# Part II. The Formation and Transformation of Operationalism

# Chapter 3. Demystifying Dimensional Analysis: The Beginning of Bridgman's Scrutiny of Theoretical Physics

Many historians do not seem to have so far attempted to clarify the reason Bridgman started his philosophical analysis of physics. It may be that Bridgman himself made several comments on the beginning of his philosophical attempt, both publicly and privately, and let historians think that they could give a sufficient account for his motivation only by quoting them. For instance, in a symposium titled "the present state of operationalism," organized in 1953 as a part of the meeting held by the American Association for the Advancement of Science, Bridgman recollected that he had started to prepare his philosophical standpoint in 1914, when he took up two advanced courses in electrodynamics and began examining the special theory of relativity.[1] He found the underlying conceptual situation "very obscure"; it caused "much intellectual distress" that he tried to alleviate as best he could. Thus, the courses in electrodynamics stimulated Bridgman to start his philosophical scrutiny of the foundations of physics.

On the same occasion, Bridgman mentioned another "cause of distress," namely the situation in dimensional analysis, which led him to philosophical reflection.

> Another cause of distress was the situation in dimensional analysis. [...] The analysis, which was essentially operational, although the word was not used, was published in 1922 (*Dimensional Analysis*, Yale Univ. Press). I think the word *operation* was first explicitly used in a discussion that I gave at the Boston meeting of the AAAS [American Academy of Arts and Sciences] in 1923 at a symposium on relativity theory.[2]

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[1] P. W. Bridgman, "The Present State of Operationalism," Philipp Frank, ed., *The Validation of Scientific Theories* (New York: Collier Books, 1961), pp. 75-80, p. 76.

[2] *Ibid.*

In 1922, before publishing his thorough analysis of relativity theory, Bridgman first published *Dimensional Analysis*,[3] in which he applied his operational method for the first time to the problem of dimensional analysis, though he did not invoke the concept "operation" in his discussion.

Together with Bridgman's other similar statements given on different occasions, Albert Moyer's detailed study [4] is helpful in comprehending the path along which Bridgman started his philosophical attempt. In 1914, upon the death of the Harvard physics professor Benjamin Osgood Peirce, Bridgman took over Peirce's courses on electromagnetic theory, including material from the special theory of relativity. The preparation for these courses made him recognize the problematic situation in physics. Though starting his own attempts to scrutinize relativity theory during World War I, he completed and published the result around 1920, as war work interrupted his effort in this direction. Meanwhile, he turned his attention to another methodological matter of physics, dimensional analysis. Thus, his first published reflections on the foundations of physics were on dimensional analysis, or "Tolman's Principle of Similitude," as the title of his first paper on this subject suggests.[5] Bridgman first took up dimensional analysis, partly because "[t]he dimensional situation proved comparatively simple."[6] Simple as it might be, his first attempt was more successful than he had expected. Bridgman could "think the situation through to [his] own satisfaction" and could raise his

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[3] P. W. Bridgman, *Dimensional Analysis* (New Haven: Yale University Press, 1922).

[4] Albert E. Moyer, "P. W. Bridgman's Operational Perspective on Physics: Part I: Origins and Development," *Studies in History and Philosophy of Science*, 22 ( 1991 ), pp. 231-258; "P. W. Bridgman's Operational Perspective on Physics: Part II: Refinements, Publication, and Reception," *Studies in History and Philosophy of Science*, 22 ( 1991 ), pp. 373-397.

[5] P. W. Bridgman, "Tolman's Principle of Similitude," *Physical Review*, 8 (1916), pp. 423-431.

[6] P. W. Bridgman, "The Present State of Operationalism," p. 76.

"intellectual morale." After the publication of *Dimensional Analysis* he could try "to see what is really involved in electromagnetic field theory, relativity theory,"[7] and quantum theory.

Furthermore, the scrutiny of dimensional analysis turned out to be the first step toward the construction of the framework of his operational perspective. In 1937 he wrote to one of his frequent correspondents Arthur Bentley of the connection between dimensional and operational analyses.

> I think my point of view with regard to dimensional analysis is very closely connected to my operational point of view, in fact I got started on the latter only after having worked through my ideas about dimensional analysis. The main point of my dimensional analysis was that it must be operational, although I did not express it in those words.[8]

Later, as we have seen, he would publicly repeat a similar comment that his examination of dimensional analysis was essentially operational, although he did not use the word explicitly. More precisely, he once explained that what he did in *The Logic of Modern Physics* was basically the same as what he did in *Dimensional Analysis*.[9]

Clearly, Bridgman's scrutiny of dimensional analysis deserves a careful study when one hopes to clarify the origins of his operational analysis. Furthermore, by examining why he picked up dimensional analysis as his first target, one can comprehend what made him plunge into a long-standing critical activity in theoretical physics. Again, Bridgman himself explained it several times.

> [Before publishing *The Logic of Modern Physics*] I had already

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[7] P. W. Bridgman, "Operational Analysis," *Philosophy of Science*, 5 (1938), p. 115.

[8] Bridgman to Bentley, Dec. 18, 1937, PWBP.

[9] P. W. Bridgman, "P. W. Bridgman's 'The Logic of Modern Physics' after Thirty Years," *Daedalus*, 88 (1959), p. 519.

> come to grips with one comparatively minor detailed situation, that presented by dimensional analysis. Here, the bulk of professional writing on the subject afforded what seemed to me a pretty sorry exhibition of "metaphysics" in its bad sense. I had been able to think the situation through, separate the wheat from the chaff, and rid the whole dimensional situation of its metaphysics.[10]

In other words the purpose of his first attempt in the philosophy of physics was to grasp what dimensional analysis actually meant, or to "demystify" dimensional analysis.

Furthermore, Bridgman pointed out another aspect of his scrutiny of dimensional analysis which has seemingly attracted less attention so far: he started the discussion on dimensional analysis by his keen concern with the situation where this theoretical method in physics "was often so expounded as to raise doubt whether experimental work was really necessary at all."[11] This statement may not be very easy to comprehend for those who know how physicists and engineers use dimensional analysis today; no one imagines that dimensional analysis can threaten the value of experimental research.

In this chapter I will try to clarify what made Bridgman start to scrutinize dimensional analysis, namely, "metaphysics" and "threat" that he mentioned to have found disagreeable in dimensional controversy, and will discuss in this light what led him to the philosophical examination of the foundations of physics. So far two historians have published detailed studies of Bridgman's concern with dimensional analysis. Their interests were, however, mainly in finding the origins of his operational perspective, paying relatively little attention to the problem of his motivation. Maila Walter has contributed one chapter of Bridgman's intellectual biography to the discussion of the role of

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[10] *Ibid.*

[11] *Ibid.*

dimensional analysis in the formation of his operational approach, describing the dimensional controversy among Bridgman's contemporary physicists and suggesting that dimensions meant "the building blocks of physical reality" to him.[12] Moyer has pointed out that the phrase "rules of operation" is frequently used in *Dimensional Analysis*, suggesting that the author had already reached the operational interpretation of physical quantities, though not self-consciously.[13] These discoveries are important in reconstructing the way in which Bridgman formed his philosophy. I will, however, suggest the significance of dimensional analysis in another way by mainly focusing on the situation which turned his attention to the foundations of physics, especially on the fact that he felt the need to advocate the value of experimental research when he started to pay attention to dimensional analysis.

## 3.1. Dimensional Analysis: Its Physics and Beyond

### 3.1.1. The Origin and Development

Though the origin of the notion of dimension went back to ancient times, it was Jean Baptiste Joseph Fourier who established the foundations of dimensional analysis.[14] In his *Théorie analytique de la*

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[12] Maila L. Walter, *Science and Cultural Crisis*, pp. 81-103.

[13] Moyer, "P. W. Bridgman's Operational Perspective on Physics."

[14] For general discussion and history of dimensional analysis, see, Enzo O. Macagno, "Historico-critical Review of Dimensional Analysis," *Journal of the Franklin Institute*, 292 (1971), pp. 391-402; Ronald Laymon, "Idealizations and the Reliability of Dimensional Analysis," in, Paul T. Durbin ed., *Critical Perspectives on Nonacademic Science and Engineering* (*Research in Technology Studies*, Vol. 4) (Bethlehem: Lehigh University Press; London and Toronto: Associated University Presses, 1991), pp. 146-180; and, E. T. Layton, Jr., "The Dimension Revolution: The New Relations between Theory and Experiment in Engineering in the Age of Michelson," in Stanley Goldberg and Roger H. Stuewer, eds., *The Michelson Era in American Science 1870-1930* (New York: American Institute of Physics, 1988), pp. 23-39.

*chaleur,*[15] published in 1822, Fourier specified the exponents for the length dimension of several physical quantities. For example, the dimension of the number of units of length is 1, and of the density -3. He also stated that every physical quantity can be expressed as a certain combination of the five fundamental physical quantities, length, time, temperature, weight, and heat, though he only left a table of physical quantities expressed as combinations of three of the five quantities, length, time, and temperature. According to the rules established by Fourier, the dimensional formula of velocity is $[L/T]$ or $[LT^{-1}]$, where L and T represent length and time, respectively. In this case the dimensions of length and time are 1 and -1. For acceleration, the dimensional formula is $[L/T^2]$ or $[LT^{-2}]$, for force $[ML/T^2]$ or $[MLT^{-2}]$, and for energy $[ML^2/T^2]$ or $[ML^2T^{-2}]$, where M represents mass.

Fourier established many other fundamental principles of dimensional, including the principle of dimensional homogeneity which states that one can add only terms that have the same dimensional formula, as well as the requirement that the dimensional formula is independent of the system of units of measurement. In fact, Fourier's main concern was dimensional homogeneity. Though giving the dimensional expressions to physical quantities and understanding their significance, Fourier did not publish any more remarkable result that could show the strength of this method as a research tool.

More than fifty years later, Lord Rayleigh developed Fourier's principles into a method in theoretical research. In his *Theory of Sound* (1877-1878) Rayleigh applied dimensional analysis successfully to several problems in physics.[16] The first example was the vibration of a

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[15] Joseph Fourier, *Théorie analytique de la chaleur* (Paris: Chez Firmin Didot, 1822).
[16] J. W. Strutt (Baron Rayleigh), *The Theory of Sound* (London: Macmillan, 1877-78), 2 vols.

mass attached to the center of a stretched string.[17] For the vibrations of small amplitude, the differential equation is

$$Md^2x/dt^2 + 2T(x/a) = 0, \qquad (1)$$

where $M$ is the mass, $x$ the displacement, $t$ time, $T$ the tension of the string, and $a$ half the length of the string. By solving this equation, the period of the vibration, $\tau$ can be given as

$$\tau = 2\pi\surd(aM/2T). \qquad (2)$$

Considering the dimensions of physical quantities involved, Rayleigh showed a way to derive the expression of $\tau$ without solving the equation. As $\tau$ should be expressed as a combination of physical quantities that appear in the equation (1), $\tau$ is a function of $a$, $M$, and $T$. Therefore,

$$\tau = f\,(a, M, T). \qquad (3)$$

The dimensional formula of $\tau$ should be [T], while that of $T$ [MLT$^{-2}$], of $a$ [L], and of $M$ [M]. The value of $\tau$ must be independent of the units of time, mass, or length, as no assumption concerning the units is made in deriving the differential equation. The only possible function of $a$, $T$ and $M$ that has the same dimensional formula as $\tau$, satisfying at the same time the condition that it should be independent of the units of measurement, is $T^{-1/2}\,M^{1/2}\,a^{1/2}$. Thus, we have obtained

$$\tau \propto T^{-1/2}\,M^{1/2}\,a^{1/2}. \qquad (4)$$

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[17] I owe much of the following discussion to Macagno, "Historico-critical Review of

This shows that one can derive the essential part of the expression of $\tau$ by dimensional analysis.

