

## 16. *Experimental Study on the Propagation of Tunami Waves. Part I.*

By Genrokuro NISHIMURA and Takeo TAKAYAMA,

Earthquake Research Institute.

(Read Dec. 20, 1933.—Received Dec. 20, 1933.)

### **Introduction.**

After the occurrence of the tunami on the Sanriku Coast of Nippon on March 3, 1933, the writers made a field investigation of the water heights attained along a part of the devastated coast of the Sanriku, and arrived at the conclusion that one of the most potent factors on the water height distribution are the geographical features along the coast, and the main object of the present paper is to discuss the effects of the topography and the configuration of the sea bottom on the propagation of the tunami waves.

For the purpose of obtaining by model experiment the wave height, the velocity of the tunami wave, the velocity of the water particle in a bay, and phenomena connected with the inundating tunami wave, the following facts must be known, namely, the height and form of the tunami at the mouth of the bay and vicinity. Of course the shape of the model of the bay must be similar to that of the actual bay to be studied.

Now the tide records, which were obtained on the day of the tunami at several tide gauge stations along the Sanriku Coast, show that the periodic motions of water, with periods 5 and 10 minutes predominating are caused in the bays by the tunami. Of course a bay is an oscillating system, therefore the periods obtained from the tide records are not necessarily those of the tunami waves propagated from the origin. Now since the mean depths of the water at the tide gauge stations are only a few meters, the apparent wave length of the waves which cause these predominating periods are generally a few kilometers. We do not know, however, whether the waves thus determined from mareogram data are of the progressive type or whether they are stationary.

It may be somewhat bold to assume that the tunami waves propagated from the origin have periods the same as that shown by mareogram data. If this assumption is within reason, then the apparent wave length of the tunami wave at the mouth of a bay may be a few kilometers, about the length of the bay measured from its mouth to head. From the results of field investigations and also from the reports of eyewitness of this tunami, the wave height of the present tunami at the mouth of a bay along the Sanriku Coast was a few meters. From these considerations we may say that the slope of the surface of the present tunami wave was about one in a few hundreds at the mouth of the bay in question.

As just stated, the propagated tunami wave may be an oscillatory wave of great wave length compared with the depth of the bay, so that the widely separated elevations (of the waves) are practically independent of one another, and therefore it may be reasonable and simple to assume that each elevation corresponds to a solitary wave. The total effects of the tunami waves on the coast, therefore, are obtained by the superposition of each effect of the solitary wave thus defined.

To study the propagation of the solitary wave thus defined for the purpose of investigating the present large scale tunami wave by tank model experiments on a small scale, we must bear in mind the law of similitude as follows:

We must, first of all, introduce a viscosity term in the  $\pi$  equation. For example, if we wish to know the actual velocity of the flow of water in a bay of the Sanriku Coast owing to the tunami, we obtain the equation

$$v = \sqrt{gh} \phi \left( \frac{l}{h_0}, \frac{\eta_0}{h_0}, \frac{\mu}{\rho \sqrt{gh_0^{3/2}}} \right), \dots \dots \dots (1)$$

where

- $v$  = velocity of flow of water,
- $\eta_0$  = wave height at the mouth of the bay to be studied,
- $h$  = water depth in the bay,
- $l$  = linear dimension,
- $g$  = acceleration due to gravity,
- $h_0$  = water depth at the mouth of the bay,
- $\mu$  = viscosity,
- $\rho$  = density,
- $\phi$  = unknown function.

If the following relations are satisfied,

$$\frac{\mu}{\rho h_0^{3/2}} = \frac{\mu}{\rho' h_0'^{3/2}} \dots (2)', \quad \frac{\eta_0}{h_0} = \frac{\eta_0'}{h_0'} \dots (2)'' \quad \text{and} \quad \frac{l}{h_0} = \frac{l'}{h_0'}, \dots (2)''' \dots \dots \dots (2)$$

we obtain

$$v' = v \sqrt{\frac{h_0'}{h_0}} \dots \dots \dots (3)$$

where the quantities with accented notations are those of the actual waves, bays, etc. Since in the present experiments, a tank a few meters in size, and a long wave about 2 meters in length, are used, we neglect the viscosity term on the assumption that viscosity has no marked effect upon the velocity of flow of water, hence use water as the liquid for the model experiment. Therefore if we adjust  $h_0$ ,  $\eta_0$ ,  $l$  to satisfy (2)', (2)'', we obtain the actual velocity of flow of water using the expression (3).

The present report<sup>1)</sup> is only preliminary to the following subjects for future research.

1. On the propagation of long waves in a bay of variable section.
2. On the study of phenomena connected with the inundating tsunami wave.
3. On the occurrence of seiches in a bay due to excitation of the propagated wave.
4. On model experiments in connection with wave propagation and the inundation of the Sanriku Coast.

### Experimental Equipment.

The wooden water-tank used in the present experiment measured 640<sup>cm</sup> × 152<sup>cm</sup> × 60<sup>cm</sup>.

One of the side walls of the tank consists of two panes of glass  $G_1$ ,  $G_2$  as shown in Fig. 1. A photographic camera  $c$  is used for taking the wave phenomenon.

To generate a solitary wave of long wave type, the wooden plate  $A$ , which is hinged to the bottom surface of the water tank, is moved from one side to the other by hand at suitable speeds and displace-

1) The model of a bay referred to in the present report consists of geometrical planes as a first approximation to the form of an actual bay. We then obtained the experimental formula for the case of a solitary wave of which the apparent length is equal to the length of the model bay being propagated in the model bay. Of course  $\eta_0/h_0$  are varied to satisfy the relation (2)'.

ments. The displacements and the speed are recorded on the drum  $d$  rotating with constant speed.

To vary the effective depth of water, the bottom of the tank is composed of double floors as shown in the figure the upper one  $B_1$  is made movable vertically; and is situated far from the plate  $A$ .

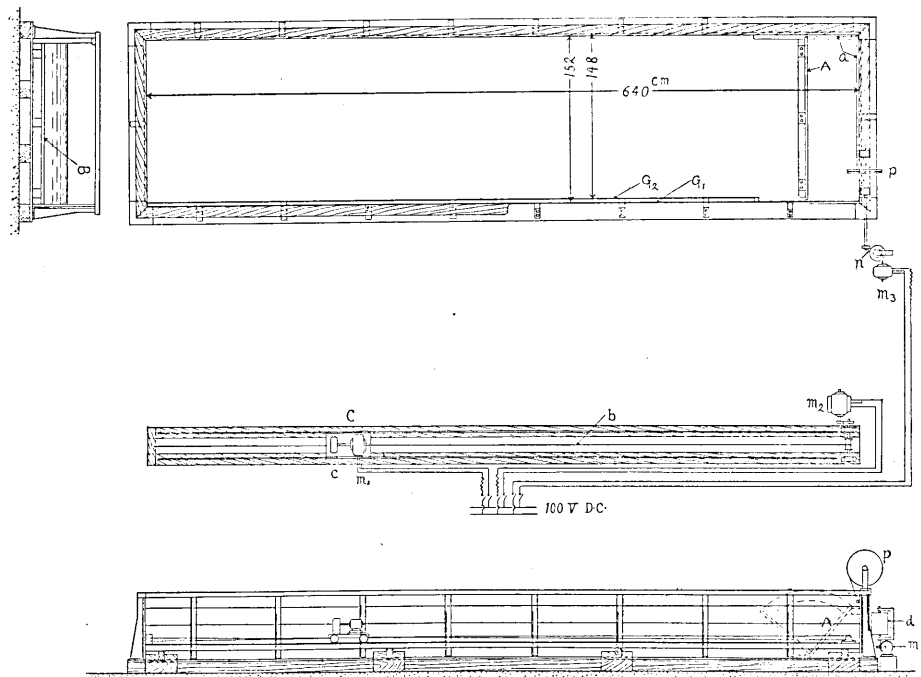


Fig. 1.

When the water in the tank rises owing to the movement of plate  $A$ , trolley  $C$ , on which are an Ica standard camera  $c$  and a  $1/16$  D. C. motor  $m$  to drive it, runs parallel with the propagation of waves on the smooth trolley rails placed 2 meters distant from the tank. The wave phenomena in the tank are then photographed by the camera. In the present experiment the camera is adjusted to take 24 pictures per second.

The wave height and the apparent wave length of the generated wave in the tank varies according to the driving speed of plate  $A$  and the amount of its displacement, respectively. It will be explained in the next section that the wave thus generated is a long wave of solitary type.

Now the water contained between the two panes of glass  $G_1$ ,  $G_2$  is virtually insulated from the main body of the water in the tank, the surface of this water remaining horizontal even though a wave is propagated in the main water. Photographing this water surface and the wave form in the film at the same time, we are able to read off the wave height and the other vertical movements of the water when a wave is propagated.

For measuring the horizontal movements of particles we paste long strips of dark paper on the surface of glass plate  $G_2$ , spaced equally apart as measured beforehand. Since these papers also go into the photograph, it is possible to measure the length of the horizontal movements of the water, etc.

With this arrangement we are ready to measure the time, and the length both horizontal and vertical.

### I. A Bay of Rectangular Section of Uniform Depth.

In the ordinary wave equation of water, we neglect terms relating to the quantities of high order, such as  $u \frac{\partial u}{\partial x}$ , where  $u$  is the horizontal velocity, but not if the wave height<sup>2)</sup> becomes so large as to be comparable with the water depth, in which case the expression for wave velocity becomes more or less complicated. Airy, Riemann, and others<sup>3)</sup> obtained theoretically the wave velocity expression for a long wave, and Scott Russell<sup>4)</sup> obtained experimentally the velocity expression  $\sqrt{g(h+\eta)}$  for solitary wave. In the above expression  $h$  and  $\eta$  are the depths of the water and the wave height respectively, and  $g$  the acceleration due to gravity.

#### 1. Wave Form and Wave Velocity.

The following two water depths were considered in the experiment.

$$h=12^{\text{cm}}, h=20^{\text{cm}}.$$

The wave generated in the tank is a solitary wave, and its form is shown in Fig. 8, in which the wave form assumed at zero second has the same form as when  $h=20^{\text{cm}}$ ,  $\eta=8^{\text{cm}}$ .

The time-distance curves for maximum wave height  $M$  and that of

2) When the tsunami approaches the coast, its wave height becomes comparable with the water depth, so that the ordinary velocity expression of long wave  $\sqrt{gh}$  becomes inapplicable.

3) See Lamb's Hydrodynamics, 6th edition, p. 260.

4) Scott Russell, *Brit. Ass. Rep.*, 1844.

wave front  $F$  are shown in the following figures. (Fig. 2, 3, 4).

In these figures, the abscissa is the time in which the unite is  $1/24$  seconds, while the ordinate corresponds to the displacement of the maximum height, front or tail of the wave, respectively. Using these curves,  $M$ , we obtained the wave velocities shown in Fig. 5.

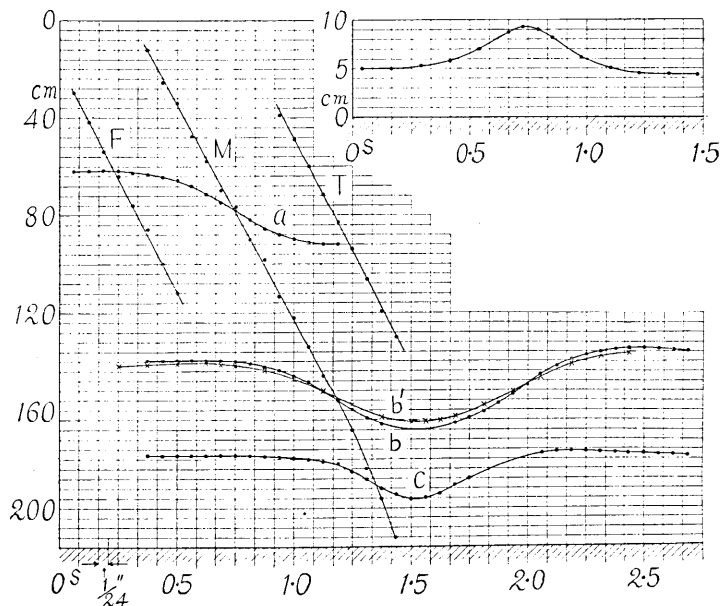


Fig. 2.  $h=20\text{cm}$ ,  $\eta=8\text{cm}$ .  $M$ : maximum wave height,  $F$ : wave front,  $T$ : rear.  $a, b, c$ : surface particles.  $b'$ : liquid particle in water.

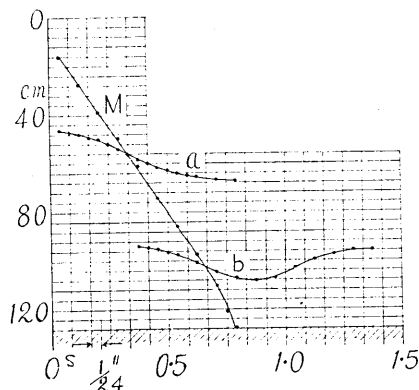


Fig. 3.  $h=12\text{cm}$ ,  $\eta=6.4\text{cm}$ .

$M$ : maximum wave height,  
 $a, b$ : surface particles.

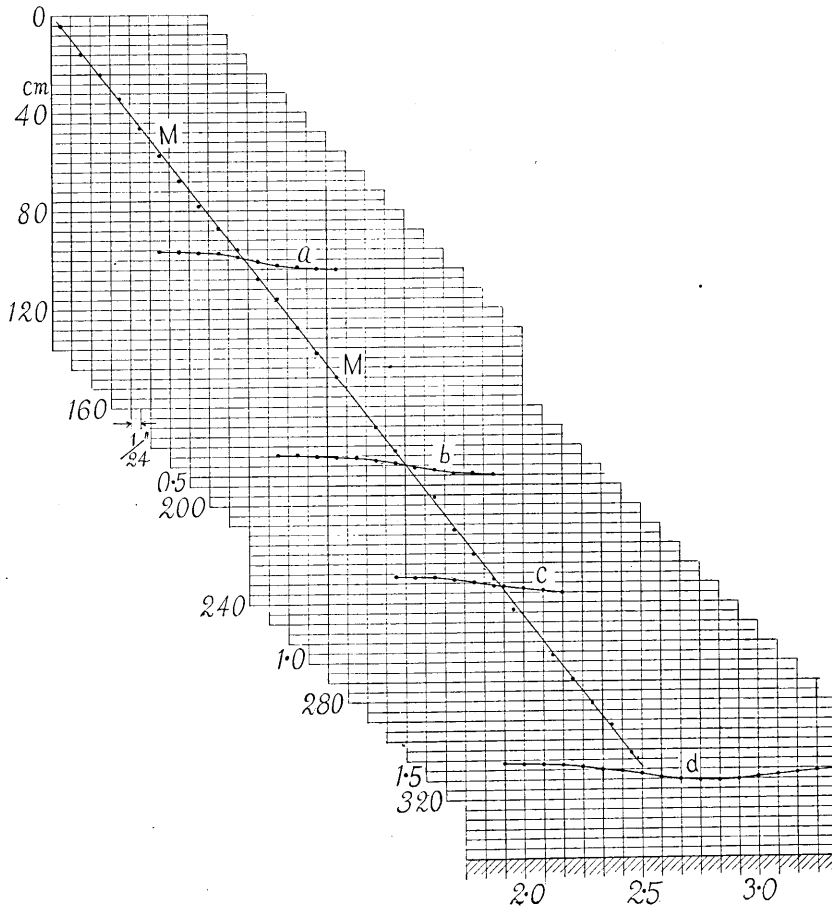


Fig. 4.  $h=12^m, \eta=2.4^m$ .  $M$ : max. wave height.  
 $a, b, c, d$ : surface particle.

