BULLETIN OF THE EARTHQUAKE
RESEARCH IHSTITUTE
UNIVERSITY OF TOKYO
Vol. 68 (1993) pp. 177–194

Implication of a Surface Wave Orbit with Stability, Drift and Azimuthal Distribution of Waves in Weakly Anisotropic Media

Takao MOMOI

(Received September 24, 1993)

Abstract

The anisotropy of elastic media, in general, causes drift and uneven distribution of waves in the azimuthal direction.

Surface waves in weakly anisotropic media are discussed, focusing attention on the implication of a wave orbit on a transverse plane with drift of waves and azimuthal distribution of wave amplitude.

When waves drift rightward (leftward), the orbit tilts leftward (rightward), where the term *rightward* (*leftward*) is used in the advancing direction of waves.

Under the assumption that uneven distribution of waves in the azimuthal direction is caused only by the anisotropy of media, we find that a surface wave orbit on a transverse plane at the surface tilts rightward (leftward) when the wave amplitude increases (decreases) rightward.

It is noted that the above-mentioned drift and uneven distribution of surface waves in the azimuthal direction in anisotropic media can be fundamentally explained by use of the solution of the characteristic equation obtained from stress-free surface conditions.

Introduction

The theory of surface wave propagation in an anisotropic half-space has been discussed by SYNGE (1957) and BUCHWALD (1961), among others. Propagation in a half-space with cubic symmetry has been investigated by STONELY (1955), and BUCHWALD and DAVIS (1963), and with orthorhombic symmetry by STONELY (1963).

In 1975, CRAMPIN (1975) calculated the particle motion of surface waves propagating in particular symmetry directions in anisotropic media and showed that propagation in some directions reveals particle motion anomalies diagnostic of the symmetry.

In 1991, we have discussed the implication of an orbit inclination of surface waves with the stability of waves (MOMOI, 1991). This paper will be referred to as *paper M* in later discussion. In *paper M*, we assumed that wave behavior only depends on the x- and z-axes (y-axis excluded) and, as a result, drift of surface waves due to the anisotropy of elastic media is excluded in the theory.

In this paper, taking dependence on y into account, we will study the effects of weak but general anisotropy of media on surface waves, focusing particular

Present address: Shakujiidai 2–25–3, Nerima-ku, Tokyo

attention on the implication of a surface wave orbit, at the free surface, with the drift and azimuthal distribution of waves. The theory was developed by using *computer algebra.*

1. Expression for Energy

A homogenous half-space model is used (see Fig. 1). The z-axis is positive downward and the free surface is situated at $z=0$ and expressed by x–y plane.

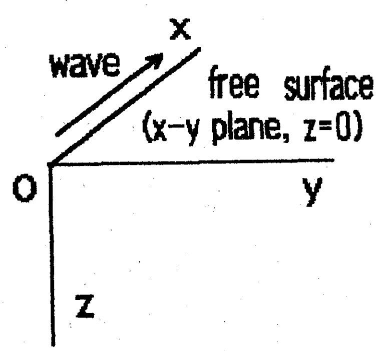

Fig. 1. Used model.

The energy equation for conservative (nondissipative) thermoelastic media (MASON, 1964) gives the thermodynamic potential, namely, the internal energy E in terms of the conjugate variables S and T, and t_{jk} *and* U_{jk}/ρ as

$$dE=TdS+(1/\rho)t_{jk}dU_{jk}, \qquad (1.1)$$

where S is the entropy, T the temperature, ρ the density of the medium, and t_{jk} are thermodynamic tensions, and U_{jk} the Lagrangian strains (THURSTON, 1964) with

$$U_{ij}=(u_{ij}+u_{ji})/2, \qquad (1.2)$$

where $u_{ij}=\partial u_i/\partial x_j$ and $\{u_1, u_2, u_3\}$ are displacement components in the directions of the Cartesian coordinate axes $\{x_1, x_2, x_3\}$. In later discussions, $\{u_1, u_2, u_3\}$ and $\{x_1, x_2, x_3\}$ will be alternatively expressed by $\{u, v, w\}$ and $\{x, y, z\}$. The potential and all extensive quantities are taken per unit mass. From the relation (1.1) the definition of the elastic coefficients for any order follows naturally. Namely, for the adiabatic and isothermal stiffnesses c of the nth order, for $n\geqq 2$,

$$c^S_{jkpq\cdots}=\rho(\partial^n E/\partial U_{jk}\partial U_{pq}\cdots)_S. \qquad (1.3)$$