Though Rayleigh developed dimensional analysis as a theoretical tool, he did not give it a mathematical formulation. This task, together with the development of more practical applications, was taken over by French engineers and physicists[18] during the last quarter of the nineteenth century. One of their notable results was the discovery and justification of the fundamental theorem of dimensional analysis, to be called the Π theorem.[19]

In the meantime, some English scientists ventured to discuss metaphysical implications of physical dimensions. For example, W. Williams, an assistant in the Physical Laboratory, Royal College of Science argued for more profound possibility of dimensional formulas.

> The dimensional formulae may be taken as representing the *physical identities* of the various quantities, as indicating, in fact, how our conceptions of their physical nature (in terms, of course, of other and more fundamental conceptions) are formed—just as the formula of a chemical compound indicates its composition and chemical identity. This is evidently a more comprehensive and fundamental view of the matter, and from this point of view the primitive numerical significance of a dimensional formula as merely a change ratio between units becomes quite a dependent

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Dimensional Analysis."

[18] For example, J. Bertrand, "Sur l'homogénéité dans les formules de physique," *Cahiers de récherche de l'Academie de Sciences*, 86 (1878), pp. 916-920; F. Lucas, "Sur les équations abstraites du fonctionnement des machines," *Bulletin de la Societé Mathématique de France*, 19 (1891), pp. 152-158; E. Carvallo, "Sur une similitude des founctions des machines," *La Lumière Électrique*, 42 (1891), pp. 506-507; and, A. Vaschy, "Sur les lois de similitude en physique," *Annales Télégraphiques*, 19 (1892), pp. 25-28.

[19] Carvalo, "Sur une similitude des founctions des machines"; and Vaschy, "Sur les lois de similitude en physique." The following roughly describes the Π theorem as stated by Vaschy: "Let $a_1$, $a_2$, ..., $a_p$ be $p$ physical quantities. Now, if one can arbitrarily choose $k$ among the $p$ units of the quantities $a_1$, $a_2$, ...$a_p$, any relationship among them which holds for any choice of the fundamental units may be reduced to a relationship among $(p-k)$ dimensionless parameters which are monomial combinations of $a_1$, $a_2$, ..., $a_p$."

> and secondary consideration.[20]

In his understanding, mass, length, time and temperature in physics played a role similar to the one the chemical elements played in chemistry. Some physicists even tried to find out which physical quantities were fundamental like the chemical elements and which were derived from the fundamental ones as chemical compounds are produced by combining some chemical elements. They considered physical quantities as blocks constituting physical reality.

On the other hand, because of its convenience in solving well-know problems in physics, some started to regard dimensional analysis as a theoretical tool that might make at least part of experimental work unnecessary. In 1915, for example, Rayleigh wrote a paper in order to suggest wider possibility of this method and emphasized its convenience and efficiency: "It happens not infrequently that results in the form of 'laws' are put forward as novelties on the basis of elaborate experiments, which might have been predicted *a priori* after a few minutes' consideration."[21] The enthusiasm over dimensional analysis was one of minor products of the theoretical dominance in physics in the first few decades of the twenties century.

### 3.1.2. The Development in America

In the United States the arrival of relativity theory ignited the discussion on dimension. Though many Americans did not immediately understand the implications of the novelty of relativity theory, some sensitively reacted to it. Among them, Gilbert N. Lewis

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[20] W. Williams, "On the Relation of the Dimensions of Physical Quantities to Directions in Space," *Philosophical Magazine*, 34 (1892), pp. 234-271, p. 237. Emphasis in original.

[21] Lord Rayleigh, "The Principle of Similitude," *Nature*, 95 (1915), pp. 66-68, p. 66.

and Richard C. Tolman went further to discuss the underlying structure of physical reality.[22] Attempting to reach a theoretical discovery as fundamental and striking as relativity, Lewis focused on physical dimensions, while Tolman took up a notion of similitude, which leads to a result similar to that of dimensional discussion.

In 1914, in a paper co-authored with Elliot Q. Adams,[23] Lewis argued that Einstein's principle of relativity[24] had enabled scientists to construct a four-dimensional geometry for kinematics in which an interval of time was a length. It follows from their discussion that the dimensions of any kinematics should be represented by a power of a single dimension. If this dimension can be denoted by [I], the dimension of area is $[I^2]$, and of volume $[I^3]$. More remarkably, they discussed, velocity would become dimensionless, while the dimension of acceleration is $[I^{-1}]$, of angular velocity $[I^{-1}]$, and of angular acceleration $[I^{-2}]$. Thus, all dimensions related to mechanical quantities can be expressed as combinations of this interval [I] and mass [M]. For example, energy and momentum have the dimension [M], force $[M][I^{-1}]$, and pressure $[M][I^{-3}]$.

Lewis and Adams went on to present bolder assumptions. In their understanding, the discovery of the principle of relativity made it possible to express all the mechanical quantities in terms of two arbitrary units of interval and mass. Apparently, if one more universal relation like the principle of relativity is found, one of these two units can be expressed in terms of the other. If two such relations are found, it will be possible "to determine absolutely both of these units" and "have a

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[22] Stanley Goldberg, *Understanding Relativity: Origin and Impact of a Scientific Revolution* (Basel, Boston, and Stuttgart: Birkhäuser, 1984).

[23] Gilbert N. Lewis and Elliot Q. Adams, "Notes on Quantum Theory: a Theory of Ultimate Rational Units; Numerical Relations between Elementary Charge, Wirkungsquantum, Constant of Stefan's Law," *Physical Review*, 3 (1914), pp. 92-102.

[24] Although Lewis and Adams only referred to the principle of relativity, obviously the invariance of the speed of light plays a crucial role in their discussion.

system of units all of which are in a sense dimensionless." Then, an "interesting question now arises."[25]

> Supposing that instead of finding two such, we found three or more such relations, would we not then have several different sets of rational units between which the choice would be purely arbitrary? Our answer to the question can best be expressed by stating our belief that these different sets of units will be dependent upon one another in a very simple way, and that if in the manner suggested we obtain the ultimate units of interval and of mass by the aid of two universal and fundamental relations, then all universal constants will prove to be pure numbers, involving only integral numbers and π, just as we have seen that in geometry several different units of angle, area and volume may be chosen, which, however, differ only by such a factor. This we shall call the *theory of ultimate rational units.*[26]

The theory of ultimate rational units was an attempt to construct a concise system of units by reducing the number of physical dimensions. To accomplish this aim, Lewis and Adams tried to find such fundamental relations as the principle of relativity. Although they did not show any further ontological assumption, their discussion revealed their belief in the mathematical simplicity of physical reality.

In the rest of the paper, the electron charge was regarded as one of the two fundamental constants to determine the ultimate units, the other one being the velocity of light. In their theoretical calculation of the constant of Stefan's law, they invoked the value of the electron charge, as well as some other assumptions, and could obtain an apparently satisfactory result. While experimental researches had suggested several different values for this constant, the authors were so confident in the power of their theoretical method as to conclude the paper by this remark: "[W]e have derived from this theory a value for the

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[25] Lewis and Adams, "Notes on Quantum Theory," p. 97.

[26] *Ibid.*

constant of Stefan's law which we believe to be far more accurate than any of the values of this quantity obtained by direct experiment."[27]

The same year, R. C. Tolman published an even more provoking paper,[28] in which he introduced the assumption called "the principle of similitude." Tolman stated his new principle thus: "The fundamental entities out of which the physical universe is constructed are of such a nature that from them a miniature universe could be constructed exactly similar in every respect to the present universe."[29] He admitted that Newton maintained "a somewhat similar hypothesis," but claimed that he developed his own postulate with the aid of the electron theory and relativity theory. Like Lewis's attempt, Tolman's is also some kind of reaction to the advent of relativity theory. In the end of the paper he called his principle "the principle of the relativity of size."[30]

In constructing his scheme, Tolman considered two observers, *O* and *O'*. While *O* is provided with the same meter sticks, clocks and other measuring apparatus as in the present physical universe, *O'* is provided with a shorter meter stick, together with correspondingly altered clocks and other apparatus. With these, he can make measurements in his miniature universe. According to Tolman's principle of similitude, he will obtain "exactly the same numerical results in all his experiments as does *O* in the analogous measurements made in the real universe."[31]

Tolman went on to derive several conclusions, applying his principle to some well-known physical laws. Now, suppose the meter stick of *O'* is shorter than that of *O* in the ratio 1 : x. If they measure the same given distance *l* and *l'*, then,

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[27] *Ibid.*, p. 102.

[28] R. C. Tolman, "The Principle of Similitude," *Physical Review*, 3 (1914), pp. 244-255.

[29] *Ibid.*, p. 244.

[30] *Ibid.*, p. 255.

[31] *Ibid.*, p. 244.

$$l' = x\,l \tag{5}$$

The theory of relativity requires that $O$ and $O'$ should measure the same value for the velocity of light. To make it possible, $O'$ should use a unit of time shorter than $O$ in the ratio of 1 : x. Therefore, if $O$ and $O'$ respectively get numbers of seconds $t$ and $t'$ by measuring the same interval of time, they should be connected by the relation,

$$t' = x\,t. \tag{6}$$

In the same manner, Tolman found the transforming formulas for velocity $v$ and acceleration $a$.

$$v' = v. \tag{7}$$

$$a' = a/x. \tag{8}$$

For an electrical charge, he derived the following formula, considering that the magnitude of a charge would be expressed as a certain number of fundamental unit of electricity, namely, the charge of the electron.

$$e' = e. \tag{9}$$

To derive the transformation formula of mass, Tolman considered the Coulomb force between two bodies with the same mass $m$, but different charges $e_1$ and $e_2$, which are separated by a distance $l$. One of the bodies will obtain the acceleration $a$, as given by the following equation:

$$ma = e_1 e_2 / l^2 . \tag{10}$$

Based on the principle of similitude, Tolman supposed that the observer $O'$ will observe the similar relation:

$$m' a' = e_1' e_2' / l'^2 . \tag{11}$$

With the help of the equations (5), (8), and (9), the equation (11) becomes,

$$m' a/x = e_1 e_2 / (x^2 l^2). \tag{12}$$

One can obtain the following relation between $m'$ and $m$ by comparing (12) with (10).

$$m' = m/x. \tag{13}$$

The transformation equations of force $f$, energy $E$, and absolute temperature $T$ will be obtained by considering their dimensions, [MLT$^{-2}$], [ML$^2$T$^{-2}$], and [ML$^2$T$^{-2}$], respectively (Tolman assumed that absolute temperature had the same dimension as energy).

$$f' = f/x^2 \tag{14}$$

$$E' = E/x \tag{15}$$

$$T' = T/x. \tag{16}$$

It is now easy to see that the following equations hold for area $S$, volume $V$, and pressure $P$.

$$S' = x^2 S \tag{17}$$

$$V' = x^3 V \tag{18}$$

$$P' = P/x^4 \tag{19}$$

Tolman then derived some important results in thermodynamics, electromagnetism, and the black body theory by applying the principle of similitude and these transformation equations. For example, the relation between the pressure volume product and the absolute temperature of an ideal gas can be obtained in the following way.