In this figure the abscissa is the value of  $\eta/h$ , namely, the ratio of the maximum height of the waves to the depth of water, while the ordinate corresponds to the ratio of the wave velocity  $v$  to  $\sqrt{gh}$ , where  $g$  is the acceleration due to gravity. From this figure we see that the wave velocity increases with the wave height  $\eta$ , and we obtain the following wave velocity expression.

$$v = \sqrt{gh} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}} \dots \dots \dots (4)$$

The curve  $A$  shown in Fig. 5 is the value of  $\sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ , and this

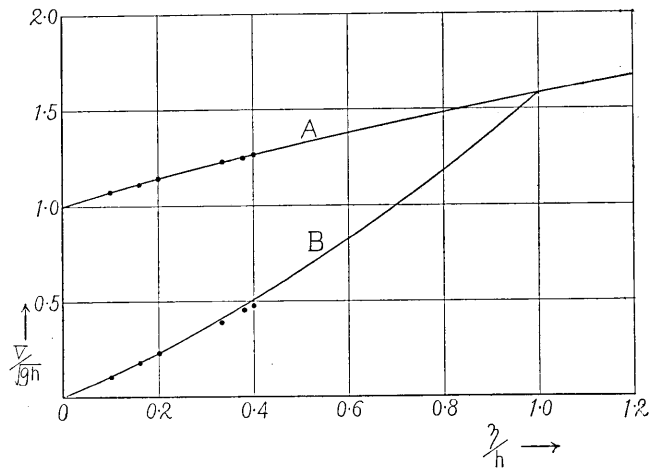


Fig. 5.  $A = \frac{v}{\sqrt{gh}} = \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ ,  $B = \frac{V}{\sqrt{gh}} = \frac{\eta}{h} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ , where

$v$  and  $V$  are the velocity of travel of maximum wave height, and the maximum horizontal velocity of surface particle respectively.

expression (4) is fairly accordant with the experimental value as shown, for example, in the following table.

Table I.

$h$	$\eta$	$\eta/h$	Experimental value	Calculated by (4)
20cm	8cm	0.40	176cm/sec	177cm/sec
20	4	0.20		
12	6.4	0.533	141.6	145.5
12	3.2	0.266	127.2	128.2
12	2.4	0.20	122.8	123.5

## 2. Motion of the Water Particle.

To see the motion of water particle in the propagated wave, a drop of a liquid<sup>5)</sup> whose density is equal to water is added to the water. This liquid particle is then suspended at a certain position in the water. A number of midget electric lamp that float on the surface of the water are connected to the city line by BS.#42 wire. When the wave

5) In the present study, we mixed machine oil and white colouring matter to the density of water. Of course the cohesion of this mixed liquid is larger than the adhesion with water.



is propagated, two particles, one of which is suspended in the water and the other floating on the surface of the water, move. These movements are photographed as shown in Fig. 6.

From this figures we can see that each of these particles,  $a$ ,  $b$  and  $b'$  far from the vertical wall supposed to be the head of the bay describes an arc, and the maximum height of the arc described by the surface particle  $b$  is equal to the wave height while that of the oil,  $b'$  suspended in water is smaller than that of the surface particle  $b$  (or  $a$ ). Since the horizontal displacements of the two particles are almost equal, we can say that the solitary wave generated in the tank is a long wave of which the apparent length is longer than the water depth.

The surface particle  $c$  near the vertical wall of the "bay" partakes of a somewhat different motion from that of particle  $b$ , that of  $c$  being caused by the wave reflected at the wall.

Needless to say, the floating lamps strictly speaking do not portray the motion of a water particle on the surface. For that purpose a drop of a mixture of oil and a colouring matter is better than the midget electric lamp. However, to simplify matters we used these lamps to study the motion of the surface particle of water throughout the course of this experiment.

Hereinafter, we shall use the words "water particle" on the surface of water and the electric midget lamp in the same sense.

Some of the time displacement curves

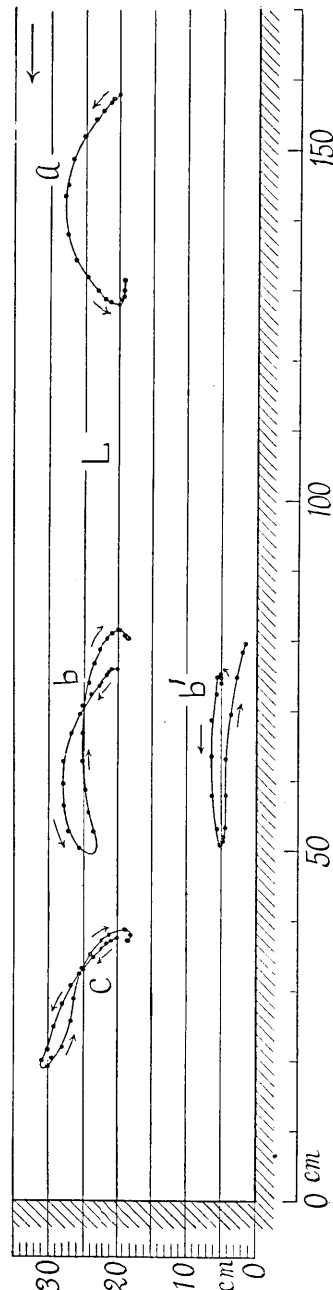


Fig. 6.  $h=20\text{cm}$ ,  $\eta=8\text{cm}$ . ( $L_1=\text{water level}$ .)

for the horizontal motion of the surface particles are shown in Fig. 2, 3, 4.

Now by the theory of infinitesimal wave motion, the horizontal total displacement  $U$  of the surface particle is expressed by

$$U = \frac{4}{\sqrt{3}} \sqrt{\eta h} \dots \dots \dots (5)$$

for the long wave of solitary type. When we take  $\frac{\eta}{h}$  as abscissa and  $\frac{U}{h}$  as ordinate, we obtain Figure 7, in which the curve  $\frac{4}{\sqrt{3}} \sqrt{\frac{\eta}{h}}$  is also shown. When  $\frac{\eta}{h}$  becomes large the expression (5) becomes inapplicable.

3. *Horizontal Maximum Velocity of the Surface Particle.*

According to the theory of hydrodynamics the maximum horizontal velocity  $V$  of a water particle when the wave motion is infinitesimal, is expressed by

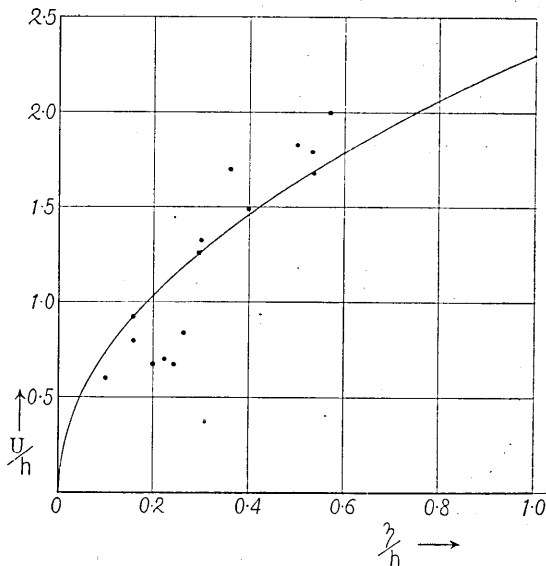


Fig. 7.

$$V = \frac{\eta}{h} \sqrt{gh} \dots \dots \dots (6)$$

When  $\frac{\eta}{h}$  becomes somewhat large, expression (6) becomes unsatisfactory, because velocity of the particle is related to the wave velocity, which is affected by the value of  $\frac{\eta}{h}$  when  $\frac{\eta}{h}$  becomes large.

T. Matuzawa<sup>6)</sup> recently obtained theoretically the following expression for the maximum horizontal velocity of a particle:

$$V = 2\sqrt{gh} \left\{ \sqrt{1 + \frac{\eta}{h}} - 1 \right\} \dots \dots \dots (7)$$

Comparing (7) with (6) we find that by Matuzawa's formula, we get a

6) T. MATUZAWA, *Disin* 5 (1933).

smaller value when  $\frac{\eta}{h}$  is large.

The time displacement curves for the surface particles in Fig. 2, 3, 4 show that the maximum horizontal velocity of a surface particle becomes large when  $\eta$  or  $\frac{\eta}{h}$  becomes large. The experimental values of horizontal velocities of the surface particles are shown in Fig. 5, in which the abscissa corresponds to the value of  $\frac{V}{\sqrt{gh}}$  and the ordinate to that of  $\frac{\eta}{h}$ .

Now the following expression of  $\frac{V}{\sqrt{gh}}$ , which is shown as *B* in Fig. 5, is fairly satisfactory for treating experimental results.<sup>7)</sup>

$$\frac{V}{\sqrt{gh}} = \frac{\eta}{h} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}} \dots\dots\dots(8)$$

If we assume the maximum velocity of the horizontal motion of the surface particle of water as being actually expressed by the expression (8), then  $v$  becomes equal to  $V$  when  $\eta=h$ ; that is, the horizontal velocity of the water particle and the maximum wave velocity both become  $1.66\sqrt{gh}$  when the wave height becomes equal to the depth of bay. Therefore if  $\eta=h$ , a solitary wave cannot be propagated in the usual way and the wave breaks. That is to say, such waves as in which the wave height is exceeds the water depth cannot be propagated in a rectangular bay of uniform depth. We might state that the present theory of wave break is based on the assumption that the particle velocity  $V$  expressed by (8) accords with the experimental results for any value of  $\frac{\eta}{h}$ .

#### 4. *Water Height and the Motion of the Surface Particle of Water near the Vertical Wall representing the Head of Bay.*

The elementary theory of infinitesimal motion of water shows that when the wave reaches a vertical wall, the water height on the vertical wall becomes twice the wave height of the propagated wave. In the present experiment also, the water height along the vertical wall is twice the wave height of the long wave propagated in the bay in the case of a moderate value for  $\frac{\eta}{h}$ , whereas when  $\frac{\eta}{h}$  becomes large, the water height

7) The horizontal velocity of the particle near the vertical wall representing the head of the bay becomes small, owing probably to the effect of the wave reflected at the vertical wall. For this reason they were not used in obtaining the experimental results plotted in Fig. 5.

on the wall becomes more than twice the wave height of the propagated wave. This is shown in Table II in which the magnitudes of the water height  $\eta$  at the wall and also the ratio of the water height at the wall to wave height  $\eta_0$  of the propagated wave in the bay are given.

The propagation of the wave form in the bay and the variation of the wave form at the vertical wall and also the generation of the reflected wave at the vertical wall, representing the head of the bay are shown in detail in Fig. 9 for the case of  $h=20\text{cm}$ ,  $\eta=8\text{cm}$ .

When the wave is propagated near the vertical wall and approaches the vertical wall which represents the head of the bay, the wave height becomes more or less larger than the wave height of that wave far from the vertical wall as the result of the reflection of the wave by the wall, so that the wave velocity for the travel of the maximum height becomes larger than when the wave is propagated from a region remote from the vertical wall. This fact can be seen from Fig. 2. (curve *M*.)

The fact that the water particle on the vertical wall has neither horizontal displacement nor horizontal velocity is shown by the

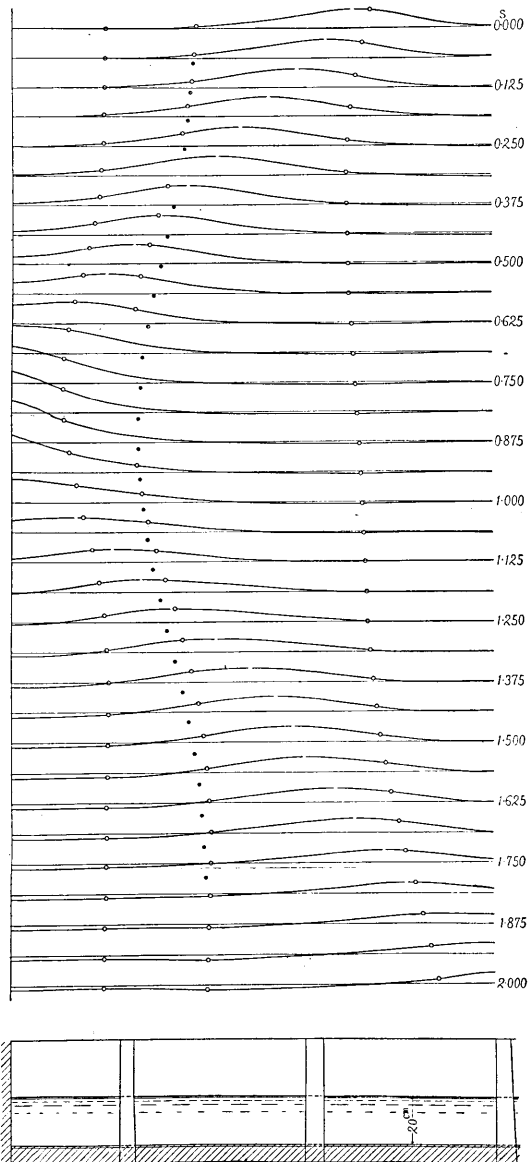


Fig. 8.  $h=20\text{cm}$ ,  $\eta=8\text{cm}$ .

Table II.

$h$	$\eta_0$	$\eta$	$\frac{\eta_0}{h}$	$\frac{\eta}{\eta_0}$
12 cm	2.4 cm	4.8 cm	0.2	2.00
12	3.2	6.4	0.267	2.00
12	6.2	13.2	0.517	2.13

elementary theory of infinitesimal wave motion. From the present experiment, this fact is shown in Fig. 2, 3, 4, in which the total horizontal displacement and the horizontal maximum velocity of the surface particle near the vertical wall becomes smaller than those remote from the vertical wall.

In Fig. 2, the total displacement of the surface particle  $b$  near the vertical wall is smaller than that of the surface particle  $a$  far from the wall, while at the same time the maximum velocity of particle  $b$  is smaller that of particle  $a$ .

## II. A Triangular Bay of Uniform Depth and with a Vertical Wall.

The dimensions of the bay and the propagated wave are given in the following table. In this table,  $x_0$  is the length of the equilateral triangular bay from the head to the mouth,  $b_0$  the width of the mouth at  $x=x_0$ ,  $h$  the water depth uniformly distributed in the bay, and  $\eta_0$  is the wave height at the mouth of the bay.

Table III.

$x_0$	$b_0$	$h$	$\eta_0$	$2\theta$
356.8 m		12 cm	3.6 cm	27°
"	"	"	5.6	"
234.4 m	"	"	2.8	46°
"	"	"	3.6	"
"	"	"	7.2	"
"	"	"	6.8	"

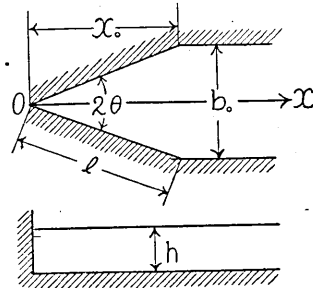


Fig. 9.

The following Figures 10, 11, 12, 13, 14, 15 give the experimental results, such as the time displacement curve for the maximum height of the propagated long wave  $M$ , and that of the front of this wave  $F$ , and also those of the surface particle. These figures also give the heights of the waves  $H$  in the bay along the center line  $x$ , as shown in Fig. 9.

