Optimization of a dc SQUID Magnetometer to Minimize the Field Resolution

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***Abstract*—The circuit parameters of a dc SQUID magnetometer have been optimized by computer simulations. Our principle of optimization is to minimize the field resolution. We have optimized the circuit parameters of a dc SQUID magnetometer, such as a Ketchen type, Drung type, directly-coupled type, etc. The only noise source was assumed to be thermal noise from the shunt resistors. For a Ketchen type SQUID, our results show that the inductance of the input coil and the washer coil should be larger than that reported previously. It is also the case for a Drung type SQUID that the inductance of the SQUID washer coil should be slightly increased. By this design method, the optimum field resolution of a Ketchen type SQUID magnetometer is improved by a factor of 2/3. Furthermore, we compared the resolution of 3 types of dc SQUIDs and concluded that the optimum type of SQUID depends on the value of the critical current and the radius of the pick-up coil. Finally, we have proposed a new design method that can optimize the field resolution of dc SQUIDs under constraints on the critical current and the spatial resolution.**

I. INTRODUCTION

The SQUID (Superconducting Quantum Interference Device) is a sensitive fluxmeter using Josephson junctions. Many excellent dc SQUIDs have been fabricated, and most of them have been designed to minimize the flux noise or energy resolution [1]. According to the literature on this subject [2], [3], the inductance parameter β equals 1 at the optimum point, where:

$$\beta \equiv \frac{2I_o L}{\Phi_o} \quad (1)$$

L = the inductance of the SQUID,
I_o = the critical current of the Josephson junctions,
Φ_o = the flux quantum.

With regard to applications for biomagnetic measurements, we propose to minimize the magnetic field (flux density) resolution, instead of the energy resolution. Our purpose is to optimize the circuit parameters of dc SQUIDs by this design principle. We derive the equations for the field resolution, and then optimize these equations in three types of SQUIDs: a Ketchen type, a Drung type, and a directly-coupled type. A new design method minimizing the field resolution is proposed based on a comparison of the results of these three kinds of SQUIDs.

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II. EQUATIONS OF THE FIELD RESOLUTION

First, we derive expressions for the sensitivity based on the field resolution. The main cause that restricts the SQUID sensitivity is the thermal noise that is generated from the shunt resistors of the SQUID. If the shunt resistors are taken to be the only noise source, it is adequate to consider the sensitivity in the white noise region only. The sensitivity is defined as the detection limit where the signal-to-noise ratio equals 1.

According to an approximation [3], the mean square value of the voltage fluctuation across the dc SQUID is on the order of

$$\left(\delta V_N^2\right) = 4k_B T \frac{R}{2} \quad . \quad (2)$$

In this equation, the factor 1/2 means that the 2 resistors are connected parallel for the output terminals. Then this is converted to the flux fluctuation.

$$\left(\delta \Phi_N^2\right) = \left(\delta V_N^2\right) / \left(\frac{\partial V}{\partial \Phi}\right)^2 \quad (3)$$

The transfer function $(\partial V/d\Phi)$ is determined by V-Φ characteristics, and is determined by the circuit parameters of the dc SQUID. However, the following approximation [4] can be used:

$$\left(\frac{\partial V}{\partial \Phi}\right) \approx \frac{I_o R}{\Phi_o/2} \cdot \frac{1}{1+\beta} \quad . \quad (4)$$

The transfer function in (3) is substituted for (4) to obtain the equation:

$$\left(\delta \Phi_N^2\right) = \frac{k_B T \Phi_o L}{I_o R} \cdot \frac{(1+\beta)^2}{\beta} \quad . \quad (5)$$

The flux noise is calculated by the square root of (5). Further, we can get the field noise, that is, the field resolution by dividing the flux noise by the area A of the pick-up loop.

$$\delta B \equiv \frac{\sqrt{(\delta \Phi_N)^2}}{A} = \sqrt{\frac{k_B T \Phi_o}{I_o R}} \cdot \sqrt{\frac{(1+\beta)^2}{\beta}} \cdot \frac{\sqrt{L}}{A} \quad (6)$$

This is the equation that determines the sensitivity of a dc SQUID. The first square root involves the ambient temperature, the $I_o R$ product of the Josephson Junctions, and the physical constants. So, we have only to consider the second and the third factors to optimize this equation. However, we cannot optimize this equation with respect to L unless we assume the geometry of the dc SQUID, because the inductance value L, the inductance parameter β, and the area A of the pick-up loop depend on the geometry and the circuit.