By definition the pressure volume product of an ideal gas depends only on the absolute temperature. Therefore,

$$PV = F(T),$$

where $F$ is an unknown function to be determined. According to the principle of similitude, $O'$ would also observe that the same relation holds in his system, and hence,

$$P'V' = F(T').$$

Substituting for the accented letters their values as given by equations (16), (18), and (19), we will get,

$$PV = xF(T/x) = F(T)$$

As $x$ may be any number, the only possible form of the function is $F(T) = kT$, where $k$ is some constant. Thus, we get the famous relation holding for an ideal gas.

$$PV = kT.$$

Tolman, however, had to admit that his method did not work in the case of gravitation, where Newton's law of the following form holds:

$$f = k\, m_1 m_2 / l^2 .$$

From the equations (5), (13), and (14), it is clear that the same law of gravitation does not hold in the system of $O'$. Instead of giving up the principle of similitude, Tolman suggested the possibility that "the gravitational attraction between two bodies is not merely a function of the masses of the bodies and the distance between them."[32] For instance, "gravitational action may really be proportional not to mass but to some quantity which is itself more or less accidentally proportional to mass," or it may be that "the attraction of gravitation does not depend merely on the masses of the attracting bodies and the distance between them, but also on the properties of some mechanism by which gravitational action is produced," such as, "the properties of some intervening medium."[33] His insistence reveals his firm belief in the principle of similitude.

After showing the above examples, Tolman concluded that "the fundamental physical entities are of such a nature that from them a

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[32] *Ibid.*, p. 253.

miniature universe could be constructed exactly similar in every respect to the present universe."[34] The transformation equations derived in his 1914 paper tell what changes will be necessary when one is constructing such a "miniature world." Furthermore, Tolman thought that his discussion uncovered a startling principle.

> If, now, throughout the universe a simultaneous change in all physical magnitudes of just the nature required by these transformation equations should suddenly occur, it is evident that to any observer the universe would appear to him the same as before, since his meter sticks would all be changed in the same ratio as the dimensions of the object, and similar considerations would apply to intervals of time, etc. From this point of view we see that it is meaningless to speak of the absolute length of an object, all we can talk about are the relative length of objects, the relative duration of intervals of time, etc., etc. The principle of similitude is thus identical with the principle of the relativity of size.[35]

Although its way of deriving relations between physical quantities was similar to a better-known theoretical tool, the principle of dimensional homogeneity, the principle of similitude had an unconventional postulate concerning the structure of physical reality. Not merely proposing a shortcut to obtain the results which had already been known, it tried to reveal an unknown principle of the universe, stating that a "miniature universe" could be constructed, or that the absolute length did not have any significance. This may be related to the fact that the concept of similitude has a different origin from that of dimensional homogeneity. It is sometimes attributed to Galileo, more often to Newton, and has wider implication in the way of understanding physical reality.

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[33] *Ibid.*, p. 254.
[34] *Ibid.*, p. 255.
[35] *Ibid.*, p. 255.

However, critics of Tolman's principle of similitude mainly pointed out the superiority of the principle of dimensional homogeneity as a theoretical tool in physics, neglecting the metaphysical implications of similitude. In fact, it is difficult for the reader to estimate how seriously Tolman believed his metaphysical conclusions. His discussion was totally based on hypothetical assumptions, but no one could tell whether they were true or not.

Edgar Buckingham, an engineer at the National Bureau of Standards, was among the first critics of Tolman's principle. In his article[36] published in the next issue of the *Physical Review,* Buckingham outlined the formulation of the principle of dimensional homogeneity and its application to several practical problems, emphasizing its strength as a research method.[37] Then he pointed out that Tolman's discussion was merely a particular case of a more general theorem of dimensional homogeneity, giving a comment on the principle of similitude that "The unnecessary introduction of new postulates into physics is of doubtful advantage."[38] He published this paper mainly to prove that no new postulate, such as that of similitude, was necessary. Perhaps baffled by the extraordinary conclusions Tolman had drown from the principle of similitude, he did not even mention its metaphysical implications. He only reservedly stated that for the purpose to which Tolman put it, the principle of similitude was, "at all events, superfluous."[39]

However, describing the principle of dimensional homogeneity as a powerful tool, Buckingham praised its strength too much. Though it is a convenient method especially when one wants to make sure of the correctness of solutions that have already been obtained in some other

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[36] Edgar Buckingham, "On Physically Similar Systems; Illustrations of the Use of Dimensional Equations," *Physical Review,* 4 (1914), pp. 345-376.

[37] Not having known Vaschy's work, Buckingham in this particle independently proved the Π theorem (Buckingham to Bridgman, May 3, 1939, HUG 4234.10).

[38] Buckingham, "On Physically Similar Systems," p. 356.

way, dimensional analysis is too general a tool that does not give any concrete clue to the understanding of each particular physical phenomenon. Nevertheless, to Buckingham it was comparable to "the methods of thermodynamics or Lagrange's method of generalized coordinates."[40] He hoped that his sample illustrations of its use would "prove interesting to physicists who have not been in the habit of making much use of dimensional reasoning."[41]

The three scientists who started dimensional discussion in the United States, Lewis, Tolman, and Buckingham were all close friends and colleagues of Bridgman. Bridgman could not miss the fray over dimension that was going on around him. At the same time, feeling intellectual distress caused by taking over courses in electromagnetism and relativity, he started to reflect on the foundations of physics. Historians have so far argued that Bridgman entered into the controversy over dimension mainly for these two reasons. However, as we will see, it was for a more poignant reason that Bridgman as an experimentalist spoke out against misinterpretations of dimensional analysis as they appeared to him.

## 3.2. Defending Experimental Research

### 3.2.1. Entering the Dimensional Debate

Bridgman could tolerate neither speculative extrapolations of dimensional analysis as were attempted by Lewis and Tolman, nor Buckingham's overemphasis of the power of the principle of dimensional homogeneity. But first, he needed to decide how to respond to Tolman's hypothesis. In 1916, he wrote to this old friend to ask what the

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[39] *Ibid.*, p. 357.
[40] *Ibid.*, p. 376.
[41] *Ibid.*

principle of similitude really meant to him. Tolman in reply repeated almost the same statement as had been put in his 1914 paper: "[Y]ou could take the actual substances, entities or materials, or whatever you wish to call them, which exist in the actual universe and build out of them a small universe which would be an exact reproduction of our actual universe when experimented with by a miniature observer."[42] Furthermore, Tolman commented that "the constructs in this miniature universe have been made from materials which belong to our own universe." Therefore, in his understanding, "these constructs will have to obey not only the laws of the miniature universe but, at the same time, the laws of our actual universe." Tolman's principle of similitude required that the same physical laws should hold in both of the universes, in the similar way that the principle of relativity required that the same laws of mechanics, electromagnetism, and optics should hold in every inertial system. Though Tolman reservedly remarked that "the fact (if it is a fact) that it is possible to construct these miniature atoms, molecules, electrons, hohlraums [black body], etc., does not mean that such miniature constructs actually do exist in our universe," he did not deny the possibility that "such miniature constructs may actually exist somewhere in our universe." For instance, he thought that "the isotopes of the elements might be of this nature," although "this does not seem to be borne out by spectrum determination." If one only reads Tolman's published papers, it is difficult to see how seriously he committed himself to "miniature universes." Buckingham, therefore, refrained from getting into this matter in his paper. Bridgman could directly ask Tolman through their private correspondence how serious he was about his speculative conclusions.

Bridgman could not accept Tolman's view. Immediately after

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[42] Tolman to Bridgman, May 9, 1916, PWBP, HUG 4234.8.

understanding Tolman's belief in his miniature universes, Bridgman published his severe criticism of the principle of similitude in the *Physical Review*.[43] The physical part of his discussion was mainly a repetition of what Buckingham had already pointed out, that "the principle of similitude contains nothing true not already contained in the theory of dimensions."[44] Tolman did not understand Bridgman's intention in this part. On February 7, 1920, Bridgman complained to Warren Weaver: "I did not make entirely plain my point of view, as I know from conversation with Tolman that I did not for him."[45] In fact, it was not only Tolman who could not grasp Bridgman's point clearly. Weaver, who was then an assistant professor of mathematics at Throop College of Technology, asked Bridgman: "In general if a certain proposition A may be reached by independent sets of arguments from two principles, does that necessarily remove the possibility of both principles being the statement of fundamental truth?"[46] The mathematical physicist Weaver did not comprehend why Bridgman should harshly criticize the principle of similitude when it was mathematically equivalent to dimensional analysis that Bridgman regarded as a legitimate theoretical tool. Replying Weaver, Bridgman clarified what he really tried to assert in his *Physical Review* paper, that he did not believe that "the principle of similitude and the principles of dimensional reasoning lead to the same proposition."[47] He pointed out that they do not lead to the same results in the cases of the Newtonian law of gravitation and the Rydberg constant of series spectra. Although in his published paper Bridgman did not emphasize this last case, he could "see no reason why the spectrum series formula for the hydrogen

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[43] P. W. Bridgman, "Tolman's Principle of Similitude."
[44] *Ibid.*, p. 431.
[45] Bridgman to Weaver, Feb. 7, 1920, PWBP, HUG 4234.8.
[46] Weaver to Bridgman, Jan. 28, 1920, PWBP, HUG 4234.8.
[47] Bridgman to Weaver, Feb. 7, 1920, PWBP, HUG 4234.8.

atom, for example, in the real and the miniature universes should not be the same, as it is not according to the principle of similitude." In short, he was trying to show that "those cases in which the principle of similitude gives correct results are precisely those cases in which the principle of dimensional reasoning says that it should." The conclusion he drew from his analysis of Tolman's principle was "that the principle of similitude is positively incorrect, as a general principle, but that it may lead to correct results in certain cases."

Bridgman's statement was only a variation of Buckingham's criticism of Tolman's principle of similitude, although Bridgman's was somewhat stronger than Buckingham's. Therefore, if one pays attention only to the technical discussion, it will seem difficult to understand why Bridgman bothered to plunge into the controversy at the time when his experimental research was producing marvelous results and kept him busy with his laboratory work.

