15~0.30 0.30~0.60 0.60~1.20 1.20~2.50 2.50~5.00 >5.00	2.1 % 9.5 20.3 36.2 25.7 6.0 0.2
		Total	100.0 %

140~160% respectively. According to the results of the tests and following to the generally recommended values, the composition and the requirements for the con- crete mix were determined as shown in table V.

Table V   Concrete

Water-cement ratio by weight (W/C × 100)	Gravel-sand ratio by weight (G/S × 100)	For 1 m³ of concrete				Ultimate compres-ive strength of 28th day	Slump
		Cement	Water	Sand	Gravel		
53	140	373 kg	210 kg	679 kg	951 kg	200 kg/cm²	8~11cm

C. The results of the preliminary experiments

In order not only to obtain an adequate design but also to make clear some fun- damental properties of the C. W.C. beams, the authors carried out various preliminary experiments, the results of which are summarized:

(1) The adhesive strength between the wood (sawed timber with rough surface) and the concrete was estimated at average 2 kg/cm² by the shearing tests; there- fore it may be neglected for the general calculation of the beam.

(2) The effect of the nails used at the connecting teeth for the purpose of rein- forcing the unification of wood and concrete was not clear. Although they were useful to increase the ultimate resistance by the shearing tests, the large amount of the bending strain in each nail and the crush of the wood fibers adjacent to the nail surfaces were observed. Hence, the reinforcing force of the nails should be rather neglected for the general calculation of the beam.

(3) The depth of the concrete-slab; t= 4.0 cm was sufficient when the maximum

grade of the gravel in the concrete was 15 mm.

(4) The span-depth ratio of the C. W. C. beams should be prefered larger than 8, because the ultimate strength by bending tests suddenly dropped when the span-depth ratio was decreased less than 8.

(5) By the bending tests of the C. W. C. beams with wood-webs of poorer flexural rigidity the failures occured by the cracks in the lower parts of the concrete-slabs. When higher flexural rigidity was given to the wood-webs, the failures due to the bending stresses were observed neither in the concrete-slabs nor in the wood-webs; in most cases, failures appeared in the form of creep or slip in the wood-teeth adjacent to the both end of the beams due to the accompanied horizontal shear.

(6) The maximum bending strength was obtained when the theoretical neutral axis calculated by the previously cited formulas, in which $n = E_w/E_c = 0.5$ was applied, fell on the web nearly coincident with the lower surface of the concrete-slab.

(7) About the shape and the dimension of the connecting teeth; the larger the length of the teeth $s = a + e$, or the larger the depth of the teeth c, the larger the deflection of the beam. When $a : e = 1 : 1 \sim 1 : 1.6$, the concrete teeth did not fail due to the horizontal shear. By smaller depth of c, the side surfaces both of the concrete and the wood teeth crushed by the compression due to the horizontal shear. When $s = 13$ cm, $a : e = 1 : 1 \sim 1 : 1.6$ and $c = 1.5 \sim 2.0$ cm, beams having rectangular cross-sections of $b = b_o$, failed by the diagonal cracks at the bottom corner of the concrete teeth. In the case of the beams with T-formed cross-sections of $b = 2b_o \sim 3b_o$ and with the same size of teeth as mentioned above, there observed no failure in the concrete, but the horizontal slips at the bottom of the wood-teeth adjacent to the both ends of the beams, which immediately followed by the failures due to the extreme fiber stresses of the wood-webs and, at the same time, the random cracks in the concrete-slabs.

(8) When a piece of air dry timber (moisture content $14 \sim 22\%$) was used in the building up of the C. W. C. beam, water in the concrete transfered into the wood, causing the decrease of strength of the concrete.

When the timber was sunk in water more than 7 days before the building up of the beam and the moisture content of the wood was increased to 50 % or more, there observed almost no change in the moisture content of the wood after the setting

·of the concrete and no influence to the water content of the cement paste.

(9) There were no remarkable difference in the strength of the beams when the ·curing of the concrete was performed either in a water pool or kept under a wet mat on which water splayed frequently.