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III. Optimization

A. Ketchen Type SQUID

A Ketchen type SQUID has a SQUID with a flux transformer coupled to the SQUID coil.

The equivalent circuit is shown in Fig. 1. From this circuit, the effective inductance L_{eff} of the SQUID [5], [6] and the effective pick-up area A_{eff} of the SQUID can be calculated.

$$L_{eff} = L_{sw} + L_w \cdot \frac{(1-k^2)L_i + L_{si} + L_p}{L_i + L_{si} + L_p}$$
$$A_{eff} \equiv \frac{\Phi_{eff}}{B_{ext}} = \frac{k\sqrt{L_w L_i}}{L_i + L_{si} + L_p} \cdot A_p \tag{7}$$

where

L_w = the inductance of the SQUID washer coil,
L_{sw} = the inductance of a strip part in the SQUID washer,
L_i = the inductance of a input coil,
L_{si} = the inductance of a strip part in the flux transformer,
L_p = the inductance of a pick-up coil,
A_p = the area of a pick-up coil,
k = the coupling constant between L_w and L_i.

However, L_{sw} and L_{si} do not influence the magnetic coupling, so we can assume:

$$L_{sw} = L_{si} = 0\ . \tag{8}$$

We can get the equation of the field resolution of a Ketchen type SQUID by replacing L and A in (6) with L_{eff} and A_{eff} in (7).

$$\delta B = \sqrt{\frac{k_B T \Phi_o}{I_o R}} \cdot \sqrt{\frac{(1+\beta_{eff})^2}{\beta_{eff}}} \cdot \frac{\sqrt{L_{eff}}}{A_{eff}}$$
$$= \sqrt{\frac{k_B T \Phi_o}{I_o R}} \sqrt{\frac{(1+\beta_{eff})^2}{\beta_{eff}}} \frac{\sqrt{\{(1-k^2)L_i + L_p\}(L_i + L_p)}}{k A_p \sqrt{L_i}} \tag{9}$$

where

$$\beta_{eff} \equiv \frac{2 I_o L_{eff}}{\Phi_o}$$

In order to optimize this equation, the following assumptions are made:

(a) The ambient temperature T is constant ($T = 4.2$ (K)).
(b) The $I_o R$ product is constant ($I_o R$ = 50 (μV)), that is, the quality of each junction is the same.
(c) The coupling constant k is constant ($k = 0.9$).
(d) The area of the pick-up coil A_p and its inductance L_p are constant (The radius is 4 (mm) and L_p = 30 (nH).), that is to say, the spatial resolution is constant.

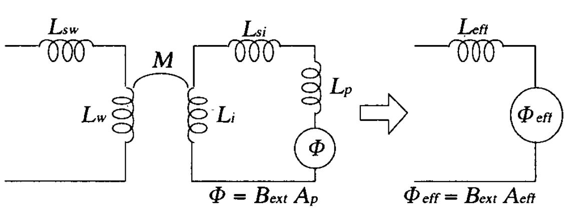

Fig. 1. A Ketchen type SQUID and the equivalent circuits.

