ON CERTAIN METHODS OF ASTATIC SUSPENSION

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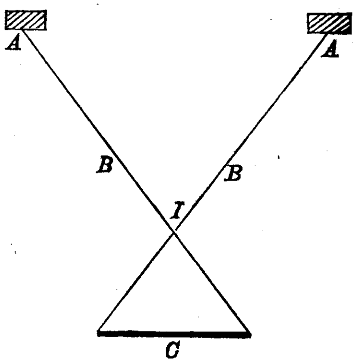
The mass whose inertia is to give steadiness in a seismometer may be hung in neutral or nearly neutral equilibrium by any system of links which produces an exactly or approximately straight-line motion, provided the links are placed so as to make its line of motion horizontal. An example of this was furnished by the model exhibited by Professor West at the last meeting of the Society, in which a heavy bar was hung by a system of links forming part of the so-called parallel motion of Watt. Or to take another example, we might suspend a mass by a pair of Peaucellier linkages, with freedom to move horizontally, and thereby obtain absolute astaticism throughout the whole range of its motion; or, by reducing the distance between the two pivot points in each linkage, we might compel the mass to move in an arc of very large radius, and thereby give it the slight stability needful in a seismometer. But this plan would be open to the same objection as the suspension by Watt's linkage proposed by Mr. West,—the multiplicity of joints would give rise to an intolerable amount of friction.

To avoid friction we should select a form of linkage with as few links as possible, and these all ties, in order that we may easily substitute flexible cords for rigid pieces with joints. No linkage satisfies these conditions better than the approximate straight-line motion of Tchebicheff, illustrated in the sketch. *A A* are fixed supports *B B* are two equal links, which, when the apparatus takes this form, may be flexible cords. They cross each other and are connected to the end of a hanging bar *C*. The vertical distance of the middle of *C*



from the line $A\,A$ is equal to the distance $A\,A$, while the length of the hanging bar is $\frac{1}{2}\,AA$. Then the middle point of the bar moves in a line which is very approximately straight and parallel to $A\,A$, provided its excursions lie within a range not greater than the distance between the fixed supports. These proportions will give sensible astaticism when AA is horizontal; but by making the depth of the hanging bar somewhat greater, or by placing the centre of gravity of the hanging bar below the line of its attachment to the links $B\,B$, we may give it any desired amount of stability.

When $B\,B$ are single cords the system is azimuthally unstable, but it is easily prevented from rotating about a vertical axis by making each of the suspending links B double, in two parts which form a V whose vertex is at the end of C, and whose base is a line through A perpendicular to the plane of the paper. Further, this prevents oscillation perpendicular to the plane of the paper, and so leaves none but the desired freedom. To increase the steadiness we may add a mass which should be as much as possible concentrated at the centre of C. This may be pivotted about a horizontal axis perpendicular to C, through its centre, and in that case the mass is equivalent to a particle concentrated there. Figs. 1 and 2 show this arrangement in elevation and plan. There the hanging bar is a light platform on which a heavy lead weight is pivotted about the axis $i\,i$ on the points of two steel screws, which press up into a conical hole and V-slot in a bar j to which the weight is rigidly attached.

Another plan is to use a pair of light suspended platforms, in line with each other, and use them to carry a massive block by three sharp feet which press into a hole and V-slot on one platform, and a V-slot parallel to these on the other. This arrangement is shown in figs. 3 and 4.

But in both of these arrangements the friction at the pivots by which the weight rests on the hanging platform is a disadvantage, which is all the more felt because the platform tilts up through a considerable angle when displaced from its mean position. For this reason the writer prefers the very simple form shown in figs. 5 and 6. There the bob or heavy mass is a piece of lead rigidly fixed to the hanging bar. The effect of this is that when a horizontal displacement of the ground occurs, in the line of the bar, the centre of the bar does not remain at rest, but moves through a small determinate distance in the same direction as the ground.

Let M be the mass of the hanging piece (including the rigidly attached bob), and let Mk^2 be its moment of inertia about its central transverse axis. When the hanging piece is displaced, its motion is one of rotation about its instantaneous axis, which is always situated at the intersection of the suspending cords (the point I in the woodcut). It is easy to show that, for any moderate displacement, this axis moves in a sensibly horizontal direction, and through the same distance as the centre of the bar. Hence the angular displacement of the bar is, very nearly, proportional to its linear displacement, and is equal to the latter divided by h, where h is the height of the instantaneous axis I above the bar. As regards its resistance to rotation, the hanging piece is therefore equivalent to a particle of mass M whose velocity-ratio relative to the centre of the bar is sensibly constant, and equal to $\frac{k}{h}$. Hence the extra inertia due to rotation is $\frac{Mk^2}{h^2}$, referred to the centre of the bar.

When a horizontal displacement of the supports occurs, we may consequently consider the whole system as consisting of a particle M together with a connected particle $\frac{Mk^2}{h^2}$, of which only the first is effective in producing steadiness, although both are constrained to share the same motion. The piece will therefore suffer an acceleration in the same direction as the acceleration of the supports, and bearing to it the ratio $k^2 : h^2 + k^2$. The centre of the bar will be displaced in the direction of the displacement of the supports, and in the same proportion; and any measurements of earthquake motion which are taken with reference to the centre of the bar as a datum-point must be multiplied by $1 + \frac{k^2}{h^2 + k^2}$ to find the true displacement of the ground. When the bob is a lump of lead whose dimensions are small compared with the length of the hanging bar, this factor differs very little from unity.

Two light bars with dense rigidly attached bobs, and suspended by silk threads so as to swing at right angles to each other, form an excellent two-component seismograph, especially suitable for the measurement of large earthquakes. The complete absence of joints makes the frictional resistance exceedingly small: in this respect the method of suspension now under examination constrasts very favourably with most other methods. Moreover the construction, and also the adjustment, is very simple. For large earthquakes the method of recording shown in fig. 5 will probably be found suitable. A light pointer *l* is forked so as to enclose the hanging bar, and is jointed to the bob at its centre. Its end *m* rests on a smoked-glass plate, the pressure being regulated by a counterpoise *n*. The plate should be set in rotation by an earthquake, by means of a seismoscope which may conveniently be somewhat wanting in sensibility, in order that it may act only when the motion becomes tolerably severe. The record will be *less* than the true motion, in the ratio given above.

Fig. 6 shows an instrument of the same kind in plan, but with a multiplying index *p*, which is pivotted to the ground on a vertical axis *o*, and receives its motion by having a bent-up end which gears into a slotted plate *q* fixed to the bob.

Fig. 7 is a sketch of a curious form of double-freedom seismometer, in which a somewhat similar method of suspension is used. The base *S*, which is fixed, is an equilateral triangle, at the corners of which there are three conical cups forming sockets for three legs *r r r* of equal length. These legs press up into three other sockets in the plate, *s*, which are placed so as to form an equilateral triangle of half the linear dimensions of the base. The height of *s* from the base is 0.866 times the distance between the base sockets. The plate *s* carries a massive bob *w*, slightly beneath it. The plate is then in nearly neutral (somewhat stable) equilibrium with respect to small motions in any azimuth. It is necessary here to invert the system and use struts instead of ties, since a plate hung by three crossed cords would be azimuthally unstable. Two of the legs are made with loops to allow the three to cross each other. By adding a multiplying pointer to record the displacements of the base with respect to the centre of the plate, we should obtain a compact form of double-freedom seismograph, whose frictional resistance (though much greater than that of the single-freedom instrument just described) could probably be kept within reasonable limits.