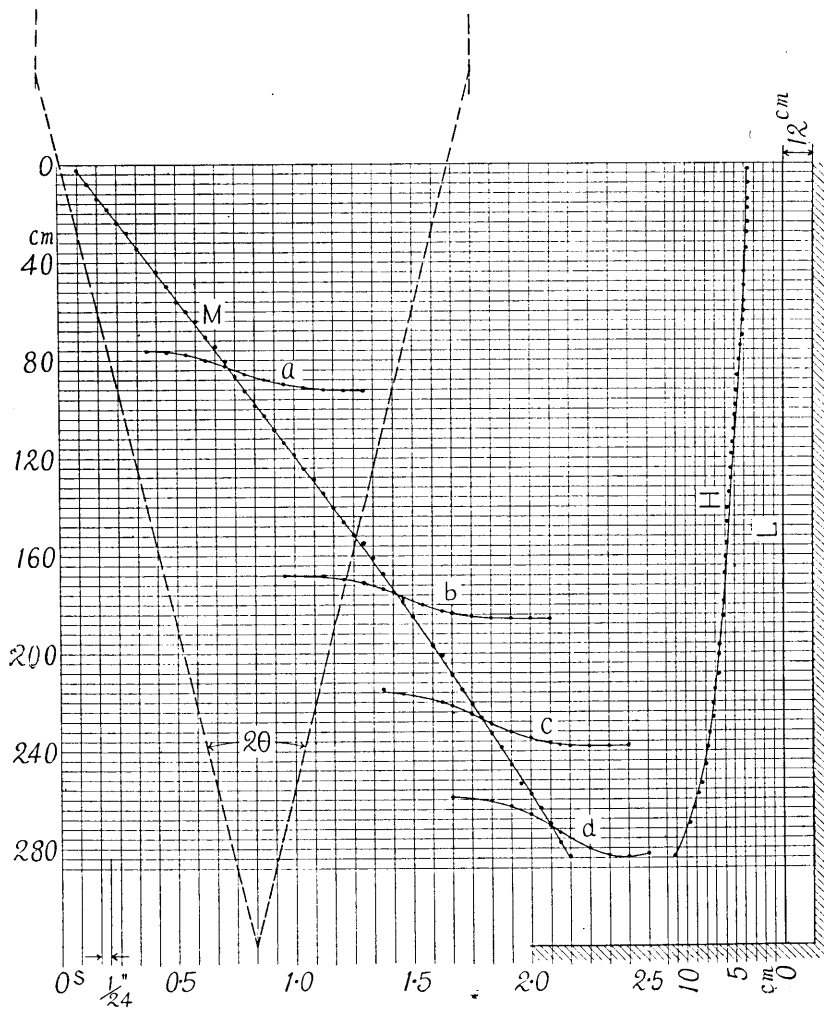


Fig. 10.  $h = 12\text{cm}$ ,  $2\theta = 27^\circ$ ,  $x_0 = 356.8\text{cm}$ ,  $\eta_0 = 3.6\text{cm}$ .

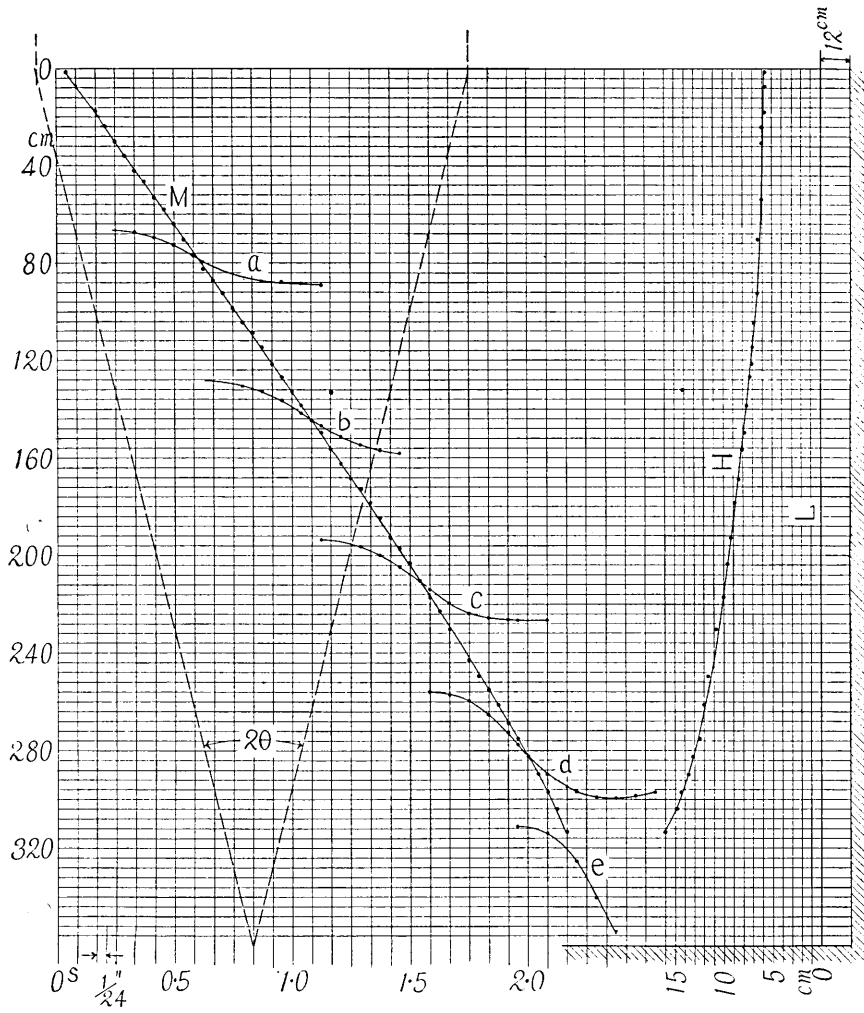


Fig. 11.  $2\theta = 27^\circ$ , depth = 12cm,  $x_0 = 356.8\text{cm}$ ,  $\gamma_0 = 5.6\text{cm}$ .





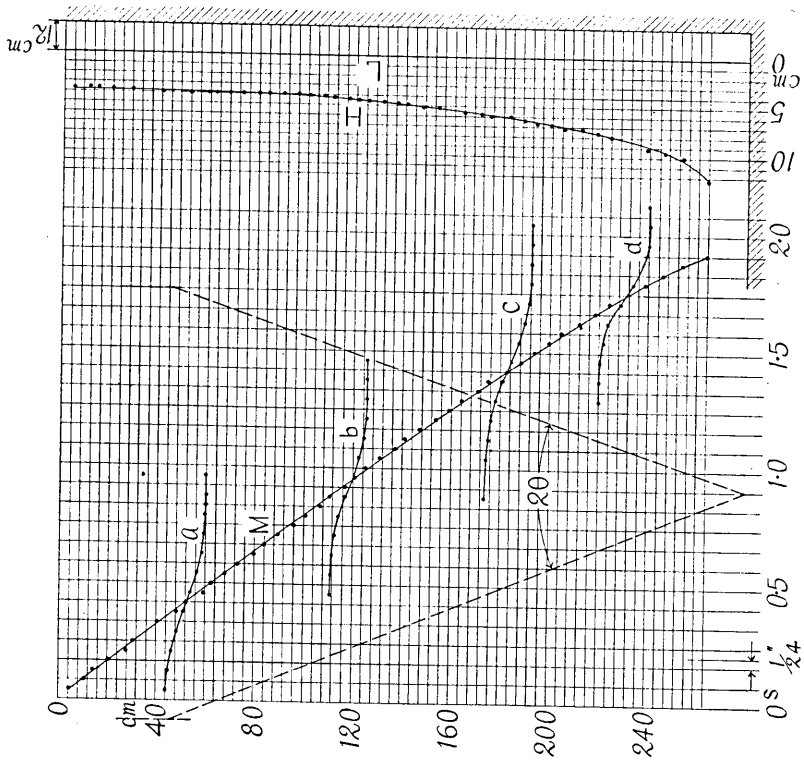


Fig. 13.  $2\theta = 40^\circ$ , depth = 12cm,  $x_0 = 234.4\text{cm}$ ,  $\eta_0 = 3.6\text{cm}$ .

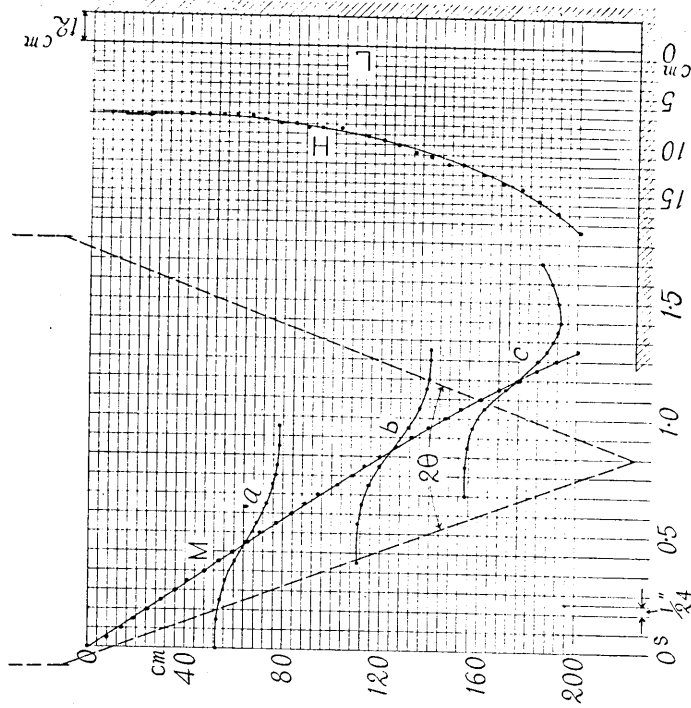


Fig. 14.  $2\theta = 40^\circ$ ,  $\eta_0 = 7.2\text{cm}$ , depth = 12cm,  $x_0 = 234.4\text{cm}$ .

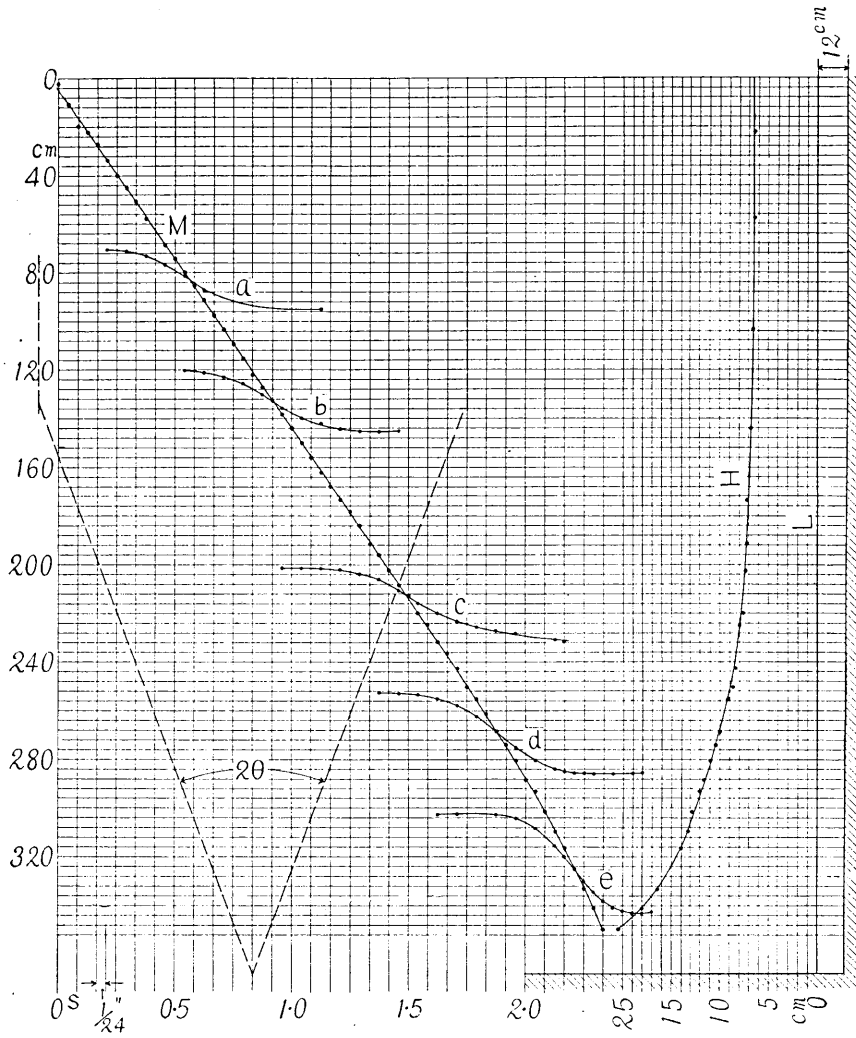


Fig. 15.  $2\theta = 40^\circ$ ,  $\eta_0 = 6.8\text{cm}$ , depth =  $12\text{cm}$ ,  $x_0 = 23.4\text{cm}$ .

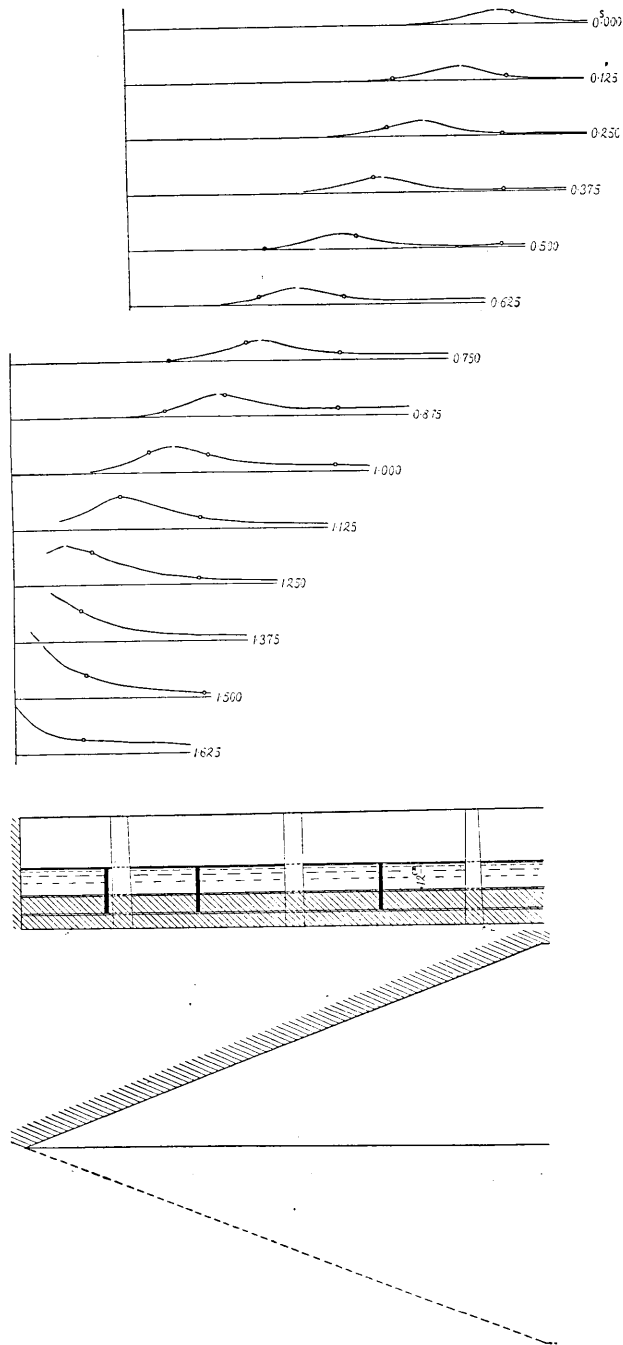


Fig. 16. water depth = 12<sup>cm</sup>  $\eta_0 = 6.4^{\text{cm}}$ ,  $2\theta = 42^\circ$ ,  $x_0 = 234.4^{\text{cm}}$

Using these figures we shall study the wave height, wave velocity and the velocity of surface particle as follows.