Besides technical analysis of the principle of similitude, however, Bridgman's *Physical Review* paper[48] contains some part which reveals an important aspect of his view of physics. He reasoned that Tolman's principle suggested the existence of the atoms precisely like, for instance, those of iron, "differing only in absolute magnitude, enjoying great stability like atoms of iron, and giving a series of spectrum lines corresponding one to one with those of iron."[49] These atoms were made of the same stuff as the actual universe and were capable of existence. Therefore, Bridgman thought that, if Tolman's hypothesis was true, there might be "a great number of such possible atoms, differing only in scale of magnitude." This was not directly mentioned in Tolman's original papers, but, as we have seen, Bridgman had "learned from correspondence with Tolman that such is his meaning." Based on

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[48] P. W. Bridgman, "Tolman's Principle of Similitude."

Tolman's answer to him, Bridgman inferred the implication of the principle of similitude.

> If the hypothesis is true the finite number of the chemical elements can, therefore, have no explanation in terms of the properties of the protean substance, but must be due to arbitrary selection. This means that every atom of the universe has been individually sorted, or as Sir John Herschel put it, bears the impress of a manufactured article. Now such a picture of the universe as this, which places the direct interposition of creative intelligence at a stage nearer to us than the utmost we can hope to reach by experiment, is certainly directly opposed to the entire spirit of physics.[50]

This statement had nothing to do with his antipathy toward religion, but it reveals Bridgman's belief that one could not reach physical reality without experimental research and that physics was, after all, experimental science. To him, direct interposition of creative intelligence, if any, should be something that might be revealed only by elaborate experimental research, but not by speculative reflection. Though Bridgman seems to have overreacted to Tolman's discussion, his response to Tolman's discussion illustrates what kind of attitude he would take towards a statement that he could interpret as implying any unreasonable limitation of experimental research.

In the middle of the 1910s when discussions on dimensional reasoning were attracting some American scientists' attention, Bridgman was in his early thirties, facing a vast field waiting to be explored by his high-pressure techniques that he had just established. He had begun to enter into the most productive years in his experimental work. In this situation, nothing might upset him more than statements that implied the inferiority of experimental work to theoretical research, all

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[49] *Ibid.*, p. 423.

[50] *Ibid.*

the more for his personal experience that he had chosen a career as an experimentalist despite his training and inclination that were rather on the theoretical and mathematical side. Moreover, as relativity and quantum theories showed, theoretical physics was rapidly broadening its territory, academically and geographically. This new trend burgeoned originally in Europe and was about to go across the Atlantic Ocean, obscuring the significance of experimental research. Lewis and Tolman in their own ways reacted to the emerging fashion in physical research, while Bridgman keenly felt it necessary to protect the value of experimental work.

Bridgman's later philosophical works on physics can be interpreted as an experimentalist's claim for the *raison d'être* against the rise of theoretical physics. His criticism of the principle of similitude was but a beginning; thereafter, he was to struggle with more formidable physical theories. While preparing for the publication of a monograph on dimensional method, *Dimensional Analysis*, he was attempting to explicate the meaning of relativity theory, probably stimulated by Arthur Eddington's 1919 solar-eclipse observations and Einstein's 1921 visit to the United States. During the early 1920s he continued his reflection on relativity theory and then quantum mechanics, which led him to publish *The Logic of Modern Physics* in 1927, the volume that made him known as a philosopher of science. Even after that, his philosophical attempt did not cease, as he found the strongest challenge from theoretical physics, the uncertainty principle of Heisenberg. Continuing experimental research, he kept on publishing papers and books in the philosophy of science. Those were the results of an experimental physicist's effort to comprehend the startling results of theoretical physics, urged by the recognition of the endangered status of experimental research.

As he published one essay after another in the philosophy of science, Bridgman's reflection on theoretical physics would become more and more sophisticated, until at last his earlier sense of growing crisis of experimental research that first led him to examination of physical theory became almost unrecognizable. In the summer of 1919, however, when he was preparing a draft on dimensional reasoning, he was consciously trying to protect the value of experimental research. This draft would be published as a series of five lectures at the Graduate Conference at Harvard in the next spring, and eventually as a primer of this theoretical method, *Dimensional Analysis*, in 1922. The second edition of this volume, with no significant revision, was published in 1931. Bridgman sent a copy of the second edition to Buckingham. He also wrote to Buckingham a letter in 1932,[51] which explains the reason Bridgman decided to write a monograph in 1919. To Bridgman, "the book is mainly a record of a personal experience which [he] went through when starting [his] career in physics."

> [Mayo D. ] Hersey [, an engineer at the National Bureau of Standards], inspired by you [Buckingham], had given us some lectures on Dimensions, and was so enthusiastic about the possibilities and power of the method that he was quite willing to believe that all the results of my program of high pressure measurement might be anticipated by arm-chair meditation. If any such catastrophe were possible it evidently was important for me to know about it before tying up my life in so futile an enterprise. Of course I did not for a moment really believe that any such clairvoyance was possible, but I found that I could not argue about the thing convincingly, and that I really had no satisfactory grasp of the whole situation.[52]

Unable to accept Hersey's overemphasis of the strength of dimensional method, Bridgman attempted to scrutinize dimensional analysis mainly

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[51] Bridgman to Buckingham, July 10, 1932, PWBP, HUG 4234.10.

to justify his choice of a career in experimental research. Although from the beginning he did not believe that dimensional analysis was such an omnipotent theoretical tool as Hersey and possibly Buckingham claimed, he noticed that he did not comprehend this method thoroughly. Thus, he first tried to "get the matter straight enough in [his] mind to be confident as to the essential nature of the situation."

> In thinking the situation through, I naturally came across many of the misconceptions of which the literature is full. Having thought the thing out sufficiently for my own purpose, it seemed to me a pity not to put the general results of my lucubrations in permanent form, since I was convinced that the average physicist was even more hazy on the subject than myself. The book is the result; I had continually in mind an audience with my own experience, and my main object was to enable this audience to handle dimensional analysis correctly as a tool, and I believe, from the comments which the book has excited, that it has justified its purpose.[53]

While trying to straighten the matter, Bridgman found many misunderstandings concerning the use of dimensional analysis. He therefore made an effort to clarify how to deal with this method properly and published the result as a book for "the average physicist."

Though the exact dates of Hersey's lectures on dimension were unclear, Bridgman mentioned Hersey's contribution in the preface of *Dimensional Analysis,* written in September 1920: "I am also much indebted to Mr. M. D. Hersey of the Bureau of Standards, who a number of years ago presented Dr. Buckingham's results to the Conference in a series of lectures."[54] In 1917, Tolman also gave lectures on the principle

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[52] *Ibid.*

[53] *Ibid.*

[54] P. W. Bridgman, *Dimensional Analysis* (New Haven: Yale University Press, 1922), "Preface."

of similitude at Harvard,[55] which Bridgman probably attended. Shortly afterwards, however, Bridgman abandoned his dimensional reflection for a while, since he left for war work at New London. In the summer of 1919, he resumed his dimensional discussion, bearing in mind its possible publication in the near future. In November, Bridgman wrote to Tolman, then at the War Department, Washington, D. C., about his confidence in his recent work:

> As for the principle of similitude, I am if possible, more orthodox than ever. This summer I spent some time trying to get straight in my own head some of the fundamental notions about dimensional reasoning, and the result of my lucubrations was some ideas which possibly I may try to publish one of these days. I am sure when you see these clear and cogent remarks you will realize only too clearly the heretical deviousness of your ways.[56]

He gave talks on dimensional reasoning at Harvard in the spring of 1920 and then spent the next summer in completing the book. In his letter to Tolman written on August 8, 1920, he again mentioned this project: "I have been writing up the talks which I gave last spring on dimensional analysis, and I am going to try to get them published, if any one will take them in these hard times."[57] As for Tolman's principle, he wrote, "I have a little to say about the Principle of Similitude, but not a great deal." He would "say more about regarding [*sic*] a dimensional formula as an expression of the essential physical nature of a quantity," namely, Lewis's type of extrapolation of dimensional formula. Bridgman's main interest seemed to be in the power of dimensional analysis as a tool in physics, but no longer in the principle of similitude. During the summer, Bridgman and Tolman exchanged several letters, only to

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[55] *Report of the President and the Treasurer of Harvard College, 1916-17* (Cambridge: Harvard University, 1918), p. 212.
[56] Bridgman to Tolman, Nov. 9, 1919, PWBP, HUG 4234.8.
[57] Bridgman to Tolman, Aug. 8, 1920, PWBP, HUG 4234.8.

convince themselves that they had "very different ways of looking at things."[58] While Tolman invented several ingenious ways to maintain that the principle of similitude holds even in the cases where it apparently does not, to Bridgman it seemed that Tolman "split up every expression into two factors, to one of which it applies and to othe [*sic*] other of which it does not, and then apply it only to the factor to which it applies."[59] By September 1920, not bothering much about Tolman's discussion, he had completed the typescript of *Dimensional Analysis*.

### 3.2.2. *Dimensional Analysis*

In his monograph on dimensional analysis, the first point that Bridgman emphasized was the importance of securing experimental knowledge before applying the principle of dimensional homogeneity to each particular problem. His discussion of the case of oscillation of a small drop of liquid illustrates this point clearly.[60] When one tries to obtain $t$ the period of oscillation of a small drop of liquid under its own surface tension, the physical quantities involved are surface tension $s$, density of liquid $d$, and radius of drop $r$, their dimensional formulas being $[MT^{-2}]$, $[ML^{-3}]$, and $[L]$, respectively. By applying the principle of dimensional homogeneity to this case, it will easily be shown that

$$t = \text{Const.}\ \sqrt{(dr^3/s)}.$$

However, a critic of this method may ask why it is sufficient to consider only those three physical quantities and why other quantities, such as the atomic structure of the liquid, are not included in the discussion, while it is widely accepted that surface tension depends on the atomic

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[58] Tolman to Bridgman, Sept. 8, 1920, PWBP, HUG 4234.8.

[59] Bridgman to Tolman, Sept. 5, 1920, PWBP, HUG 4234.8.

structure. "To which," Bridgman explained, "we would be forced to reply that we have indeed had a wider experimental experience than our critic." This experience ensures that those three physical quantities are sufficient to analyze the matter.

> We shall thus ultimately be able to satisfy our critic of the correctness of our procedure, but to do it requires a considerable background of physical experience, and the exercise of a discreet judgment. The untutored savage in the bushes would probably not be able to apply the methods of dimensional analysis to this problem and obtain results which would satisfy us.[61]

Behind each application of dimensional analysis, there is an accumulation of experimental knowledge. Bridgman repeated a similar statement in discussing how to apply the method to the problem of heat conduction.

> For instance, why are we justified in neglecting the density, or the viscosity, or the compressibility, or the thermal expansion of the liquid, or the absolute temperature [in dealing with the problem of the conduction of heat] ? We will probably find ourselves able to justify the neglect of all these quantities, but the justification will involve real argument and a considerable physical experience with physical systems of the kind which we have been considering. The problem cannot be solved by the philosopher in his armchair, but the knowledge involved was gathered only by someone at some time soiling his hands with direct contact.[62]

Bridgman argued that dimensional analysis could not lead to correct results unless one had sufficient experimental knowledge of the phenomena to which the method was applied, and that the method essentially depended on experimental research. Physicists sitting in

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[60] Bridgman, *Dimensional Analysis*, pp. 3-5.
[61] *Ibid.*, p. 5.
[62] *Ibid.*, pp. 11-12.

their “armchair” cannot handle dimensional analysis properly.

