A SEISMOMETER FOR VERTICAL MOTION.

BY

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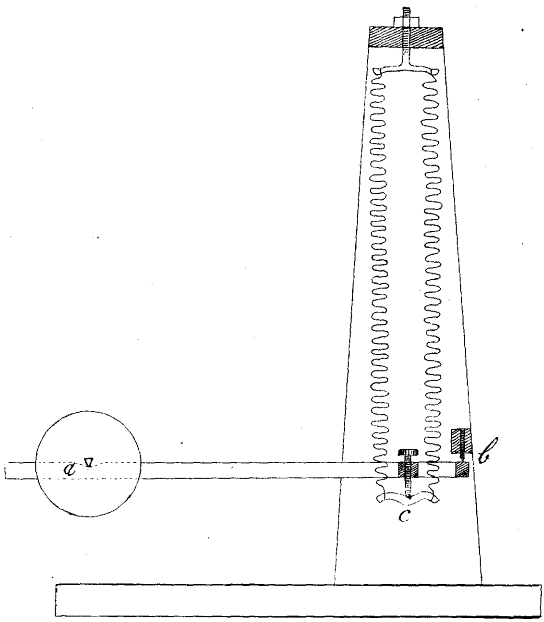
At a recent meeting of the Society Mr. T. Gray described a seismometer for vertical motion, in which the problem of supporting a heavy mass so that it should be free to move vertically and yet remain in neutral equilibrium was for the first time (so far as I am aware) satisfactorily solved. In Mr. Gray's instrument the mass is placed at the long end of a horizontal lever which is free to move in a vertical plane only, and the weight of the mass is supported by the upward pull of a stretched spiral spring attached to a point in the lever comparatively near the fulcrum. Mr. Gray has pointed out the advantage gained by applying the upward pull of the spring not directly to the heavy mass, but through a lever, by which a longer period of oscillation is obtained than if the weight were directly borne by a spring elongated to the same extent. This arrangement, however, still leaves too large an amount of stability (especially when the length through which the spring is extended is moderate), and Mr. Gray has supplemented it by an ingenious device which is equivalent to increasing the weight at the long end of the lever when it is displaced downwards and diminishing the weight when it rises, the increase and diminution being proportioned so that in all positions throughout a limited but sufficient range, the tension of the spring just suffices to balance the weight. This Mr. Gray has done by adding in the first instance a siphon, and subsequently a horizontal tube containing a fluid, so


a
b
c

placed that when the weight goes down* the fluid runs over towards it and increases the weight, and *vice versa.*

I have recently devised another method by which this compensation may be effected, not by a change of the mass at the end of the lever, but by a change of the leverage at which the spring acts. If we apply the spring not at a point in the horizontal line in which the centre of the heavy mass and the fulcrum of the lever lie, but at some distance below that line, it is clear that when the mass goes down the leverage at which the spring acts is reduced while the pull of the spring is of course increased, and when the mass goes up the leverage is increased and the pull of the spring diminished. To secure neutral equilibrium we must have the product of these quantities, or in other words the moment of the pull of the spring about the fulcrum, equal and opposite to the moment of the weight. In small displacements the moment of the weight remains sensibly constant, and we must therefore make the moment of the pull of the spring constant.

In the annexed sketch $a\,b$ is a horizontal lever carrying a heavy mass at a and pivotted on a horizontal axis at b. c is the point of application of the spring (or rather pair of springs acting as one), the pull of which is exerted vertically upwards.

Let h be the *horizontal* distance of c from b, and let v be the vertical distance of c from the horizontal line through $a\,b$. Then if we suppose the lever to be displaced downwards through any very small angle $d\theta$, the point c is displaced vertically down through a distance $h\,d\,\theta$ and horizontally towards the fulcrum through a distance $v\,d\,\theta$.

Now if we call e the distance through which the spring is stretched in the normal position of the lever, it follows from Hooke's Law that by the supposed displacement the upward pull of the spring is increased in the ratio

$$\frac{e + h\,d\,\theta}{e}$$

* *That is, relatively to surrounding objects. In reality, of course, during an earthquake the weight does not move, but other objects go up.*

and the leverage at which the spring acts is diminished in the ratio

$$\frac{h - v\, d\, \theta}{h}$$

The product of the pull into the leverage is to be constant, being equal to the moment of the weight, and hence we must have

$$e\, h = (e + h\, d\, \theta)(h - v\, d\, \theta)$$

From this, neglecting the term which involves $d\, \theta^2$, we have

$$e\, v = h^2$$

$$\text{or} \qquad v = \frac{h^2}{e}$$

an equation which determines the distance v below the horizontal line of the lever of the point at which the tension of the spring is to be applied, so as to give neutral equilibrium for infinitesimally small displacements.

In practice v should be made somewhat less than this, in order to leave a small margin of stability, and to prevent the lever from becoming unstable for such displacements as are liable to occur during an earthquake. It is easy to make the equilibrium sensibly neutral throughout a more than sufficiently wide range. In large displacements the equilibrium becomes unstable.

In the model now exhibited to the Society, the lever $a\, b$ is a light stiff wooden frame, of which a is the axis of rotation corresponding to b as axis of percussion. The heavy mass is pivotted (not fixed) at a. The fulcrum consists of two steel points fixed in a stout upright frame, and standing one in a V groove and the other in a conical hole at the end of the lever. There are two parallel springs the height of whose top support is adjustable, while their lower ends are attached to a hanging cross bar, the middle point of which presses up against an adjustable set-screw in the lever at c.

The vertical motion of the earth relatively to the mass at a requires to be multiplied greatly by means of an independent pointer, which should possess the same kinetic property (as to its axes) as that mentioned above. It is intended to record the vertical motion on the continuously revolving glass plate which is already used to receive records of horizontal motion.