Since the strains and the thermodynamic tensions are symmetric, i.e., $\partial U_{jk}=\partial U_{kj}$ and $\partial t_{jk}=\partial t_{kj}$, only six of each set of nine variables are independent, and it is customary to introduce the VOIT (1928) notation: $\{ij\sim J\}$

$$\{11\sim 1,\ 22\sim 2,\ 33\sim 3,\ 23\sim 4,\ 13\sim 5,\ 12\sim 6\}$$

and the convention

$$U_{ij}=(1/2)(1+\delta_{ij})U_J \qquad (1.4)$$

where δ_{ij} is the Kronecker delta with suffixes i, j.

Considering the potentials now as functions of the single-subscript variables, (1.3) become

$$c_{JP}=\partial^n E_n/\partial U_J\partial U_P, \qquad (1.5)$$

where the subscript S is omitted and $E_n=\rho E$.

Expanding the internal energy about the state of zero strain one obtains for

$$E_n(U)=E_n(S,U)-E_n(S,0)$$

the form up to second order in U_J,

$$E_n(U)=(1/2)c_{JP}U_JU_P, \tag{1.6}$$

where summations over repeated indices $\{J,P\}$ are implied (this convention will be used, unless stated otherwise), and the c_{JP} are the elastic coefficients defined by K. BRUGGER (1964).

In later discussion, the elastic coefficients c_{JP} normalized by $\rho\omega^2$ will be used, i.e.,

$$C_{JP}=c_{JP}/(\rho\omega^2), \tag{1.7}$$

where ω is the angular frequency of surface waves.

In a weakly anisotropic case, the above normalized elastic coefficients are expressed as

$$\begin{aligned}
&C_{11}=L_{2m}+d_{11}, \quad C_{22}=L_{2m}+d_{22}, \quad C_{33}=L_{2m}+d_{33},\\
&C_{12}=L_a+d_{12}, \quad C_{13}=L_a+d_{13}, \quad C_{23}=L_a+d_{23},\\
&C_{44}=m_u+d_{44}, \quad C_{55}=m_u+d_{55}, \quad C_{66}=m_u+d_{66},\\
&C_{14}=d_{14}, \quad C_{15}=d_{15}, \quad C_{16}=d_{16}, \quad C_{24}=d_{24}, \quad C_{25}=d_{25},\\
&C_{26}=d_{26}, \quad C_{34}=d_{34}, \quad C_{35}=d_{35}, \quad C_{36}=d_{36}, \quad C_{45}=d_{45},\\
&C_{46}=d_{46}, \quad C_{56}=d_{56},
\end{aligned}$$

where $L_a=\lambda/(\rho\omega^2)$ and $m_u=\mu/(\rho\omega^2)$ are the elastic coefficients normalized by $\rho\omega^2$ in the case of an isotropic medium, $L_{2m}=L_a+2m_u$, and d_{ij} (i,j: integers) is the weak deviation of the anisotropic elastic coefficients from the isotropic ones.

2. Equations

The stress tensor S_{ij} is related to the energy function (1.6)

$$S_{ij}=\partial E_n/\partial u_{ij}, \tag{2.1}$$

By use of the above relation, governing equations can be expressed (LANDAU and LIFSHITZ, 1985) by

$$\rho\partial^2 u_i/\partial t^2=\partial S_{ij}/\partial x_j \quad (i=1,2,3), \tag{2.2}$$

where t is time.