After pointing out the limit of the power of dimensional analysis in the first chapter, Bridgman went on to discuss the meaning of dimensional formula. Clarifying the meaning of symbols appearing in dimensional formula, Bridgman maintained that “[t]he purpose of dimensional analysis is to give certain information about the relations which hold between the measurable quantities associated with various phenomena.”[63] The relations can be expressed, for example, in the following form.

$$x_1 = f(x_2, x_3, x_4\text{ , etc. })$$

Bridgman stressed that “$x_1$, $x_2$, etc., stand for the numbers which are the measures of particular kinds of physical quantity,” not the physical quantities themselves. He did not keenly stick to this distinction between the physical quantities and their measures, abbreviating for the rest of the discussion the “long-winded” description into saying that $x_1$ is, for example, a velocity. For him, however, “of course it really is not [a velocity], but is only a number which measures velocity.”[64] This comment gives some clue to understanding Bridgman’s peculiar view of the status of measurement in physics. What he meant was that dimensional analysis, or algebraic operations in physics in general, only dealt with the numbers obtained by measuring physical quantities, not physical quantities themselves. To him, physical quantities meant far more than plain numbers. Dimensional analysis, therefore, could discuss only a limited aspect of physical phenomena.

Furthermore, Bridgman’s insistence on a clear distinction between physical quantities and their measured numbers implies that he

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[63] *Ibid.*, p. 17.

maintained that there existed certain "incommensurability" between different kinds of physical quantities. For example, despite a usual understanding that "a velocity is measured by definition by dividing a certain length by a certain time," Bridgman assumed that "this really means dividing the number which is the measure of a certain length by the number which measures a certain time."[65] For him, "it is meaningless to talk of dividing a length by a time."[66] One cannot arbitrarily relate one kind of physical quantity to another, since, as he discussed later in the book, each physical quantity is measured in its own particular way. The boundary between different quantities does not blur easily. Thus, algebraic operations are possible only on the measuring numbers of physical quantities, but not on physical quantities themselves.

The contemporary scientists did not miss this apparently strange habit of distinguishing the physical quantities and their measuring numbers, although they did not further discuss Bridgman's way of understanding the meaning of letter-symbols. The reader appointed by the Council's Committee on Publication of Yale University, who had reviewed the draft of *Dimensional Analysis* before they made a decision on its publication, reported that he could find only one point on which he would dissent from Bridgman's position. This point was Bridgman's "contention that not physical quantities, but only their numerical values are susceptible of combination according to the rules of ordinary algebra."[67] Moreover, Bridgman's former students and colleagues, Edwin C. Kemble and Francis Birch, who probably had listened to Bridgman's talks on dimensional analysis delivered at Harvard,

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[64] *Ibid.*

[65] *Ibid.*, p. 23.

[66] Edwin C. Kemble and Francis Birch, "Percy Williams Bridgman, April 21, 1882-August 20, 1961," *National Academy of Sciences of the United States of America, Biographical Memoirs*, 41 (1970), p. 40.

recognized this same point as a notable character of his reflection on dimensional analysis:

> [H]e rejected the widely held view that the letter symbols for the different physical quantities in the equations of physics should be considered to represent corresponding physical quantities. It suffices to suppose that the symbols are place-holders for the numerical measure-values of the physical quantities. He recognized the practical convenience of the custom of treating the equations of physics as relations between numbers carrying dimensional unit labels, but considered that to call such labeled numbers "physical quantities" is to introduce an unwarranted and misleading verbalism.[68]

From their description, one can understand that he then stated that dimensions had "no absolute significance whatever," but were "defined merely with respect to that aspect of the rules of operation by which we obtain the measuring numbers associated with the physical phenomenon."[69] In his understanding, dimensions represent only the numerical aspect of physical phenomena. Thus, "an equation" is merely "an equation between numbers which are the numerical measures of certain physical quantities."[70] "The dimensional formula need not even suggest certain essential aspects of the rules of operation."[71] For example, the dimensional formula of force as mass times acceleration does not show that acceleration, and therefore force, are actually vectors. Bridgman could not accept "the view that regards a dimensional formula as an expression of operations on concrete physical things,"[72] or the view "which regards a dimensional formula as an expression of the

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[67] Day to Bridgman, Dec. 18, 1921, PWBP, HUG 4234.8.
[68] Kemble and Birch, "Percy Williams Bridgman," p. 40.
[69] Bridgman, *Dimensional Analysis*, p. 23.
[70] *Ibid.*, p. 41.
[71] *Ibid.*, p. 23.
[72] *Ibid.*, p. 46.

'essential physical nature' of a quantity."[73]

To the Bridgman of *Dimensional Analysis*, what appeared most fundamental in physics was not dimensions, symbols, or even equations, but measurements of physical quantities. While the numbers obtained by measurements may be important in describing the numerical relations between physical quantities, one may miss many points vital to physical phenomena, if focusing only on their numerical aspects. In a similar manner, theoretical part of physics does not and cannot comprehensively describe its counterpart, experimental research. Although Bridgman did not clearly argue for these points, they were tacit assumptions of his understanding of dimensional analysis.

The discussion of the nature of measurement played an essential role in Bridgman's scrutiny of dimensional analysis. Bridgman stated the definition of measurement thus: "for each different kind of quantity we have a different rule of operation by which we measure it, that is, associate the quantity with a number."[74] To him, measurement meant associating a physical quantity with a number by following a rule of operation specific to each quantity. Then he split up physical quantities into two groups, primary and secondary. Primary quantities are the ones "which, according to the particular set of rules of operation by which we assign numbers characteristic of the phenomenon, are regarded as fundamental and of an irreducible simplicity."[75] For example, in mechanics, they are mass, length, and time. Secondary quantities, on the other hand, are the quantities whose numerical measures are obtained "by making measurements of certain quantities of the first kind associated with the quantity under consideration, and then combining the measurements of the associated primary quantities

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[73] *Ibid.*, p. 41.
[74] *Ibid.*, p. 17.
[75] *Ibid.*, p. 18.

according to certain rules which give a number that is defined as the measure of the secondary quantity in question."[76] For example, a velocity can be defined as a secondary quantity, whose measure is obtainable "by measuring a length and the time occupied in transversing this length (both of these being primary quantities), and dividing the number measuring the length by the number measuring the time."[77]

Bridgman, however, did not hint any philosophical implication by the separation between two categories of operation, considering that the distinction was "to a large extent arbitrary" and dependent on "the particular set of rules of operation which we find convenient to adopt in defining our system of measurement."[78] "For instance," he explained, in the ordinary system of mechanics, "force is a secondary quantity, and its measure is obtained by multiplying a number which measures a mass and the number which measures an acceleration (itself a secondary quantity)," while in some other systems, "force is well adopted to be used as a primary quantity."[79] Bridgman did not regard the distinction between primary and secondary quantities to be as rigid as that between chemical elements and compounds, considering no physical quantity as specially fundamental in the structure of physical reality. Furthermore, Bridgman admitted that scientists could change the rules of operation at their convenience.

> [I]n our measurements of nature, the rules of operation are in our control to modify as we see fit, and we would certainly be foolish if we did not modify them to our advantage according to the particular kind of physical system or problem with which we are dealing. We shall [...] find many problems in which there is an advantage in choosing our system of measurement, that is, our rules of operation, in a particular way for the particular

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[76] *Ibid.*, p. 19.
[77] *Ibid.*
[78] *Ibid.*, p. 20.
[79] *Ibid.*

> problem. Different system of measurement may differ as to the kinds of quantity which we find it convenient to regard as fundamental and in terms of which we define the others, or they may even differ in the number of quantities which we choose as fundamental. All will depend on the particular problem, and it is our business to choose the system in the way best adapted to the problem in hand.[80]

Bridgman maintained that in each case physicists should choose the most convenient and most suitable set of rules of operation for their specific purposes. No single system of measurement would be potent enough to cover the entire realm of physical phenomena. Each phenomenon is to be described by a specific set of primary physical quantities. Physical analysis is, therefore, relative to the choice of the system of measurement. One physical quantity can have several forms of dimensional formula that differs from system to system. It follows from this understanding that it is pointless to discuss the dimensions without specifying the system of measurement. Thus, Bridgman rejected the view which was "commonly held and frequently expressed," that "a dimensional formula has some esoteric significance connected with the 'ultimate nature' of an object," or that "we are in some way getting at the ultimate nature of things in writing their dimensional formulas."[81]

As Kemble and Birch described almost half a century later, Bridgman's scrutiny of dimensional analysis revealed "the limitations of dimensional reasoning" and also removed "the mystery of its power."[82] He claimed the necessity and priority of experimental research, denying the arbitrary philosophical extrapolation of dimensional analysis by pointing out two facts: that detailed experimental knowledge is always necessary in order to obtain a correct result by applying dimensional

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[80] *Ibid.*, p. 24.
[81] *Ibid.*

analysis; and that letter symbols represents only a numerical part of physical quantity, implying nothing about its actual measurement. In short, his purpose of writing *Dimensional Analysis* was to demystify dimensional reasoning by scrutinizing the method from an experimentalist's point of view. Apart from this rather negative aspect, his reflection on physical quantity reveals a positive part of his view of physics, namely, his belief that measurement was essential in describing phenomena physically.

Not just helping Bridgman deepen his personal reflection on physics, this monograph contributed to the establishment of the technical foundations of dimensional analysis mainly through two discoveries: the presentation of a new proof of the Π theorem; and the recognition of the "absolute significance of relative magnitude" of physical quantities. While several others deserve credit for the former, Bridgman was the first to state the latter clearly. He described the absolute significance of relative magnitude: "This requirement for measurement of length, for example, is that if a new unit of length is chosen let us say half the length of the original unit, then the rules of operation must be such that the number which represents the measure of any particular concrete length in terms of the new unit shall be twice as large as the number which was its measure in terms of the original unit."[83] Bridgman pointed out that this is a "tacit requirement in selecting the rules of operation by which primary quantities are measured in terms of quantities of their own kind."[84] From this requirement, it follows that the ratio of the measures of, for example, any two particular lengths, "is independent of the size of the unit with which they are measured."[85] The absolute significance of relative magnitude is not only "essential to

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[82] Kemble and Birch, "Percy Williams Bridgman," p. 41.
[83] Bridgman, *Dimensional Analysis*, p. 18.
[84] *Ibid.*

all the systems of measurement in scientific use," but also "an absolute requirement if the method of dimensional analysis is to be applied to the results of the measurements." [86] Bridgman went on to prove mathematically that "[e]very secondary quantity, [...], which satisfies the requirement of the absolute significance of relative magnitude must be expressible as some constant multiplied by arbitrary powers of the primary quantities."[87]

Bridgman's lucubration of dimensional analysis was firmly based on his analysis of measurement. While writing *Dimensional Analysis*, however, he did not develop his reflection on measurement into a more comprehensive one, only applying it to the particular case of dimensional analysis. One can vaguely see the fundamental ideas underlying his analysis of dimensional method only by reading between lines not to miss any hint or implication. On one hand, he agreed that by theoretical and mathematical reasoning one could seek for correlation among measures of physical quantities. On the other hand, dimensional analysis appeared to him to describe only the numerical aspect of nature, leaving unexamined many other properties connected to actual measurement. While applying a theoretical tool, one should not forget that mathematical analysis is dependent on measurement, as numerical part of physical quantities is produced by measurement. Bridgman thus pointed out that the kind of physical quantities taken into description and analysis of phenomena were determined by the choice of a system of measurement.