Ⅲ The bending test of the composite wood-concrete T-beam bridge-models

Considering the capacity of our testing machine and according to the previous investigations mentioned above, the authors prepared bridge-models composed of the C. W. C. beams, the effective span of which chosen at 105 cm. Each model was built of three C. W. C. T-beams, and the continuous concrete-slab unified them to a solid bridge similar to an ordinary reinforced concrete T-beam-bridge. The design and the dimension of the bridge-models are shown in Fig. 2.

Fig. 2 C. W. C. T-beam bridge model
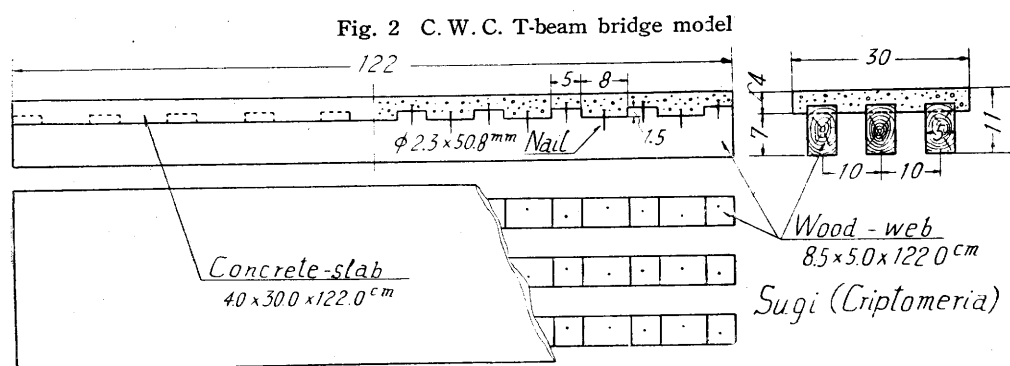
The same materials and concrete mix as described in II. A. B, were used and cured in the water pool during 20 days while temperature of water kept at 20° C.

Fig. 3 Method of loading
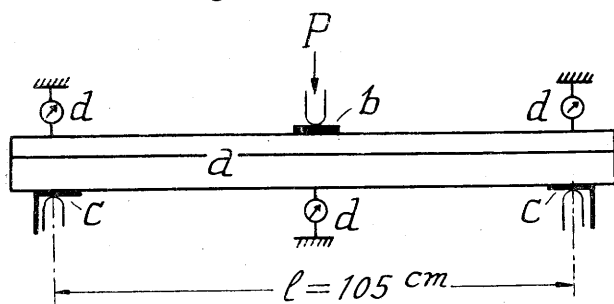
P; Statical concentrated load at the center of the span
a; Test bridge model
b,c; Steel plate preventing the damage of the surface
d; Dial gauge
l; Effective span

The bending tests were performed 20 days after the setting of the concrete by the AMSLER-type 30 ton universal testing machine[1] equiped in the Laboratory of Forest Utilization attached to the Division of Forestry, Faculty of Agriculture, Tokyo University. The method of loading is shown in Fig.3. The load

(1) Manufactured by Tokyo Koki Co. (東京兩機株式會社)

was increased in each case in the ratio of 5~10 kg/sec until breaking. The amount of the displacement of the three definite points, e. i. δ_1 at the span center and δ_2, δ_3 at the both supports, were measured by the dialguages attached to the beam.

Then, the real amount of the central deflection of the beam was calculated

$$\delta = \delta_1 - (\delta_2 + \delta_3)/2$$

For the purpose of comparing the strength and the flexural rigidity of the C.W. C. beam-bridges to those of the wooden beam-bridges, the bridge-models, each of which with three wooden beams, were also tested in the same manner. The same dimensions were given to these wooden bridge-models as in the C. W. C. models, while the slabs were made of wooden boards simply nailed upon the beams in such a manner that they never resist any bending moment.