By use of (2.1) and the expressions for energy in the foregoing section, the above equations are reduced to the following.

$$\rho u_{t2}=q_1, \quad \rho v_{t2}=q_2, \quad \rho w_{t2}=q_3, \tag{2.3}$$

where

$$q_1=u_{x2}c_{11}+v_{xy}c_{12}+v_{y2}c_{26}+v_{yz}c_{25}+w_{xz}c_{13}+w_{yz}c_{36}+w_{z2}c_{35}+c_{16}P_{a2}$$
$$+c_{15}P_{a3}+c_{66}P_{a4}+c_{56}P_{a6}+c_{55}P_{a8}+c_{14}P_{a7}+c_{46}P_{a11}+c_{45}P_{a13}\,,$$

$$q_2=u_{x2}c_{16}+u_{xy}c_{12}+u_{xz}c_{14}+v_{y2}c_{22}+w_{xz}c_{36}+w_{yz}c_{23}+w_{z2}c_{34}+c_{26}P_{b6}$$
$$+c_{25}P_{b8}+c_{66}P_{b2}+c_{56}P_{b4}+c_{46}P_{b7}+c_{45}P_{b9}+c_{24}P_{b12}+c_{44}P_{b13}\,,$$

$$q_3=u_{x2}c_{15}+u_{xy}c_{14}+u_{xz}c_{13}+v_{xy}c_{25}+v_{y2}c_{24}+v_{yz}c_{23}+w_{z2}c_{33}+c_{36}P_{c9}$$
$$+c_{35}P_{c10}+c_{56}P_{c2}+c_{55}P_{c4}+c_{46}P_{c6}+c_{45}P_{c8}+c_{34}P_{c14}+c_{44}P_{c12}\,,$$

with

$$P_{a2}=2u_{xy}+v_{x2}\,,\quad P_{a3}=2u_{xz}+w_{x2}\,,\quad P_{a4}=u_{y2}+v_{xy}\,,\quad P_{a6}=2u_{yz}+v_{xz}+w_{xy}\,,$$
$$P_{a7}=v_{xz}+w_{xy}\,,\quad P_{a}=u_{z2}+w_{xz}\,,\quad P_{a11}=v_{yz}+w_{y2}\,,\quad P_{a13}=v_{z2}+w_{yz}\,,$$
$$P_{b2}=u_{xy}+v_{x2}\,,\quad P_{b4}=u_{xz}+w_{x2}\,,\quad P_{b6}=u_{y2}+2v_{xy}\,,\quad P_{b7}=u_{yz}+2v_{xz}+w_{xy}\,,$$
$$P_{b8}=u_{yz}+w_{xy}\,,\quad P_{b9}=u_{z2}+w_{xz}\,,\quad P_{b12}=2v_{yz}+w_{y2}\,,\quad P_{b13}=v_{z2}+w_{yz}\,,$$
$$P_{c2}=u_{xy}+v_{x2}\,,\quad P_{c4}=u_{xz}+w_{x2}\,,\quad P_{c6}=u_{y2}+v_{xy}\,,\quad P_{c8}=u_{yz}+v_{xz}+2w_{xy}\,,$$
$$P_{c9}=u_{yz}+v_{xz}\,,\quad P_{c10}=u_{z2}+2w_{xz}\,,\quad P_{c12}=v_{yz}+w_{y2}\,,\quad P_{c14}=v_{z2}+2w_{yz}\,,$$

and

$$u_{t2}=\partial^2u/\partial t^2\,,\quad u_{x2}=\partial^2u/\partial x^2\,,\quad u_{xy}=\partial^2u/\partial x\partial y\,,\quad u_{xz}=\partial^2u/\partial x\partial z\,,$$
$$u_{y2}=\partial^2u/\partial y^2\,,\quad u_{yz}=\partial^2u/\partial y\partial z\,,\quad u_{z2}=\partial^2u/\partial z^2\,,$$
$$v_{t2}=\partial^2v/\partial t^2\,,\quad v_{x2}=\partial^2v/\partial x^2\,,\quad v_{xy}=\partial^2v/\partial x\partial y\,,\quad v_{xz}=\partial^2v/\partial x\partial z\,,$$
$$v_{y2}=\partial^2v/\partial y^2\,,\quad v_{yz}=\partial^2v/\partial y\partial z\,,\quad v_{z2}=\partial^2v/\partial z^2\,,$$
$$w_{t2}=\partial^2w/\partial t^2\,,\quad w_{x2}=\partial^2w/\partial x^2\,,\quad w_{xy}=\partial^2w/\partial x\partial y\,,\quad w_{xz}=\partial^2w/\partial x\partial z\,,$$
$$w_{y2}=\partial^2w/\partial y^2\,,\quad w_{yz}=\partial^2w/\partial y\partial z\,,\quad w_{z2}=\partial^2w/\partial z^2\,,$$