By comparing with the views of other participants of the dimensional controversy, one can characterize Bridgman's stance clearly. They exchanged their ideas more freely in their private correspondence.

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[85] *Ibid.*, p. 19.
[86] *Ibid.*, p. 20.
[87] *Ibid.*, p. 21f.

In November 1922, having read *Dimensional Analysis*, published the same year, Buckingham blamed Bridgman's discussions for being "too mathematical."[88] Recognizing that Bridgman's book was "addressed to a class of readers whom [he] should not venture to try to instruct," Buckingham admitted that he lacked sufficient mathematical knowledge. He found out that Bridgman could use "abstract ideas and mathematical arguments" more freely than he "should ever attempt to do," but complained that Bridgman's argument was "too remote from the range of ideas that we get directly from our senses"-- too remote "for [Bridgman's] own good."

> In other words, it seems to me that you do not, in your own mind, ascribe enough importance to physical instinct or common sense, which can not in reality be replaced by any amount of logic. For example: to treat physical quantities impartially is to ignore a vitally essential fact, namely, that there are in reality only a very few physical magnitudes of which we can form any conception as measurable quantities. The rest of our quantities, including all the electromagnetic and all the thermal except temperature, are artificial combination of simpler ideas, invented for the sake of abbreviating and condensing descriptions of phenomena which would be unmanageably complicated if we tried to describe them in terms of force, time, temperature, and the geometrical quantities.[89]

To Buckingham, even Bridgman's painstaking analysis of measurement seemed insufficient. He observed that but a few physical quantities were directly measurable, while others were their combination. Also in other letters, Buckingham repeated the same observation that Bridgman's view was mathematical, comparing it with his own "physical" one.

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[88] Buckingham to Bridgman, Nov. 13, 1922, PWBP, HUG 4234.8.

[89] *Ibid.*

In reply, Bridgman could not help expressing his dismay: "The one thing that most impressed me in copying the notes was your continued emphasis of the fact that your point of view is 'physical' while mine is 'mathematical.'"[90] He emphasized that he was "the last person in the world to take the attitude of Berkeley which neglects too much the contribution of the outside world to our mental processes." For example, he always had "vividly in mind that we are compelled to measure lengths with meter sticks in the way we do because of some characteristic of the things outside of us." He then clarified the difference between his standpoint and Buckingham's.

> I think that having once made our measurements in the way which nature compels us to, this is an end of it, and from here on everything is purely formal. You on the other hand want to recognize that the properties of nature which compel measurements by certain methods continue to influence the results after the measurements have been obtained.[91]

Bridgman split up physical research into two parts: actual measurement and "purely formal" operation of numbers obtained by measurement. Recognizing the importance of theoretical and mathematical physics, he could not take the attitude of such a die-hard empiricist, Buckingham, an engineer-physicist at the National Bureau of Standards. Bridgman admitted that physics had a part where things went purely formal, although he always insisted that it was inseparably connected to the outside world through measurement.

In 1934, Tolman asked Bridgman for an opinion concerning Tolman's replies to the questions of F. W. Warburton, a professor at the University of Kentucky. Bridgman did not miss this opportunity to tell an old friend of his about his standpoint in *Dimensional Analysis.*

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[90] Bridgman to Buckingham, April 8, 1923, PWBP, HUG 4234.8.

Having published *The Logic of Modern Physics* in 1927, Bridgman was self-conscious of his operational stance, which he had only vaguely implied in *Dimensional Analysis.* Over a decade after the publication of the first edition of *Dimensional Analysis* in 1922, Bridgman realized that his scrutiny of dimensional method meant to him more than it could only as the starting point of his fundamental reflection on physics. Bridgman first explained to Tolman that he was "in much the same position" as before, since he "had not been stimulated to do any fresh thinking on the subject."[92] Mentioning that he could "signify only the present state of the precipitate left in [his] mind from a previous condition of activity," Bridgman started to rebut Tolman. To him, Tolman seemed to consider that Bridgman took "the attitude which regards a dimensional formula as a compact way of summarizing certain operations which have been performed on numbers." On the other hand, Bridgman had found something more significant in dimensions.

> [M]y attitude in the face of dimensional formula contains much more than a consciousness of the way in which certain numbers were obtained--it contains also as a vital part of the background a consciousness of the physical operations involved in the measurements and also a realization, obtained from "the experience of all the ages", that it is useful to describe our experience in terms of these operations.[93]

He admitted that he had an idea to describe nature in terms of operations when he faced dimensional formulas. In *Dimensional Analysis,* because of the character of the subject, the focus was mainly on the relation between physical quantities and measurements; he did not give much consideration to other types of concepts in physics which are not totally of the quantitative nature. However, reflection on

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[91] *Ibid.*

[92] Bridgman to Tolman, June 14, 1934, PWBP, HUG 4234.10.

dimensional analysis made him aware of the basis of his view of physics. The further development and broadening of his perspective would follow his intellectual struggles with relativity theory and quantum mechanics.

Furthermore, by 1934, Bridgman had realized that "the experience of all the ages" had selected rules of operation for measurement that were useful to describe physical reality. The encounter with Hugo Dingler's *Das Experiment*,[94] which Bridgman reviewed in 1928, had led him to the recognition that "long and arduous" mental preparation was necessary before human beings were ready for the experimental method.[95] Later in 1950, he would reiterate almost the same point, stating that it required "long experience" to find appropriate operations for measurements that directly refer to standards and make the resulting measured numbers combinable according to the commutative or associative rules for mathematical addition.[96]

Despite the author's enthusiasm, the negotiation for the publication of *Dimensional Analysis* did not proceed smoothly. Bridgman had to write to a number of publishers letters of application for the publication of his first book. On December 5, 1920, he sent his manuscript to Ginn and Company in Boston. Two days later, they replied that they were obliged to regard the expense of its publication as prohibitive in the difficult economic situation, though recognizing the intrinsic value of Bridgman's attainments.[97] On December 22, the McGraw-Hill Book Co., to which he had sent only the Preface on December 19, gave him a

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[93] *Ibid.*

[94] Hugo Dingler, *Das Experiment: sein Wesen und seine Geschichte* (München: Verlag Ernst Reinhardt, 1928).

[95] P. W. Bridgman, review of Hugo Dingler's *Das Experiment*, *Physical Review*, 32 (1928), pp. 316-317.

[96] P. W. Bridgman, "Dimensional Analysis," in *Encyclopædia Britannica: A New Survey of Universal Knowledge* (Chicago, London, Toronto: Encyclopædia Britannica, Ltd., 1950), Vol. 7.

[97] Bridgman to Ginn and Company, Dec. 5, 1920; Ginn and Company to Bridgman, Dec. 7, 1920, PWBP, HUG 4234.8.

similar reply.[98] On December 24, he sent a manuscript to John Wiley and Sons, Inc. in New York, but three weeks later, he again received a disappointing letter.[99] In this case, all of the experts who were consulted to evaluate the book praised the manuscript very highly. One of the reviewers wrote: "The work is a very important one. It is on the firing line, so to speak, and would be the first book of formal exposition in that line." However, while they believed that it was "the kind of book in which there would be an increased demand," they doubt whether the publisher "could sell over 1,500 copies in the next three or five years." None of the three publishers to which he wrote for publication in 1920 and 1921, the Macmillan Company, Longman, Green & Co., as well as the Harvard University Press, accepted his proposal.[100] In the autumn of 1921, Bridgman had an interview with G. P. Day, the President of the Yale University Press, at whose suggestion he sent the manuscript to Yale.[101] In December, Day wrote him that the Council's Committee on Publications of Yale University had formally sanctioned the publication of his draft by the Yale University Press.[102] Thus, *Dimensional Analysis* was published in 1922.

Upon the publication Bridgman received many favorable reactions to *Dimensional Analysis*. M. D. Hersey at the National Bureau of Standards, who introduced Buckingham's discussion to Bridgman, praised him as a writer. He wrote, "I have hardly ever read a book with so much satisfaction and hope that you will continue to publish something along this line from time to time for the interest of others."[103]

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[98] Bridgman to the McGraw-Hill Book Co., Dec. 19, 1920; the McGraw-Hill Book Co. to Bridgman, Dec. 22, 1920, PWBP, HUG 4234.8.

[99] Bridgman to John Wiley and Sons, Inc., Dec. 24, 1920; John Wiley and Sons, Inc. to Bridgman, Jan. 14, 1921; PWBP, HUG 4234.8.

[100] The Macmillan Company to Bridgman, Dec. 30, 1920; Longman, Green & Co. to Bridgman, June 3, 1921; Hersey to Bridgman, Nov. 15, 1922; PWBP, HUG 4234.8.

[101] Bridgman to Deane, Oct. 8, 1921, PWBP, HUG 4234.8.

[102] Day to Bridgman, Dec. 18, 1921, PWBP, HUG 4234.8.

[103] Hersey to Bridgman, Nov. 15, 1922, PWBP. HUG 4234.8.

Hersey's comment on Bridgman's distinction between physical quantities and their numerical measures was almost the contrary to Buckingham's, perhaps representing a common view of most physicists.

> [Y]ou seem to imply that people in general may at times overlook the distinction and deceive themselves into imagining that you can operate on physical quantities in the sense of actual realities and not mere numbers. For myself the possibility that the word "quantity" in physical discussions could mean anything other than a numerical magnitude, never entered my head until reading your book. I always regarded the word quantity as synonymous with numerical magnitude on account of its suggested contrast with the word "quality".[104]

Maintaining the conventional distinction between quality and quantity, Hersey did not recognize any difference between physical quantities and their numerical magnitudes, while Buckingham, his colleague at the National Bureau of Standards, agreed with Bridgman that physical quantities mean something beyond measured numbers. Furthermore, Tolman and Lewis had been striving to prove that physical quantities had special qualities that presented the key to comprehending the construction of physical reality. However, Hersey's comment suggests the possibility that such philosophical attempts as Tolman's and Lewis's were unusual among American scientists.

Many scientists appreciated his 1922 monograph as a rebuttal to Lewis's and Tolman's theories, as Ludwik Silberstein, an engineer at Eastman Kodak, wrote to Bridgman: "Your abstinence throughout the book from 'mystical' elements and from hypostasy of the products of the human brain is a very welcome feature."[105] However, most of them, like Hersey, do not seem to have bothered about Bridgman's peculiar

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[104] *Ibid.*

[105] Silberstein to Bridgman, Jan. 1923, cited in Walter, *Science and Cultural Crisis*, p. 96.

discussion of measurement, such as his careful distinction between physical quantities and their numerical values.