1. *Wave Form and Wave Height.*

From the standpoint of the continuity of flow of water in wave phenomenon, we are able to conceive that the wave height in the bay becomes greater when a wave is propagated in a triangular bay, and that the slope of the wave becomes steeper in the bay. Fig. 16 and Fig. 17 show the variation in the wave form in a bay when solitary wave, whose initial wave height  $\eta_0$  is 6.4 cm, is propagated in a bay of this form.

It was shown in the preceding section that in a rectangular bay of uniform depth, the wave form breaks when its height exceeds the water depth. While in a triangular bay of uniform depth, the wave form varies as the wave is propagated, and the wave form does not break even if the wave height becomes greater than the depth. From the wave height in the bay shown in Figs. 10, 11, 12, 13, 14, 15 we obtain the following Figs. 18, 19 in which the abscissa corresponds to  $\frac{x}{x_0}$  and the ordinate shows the ratio of the wave height  $\eta$  at  $x=x$  to that at the mouth of the bay  $\eta_0$ , the angle  $2\theta$  being respectively equal to  $27^\circ, 40^\circ$ .

The theory of infinitesimal wave motion of water shows the following relation of the wave height in a triangular bay of uniform depth:

$$\frac{\eta}{\eta_0} = \sqrt{\frac{x_0}{x}} \dots\dots\dots (9)^8)$$

This expression is also shown in Fig. 18 and Fig. 19. It will be seen that the experimental results are approximately accordant with the theoretical result, whence we conclude that the expression (9) is applicable even if the angle  $2\theta$  should become as large as  $40^\circ$ .

2. *Wave Velocity.*

Since, as already explained, when a wave is propagated in a triangular bay of uniform depth, the wave height gradually approximates the expression (9), we can conceive that the wave velocity of the travel of maximum wave height becomes gradually large in this bay.

From the time displacement curve for the maximum height of the propagated wave, we obtain the velocity  $v$  of propagation of the maximum height of the wave.

---

8) We assume, of course, that  $2\theta$  theoretically is small in order to enable equation (9) to be reduced. Recently Arakawa obtained a generalized treatment of Green's theorem on long wave. We shall deal with Arakawa's theorem on another occasion.

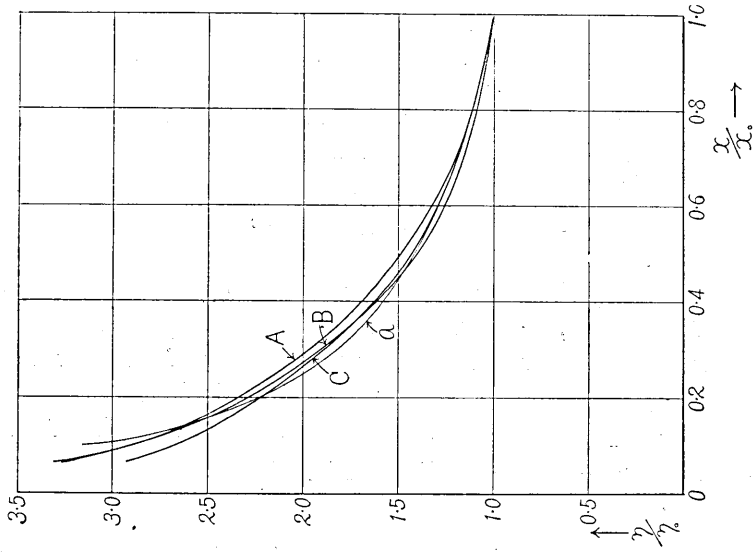


Fig. 18.  $\alpha = \sqrt{\frac{x_0}{x}}$ .  
 A:  $\frac{\gamma_0}{h} = 0.30$ , B:  $\frac{\gamma_0}{h} = 0.5233$ ,  
 C:  $\frac{\gamma_0}{h} = 0.242$ .  $2\theta = 27^\circ$ .

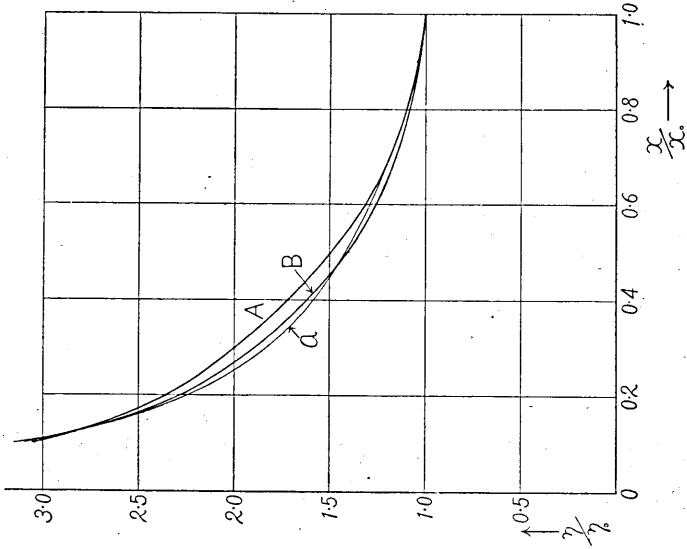


Fig. 19.  $\alpha = \sqrt{\frac{x_0}{x}}$ .  
 A:  $\frac{\gamma_0}{h} = 0.5$ ,  
 B:  $\frac{\gamma_0}{h} = 0.3$ .  $2\theta = 40^\circ$ .

The velocity  $v$  thus obtained are fairly accordant with the expression (4) obtained in Section 2, namely,

$$v = \sqrt{gh} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$$

is applicable even if the value of  $\frac{\eta}{h}$  exceeds 1.6.

If we assume that the wave height in the triangular bay is fairly expressed by (9), wave velocity for the travel of maximum height is expressed by

$$\left. \begin{aligned} v &= \sqrt{gh} \sqrt{\left\{1 + \frac{3}{2} \frac{\eta_0}{h} \sqrt{\frac{x_0}{x}}\right\}}, \\ \frac{v}{v_0} &= \frac{\sqrt{\left\{1 + \frac{3}{2} \frac{\eta_0}{h} \sqrt{\frac{x_0}{x}}\right\}}}{\sqrt{1 + \frac{3}{2} \frac{\eta_0}{h}}}, \end{aligned} \right\} \dots\dots\dots(10)$$

where  $v_0$  is the velocity at the mouth of the bay. The curves  $a, c, e, f$  and  $g$  in Fig. 20 are calculated from the expression (10). The parameter of the curves in the figure is the value of  $\frac{\eta_0}{h}$ . The curves  $E$  and  $C$  in this figure are the experimental results for the cases of  $\frac{\eta_0}{h} = 0.3, 0.5$  respectively. We see that the expression (10) is fairly accordant with the experimental results.

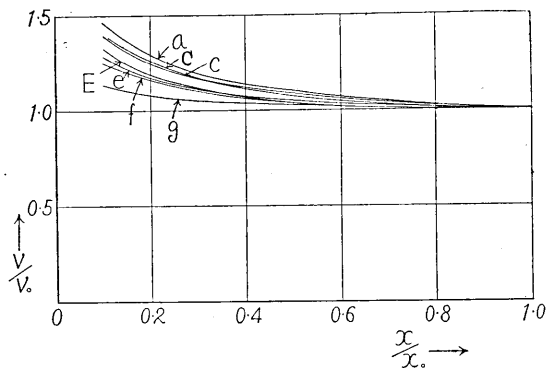


Fig. 20.  $C: \frac{\eta_0}{h} = 0.5, a: \frac{\eta_0}{h} = 0.8, f: \frac{\eta_0}{h} = 0.242,$   
 $E: \frac{\eta_0}{h} = 0.3, c: \frac{\eta_0}{h} = 0.5, g: \frac{\eta_0}{h} = 0.10, e: \frac{\eta_0}{h} = 0.3.$

3. *The Motion of the Surface Particle.*

From Figs. 10, 11, 12, 13, 14, 15, in which the time displacement curves for the surface particles are shown for the respective wave heights at the mouth, we find that the total horizontal displacements and the maximum horizontal velocities of the surface particles near the head

of the triangular bay are larger than those of the surface particles remote from the head of the bay.

For example, the total displacements in the bay are shown in Fig. 21, where the abscissa corresponds to the value of  $\frac{x}{x_0}$ , the ordinate being the ratio of the total displacement  $U$  of the surface particle in the bay to that  $U_0$  at mouth. The parameter of these curves corresponds to the value of  $\frac{\eta_0}{h}$ .

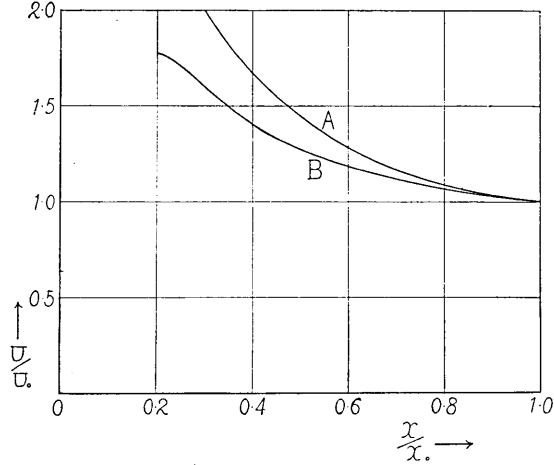


Fig. 21. A:  $\frac{\eta_0}{h}=0.5$ , B:  $\frac{\eta_0}{h}=0.3$ ;  $2\theta=27^\circ$ ,  
 $h=12\text{cm}$ ,  $x_0=356.8\text{cm}$ .

Now if we adopt the expression (8) as the maximum horizontal velocity of the surface particle in the triangular bay, we obtain the following expression for the maximum horizontal velocity of surface particle in this bay:

$$V = \sqrt{gh} \left( \frac{\eta_0}{h} \right)^{\frac{3}{2}} \sqrt{\frac{x_0}{x}} \sqrt{\left\{ \frac{h}{\eta_0} + \frac{3}{2} \sqrt{\frac{x_0}{x}} \right\}}, \dots\dots\dots (11)$$

and assuming the maximum horizontal velocity of the surface particle at the mouth of the bay to be  $V_0$ , we obtain

$$\frac{V}{V_0} = \frac{\sqrt{\frac{x_0}{x}} \sqrt{\left\{ \frac{h}{\eta_0} + \frac{3}{2} \sqrt{\frac{x_0}{x}} \right\}}}{\sqrt{\left\{ \frac{h}{\eta_0} + \frac{3}{2} \right\}}}, \dots\dots\dots (12)$$

which is also shown in Fig. 22. The value of the parameter of all the curves in this figure is  $\frac{\eta_0}{h}$ . From these curves we see that the horizontal velocity of the surface particle in the bay increases with the value of  $\frac{\eta_0}{h}$ . The horizontal velocity of the surface particle near the head of a bay is obviously larger than that of the surface particle remote from the head of that bay. On plotting the experimental results, we can see that they are not accordant with the curve expressed by (12).

This fact may seem strange, but it is understandable if we remember that the velocity expression (12) is derived from the velocity expression (8) which is applicable to a rectangular bay of uniform depth, and also that the maximum horizontal velocity of the surface particle greatly differs from that of a rectangular bay of uniform depth and that velocity in triangular bay is much affected by the presence of vertical side walls.

Figs. 14, 15, 11, in which the wave heights in the bay are given, show that even though the wave height may exceed the water depth of the

bay, the wave form of the propagated wave does not break, and that the maximum horizontal velocity of the surface particle is smaller than the velocity of travel of maximum height of the propagated wave in the bay, even were the wave height  $\eta$  to exceed the depth of the water, and as well as when eventually the value of  $\frac{\eta}{h}$  becomes about 1.7. The two velocities above discussed become equal when  $\frac{\eta}{h} \doteq 1.7$ . This fact shows that the maximum horizontal velocity of the surface particle in a triangular bay of uniform depth is smaller than that of the surface particle in a rectangular bay of uniform depth.

Now, plotting the value of  $\frac{V}{\sqrt{gh}}$  as ordinate and that of  $\frac{\eta}{h}$  as abscissa, we obtain the curves C, D, E and F in Fig. 23. The values of parameter  $\frac{\eta_0}{h_0}$  of these curves are 0.528, 0.50, 0.30 and 0.242 respectively. Curve B is the velocity curve corresponding to the surface particle in

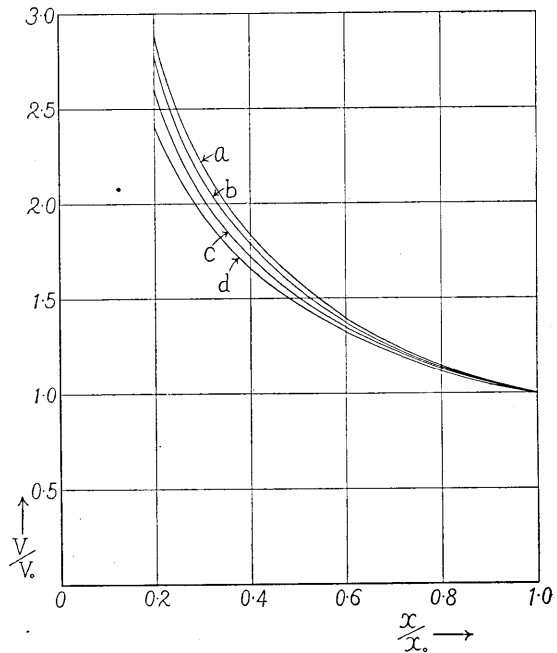


Fig. 22.  $a: \frac{\eta_0}{h} = 0.80, c: \frac{\eta_0}{h} = 0.30,$   
 $b: \frac{\eta_0}{h} = 0.528, d: \frac{\eta_0}{h} = 0.10.$



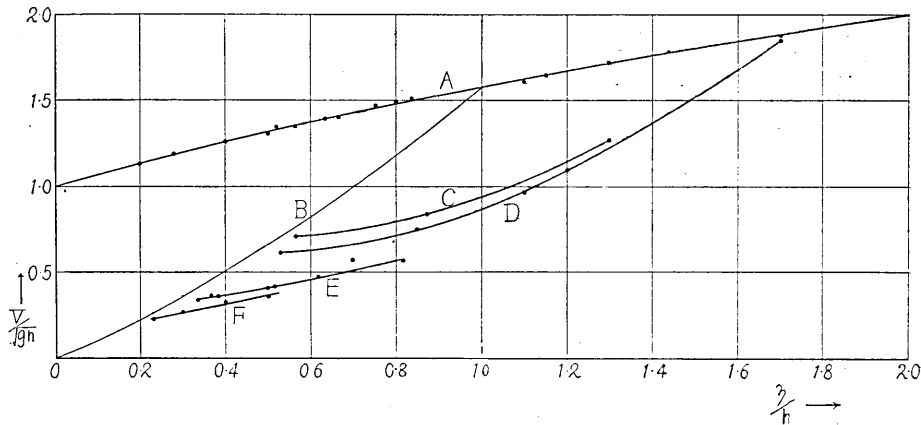


Fig. 23.  $A = \frac{v}{\sqrt{gh}} = \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ ,  $B = \frac{V}{\sqrt{gh}} = \frac{\eta}{h} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$   
 $C: \frac{\eta_0}{h_0} = 0.523$ ,  $D: \frac{\eta_0}{h_0} = 0.50$ ,  $E: \frac{\eta_0}{h_0} = 0.30$ ,  $F: \frac{\eta_0}{h_0} = 0.242$ .

a rectangular bay of uniform depth, while curve *A* is the expression for velocity of travel of maximum wave height in the bay. From this figure we see that the maximum horizontal velocity of the surface particle at the mouth of the bay is fairly expressed by curve *B*, namely by the expression (8). All the curves *C*, *D*, *E* and *F* show that the maximum horizontal velocity of the surface particle increases in the bay, but that the rate of this increase in the bay is smaller than that of curve *B*. Curves *A* and *D* show that when  $\frac{\eta}{h} = 1.7$ , the velocity of travel of maximum wave height in the bay becomes equal to the maximum horizontal velocity of the surface particle in the triangular bay, namely  $v$  and  $V$  both become  $1.8\sqrt{gh}$ . It is in this way that the form of the propagated wave in the bay breaks. As we have discussed in Section 3, however, when  $v$  and  $V$  both become  $1.6\sqrt{gh}$  in the rectangular bay, the form of the propagated wave breaks. We can see from this that the wave velocity in a triangular bay of uniform depth can become  $0.2\sqrt{gh}$  higher than the wave velocity in a rectangular bay of uniform depth.

