

*ON A SEISMOGRAPH FOR REGISTERING  
VERTICAL MOTION.*

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Of the different motions, which take place in an earthquake, the vertical is perhaps the most difficult to get a satisfactory record of. I have contrived various methods of obtaining such a record and most of these methods have been mentioned in papers communicated to this society, either by myself, or by Mr. Milne who has for the most part used them.

The present instrument is different in some respects from those above referred to and I intend briefly to point out its peculiarities. The instrument will be readily understood from the annexed sketch (see fig.). In the sketch *S* is a spiral spring, about 25 centimetres long when unstretched and about 50 centimetres when stretched. This spring supports an arm *A*, which turns round a knife edge *K* at one end and carries at the other the mass *R* (about 600 grammes). In front of the mass *R* a small tube *t* is fixed to the arm *A* and put in communication with the beaker *b* by means of a siphon *s*. The bore of this siphon should be as large as possible consistent with free up and down motion of the tube *t*, and it should be made as short as possible. The beaker and tube are partly filled with a liquid and the surface in the two kept at the same level by means of the siphon, which is for that purpose kept full.

If we now suppose the mass *R* to be raised or lowered it is easy to see that the liquid in the tube *t* will be diminished or increased and hence that if the width of the tube and beaker, and the density of the liquid, be properly arranged the mass will have no tendency to return to its old position. In

same way if the top of the spring be raised or lowered the inertia of the mass R in the first place, and flow of liquid in the second place, will tend to keep the mass at its old position in space. By this simple arrangement, then, or a modification of it an astatic vertical motion seismograph may be constructed. In the instrument now exhibited the siphon and tubes are replaced by a small tube resting on the short arm of the recording lever.

When this tube is partly filled with mercury and properly adjusted it is quite sufficient to produce compensation through a considerable angle of motion.

With regard to the form of this instrument, it is possible that a flat spiral spring may be substituted for the long spiral now used with advantage in the way of compactness, but the manner the spring has to move in that case is an objection.

Again the method of compensation may be readily applied to the spring without the arm A. It must be borne in mind however that the arm A allows the period of free vibration to be considerable without having a long spring and without compensation. This period is necessary in order to allow the mercury time to act before the mass R moves sensibly.

That the arm A increases the period will be readily seen from the following investigation

Let  $\varphi$  = angle turned through by A from its normal position at time  $t$

$e$  = consequent elongation of the spring

$E$  = normal total elongation of spring

$l$  = length of long arm of lever

$l_1$  = length of short arm

$g$  = force of gravity on unit mass

Then we have for the equation of motion, supposing  $\varphi$  small and neglecting the mass of the spring,

$$\begin{aligned} \frac{d^2\varphi}{dt^2} &= \frac{R g \frac{l_1}{l} e}{E R l_1} = \frac{g e}{E l} \\ &= \frac{g \varphi l_1}{E l} \end{aligned}$$

$$\therefore \frac{d^2\varphi}{dt^2} \cdot \frac{1}{\varphi} = \frac{g}{E} \frac{l_1}{l}$$

and therefore by integration

$$T = 2\pi \sqrt{\frac{E}{g} \frac{l}{l_1}}$$

From this we see that the period increases as the root of  $\frac{l}{l_1}$  and consequently an advantage in length of period is gained. A disadvantage is, of course, the diminution of mass which is directly as  $\frac{l}{l_1}$  and is therefore greater than the increase of period.