As the first comprehensive monograph on the subject, *Dimensional Analysis* attracted many scientists' attention. Hersey wrote how popular the book was:

> It [*Dimensional Analysis*] arrived a few minutes before I was called up to a conference of section chiefs, and I took it along to read to myself while the other people were doing the talking, but was improvident enough to pass it over to another gentleman across the table just before the meeting was called to order, Mr. John Blizard, chief of the Fuels Section [of Experiment Station of Bureau of Mines, Department of the Interior], and Mr. Blizard would not let it get out of his hands during the meeting. I think he must have read it half through during the session for he inquired afterward if he might have it back to finish later, and as he wouldn't pass it over to me during the meeting, I was deprived of that pleasure until later in the evening.[106]

It was not only Americans who found Bridgman's monograph attractive. Henry Crew, the chairman of the Department of Physics at Northwestern University, wrote to Bridgman: "Professor [Arnold] Sommerfeld, who happened to be here last week giving a couple of lectures, was greatly interested in turning the pages of your book."[107] Crew added his own comment: "If Lord Rayleigh were alive, I am sure the work would draw from him also high praise." It is easy to imagine that this favorable response to his first attempt to publish his own discussion on the foundations of physics encouraged him to broaden his activity along this line. After publishing *Dimensional Analysis*, he went on to examine relativity theory, and then quantum theory.

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[106] Hersey to Bridgman, Nov. 15, 1922, PWBP, HUG 4234.8.

[107] Crew to Bridgman, Nov. 21, 1922, PWBP, HUG 4234.8.

### 3.3. After *Dimensional Analysis*

While refining his thought on the methodology of physics, Bridgman kept paying attention to the controversy on dimensional analysis. Immediately after Bridgman published *Dimensional Analysis*, a new paper by Lewis on the theory of ultimate rational units appeared in the 1923 volume of the *Philosophical Magazine*.[108] Assuming that "there is nothing nature abhors more than an arbitrary number which possesses no intrinsic meaning," Lewis proposed to call natural constants "unnatural constants," reserving "the term natural constants for simple numbers like 2 or π." Thus, he considered every physical constant as indicating some flaws in the system of units. The purpose of the paper was to show how to progress toward "a more natural or 'rational' set of physical units." Lewis suggested the following procedure: eliminating physical constants by "a mere change in the units of measurement," discovering new constants, and then "reducing these constants to unity or simple numbers, by a change in the system of units."

The editor of *Philosophical Magazine*, O. J. Lodge, put his special comment on this apparently extraordinary article: "The publication of a paper does not mean agreement with its contention."

> I for one hold that an attempt to unify essentially different physical quantities, and obliterate the ratios or constants connecting them, can only result in confusion. *A unit is not unity.* Length and Mass and Time and Energy and Momentum are not the same. Nor is Temperature identical with Energy. [109]

To Lodge, "the arbitrary elimination of constants, or the attempt to mask them and replace them prematurely by pure numbers" seemed

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[108] G. N. Lewis, "Physical Constants and Ultimate Rational Units," *Philosophical Magazine*, 45 (1923), pp. 266-275.

[109] O. J. Lodge, "Note by O. J. L.," *Philosophical Magazine*, 45 (1923), p. 275. Emphasis

"retrograde." He argued that although choice of units was "merely a matter of convenience," the choice which Lewis suggested "would assuredly not be convenient."

It was not only Lodge who found Lewis's theory irrational, rather than rational. The English physicist-philosopher Norman Campbell published a critical remark on the theory of ultimate rational units in the January 1924 issue of *Philosophical Magazine,*[110] commenting that "the theory is demonstrably false." Campbell had once attacked Lewis's theory in his *Physics, the Elements,*[111] published in 1920. Having since observed that no one took Lewis's theory seriously, Campbell had not discussed this matter until he realized that "the very simple fallacy on which the theory is based"[112] had not actually been understood generally. He started his criticism by blaming the ambiguity of the terms Lewis adopted, especially the word "simple." Admitting that "it is impossible to prove that U. R. U. [ultimate rational units] is not true" because of "the vagueness of the term involved," he pointed out that "if it [the theory of ultimate rational units] is true, it is equally true when for 'simple' we write 'simple or not simple.'" Campbell then indicated that "there were infinitely many sets of values assigned" to Lewis's constants "which would enable him to determine from them a set of units," while the theory of ultimate rational units required that "the set ought to be unique." This fact, he discussed, might have been observed by Lewis himself, "if he had not been obsessed by his belief in U.R.U." Campbell sarcastically concluded: "The theory will be true,--but it will be useless."[113]

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in original.

[110] Norman Campbell, "Ultimate Rational Units," *Philosophical Magazine*, 47 (1924), pp. 159-172.

[111] Norman Campbell, *Physics, the Elements* (Cambridge: Cambridge University Press, 1920).

[112] Campbell, "Ultimate Rational Units," p. 160.

[113] *Ibid.*, p. 172.

In the March 1924 issue of the *Philosophical Magazine*, Campbell then took up dimensional analysis as an object of his criticism.[114] Campbell understood that dimensional analysis was applicable only "if the laws determining the behavior of the system are completely known, and are actually expressible in the form of differential equation, the integration of which by purely mathematical processes would give a complete solution of the problem."[115] If, however, the system is so known, "all the formalities which various writers have elaborated are useless."[116] This devastating statement was again his conclusion. In his view, dimensional reasoning not based on differential equations would easily lead to errors. He warned that one should accept dimensional analysis only on the basis of the physical soundness of the assumptions involved, not on that of "the formal logical process employed."[117]

Around the same time, Bridgman and Lewis happened to notice a paper published in the 1916 volume of *Mathematische Annalen* by another participant in the controversy, Tatiana Ehrenfest-Afanassjewa.[118] Bridgman had known her paper "for some time from abstracts,"[119] having tried to obtain a copy before writing *Dimensional Analysis*. However, as "a number of periodicals were very late in reaching" the United States after World War I, he had not been able to read it until Ehrenfest-Afanassjewa sent its reprint to him in the spring of 1924.[120] A few days after Bridgman received it, Lewis asked

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[114] Norman Campbell, "Dimensional Analysis," *Philosophical Magazine*, 47 (1924), pp. 481-494.

[115] *Ibid.*, p. 486.

[116] *Ibid.*, p. 494.

[117] *Ibid.*

[118] Tatiana Ehrenfest-Afanassjewa, "Der Dimensionsbegriff und der analytische Bau physikalischer Gleichungen," *Mathematische Annalen*, 77 (1916), pp. 259-276.

[119] Bridgman to Lewis, March 24, 1924, PWBP, HUG 4234.8.

[120] It is not clear when Bridgman came to notice her name and her articles. Although she published another mathematical examination of Tolman's principle of similitude in

Bridgman for his comment on it,[121] since he found her article helpful in clarifying "the intangible doubts" which he had had regarding the dimensional method. As Bridgman was just about to leave for Brussels to attend the Fourth Solvay Congress, he could not carefully read or write much about Ehrenfest-Afanassjewa's paper. He promised Lewis to read it on the boat and discuss the matter directly with her during his visit to Leiden that he was planning. In Leiden Bridgman talked with her for a couple of hours. Their talk, however, did not bring "any very definite result," as he wrote to Lewis after he came back to Harvard.

> In the first place there was a language difficulty, she speaking no English, and I having to stumble along as best I could with my very poor German. Secondly, also because of her unfamiliarity with English, she had given only the most casual examination to my book. It was evident that in many points we thought in the same way, and also evident that she was surprised that I could agree with her on many points. We are both fully agreed that dimensional analysis as ordinarily presented and used in unsound in many particulars. I am not wuite [*sic*] clear in my own mind whether there is any essential difference between us, or whether it will eventually come down to a matter of words.[122]

Despite the fact that she had published a paper in English, Bridgman noticed her poor command of English. She may have written it with the aid of her husband, Paul Ehrenfest, a well-known theoretical physicist and successor of Heike A. Lorentz in Leiden. Bridgman had to write to Lewis: "I have not yet any definitely formulated opinion to give you" concerning Ehrenfest-Afanassjewa's mathematical treatment of dimensional analysis.

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the 1916 volume of the *Physical Review* (Tatiana Ehrenfest-Afanassjewa, "On Mr. R. C. Tolman's 'Principle of Similitude," *Physical Review*, 8 (1916), pp. 1-7), where she referred to her *Mathematische Annalen* paper, apparently Bridgman had paid no attention to either of them until several years later. He did not mention her name in his 1916 paper, nor in the first edition of *Dimensional Analysis*.

[121] Lewis to Bridgman, March 14, 1924, PWBP, HUG 4234.8.

In fact, there was no noticeable difference between Ehrenfest-Afanassjewa's and Bridgman's standpoints at that time. Ehrenfest-Afanassjewa understood that by the transformation to the miniature universe Tolman actually constructed his own system of units, although this was not his intention. In composing his system, Tolman adopted two extra assumptions that the C. G. S. system did not—the assumption of the constancy of velocity of light and that of the unit of electric charge--, thus reducing the number of degrees of freedom for the model transformation from three to one. Ehrenfest-Afanassjewa pointed out that if one added another assumption, namely, the equation of gravitation, to Tolman's list of fundamental equations, the number of degrees of freedom for transformation would become zero: "[I]n other words the equations of physics are of such a nature that no model transformation exists in which all the universal constants have the same values as in our universe."[123] She also mentioned G. Nordström's attempt to maintain a model universe "at all costs," for example, by "correcting" all physical laws in such a way that they satisfied the principle of similitude. It is understandable that Bridgman found no special objection to her argument during their talk in Leiden. Two years later, however, she would discuss the matter differently to the extent that Bridgman felt the need to restate his standpoint by sending his last remarks on this subject to *Philosophical Magazine*.

As for Campbell's criticism, Bridgman asked Lewis, "Have you, or do you intend, to reply to Norman Campbell's article in the Phil. Mag. on ultimate rational units?"[124] Lewis replied, "I did send to the Philosophical Magazine an article called forth by Norman Campbell's

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[122] Bridgman to Lewis, May 27, 1924, PWBP, HUG 4234.8.

[123] Tatiana Ehrenfest-Afanassjewa, "On Mr. R. C. Tolman's 'Principle of Similitude," *Physical Review*, 8 (1916), pp. 1-7.