3. Surface Conditions

By use of relation (2.1), surface conditions are expressed as

$$S_{13}:\ u_{11}C_{15}+u_{21}C_{56}+u_{22}C_{25}+u_{12}C_{56}+u_{23}C_{45}+u_{31}C_{55}+u_{13}C_{55}+u_{32}C_{45}+u_{33}C_{35}=0\,,$$
$$S_{23}:\ u_{11}C_{14}+u_{21}C_{46}+u_{22}C_{24}+u_{12}C_{46}+u_{23}C_{44}+u_{31}C_{45}+u_{13}C_{45}+u_{32}C_{44}+u_{33}C_{34}=0\,,$$
$$S_{33}:\ u_{11}C_{13}+u_{21}C_{36}+u_{22}C_{23}+u_{12}C_{36}+u_{23}C_{34}+u_{31}C_{35}+u_{13}C_{35}+u_{32}C_{34}+u_{33}C_{33}=0\,,$$
$$\text{at}\quad z=0\,. \tag{3.1}$$

4. Splitting of Lobe of the Surface Waves

In order to obtain expressions for surface waves, we assume the following for the displacements:

$$u=A_uE_p\,,\quad v=A_vE_p\,,\quad w=A_wE_p\,, \tag{4.1}$$

where A_u, A_v and A_w are amplitudes of displacements u, v and w, respectively, and

$$E_p = \exp(-ik_x x - ik_y y - ab \cdot z)$$

with k_x, k_y and ab, in general complex, being the x-, y- and z-wave numbers. In the above expression, the time factor $\exp(i\omega t)$ is omitted. This convention will be followed in the subsequent discussion.

Assume that surface waves in the case of an isotropic medium are propagated in the direction of the positive x-axis. Since a weakly anisotropic medium is considered in this paper, wave numbers k_x, k_y and ab are expressed as

$$k_x = k_r + dk_r, \quad k_y = dk_y, \quad ab = ab_0 + dab, \tag{4.2}$$

where k_r is the wave number of surface waves in an isotropic medium, $ab_0 = a_0(=\alpha)$ or $b_0(=\beta)$ with

$$a_0^2 = k_r^2 - 1/(L_a + 2m_u) \quad \text{and} \quad b_0^2 = k_r^2 - 1/m_u, \tag{4.3}$$

dk_r, dk_y and dab indicate the deviation of each wave number (x-, y- and z-component, respectively) in a weakly anisotropic medium from an isotropic one.

Substituting (4.1) into Eqs. (2.3) and taking the first order of d_{ij}, we have the deviations $dab = \{da, db_1 \text{ and } db_2\}$ of ab from a_0 and b_0 in an isotropic case, i.e.,

$$da = ab - a_0, \quad db_1 \text{ or } db_2 = ab - b_0,$$

as follows.

$$\begin{aligned} da = {} & k_r^4/(2a_0)d_{11} + a_0^3/2d_{33} - 2a_0k_r^2d_{55} - a_0k_r^2d_{13} \\ & -i2k_r^3d_{15} + i2a_0^2k_rd_{35} + k_r/a_0dk_r \end{aligned} \tag{4.4}$$

and

$$db_1 = -dM + dN, \quad db_2 = -dM - dN, \tag{4.5}$$

where

$$dM = b_0/(2m_u)d_{44} - k_r^2/(2m_ub_0)d_{66} + ik_r/m_ud_{46} - k_r/b_0dk_r - 1/(2m_ub_0)dQ,$$

$$dN = (dP^2/m_u + dQ^2)^{(1/2)}/(2b_0m_u),$$

$$dP = -b_0^2k_rm_ud_{14} + b_0^2k_rm_ud_{34} - C_fk_rd_{56} - ib_0k_r^2m_ud_{16} + ib_0k_r^2m_ud_{36} + iC_fb_0d_{45},$$