The correlation of the inductance and the area depends on the geometry of the coil. Now, we assume that the coil is "single circular coil," and then the inductance and the area are written as a function of the radius of the pick-up coil r and the radius of the wire c (c = 10 (μm)).

$$L = \mu_o r \ln\left(\frac{r}{c}\right) \quad , \quad A_p = \pi r^2 \tag{10}$$

Finally, there remains only the two independent variables, L_w and L_i. The optimum condition can be calculated analytically.

$$\beta_w \equiv \frac{2 I_o L_w}{\Phi_o} = \frac{1}{\sqrt{1-k^2}} \quad , \quad L_i = \frac{L_p}{\sqrt{1-k^2}} \tag{11}$$

On the other hand, the optimum point reported previously [7] is written as follows:

$$\beta_w \equiv \frac{2 I_o L_w}{\Phi_o} = 1 \quad , \quad L_i = L_p\ . \tag{12}$$

Then we conclude that the inductance of the washer coil and the input coil in a Ketchen type SQUID should be designed larger than previously reported. The ratio of the sensitivities by the two methods is calculated by substituting (11) and (12) to (9) respectively.

$$\frac{\delta B(new)}{\delta B(old)} = \frac{2\left(\sqrt{1-k^2}+1\right)}{4-k^2} \tag{13}$$

By our design method, the field resolution of a Ketchen type SQUID magnetometer will be improved to 2/3 when the coupling constant k equals 1 in the ideal case.

Incidentally, if the coupling constant k is zero in (11), it becomes the same as (12). So, it is also found that (11) is an extension of (12), and that (12) is the case where the dc SQUID and the flux transformer have been optimized separately.

B. Drung Type SQUID

A Drung type SQUID has a multi-turn coil connected in parallel instead of a flux transformer. Although the total area of its pick-up coil is very big, its inductance is quite small because of the multi-turn structure.

The equivalent circuit is shown in Fig. 2. From this circuit, the effective inductance L_{eff} and the effective pick-up area A_{eff} can be calculated.

$$L_{eff} = \frac{L(A_p / n)}{n} \quad , \quad A_{eff} \equiv \frac{\Phi_{eff}}{B_{ext}} = \frac{A_p}{n} \tag{14}$$

where

A_p = the total area of a pick-up coil,
n = the division number,
$L(x)$ = the inductance of the coil whose area is x.

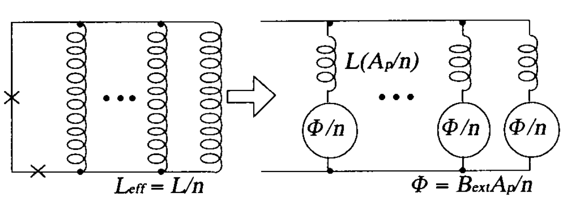

Fig. 2. A Drung type SQUID and the equivalent circuit.

We assume that the total area of the pick-up coil A_p is constant, that is the same as for the Ketchen type SQUID. So, if the coil is divided, the area of each coil becomes small. It means that the spatial resolution should be constant. We can get the equation of the field resolution of a Drung type SQUID by replacing L and A in (6) with L_{eff} and A_{eff} in (14).

$$\delta B = \sqrt{\frac{k_B T \Phi_o}{I_o R}} \cdot \sqrt{\frac{\left(1+\beta_{eff}\right)^2}{\beta_{eff}}} \cdot \frac{\sqrt{n \cdot L\left(A_p / n\right)}}{A_p} \qquad (15)$$

In this equation, the inductance L is a function of the area A, described by (10). Therefore, there remain only the two independent variables, the critical current I_o and the division number n. The calculated resolution as a function of I_o and n is shown in Fig. 3.

The curve shifts to the upper right side according to the increase of the division number n. The right shift means that the large critical current is permitted because of the decrease of the inductance, and the upper shift means the degradation of the field resolution because of the decrease of the pick-up area. In this figure, the curve where n equals 1 corresponds to a traditional "dc SQUID" that has neither a flux transformer nor a multi-turn coil. There is one optimum point on each curve, and that is its bottom where the inductance parameter is 1 as easily calculated.