The results of these experiments may be explained as follows. (See also Fig. 4)
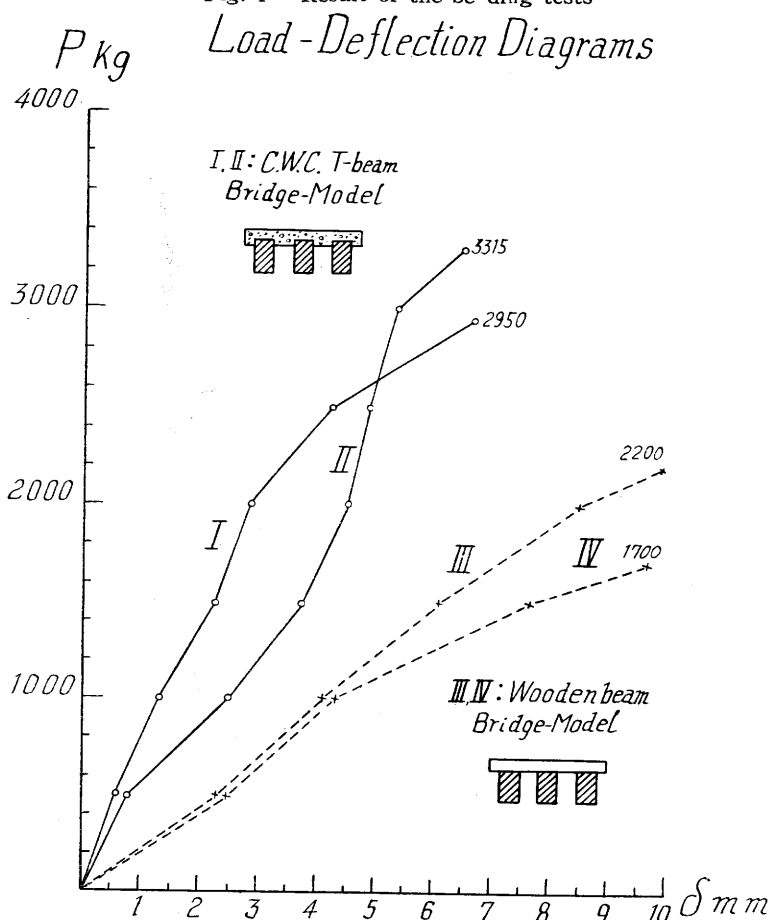
(1) The ultimate strength of the C.W. C. T-beam-bridge was more than $1\frac{1}{2}$ times higher than that of the wooden beam-bridge of the same size.

(2) The flexural rigidity of the C. W. C T-beam-bridge was far greater than that of the wooden beam-bridge. For the amount of the maximum deflection equals to 1/300 of the span length,[1] e. i. $\delta = 3.5$ mm, the ratio of the cor-

Fig. 4   Result of the be ding tests

Load - Deflection Diagrams

P kg

I, II: C.W.C. T-beam Bridge-Model

III, IV: Wooden beam Bridge-Model

δ m m

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[1] "The maximum deflection=1/300 of the span length" is usually specified for the allowable limit of the deflection of the teams of the wooden bridges.

responding loads, P_c and P_w of the C. W. C. bridge and wooden bridge respectively, was

$$P_c : P_w = 1^3/_4 \sim 2^3/_4 : 1$$

(3) It was proved by the experiments that the theoretical formulas for the C. W. C. beams were practically reliable. For instance, if we take [1] $n = E_u/E_c = 0.5$, $t_o = 2.5\text{cm}$, $b = 10$ cm, $b_o = 5$ cm, $h = 11$ cm, $h_o = 7$ cm in eq. (1), (2), then,

$$x = 3.8 \text{ cm}, \quad I = 206.69 \text{ cm}^4$$

Substituting these values to eq. (3) and taking allowable stresses $\sigma_{ca} = 50$ kg/cm and $\sigma_{wa} = 60$kg/cm, we obtain the theoretical resisting moment of the single C. W. C. beam and the theoretical extreme fiber stress of the wood,

$$M = 2720 \text{ kg/cm}, \qquad \sigma_w = 47 \text{ kg/cm} < \sigma_{wa}$$

So that the amount of the allowable load, which assumed to be a single concentrated load Q_1 at the center of the span of length $l = 105$ cm, for each one of the three beams is

$$Q_1 = \frac{4M}{l} = \frac{4 \times 2720}{105} = 104 \text{ kg}$$

While, the mean value of the real breaking load Q_2 borne by each beam is approximately,

$$Q_2 = P_b/3 = \frac{2950}{3} = 983 \text{ kg} \qquad \text{where, } P_b = \text{Total breaking load}$$