$$\begin{aligned} dQ = {} & -k_r^2m_ub_0^2/2d_{11} - k_r^2m_ub_0^2/2d_{33} + b_0^2/2d_{44} + C_f^2/(2m_u)d_{55} - k_r^2/2d_{66} + k_r^2m_ub_0^2d_{13} \\ & + iC_fk_rb_0d_{15} - iC_fk_rb_0d_{35} + ik_rb_0d_{46}, \end{aligned}$$

with

$$C_f = 2k_r^2m_u - 1.$$

The deviation of b_0 splits even in the case of the theory including the effect of surface wave drift (effect of the y-component). This splitting of b_0 has already been discussed in *paper M*, where drift of waves is not taken into consideration. Expressions (4.4) and (4.5) involve the imaginary terms, so that the lobes of surface waves are moving with time.

5. Expressions for Surface Waves

Since we have three components of ab as discussed in the foregoing section, surface waves in an anisotropic medium are expressed as

$$\begin{aligned} u&=E_{aL}A_u+E_{b1}B_{u1}+E_{b2}B_{u2}, \\ v&=E_{aL}A_v+E_{b1}B_{v1}+E_{b2}B_{v2}, \\ w&=E_{aL}A_w+E_{b1}B_{w1}+E_{b2}B_{w2}, \end{aligned} \tag{5.1}$$

where

$$a=a_0+da, \quad E_{aL}=\exp(-ix_n-za),$$
$$b_j=b_0+db_j, \quad E_{bj}=\exp(-ix_n-zb_j) \quad (j=1,2),$$
$$x_n=k_xx+k_yy,$$

and $A_u, A_v, A_w, B_{u1}, B_{v1}, B_{w1}, B_{u2}, B_{v2}, B_{w2}$ are the amplitudes determined by the surface conditions.

6. Characteristic Equation

In this section, the two deviations, dk_r and dk_y, of wave number in (4.2) will be obtained.

Substituting (5.1) into (3.1) and taking the terms up to first order in d_{ij}, we have the following characteristic equation.

$$f_{dkr}f_{dky}=0,$$

where

$$f_{dkr}=dk_r-k_r\theta_r-ik_r\theta_x \tag{6.1}$$

and

$$f_{dky}=dk_y-k_r\theta_d-ik_r\theta_y, \tag{6.2}$$

where θ_r, θ_x, θ_d and θ_y are given by (6.1.2), (6.1.3), (6.2.2) and (6.2.3), respectively.

As shown above, the characteristic equation is expressed in a product form with factors f_{dkr} and f_{dky}, so that we can obtain *two* solutions dk_r and dk_y from *one* characteristic equation by putting $f_{dkr}=0$ and $f_{dky}=0$, respectively. It must be noted here that the drift phenomenon (due to the existence of dk_y) of surface waves can be explained by a characteristic equation in anisotropic media as its fundamental solution.

Solving the above characteristic equation, we have the following two solutions.

From $f_{dkr}=0$,

$$dk_r=k_r\theta_r+ik_r\theta_x, \tag{6.1.1}$$

$$\theta_r=d\theta_{11}+d\theta_{33}+d\theta_{55}+d\theta_{13}, \tag{6.1.2}$$

$$\theta_x=d\theta_{15}+d\theta_{35}, \tag{6.1.3}$$

with

$$d\theta_{11}=d_{11}\Theta_{11},\quad d\theta_{33}=d_{33}\Theta_{33},\quad d\theta_{55}=d_{55}\Theta_{55},$$

$$d\theta_{13}=d_{13}\Theta_{13},\quad d\theta_{15}=d_{15}\Theta_{15},\quad d\theta_{35}=d_{35}\Theta_{35},$$

$$\Theta_{11}=-(64b_m{}^{10}+64b_m{}^8-104b_m{}^6-164b_m{}^4-62b_m{}^2-3)(4b_m{}^2+3)b_m{}^3k_{rm}/(4\cdot Dk_r),$$