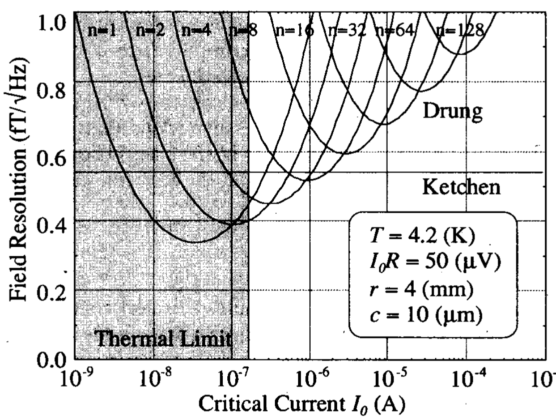

Fig. 3. The field resolution of a Drung type SQUID vs. I_o and n. A Ketchen type SQUID is also plotted. (It doesn't depend on I_o.)

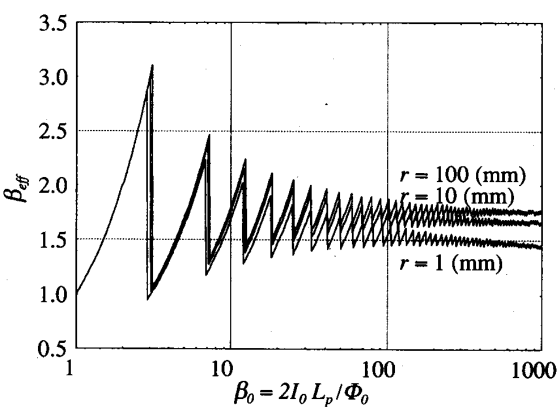

Fig. 4. The optimum β_{eff} of a Drung type SQUID.

However, this point is not the overall optimum. The fact that all the points are located above the envelope means that the point on the envelope is the overall optimum. The optimum condition on that envelope cannot be obtained analytically, so we calculated this solution numerically by computer simulation. The result is shown in Fig. 4. where $\beta_o \equiv 2I_o L_p / \Phi_o$ is the inductance parameter of undivided coil. The optimum β_{eff} doesn't change continuously because the division number n is an integer. It depends on the critical current and its pick-up area. But ordinarily, β_o is more than 100, the optimum β_{eff} is approximately

$$\beta_{eff} \approx 1.6 - 1.7 \,. \qquad (16)$$

We conclude that in the design of a Drung type SQUID, the division number should be decreased somewhat and the area of divided pick-up coil should be increased a little even though the inductance parameter becomes large to some extent.

Incidentally, the gray region in Fig. 3 means a thermal limit [2] derived from $\Gamma \equiv 2\pi k_B T / I_o \Phi_o = 1$. The value is 0.18 (μA) at 4.2 (K). A Drung type SQUID has the maximum sensitivity when the division number n equals 1, but this is impossible because the necessary critical current is far below the thermal limit.

C. Directly-Coupled Type SQUID

A directly-coupled type SQUID means a SQUID that has a pick-up coil connected to the SQUID coil directly. Therefore, the area of the pick-up coil can be large, although the inductance of the coil is quite small.

Its equivalent circuit is shown in Fig. 5. From this circuit, it is easily found that a directly-coupled SQUID is a kind of a Drung type SQUID whose division number is 2 and whose coils are unbalanced. We define an unbalance factor α, which takes a value between 0 and 1, and then we can calculate the inductance of the two pick-up areas.

$$L_1 = \mu_o r\sqrt{\alpha} \cdot \ln\frac{r\sqrt{\alpha}}{c}$$
$$L_2 = \mu_o r\sqrt{1-\alpha} \cdot \ln\frac{r\sqrt{1-\alpha}}{c} \qquad (17)$$

From these equations, the effective inductance L_{eff} and the effective pick-up area A_{eff} can be calculated.

$$L_{eff} = \frac{L_1 L_2}{L_1 + L_2} \;, \; A_{eff} = \frac{L_2\alpha + L_1(1-\alpha)}{L_1 + L_2} \cdot A_p \qquad (18)$$

The optimum condition is that the unbalance parameter α equals 0.5, that is to say, it is a Drung type SQUID with a 2-turn coil. So, we conclude that a directly-coupled SQUID has a worse sensitivity than a Drung type SQUID. However, one advantage of a directly-coupled type SQUID is ease of fabrication.