When the breadth at the mouth  $b_0$  and the depth  $h$  are constant, the wave height in a triangular bay of uniform depth is expressed by

$$\frac{\eta_1}{\eta_2} = \sqrt{\frac{\tan \theta_2}{\tan \theta_1}}, \dots\dots\dots(13)$$

which shows that as the length of the bay  $l$  becomes longer, the wave height in the bay becomes greater.

When the depth  $h$  and the length of the two side walls  $l$  are constant,

$$\frac{\eta_1}{\eta_2} = \sqrt{\frac{\cos \theta_1}{\cos \theta_2}}, \dots\dots\dots (14)$$

showing also that when the angle  $\theta$  is smaller, the wave height in the bay becomes greater. The above expressions (13), (14) are obtained of course from the theory of infinitesimal wave motion of the water, and  $\eta_1$  is the wave height in a bay, the angle of whose sides is  $\theta_1$ , and  $\eta_2$  corresponds to that when the angle is  $\theta_2$ .

The following two Figures show the maximum height of the water at the head of the triangular bay, the angle of whose sides varies from  $7^\circ$  to  $72^\circ$  and the length of the side wall is constant.

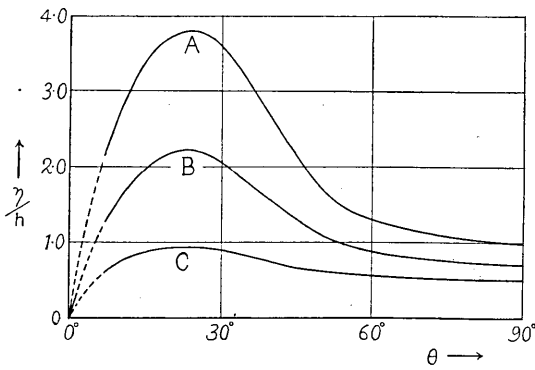


Fig. 24. A:  $\frac{\eta_0}{h} = 0.459$ , B:  $\frac{\eta_0}{h} = 0.383$ ,  
C:  $\frac{\eta_0}{h} = 0.282$ .

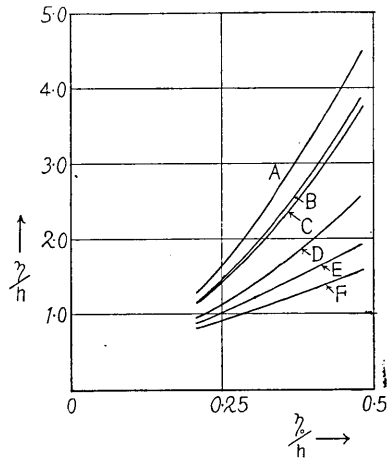


Fig. 25. A:  $\theta = 25^\circ$ , B:  $\theta = 13^\circ$ ,  
C:  $\theta = 36^\circ$ , D:  $\theta = 7^\circ$ ,  
E:  $\theta = 53^\circ$ , F:  $\theta = 72^\circ$ .

These figures show that there is an angle of a triangular bay of uniform depth which should give the maximum height of the water at the head of the bay. That angle in this case is about  $45^\circ$ .

In studying the height of the wave in a triangular bay we shall also study the maximum height of the water at the head of a trapezoidal bay of uniform depth.

When the breadth at the mouth of trapezoidal bay of uniform depth is constant, the maximum water height at the head of that bay is given in the following two figures. To obtain these figures, the angle between the two sides of the vertical wall is constant and equal to  $40^\circ$ .

From these figures we can see that the greater the length of the bay from the head to the mouth, the greater the water height at the head of the bay.

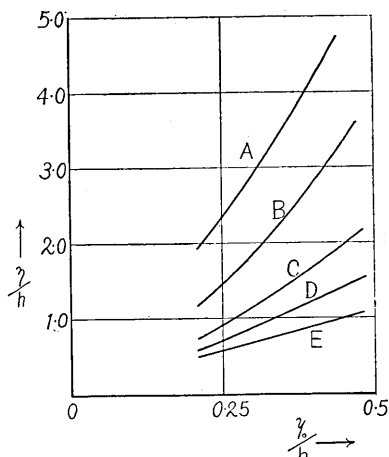


Fig. 26. A:  $\frac{x}{x_0} = 0$ , D:  $\frac{x}{x_0} = 0.621$ ,  
 B:  $\frac{x}{x_0} = 0.154$ , E:  $\frac{x}{x_0} = 0.804$ ,  
 C:  $\frac{x}{x_0} = 0.382$ .

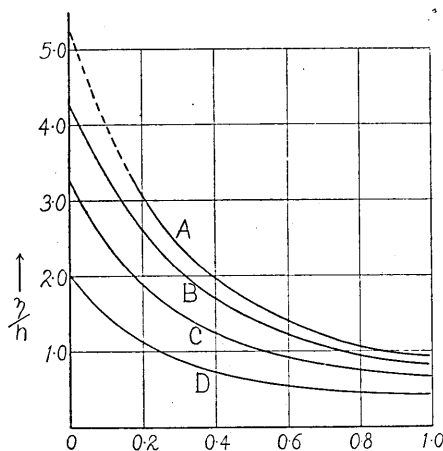


Fig. 27 A:  $\frac{\eta_0}{h} = 0.458$ , C:  $\frac{\eta_0}{h} = 0.333$ ,  
 B:  $\frac{\eta_0}{h} = 0.418$ , D:  $\frac{\eta_0}{h} = 0.208$ .

### III. A Bay Whose Depth Varies Uniformly as the Distance from the Head to the Mouth, the Breadth being Uniform, the Bay Having a Vertical Wall At its Head.

We shall study the wave motion in a bay in which the water depth increases linearly from zero to  $h_0$  as the distance increases from the vertical wall  $x=0$  supposed to be the head of the present bay to the mouth  $x_0$ . See Fig. 28.

The following Table shows the dimensions of the bays used in the present experiment.

Table IV.

$h_0$	$\theta$
20 cm	7.5°
12	5.0°
20	3.0°
12	2.0°

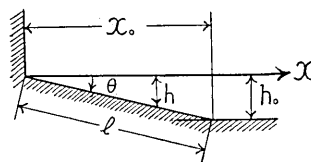


Fig. 28.

The time displacement curves for the maximum heights, those for the front of the propagated wave, and the variation of wave height in the bay, and also the time displacement curves for the surface particles of the water in the bay, are shown in Figs. 29, 30, 31, 32.

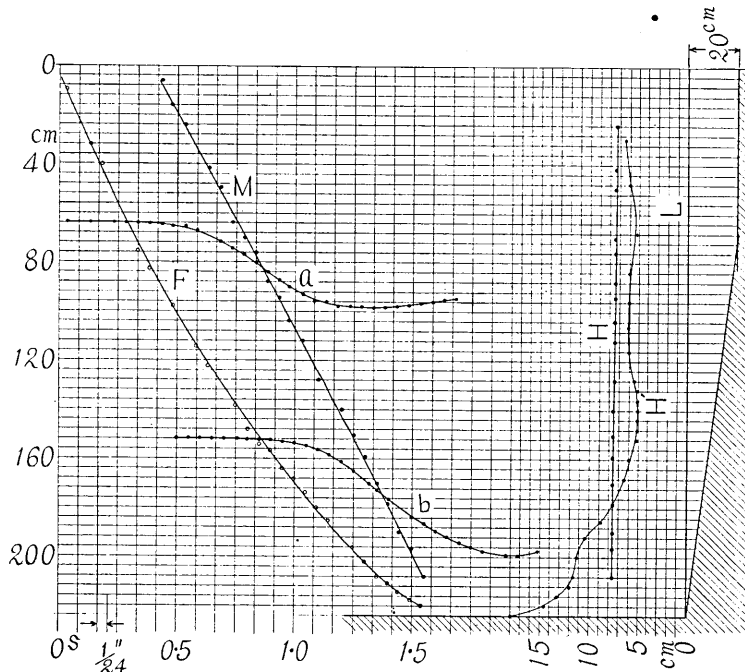


Fig. 29.  $h_0=20\text{cm}$ ,  $\theta=7.5^\circ$ ,  $x_0=152.8\text{cm}$ ,  $\eta_0=7.2\text{cm}$ .

1. *Wave Height and Wave Form in the Bay.*

We have already discussed in Section I the case in which angle  $\theta$  is equal to  $90^\circ$ . When angle  $\theta$  becomes smaller than  $90^\circ$ , the wave in the bay breaks and curls over and the form of the wave form collapses when the height  $\eta$  and the depth  $h$  bears a certain relation to each other.

(1)  $\theta=7.5^\circ$ ,  $h_0=20\text{cm}$ .

In the foregoing case, it is not possible to witness such phenomenon as the breaking or curling of the wave form of the propagated wave in a bay even though the magnitude of  $\eta_0$  becomes as large as  $8\text{cm}$ . Of course when  $\eta_0$  is smaller than  $8\text{cm}$ , there is no phenomenon of the curling of the wave. Fig. 33 shows the variation in the forms

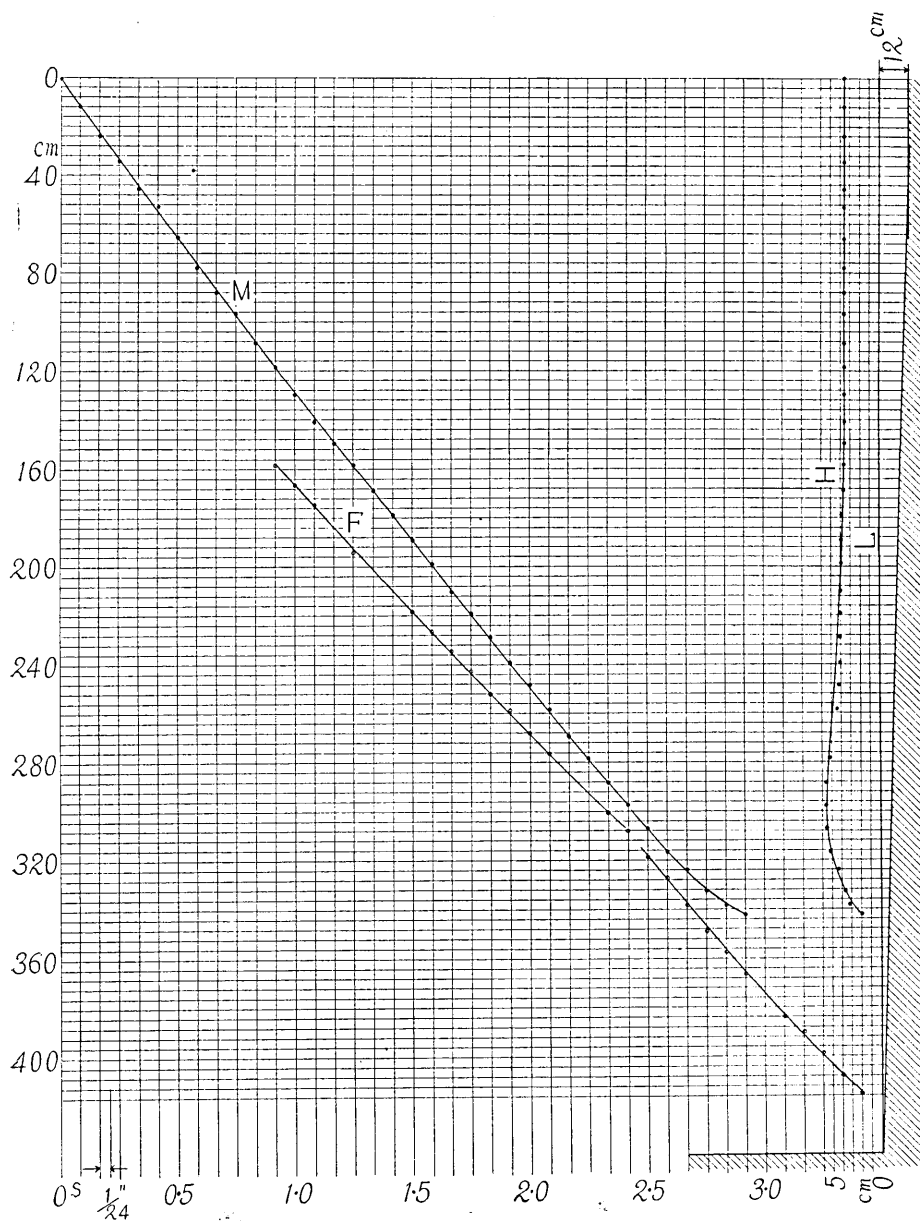


Fig. 30.  $\theta=2^\circ$ ,  $x_0=405.6\text{cm}$ ,  
 $h_0=12\text{cm}$ ,  $\eta_0=3.6\text{cm}$ .