Comparing Q_1 with Q_2, we know that, $\quad Q_2 = 9.4 \, Q_1$

Which means the factor of safety for bending of the C. W. C. T-beam is approximately 9.4 and is fortunately larger than the factor of safety in regard to the allowable stresses used for the theoretical calculation; so that the theoretically designed beam is safe enough by statical bending.

IV Conclusion

Considering the results of the investigations mentioned above, it may be concluded that,

(1) The strength and the flexural rigidity of the C.W.C. beam-bridges are considerably higher than those of the wooden beam-bridges of the same size.

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[1] See also, 伊福部宗夫 (Muneo IFUKUBE)：「木コンクリート橋の設計に關する二，三の考察」北海道土木試驗所彙報第6號，昭25，5月 (″Studies on Wood-Concrete T Beam Bridge″, Report of the Civil Engineering Experimental Station of Hokkaido. May, 1950, p.p.5,6.)

(2) The theoretical formulas are practically applicable for the proper design of the C. W. C. beams.

(3) Adequately designed C. W. C. beam-bridges composed of good materials might be safely applied for the practical purposes if they were carefully erected.

Moreover, it is also evidently foreseen that the durability[1] of the C. W. C. beam-bridges may be far more excellent than that of the ordinary wooden beam-bridges, because the wooden parts of the former beams are covered by the concrete-slabs and thus weatherproved.

The authors have now the opinion that the extended use of the C.W.C. beam-bridges for the forest-roads instead of the wooden-bridges may be successfully expected not only from the view point of the structural mechanics but also for their economical advantages, especially when and where adequate steel materials are not available.

木・コンクリート集成T桁橋の研究 （摘　要）

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本文は木・コンクリート集成桁の理論と集成桁を使用した橋梁の模型實驗の結果を比較檢討した研究報告である。橋梁模型はスギ材とコンクリート（秩父シリカセメント及荒川産骨材使用）を矩形齒形により集成したT桁3個でコンクリート・スラブの幅 30cm, 長さ122cm に作製し, 有効徑間 105cm としアムスラー型試驗機で中央1點集中荷重による曲げ破壊試驗を行つた。この模型實驗に先立ち各般の豫備實驗を行つてコンクリートの配合及T桁各部の寸法等を最も合理的に定めた。模型實驗の結果

（1）　木・コンクリート集成T桁橋の破壞強度及彈性剛度は同一寸法の木桁橋のそれらに比べて相當大であること

（2）　木・コンクリート集成桁に關する理論公式は實際の橋梁設計に應用して差支ないこと

（3）　適當な設計により, 良好な材料で構築した木・コンクリート集成桁橋は, その架設方法に注意すれば十分實用に供し得ること

が認められた。以上の外耐久力の點では明かに有利であることが豫想されるから, 林道用橋梁に木・コンクリート集成T桁を用いることは構造力學上並に經濟上の見地よりして極めて有効であると思はれるが, 今後の實驗によりそれを確認し度い。尚この實驗は文部省科學研究費の補助を受けて行つたものである。

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(1) The authors are now preparing the execution of the durability experiments and impact tests of the C. W. C. beam-bridges. See also, 北村幸治 (Koji KITAMURA)：「木コンクリート桁橋の調査報告」北海道土木試驗所報告第7號, 昭25, 9月 ("Report on the Investigation of Wood-Concrete Beam Bridges", Report of the Civil Engineering Experimental Station of Hokkaido, Sept., 1950, p.p 20～27)