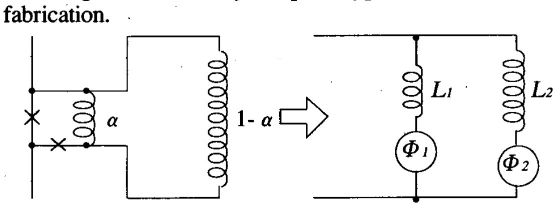

Fig. 5. A directly-coupled SQUID and the equivalent circuits.

IV. Comparison

In Fig. 3, it is remarkable that a Ketchen type SQUID is better than a Drung type in the range of more than about 2 (μA). On the contrary, below 2 (μA), a Drung type has the advantage, and under some circumstances, a simple dc SQUID is preferable. The cause of this preference is the flux transformer. A Drung type SQUID (including a single dc SQUID) can detect a magnetic field without any flux losses in the transformer, but it cannot absorb much difference of the inductance.

Each SQUID has advantages and disadvantages that are determined by the ranges of the critical current and the radius of the pick-up coil. Therefore, we have calculated the range over which each type of SQUID is optimal. The result is shown in Fig. 6. This figure shows which type is the best under constraints on the critical current I_o and the radius of the pick-up coil *r*. In this figure, we plotted I_o and *r* of dc SQUID reported previously [7]-[11]. The general tendency is that if I_o and *r* are large, a Ketchen type is better than others, and that a Drung type or a simple dc SQUID is preferable according to decreasing of I_o and *r*.

V. New Design Method

From the above results, we propose a new design method that optimizes the field resolution. The method is as follows.

(1) Decide the radius of pick-up coil. (It determines the spatial resolution.)
(2) Decide the magnitude of the critical current of the Josephson junctions that can be fabricated.

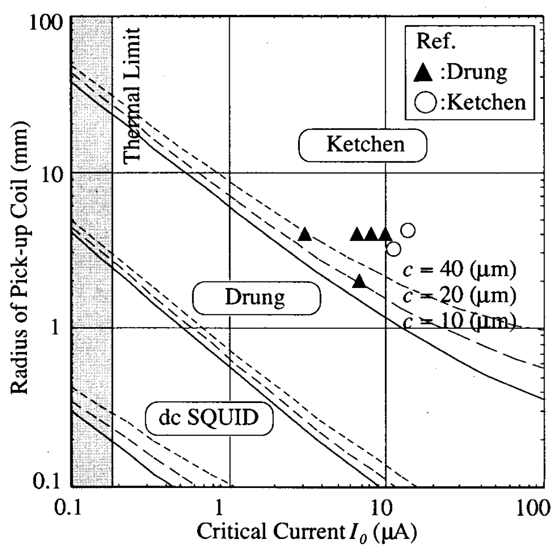

Fig. 6. Comparison of three types of dc SQUID. The circles in this figure are Ketchen type SQUIDs [7], [8], and the triangles are Drung type SQUIDs [9]-[11].

(3) Judge which type is the best SQUID from Fig. 6.
(4) If a single dc SQUID, $\beta_{eff} = 1$.
If a Drung type, $\beta_{eff} \approx 1.6 - 1.7$.
If a Ketchen type, $\beta_w = 1/\sqrt{1-k^2}$, $L_i = L_p/\sqrt{1-k^2}$.

VI. Conclusion

Using analytical calculations and computer simulations, we propose a new design method that can optimize the field resolution of dc SQUIDs. It has been found that the inductance of a SQUID washer should be larger than that previously reported. Furthermore, it has been also found that you should select the type of dc SQUID depending on the spatial resolution that you need and the magnitude of the critical current that you can fabricate. This design method is effective particularly in a multi-channel SQUID system where many pick-up coils are located without any open space.

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