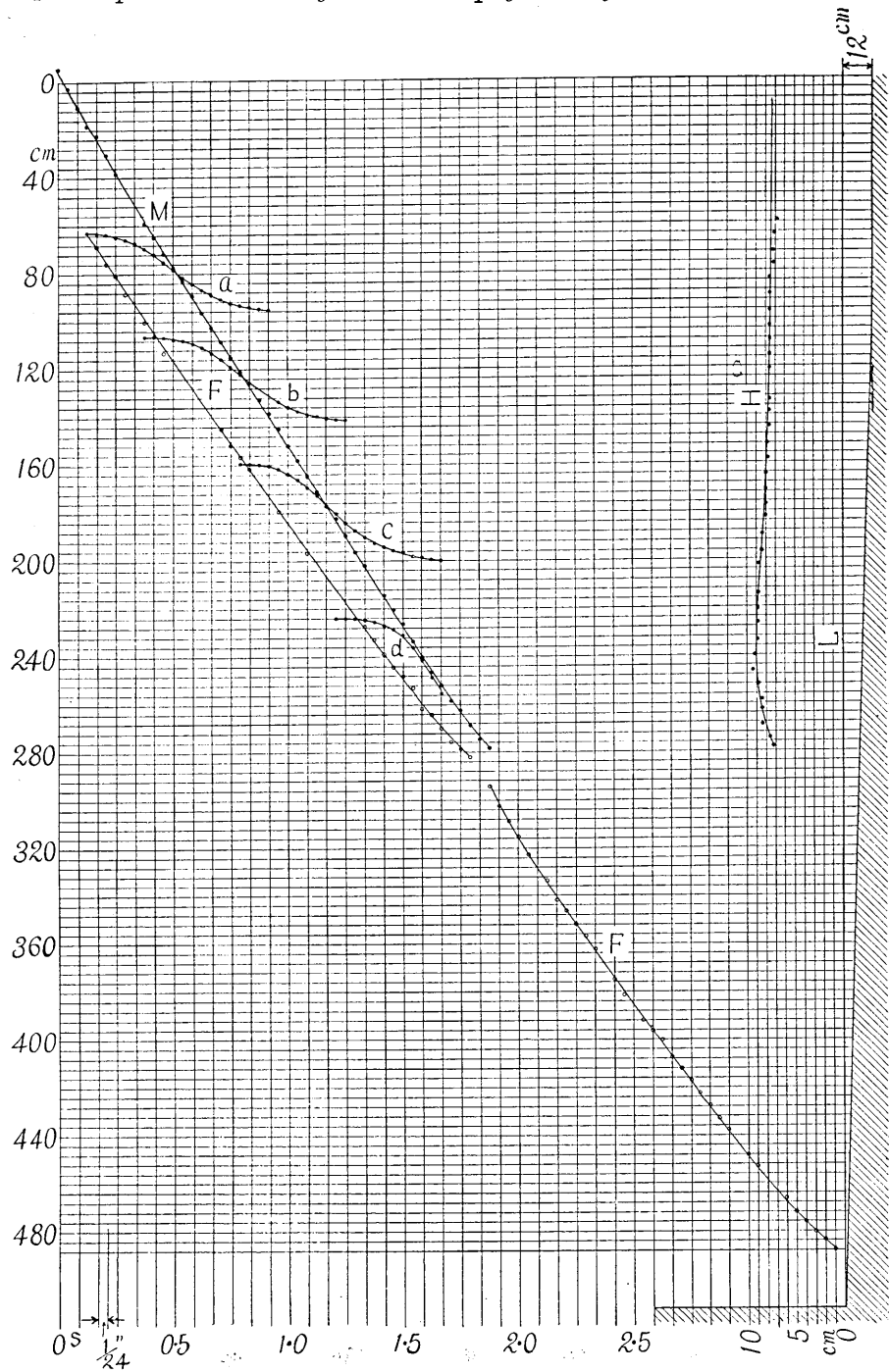


Fig. 31.  $\theta = 2^\circ$ ,  $x_0 = 405.6\text{m}$ ,  
 $h_0 = 12\text{cm}$ ,  $\eta_0 = 7.6\text{cm}$ .

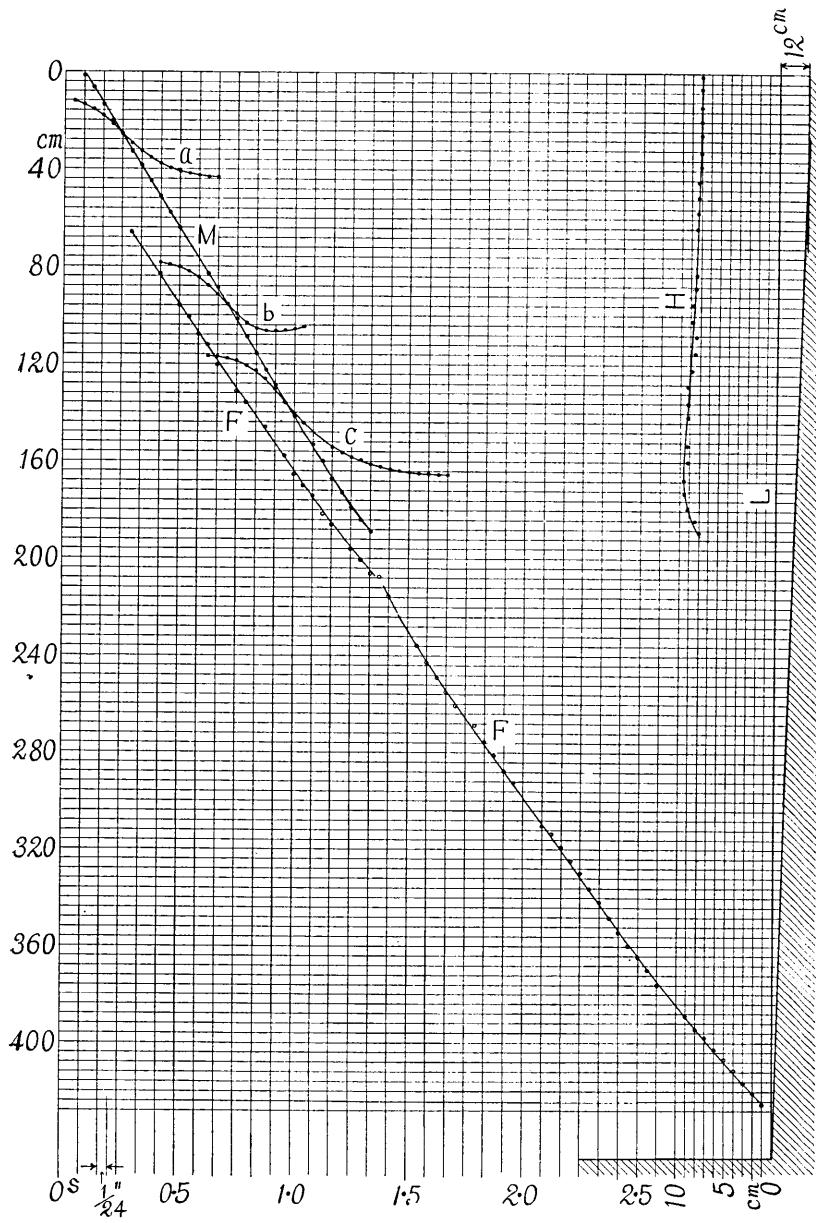


Fig. 32.  $\theta = 2^\circ$ ,  $x_0 = 405.6 \text{ cm}$ ,  
 $h_0 = 12 \text{ cm}$ ,  $\eta_0 = 8 \text{ cm}$ .

of wave propagated in a bay whose angle  $\theta$  is  $7.5^\circ$ . The magnitudes of  $\eta_0$  is  $7.2^{\text{cm}}$ .

As the wave is propagated in the bay, the velocity of the propagation of the wave front becomes small, while the velocity of travel of the maximum wave height becomes a little larger than that at the mouth of the bay. Consequently the slope of the front part of the wave form becomes steep, while that of the rear part becomes gently inclining. Finally, even if the ratio of  $\eta$  to  $h$  becomes 4 as shown in Fig. 33, the wave does not break in reaching the vertical wall. The height of the propagated wave is almost constant in the bay and shows a little increase when the wave approaches the vertical wall representing the head as shown in this figure and in Fig. 29. When the wave reaches the vertical wall, the water height on that wall becomes two or three times the initial wave height  $\eta_0$ , and the wave reflected at that wall begins to be propagated. The wave form varies as

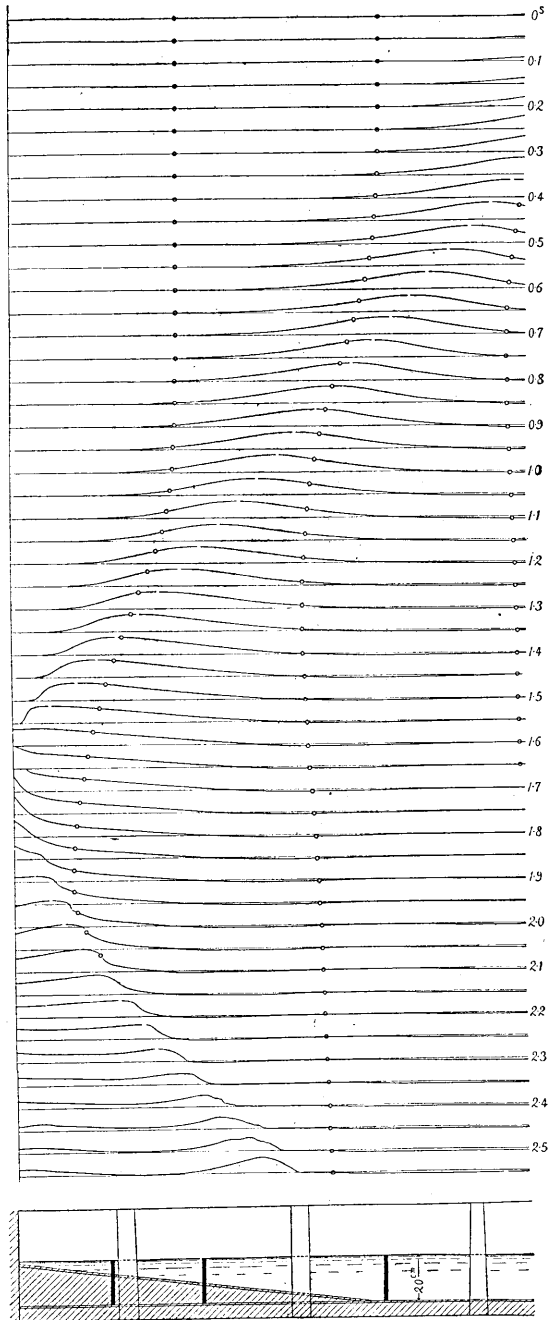


Fig. 33.  $\theta = 7.5^\circ$ ,  $x_0 = 152.8^{\text{cm}}$ ,  $\eta_0 = 7.2^{\text{cm}}$ ,  $h_0 = 20^{\text{cm}}$ .



this wave recedes from the head as shown in the Figure. The wave height  $H'$  of the reflected wave are also shown in Fig. 29.  $H'$ , which is higher than  $H$  near the head, gradually becomes smaller than  $H$  remote from the head.

Fig. 34 shows the variation of the wave height in the bay. From this Figure we can see that the larger the value of  $\frac{\eta_0}{h_0}$ , the larger becomes the value of  $\frac{\eta}{\eta_0}$ .

Studying curve,  $F$ , which shows the time displacement curve of the wave front in Fig. 29, we find that when the water depth becomes shallow in the bay, the velocity of the wave front becomes small. Curve  $M$ , which corresponds to the time displacement curve of the maximum height of the propagated wave, shows that when the depth becomes shallow in the bay, the velocity of the maximum height becomes large. The velocity of the maximum height of the wave is accordant with the expression (4).

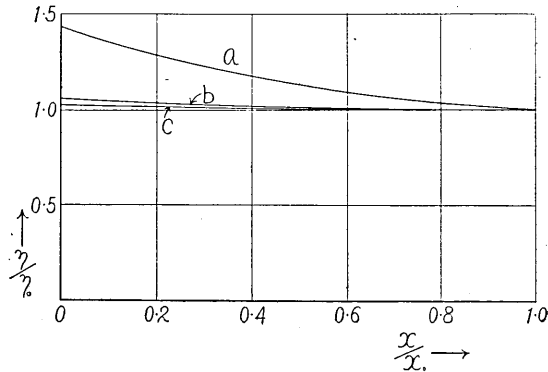


Fig. 34.  $a: \frac{\eta_0}{h_0}=0.16$ ,  $b: \frac{\eta_0}{h_0}=0.35$ ,  $c: \frac{\eta_0}{h_0}=0.37$ .

$$(2) \theta=3^\circ, h_0=20\text{cm}.$$

In the foregoing case, the wave in the bay breaks and the crest curls when the wave height and the depth become approximately equal. The variation of the wave form in the bay is shown in Fig. 35. The wave velocity  $v$  in this case decreases when the wave is propagated in the bay. Both the maximum horizontal velocity of the surface particle and the total displacement of that particle in the bay also increase in this case.

In the following Fig. 36<sup>9)</sup> the values of  $\frac{\eta}{\eta_0}$  corresponding to  $\frac{x}{x_0}$  are shown in the respective cases of  $\frac{\eta_0}{h_0}=0.16$ , 0.36, 0.115 and 0.10. We

9) Fig. 37 shows the values of  $\eta/\eta_0$  when  $\theta=5^\circ$ ,  $h_0=12\text{cm}$ .

can see that, before the wave breaks and curls over, the value of  $\frac{\eta}{\eta_0}$  in the bay becomes maximum when  $\frac{\eta}{h}$  becomes equal to a certain value. Before the curling of the wave, the value of  $\frac{\eta}{\eta_0}$  suddenly increases, and the space derivative of  $\frac{\eta}{\eta_0}$  changes its sign when the wave breaks.

(3)  $\theta=2^\circ$ ,  $h_0=12\text{cm}$ .

Figs. 30, 31 and 32 show the time displacement curves of the maximum height, that of the wave front, and also that of the surface particle. In these figures are represented also the variations of the maximum height of the propagated wave in the bay.

a) Wave Height.

Using the wave height in the bay shown in Figs. 30, 31 and 32, we obtain Fig. 38 in which the ordinate shows the

values of  $\frac{\eta}{\eta_0}$  corresponding to the respective values of  $\frac{\eta_0}{h_0}$ . As discussed in (2), the value of  $\frac{\eta_0}{h_0}$  is smaller, the larger the value of  $\frac{\eta}{\eta_0}$ .

When the value of  $\frac{\eta}{\eta_0}$  becomes maximum and begins to decrease, the wave breaks and curls over. Therefore a sudden increase in  $\frac{\eta}{\eta_0}$  be-

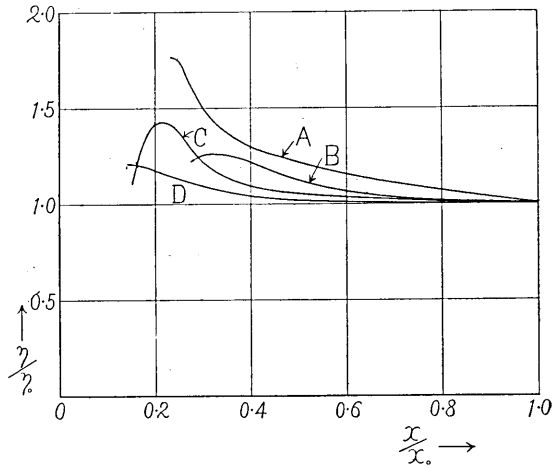


Fig. 36.  $\theta=3^\circ$ ,  $h_0=20\text{cm}$ ,  $x_0=400\text{cm}$ .

A:  $\frac{\eta_0}{h_0}=0.16$ , B:  $\frac{\eta_0}{h_0}=0.36$ ,

C:  $\frac{\eta_0}{h_0}=0.115$ , D:  $\frac{\eta_0}{h_0}=0.10$ .

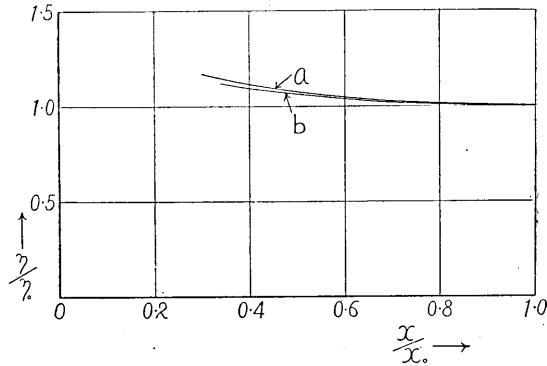


Fig. 37.  $\theta=5^\circ$ ,  $h_0=12\text{cm}$ ,  $x_0=160\text{cm}$ .

a:  $\frac{\eta_0}{h_0}=0.334$ , b:  $\frac{\eta_0}{h_0}=0.567$ ,

fore it decreases, precedes the break of a wave. When  $\frac{\eta}{\eta_0}$  assumes its maximum value,  $\frac{\eta}{h}$  becomes nearly equal to 1. That is to say, when  $\frac{\eta}{h}=1$ , approximately, the wave begins to break.

The position of the maximum value of  $\frac{\eta}{\eta_0}$ , namely the part of the wave where it breaks, varies with the value of  $\frac{\eta_0}{h_0}$ .