[124] *Ibid.*

effusion, and I shall be glad to have your comments on it."[125] He therewith enclosed his second copy of the paper.[126] In it, Lewis tried to rebut not only Campbell but also Bridgman, who wrote in *Dimensional Analysis* that "[t]he justification of this point of view [the theory of ultimate rational units] at present is not to be found in any accurate results of measurement, but is rather quasi-mystical in its character."[127] Observing that "the question of how we may logically define a 'simple number' has greatly troubled both Professor Bridgman and Dr. Campbell," Lewis tried to explicate what he meant by "simple."[128] He first criticized Bridgman for employing Einstein's idea that "a suspected physical relation obtained by considerations of dimensionality becomes probable if the numerical coefficients involved are found to be small numbers"[129]; Bridgman, who pointed out the vagueness of the word "simple," used the term "small" a little carelessly. Lewis found it equally "difficult to explain just what is to be meant by a small number."[130] As for Campbell's discussion that "no matter how narrow the experimental limits of error may be, there always remains within these limits an infinite number of values consistent with the experiments," Lewis contended that "this is not a disproof of the theory of ultimate rational units, but rather a statement that from a purely logical standpoint no proof of any physical theory can ever be obtained."[131] Other than the new rebuttals to the critics' points, Lewis's fundamental ideas had not substantially changed since his first paper on the theory of ultimate rational units. What he did in the paper was basically the repetition of

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[125] Lewis to Bridgman, June 4, 1924, PWBP, HUG 4234.8.

[126] G. N. Lewis, "Ultimate Rational Units and Dimensional Theory," *Philosophical Magazine*, 49 (1925), pp. 739-750.

[127] Bridgman, *Dimensional Analysis*, p. 105.

[128] Lewis, "Ultimate Rational Units and Dimensional Theory," p. 743.

[129] *Ibid.*, p. 744.

[130] *Ibid.*

[131] *Ibid.*, p. 742.

his long-standing belief that nature was mathematically simple.

Bridgman could share the belief that nature is simple, but not in Lewis's sense. In his comment on Lewis's paper,[132] Bridgman precisely described the difference between their understandings of "the simplicity of nature."

> You believe that all the laws of nature of wide applicability are simply connected with each other. By "simple" you mean simply expressible in terms of human mathematical analysis. It would seem to me very strange if our mathematics, which for many phenomena is such an extremely clumsy tool, should turn out to be so peculiarly well adapted for the correlation of *all* the broad universal relations. My feeling is that the simplicity of nature is a much more restricted thing; that there are different domains within which the relations are simple, but that the different domains need not necessarily be simply connected in terms of ordinary mathematical analysis with each other. I recognize that apparent simplicity may arise in different ways; we may have such simplicity as we have in the behavior of perfect gases arising from the statistical effect of enormous numbers, or we may have (perhaps) an entirely different sort of simplicity such as the inverse square law between two electrons. I see no reason why the simple relations in one of these domains should be simply related to those of the other. [...] For each of these domains there would be a set of "rational" units, but the sets of units would be different in the different domains and there would be no such thing as "*ultimate*" rational units.[133]

Though admitting that there is certain simplicity in each realm in physics, Bridgman did not agree that different domains should necessarily be connected with each other in a mathematically simple way. Moreover, he pointed out the fact that Lewis's theory had yielded only two results, writing to Lewis, "I should expect it to be very much more fruitful if correct, in fact, a veritable open sesame."

When Lewis's paper was published in the *Philosophical Magazine*,

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[132] Bridgman to Lewis, June 21, 1924, HUG 4234.8.

the editor Oliver Lodge again attached a critical comment. He asked Lewis whether he was not "attaching undue importance to the numerical part of physical quantities, and trying to regard actual things as if they were or might be pure numbers."[134] Lodge added a judgment similar to Bridgman's: "Every physical quantity has a nature of its own, and it is the reverse of helpful to mask or disregard it." "We do not know," he continued, "what an electric charge is, but we know that it is something real and not an abstraction; no mere number can express it."[135] Like Bridgman, Lodge believed that physical quantity was something more than a mere measured number, rejecting any speculation that was based only on numerical part of physical quantity.

In 1926, encouraged by Campbell's suggestion that "dimensional analysis is intrinsically nothing less than rejected," Ehrenfest-Afanassjewa published another paper.[136] She argued for the necessity of distinction between dimensional analysis and the theory of similitude, preferring the latter as a method which "yields results essentially dependable." [137] Although admitting that dimensional analysis could be more useful when the fundamental equations were unknown, she warned that even in such cases "one advances only gropingly," and that "without experimental or theoretical proof from another quarter one can never be completely certain of the results."[138]

Campbell agreed with her, "against the 'dimensional analysts,"[139] as to the conditions in which dimensional analysis could apply: that the

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[133] *Ibid.*

[134] Oliver Lodge, "Units and Dimensions," *Philosophical Magazine*, 49 (1925), pp. 751-752.

[135] *Ibid.*, p. 751.

[136] T. Ehrenfest-Afanassjewa, "Dimensional Analysis Viewed from the Standpoint of the Theory of Similitudes," *Philosophical Magazine*, 1 (1926), pp. 257-272.

[137] *Ibid.*, p. 272.

[138] *Ibid.*

[139] Norman Campbell, "Dimensional Analysis," *Philosophical Magazine*, 1 (1926), pp. 1145-1151.

systems should be similar; and that the laws governing those systems' behaviors should be completely known.[140] Nevertheless, he did not think that such mathematical discussion as she had published was necessary. He considered her mathematical analysis as "elegant irrelevancies"[141] that distracted attention from essential points. In Campbell's understanding, "very few, if any, systems are *completely* similar, and *complete* similarity is not actually required."[142] Thus the judgment as to the applicability of dimensional analysis should be made on the basis of physical consideration, not of mathematical relations. It should be made individually for each case and for the immediate purpose. As the only criterion for similarity that can be made explicit, he reiterated the strong requirement stated in his previous paper.[143]

Provoked by papers of Ehrenfest-Afanassjewa and Campbell, Bridgman published his final comment on dimensional analysis.[144] Although Bridgman thought that after so much discussion on dimensional analysis "one would like to see it die a natural death,"[145] he could not overlook Ehrenfest-Afanassjewa's argument and Campbell's comment. He began his paper by the description of his position in *Dimensional Analysis*: he wanted to "put it on a thoroughly sound basis" and to show the method had nothing esoteric. Bridgman took it for granted that "[n]o refinement of mathematical argument will give correct results if our physical grasp of the broad features of the problem is inadequate."[146] Nevertheless, "broad physical grasp necessary to an application of the method" can be found expressions "in a knowledge of

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[140] *Ibid.*, p. 1146.

[141] *Ibid.*

[142] *Ibid.*, p. 1149. Emphases in original.

[143] Campbell, "Dimensional Analysis," *Philosophical Magazine*, 47 (1924), pp. 481-494.

[144] P. W. Bridgman, "Dimensional Analysis Again," *Philosophical Magazine*, 2 (1926), pp. 1263-1266.

[145] *Ibid.*, p. 1263.

[146] *Ibid.*, p. 1264.

the *general character* of the equations which govern the motion of the system."[147] Thus, to him, Ehrenfest-Afanassjewa's discussion seemed to require too detailed knowledge of equations. He did not understand how her method could apply to the problems of the airplane, the most important application of dimensional analysis, in which "it is hopeless to attempt to write the equation in detail."[148] His conclusion was clear: "I believe that the dimensional method *can* be made rigorous, that it is *not* necessary to replace it by the mathematical method of similitudes which she offers, that the dimensional method is much easier for the physicist to apply, and in many important practical cases is more powerful."[149] He also mentioned his bafflement to be called the "dimensional analyst" to whom "a miscellaneous assortment of errors" had been ascribed.[150]

On Campbell's criticism, Bridgman made a short comment. By pointing out the case of deducing the relation among the mass of an electron, its charge, radius, and velocity of light, he showed that in order to apply dimensional analysis it was not necessary to assume that "more than one kind of electron is physically possible." Physical similarity is, therefore, unnecessary when one applies dimensional analysis: "if as deep an analysis as [Campbell's] were necessary, few physicists would ever have the courage to attempt a dimensional analysis at all."[151]

After the 1926 paper, Bridgman lost interest in discussing dimensional analysis publicly. Although as the author of *Dimensional Analysis* he sometimes received letters and papers from scientists interested in the matter, he was not enthusiastic in answering them. To the second edition of *Dimensional Analysis,* published in 1931, Bridgman only added a short comment. In the early 1930s, he noticed

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[147] *Ibid.*

[148] *Ibid.*, p. 1265.

[149] *Ibid.*, p. 1266.

[150] *Ibid.*, p. 1264.

[151] *Ibid.*, p. 1266.

that there was a "furor of discussion" on this topic in the *American Physics Teacher*. Around the same time, some engineering journals also took up the matter in connection with the International Committee to adopt some systematic scheme of units. Bridgman, however, observed that "there has been a great deal of bunk in this engineering discussion," as he wrote to a friend of his in 1935.[152]

Immediately after he published *Dimensional Analysis*, Bridgman started to write on the foundations of relativity theory. He went on to generalize his reflection and published it in 1927 as *The Logic of Modern Physics*. Since then, encouraged by the success of his first publication in the philosophy of science, he seems to have been more attracted to applying his "operational analysis" to the marvel results of theoretical physics than reexamining the starting point of his critical activity, the scrutiny of dimensional analysis.

As his first attempt to discuss the foundations of physics, Bridgman's examination of dimensional analysis played a crucial role in the formation of his operational perspective. Through the scrutiny of dimensional analysis he tried to refute the overemphasis of the strength of this method as a research tool, which he recognized in Mayo Hersey's lectures at Harvard. He was also aware that scientists who, stimulated by the advent of relativity theory, were making speculative extrapolation of dimensional reasoning tended to underestimate the role of experimental research. Confronted with this situation, Bridgman, who had just entered into his career as an experimental physicist, felt the need to clarify the meaning of experiment in physics. Thus, he launched to embark in the scrutiny of the foundations of dimensional analysis that would eventually lead him to more profound reflection.

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[152] Bridgman to Bentley, Dec. 17, 1935, PWBP, HUG 4234.10.

Though his sense of the endangered status of experimental research would gradually become invisible, the entire scheme of his unique perspective on the methodology of physics can be regarded as stemming from an experimentalist's cry for the recognition of their business in the era of relativity theory and quantum mechanics.

While examining the detail of dimensional analysis, Bridgman could have a chance to recognize the role of experiment in physics. He became sure that one needs experimental knowledge to apply dimensional analysis correctly. Moreover, in his understanding, experimental research determines which physical quantities are taken to describe each particular phenomenon and how they are measured, while theoretical research can only establish relations between their measured numbers, leaving many other aspects of physical quantities unexamined. From this point of view, Bridgman criticized misuses of dimensional analysis, such as Lewis's theory of ultimate rational units or Tolman's theory of similitude. On the other hand, Bridgman fully appreciated the value of theoretical research, as his discussion with another participant in the dimensional controversy, Edgar Buckingham, illustrates. What Bridgman attempted to do was to settle the function and limitation of a theoretical tool by analyzing how it is based on experiment.

It has been pointed out that Bridgman's main work in the philosophy of science, *The Logic of Modern Physics*, does not contain much discussion on the meaning of the word "operation," despite the fact that it is considered as a manifesto of operationalism. Furthermore, some may even wonder how Bridgman as an experimentalist is reflected in the statement of operationalism. To answer these questions, it is necessary to comprehend how and why Bridgman felt the need to scrutinize dimensional analysis. His analysis of dimension shows us a unique attempt by an experimentalist to assimilate the meaning of the rising theoretical pursuit and secure at the same time the value of traditional experimental activity.