$$\Theta_{33}=-(512b_m{}^{16}+1152b_m{}^{14}+128b_m{}^{12}-1712b_m{}^{10}-1856b_m{}^8-784b_m{}^6$$
$$-136b_m{}^4-10b_m{}^2-1)C_f^3/(64b_m\cdot Dk_r\cdot k_{rm}{}^2),$$

$$\Theta_{55}=-(8b_m{}^4+12b_m{}^2+5)(4b_m{}^2+3)b_mC_f^5/(4\cdot Dk_r),$$

$$\Theta_{13}=-(96b_m{}^{10}+232b_m{}^8+192b_m{}^6+59b_m{}^4+5b_m{}^2+1)b_mC_f^3/(2\cdot Dk_r),$$

$$\Theta_{15}=-k_{rm}{}^{(1/2)}(32b_m{}^6+64b_m{}^4+40b_m{}^2+7)b_m{}^2C_f^3k_{rm}/Dk_r,$$

$$\Theta_{35}=(64b_m{}^{10}+160b_m{}^8+144b_m{}^6+54b_m{}^4+8b_m{}^2+1)C_f^3k_{rm}/(2k_{rm}{}^{(1/2)}Dk_r),$$

$$Dk_r=(16b_m{}^8+28b_m{}^6+18b_m{}^4+7b_m{}^2+2)(8b_m{}^4+8b_m{}^2+1)(4b_m{}^2+3)b_mm_u.$$

From $f_{dky}=0$,

$$dk_y=k_r\theta_d+ik_r\theta_y, \tag{6.2.1}$$

$$\theta_d=d\theta_{16}+d\theta_{36}+d\theta_{45}, \tag{6.2.2}$$

$$\theta_y=d\theta_{14}+d\theta_{34}+d\theta_{56}, \tag{6.2.3}$$

with

$$d\theta_{16}=d_{16}\Theta_{16},\quad d\theta_{36}=d_{36}\Theta_{36},\quad d\theta_{45}=d_{45}\Theta_{45},$$

$$d\theta_{14}=d_{14}\Theta_{14},\quad d\theta_{34}=d_{34}\Theta_{34},\quad d\theta_{56}=d_{56}\Theta_{56},$$

$$\theta_{16}=-8k_{rm}{}^3b_m{}^2/(m_uuuu),$$

$$\theta_{36}=8k_{rm}{}^2b_m{}^2a_m{}^2/(m_uuuu),$$

$$\theta_{45}=16k_{rm}{}^2b_m{}^4/(m_uuuu),$$

$$\theta_{14}=8k_{rm}{}^{(5/2)}b_m{}^4/(m_uuuu\cdot a_m),$$

$$\theta_{36}=-8k_{rm}{}^{(3/2)}b_m{}^4a_m/(m_uuuu),$$

$$\theta_{56}=16k_{rm}{}^{(5/2)}b_m{}^2a_m/(m_uuuu),$$

where

$k_{rm}=k_r^2m_u,\quad a_m=a_0m_u{}^{(1/2)},\quad b_m=b_0m_u{}^{(1/2)},\quad$ (for a_0 and b_0, refer to (4.3)).

$C_f=2b_m{}^2+1,\quad uuu=8b_m{}^4+8b_m{}^2+1$

and further a_m and k_{rm} are expressed by b_m as

$$k_{rm}=b_m{}^2+1,\quad a_m=(2b_m{}^2+1)^2/(4(b_m{}^2+1)b_m).$$

In the derivation of the above, the characteristic equation in the case of an isotropic medium

$$(2k_{rm}-1)^2-4k_{rm}a_mb_m=0 \tag{6.3}$$

with

$$a_m = (k_{rm} - 1/(2+L_m))^{(1/2)}$$

was used in order to eliminate the ratio $L_m(=\lambda/\mu)$.