Now the ratio of the wave height  $\eta$  to that at the mouth  $\eta_0$  is given by the following expression

$$\frac{\eta}{\eta_0} = \sqrt[4]{\frac{x_0}{x}} \dots \dots \dots (15)$$

which is derived from the theory of infinitesimal wave motion.

Plotting the value of  $\frac{\eta}{\eta_0}$  expressed by (15), we obtain the curve expressed by *a* in Fig. 38, which is approximately accordant with the experimental values, which are smaller than those corresponding to the occurrence of the break of the wave in the bay, especially when the value of  $\frac{\eta_0}{h_0}$  is small. These results show that when angle  $\theta$  becomes as small as  $2^\circ$ , and when  $\frac{\eta_0}{h_0}$  is also comparatively small, the expression (15) is applicable in giving the height of the wave before it breaks in the bay.

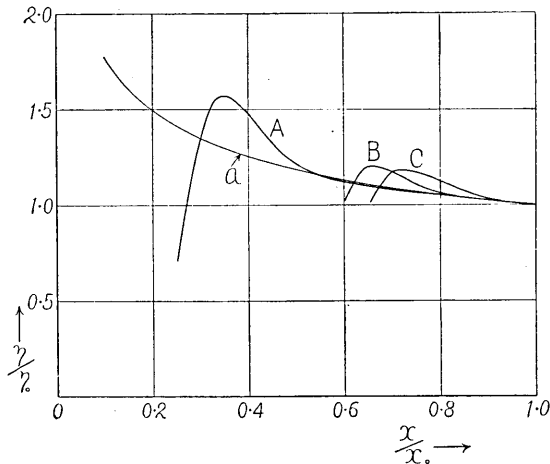


Fig. 38.  $\theta = 2^\circ$ ,  $h_0 = 12\text{cm}$ ,  $x_0 = 405.6\text{cm}$ .

A:  $\frac{\eta_0}{h_0} = 0.30$ ; B:  $\frac{\eta_0}{h_0} = 0.634$ ; C:  $\frac{\eta_0}{h_0} = 0.667$ ,  
 $a = \sqrt[4]{\frac{x_0}{x}}$ .

b) Wave Velocity.

From curves *M* and *F* shown in Figs. 30, 31 and 32, we see that the wave velocity of the maximum height and that of the wave front gradually diminishes in the bay before the wave breaks. Moreover, the rate of decrease in velocity of the front is greater than that of the maximum height of the wave.

Obtaining the wave velocity of the maximum height of the propagated wave for every part of the bay, we see that the value obtained from the velocity expression

$$\frac{v}{\sqrt{gh}} = \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$$

is also accordant with the result from experiment.

We now know that the wave height in the bay before the wave breaks and curls over is expressed by (12), whence we obtain the following expressions for the velocity in the bay.

$$v = \sqrt{gh_0} \sqrt{\frac{x}{x_0} + \frac{3}{2} \frac{\eta_0}{h_0} \sqrt{\frac{x_0}{x}}}, \dots\dots\dots (16)$$

and

$$\frac{v}{v_0} = \frac{\sqrt{\frac{x}{x_0} + \frac{3}{2} \frac{\eta_0}{h_0} \sqrt{\frac{x_0}{x}}}}{\sqrt{1 + \frac{3}{2} \frac{\eta_0}{h_0}}}, \dots\dots\dots (17)$$

where  $v_0$  is the wave velocity at the mouth of the bay. Curves  $a, b, c$  and  $d$  in Fig. 39 are the calculated values of  $\frac{v}{v_0}$  when  $\frac{\eta_0}{h_0} = 1.0, 0.34, 0.30$  and  $0$  respectively. Fig. 39 shows that the wave velocity decreases in the bay and that the rate of decrease of the velocity in the bay is large when the value of  $\frac{\eta_0}{h_0}$  is small.

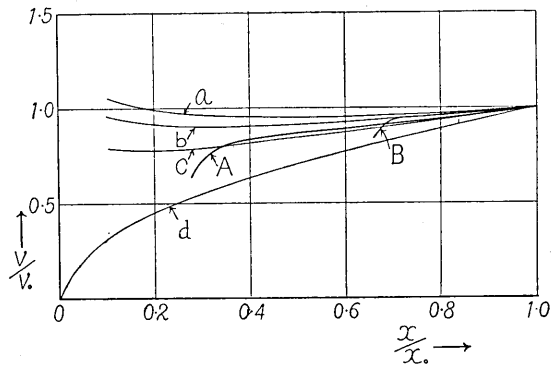


Fig. 39.  $a: \frac{\eta_0}{h_0} = 1.00, b: \frac{\eta_0}{h_0} = 0.634, c: \frac{\eta_0}{h_0} = 0.30,$   
 $d: \frac{\eta_0}{h_0} = 0, A: \frac{\eta_0}{h_0} = 0.30, B: \frac{\eta_0}{h_0} = 0.634.$

Plotting the experimental results  $A, B$  in Fig. 39, we can see that the expression (17) is applicable in obtaining the value of a velocity that is smaller than that corresponding to the maximum value of  $\frac{\eta}{\eta_0}$ .

(3) Particle Velocity.

The horizontal time displacement curves of the surface particles in Fig. 30, 31, 32 show that when the depth of the water in the bay be-

comes small, the maximum horizontal velocity of the surface particle becomes large.

If we assume that the maximum velocity of the surface particle in the bay is also expressed by (8), we obtain the following expression.

$$V = \sqrt{gh_0} \frac{\eta_0}{h_0} \left\{ \sqrt[4]{\frac{x_0}{x}} \right\}^3 \sqrt{1 + \frac{3}{2} \frac{\eta_0}{h_0} \left( \sqrt[4]{\frac{x_0}{x}} \right)^5}, \dots\dots\dots (18)$$

therefore

$$\frac{V}{V_0} = \frac{\left( \sqrt[4]{\frac{x_0}{x}} \right)^3 \sqrt{1 + \frac{3}{2} \frac{\eta_0}{h_0} \left( \sqrt[4]{\frac{x_0}{x}} \right)^5}}{\sqrt{1 + \frac{3}{2} \frac{\eta_0}{h_0}}}, \dots\dots\dots (19)$$

where  $V_0$  is the maximum horizontal velocity of the surface particle at the mouth of the bay.

In Fig. 40, the calculated results of  $\frac{V}{V_0}$  as expressed by (19) are shown for the cases when  $\frac{\eta_0}{h_0} = 0, 0.30, 0.634$  and  $1.0$ . From these results we can see that the particle velocity increases in the bay and that the value of  $\frac{V}{V_0}$  increases with the value of  $\frac{\eta_0}{h_0}$ . Plotting the experimental result for the case when  $\frac{\eta_0}{h_0} = 0.634$ , we obtain curve  $A$ , which however is not in good accord with the calculated curve  $b$ .

This shows that the expression (8) for the maximum horizontal velocity of the surface particle is not

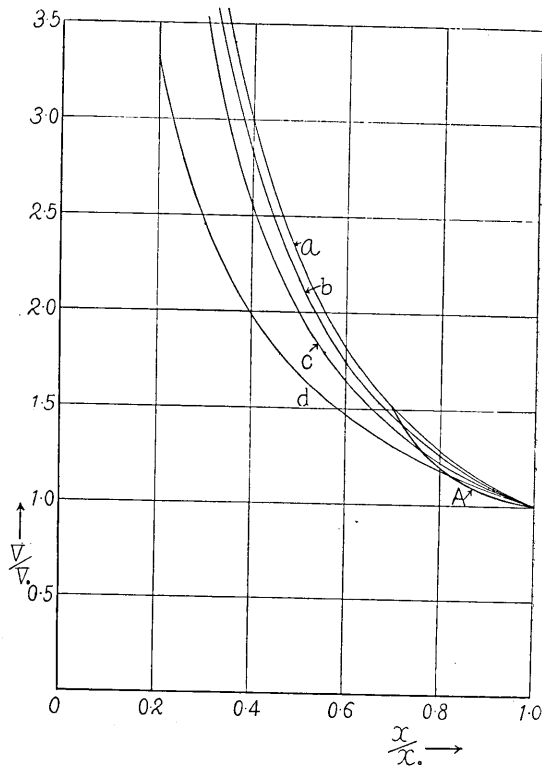


Fig. 40.  $A: \frac{\eta_0}{h_0} = 0.634$   $a: \frac{\eta_0}{h_0} = 1.00$   $b: \frac{\eta_0}{h_0} = 0.634$   
 $c: \frac{\eta_0}{h_0} = 0.30$   $d: \frac{\eta_0}{h_0} = 0$

applicable to the kind of bay now under consideration.

Plotting the values for  $\frac{V}{\sqrt{gh}}$ , we obtain the following curves, (C, D, E, F, G), the values of the parameter  $\frac{\eta_0}{h_0}$  of which are 0.667, 0.634, 0.38, 0.333 and 0.16 respectively. (Fig. 41).

The velocity  $v$  of the maximum height of the wave in a bay of form now being considered, as well as the maximum horizontal velocity  $V$  of the surface particle in a bay of rectangular form and of uniform depth, are shown in Fig. 41.<sup>10)</sup> From this figure, as in the case of the triangular bay, we can see that for the same values of  $\frac{\eta_0}{h_0}$  the maximum horizontal velocities of the surface particles in the bay now being considered, differ from those expressed by (8) and are smaller than those in a rectangular bay of uniform depth.

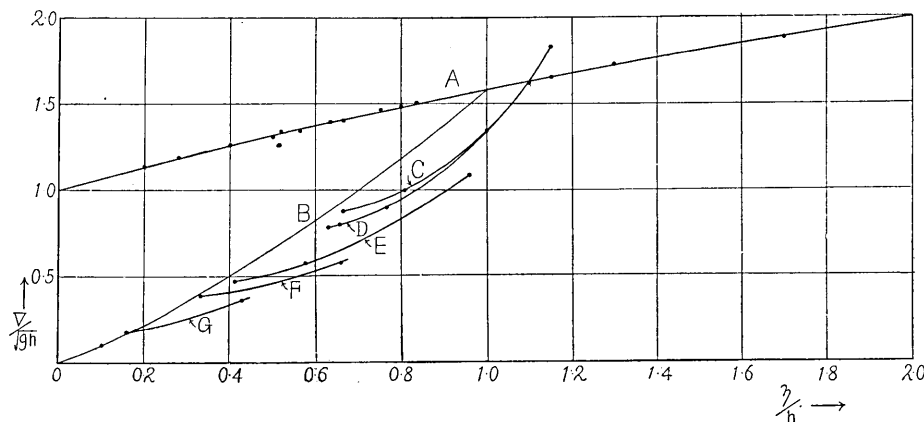


Fig. 41.  $A: \frac{v}{\sqrt{gh}} = \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ ,  $B: \frac{V}{\sqrt{gh}} = \frac{\eta}{h} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ ,  $C: \frac{\eta_0}{h_0} = 0.667$   
 $D: \frac{\eta_0}{h_0} = 0.634$ ,  $E: \frac{\eta_0}{h_0} = 0.38$ ,  $F: \frac{\eta_0}{h_0} = 0.333$ ,  $G: \frac{\eta_0}{h_0} = 0.16$ .

The curves A and C in this figure show that both the surface particle and the maximum height of the wave in a bay now being considered have the same velocities when  $\frac{\eta}{h} \approx 1.1$ . This shows that the wave in this bay breaks and curls over when  $\frac{\eta}{h} \approx 1.1$ . Comparing this re-

<sup>10)</sup> Curve A shows  $\frac{v}{\sqrt{gh}} = \sqrt{1 + \frac{3}{2} \frac{\eta}{h}}$ , B corresponds to the expression (8)  $\left( \frac{V}{\sqrt{gh}} = \frac{\eta}{h} \sqrt{1 + \frac{3}{2} \frac{\eta}{h}} \right)$ .

sult with that in the case of the triangular bay of uniform depth, we see that the maximum horizontal velocity of the surface particle in this case is larger than that in the case of the triangular bay of uniform depth for the same value of  $\frac{\eta_0}{h_0}$ .

(4) We shall now study the velocity of the wave at the instant of and after the curl of the wave. The time displacement curve for the maximum height and that for the front of the wave in the bay show that at the curling of the wave, the velocities, both of the maximum height of the wave and of the wave front, suddenly decrease. After the curling of the wave, another type of wave is generated in the water, the velocity of wave front of which suddenly increases at the initial stage of the generation of this type of wave, but gradually decreases with advance to the shore. These phenomena are shown in the Fig. 42, in which the variations of the wave velocities from the mouth to the head of the bay are shown for the cases of  $\frac{\eta_0}{h_0} = 0.634, 0.30$ . When  $\frac{\eta_0}{h_0} = 0.634$  and  $0.30$ , the abrupt changes in the velocity of the front and in that

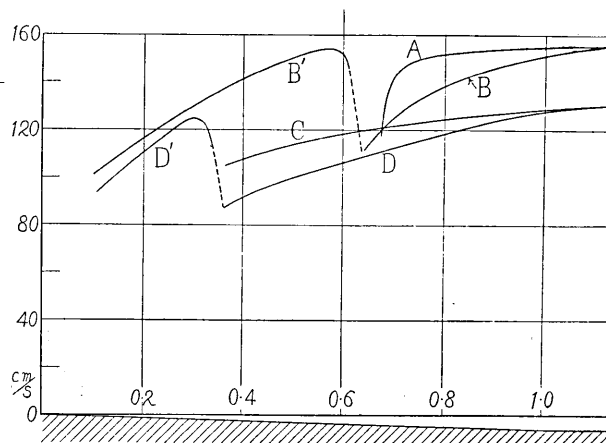


Fig. 42.  $\theta = 2^\circ$ ,  $h_0 = 12\text{cm}$ ,  $x_0 = 405.6\text{cm}$ .

- A: velocity of maximum height for  $\frac{\eta_0}{h_0} = 0.634$ ,
- B: velocity of the wave front for  $\frac{\eta_0}{h_0} = 0.634$ ,
- C: velocity of maximum height for  $\frac{\eta_0}{h_0} = 0.30$ ,
- D: velocity of the wave front for  $\frac{\eta_0}{h_0} = 0.30$ ,
- B': velocity of wave front after the curling of the wave for  $\frac{\eta_0}{h_0} = 0.634$ ,
- D': velocity of front after the curling of the wave for  $\frac{\eta_0}{h_0} = 0.30$ .

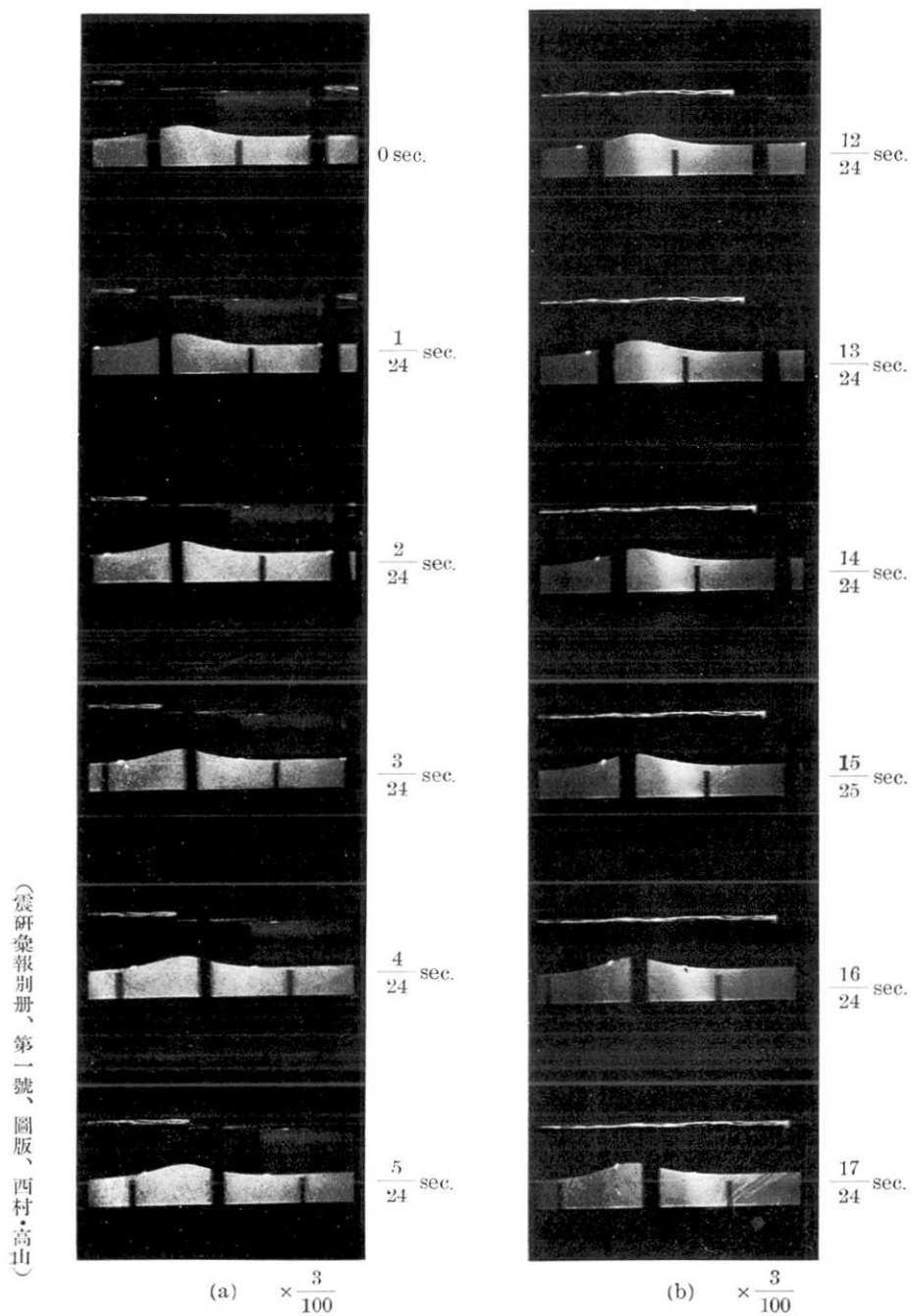
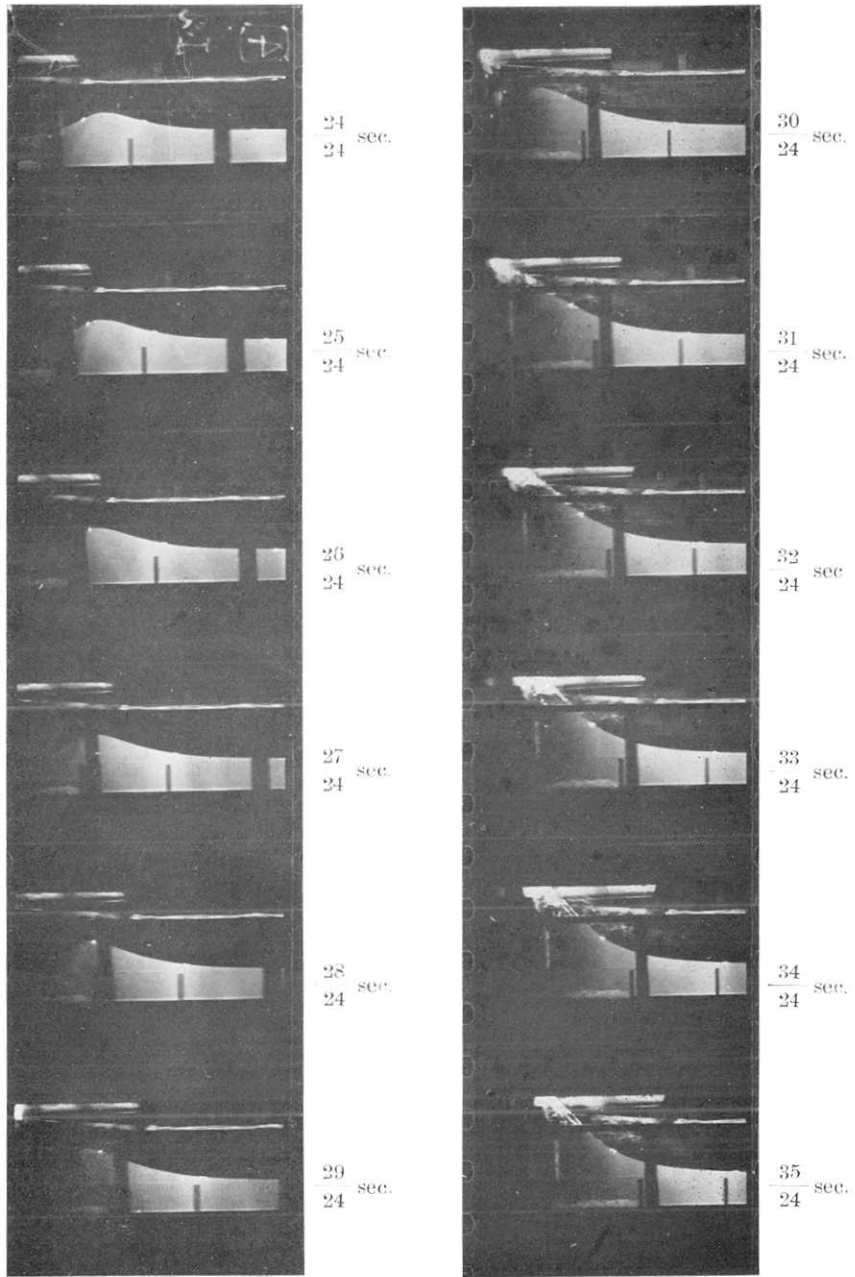


Fig. 17 a. Triangular bay of uniform depth.

$h = 12 \text{ cm}, \quad \gamma_0 = 6.4 \text{ cm}, \quad 2\theta = 42^\circ.$



(震研彙報別冊、第一號、圖版、西村・高山)



(c)  $\times \frac{3}{100}$

(d)  $\times \frac{3}{100}$

Fig. 17 b. Triangular bay of uniform depth.

$h = 12 \text{ cm}, \quad \eta_0 = 6.4 \text{ cm}, \quad 2\theta = 42^\circ.$

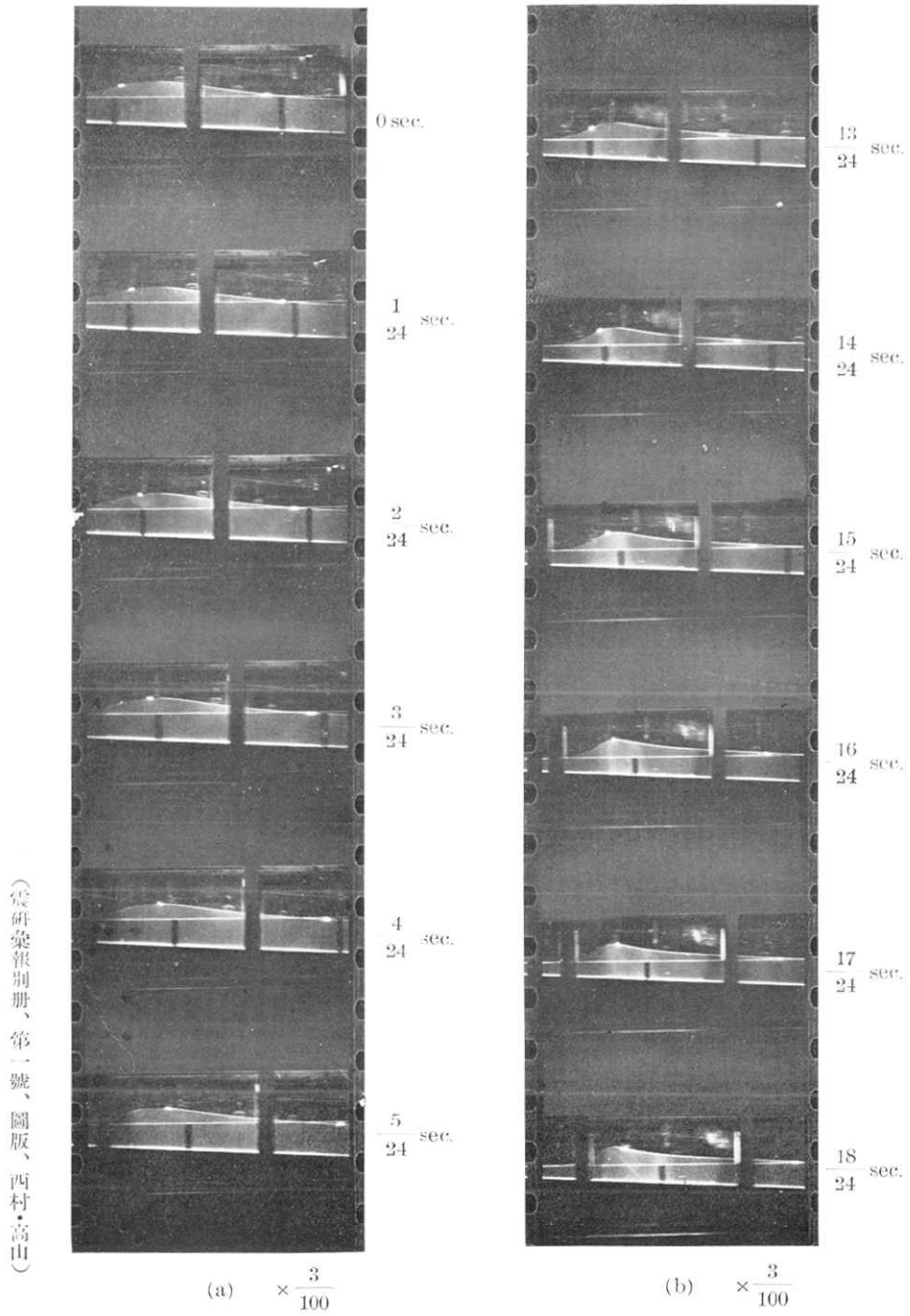


Fig. 35 a.  $\theta = 3^\circ$ ,  $h_0 = 20$  cm.  $\gamma_0 = 4$  cm.

(震研彙報別冊、第一號、圖版、西村・高山)

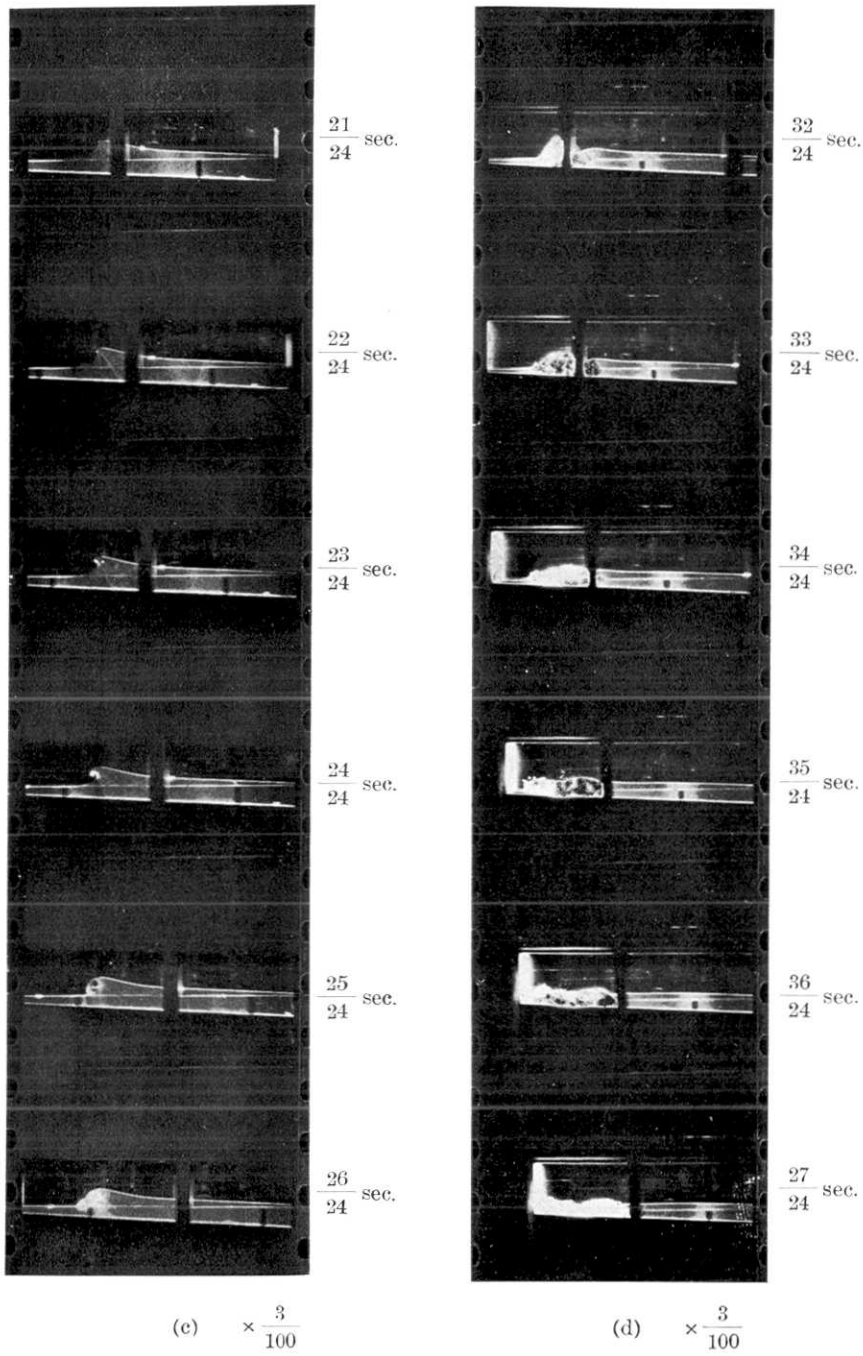


Fig. 35 b.  $\theta = 3^\circ$ ,  $h_0 = 20$  cm,  $\eta_0 = 4$  cm.

of the maximum height of the wave occur at  $\frac{x}{x_0} \approx 0.65$  and  $0.35$  respectively where the curlings of the waves take place.

## 16. 津浪傳播に關する實驗的研究 (其の 1)

地震研究所 { 西村源六郎  
                  高山威雄

昭和 8 年 3 月 3 日三陸津浪の後、これに關係して、海岸及び海底地形の津浪傳播に及ぼす影響を明かにする目的で模型實驗を始めた。

模型實驗で先づ第 1 に考へなければならぬのは similitude の法則を如何に適用するかと云ふ事である。灣の形、海岸や海底の様子は全然 similar にしなければならない事は勿論であるが、この他に灣の入口での波の形も similar にしなければならぬ。この 2 つの事柄に注意して所謂  $\pi$ -方程式より、求める目的物に關する實物と模型との或る關係式を求める事が出来る。それを利用して實驗結果より津浪當時の海水の流れの早さ、水の流れ模様或は海水の津浪の爲め溢れる狀況等に關する量的の關係を求める事が出来る。

本實驗報告は僅か 1 ケ月の豫備的な實驗結果の報告であつて、今後の模型實驗に對する準備實驗である事を述べておく。