In the expressions (6.1.1), (6.1.2), (6.1.3), (6.2.1), (6.2.2) and (6.2.3), $d\theta_{11}$, $d\theta_{33}$, $d\theta_{55}$, $d\theta_{13}$, $d\theta_{15}$ and $d\theta_{35}$ indicate the rates of increase of wave number $k_x(=k_r + dk_r)$ associated with the anisotropic elastic coefficients d_{11}, d_{33}, d_{55}, d_{13}, d_{15} and d_{35} and also $d\theta_{16}$, $d\theta_{36}$, $d\theta_{45}$, $d\theta_{14}$, $d\theta_{34}$ and $d\theta_{56}$ are those of wave number $k_y(=dk_y)$ associated with d_{16}, d_{36}, d_{45}, d_{14}, d_{34} and d_{56}, respectively.

As found from (6.1.1) and (6.2.1), the deviation terms dk_r and dk_y include the imaginary terms, $ik_r\theta_x$ and $ik_r\theta_y$. This implies that the surface waves in an anisotropic medium always face a stability problem (for dk_r) and are subject to a transversely uneven distribution of waves (for dk_y) depending on the sign of the imaginary terms.

7. Surface Waves on the Absolute Coordinates

In this section, surface waves at the free surface on the absolute coordinates will be discussed.

7.1. Displacements at the free surface on the absolute coordinates

Substituting (5.1) into (3.1), we can obtain the following expressions for surface waves at the free surface on the absolute coordinates.

$$\begin{aligned} u &= A_u E_{\mathrm{van}} \sin(\Theta_{xn} + P_u)\,, \\ w &= A_w E_{\mathrm{van}} \cos(\Theta_{xn} + P_w)\,, \\ v &= A_v E_{\mathrm{van}} \cos(\Theta_{xn} + P_v)\,, \end{aligned} \tag{7.1.1}$$

with

$$E_{\mathrm{van}} = \exp(\theta_x k_r x + \theta_y k_r y)\,, \quad \Theta_{xn} = \omega t - (1+\theta_r)k_r x - \theta_d k_r y\,, \tag{7.1.2}$$

where θ_r, θ_x, θ_d and θ_y are given by (6.1.2), (6.1.3), (6.2.2) and (6.2.3), respectively.

Let θ_{drift} be the drift angle of surface waves due to the anisotropy of the medium. From (7.1.2), it is expressed as

$$\theta_{\mathrm{drift}} = \theta_d/(1+\theta_r) \tag{7.1.3}$$

In the case of a weakly anisotropic medium, the above expression is reduced, by use of (6.1.2) and (6.2.2), to

$$\begin{aligned} \theta_{\mathrm{drift}} &= \theta_d\,, \\ &= d\theta_{16} + d\theta_{36} + d\theta_{45}\,, \end{aligned} \tag{7.1.4}$$

since θ_d and θ_r are of the order of d_{ij}.

For amplitudes and phases of u- and w-components in (7.1.1), we have

$$\begin{aligned} A_u &= a_m R_{Lam}/(2k_{rm}^{(1/2)}) + a_{u11}d\theta_{11} + a_{u33}d\theta_{33} + a_{u55}d\theta_{55} + a_{u13}d\theta_{13}\,, \\ A_w &= a_m^2 R_{Lam}/C_f + a_{w11}d\theta_{11} + a_{w33}d\theta_{33} + a_{w55}d\theta_{55} + a_{w13}d\theta_{13}\,, \end{aligned} \tag{7.1.5}$$

$$P_u = 2k_{rm}^{(1/2)}/(a_m R_{Lam})(a_{u15} d\theta_{15} + a_{u35} d\theta_{35}),$$
$$P_w = C_f/(a_m^2 R_{Lam})(a_{w15} d\theta_{15} + a_{w35} d\theta_{35}), \tag{7.1.6}$$

where $\{a_{u11}, a_{u33}, a_{u55}, a_{u13}, a_{w11}, a_{w33}, a_{w55}, a_{w13}\}$ and $\{a_{u15}, a_{u35}, a_{w15}, a_{w35}\}$ are given in Appendix A, and

$$R_{Lam} = 1 + L_m, \quad R_{L2m} = 2 + L_m, \quad C_f = 2b_m^2 + 1.$$

Assume a complex expression for the amplitude of v, i.e.,

$$v = v_{\text{amp}} \exp(i\omega t - ix_n)$$

with

$$v_{\text{amp}} = R_v + iI_v \quad \text{(in complex expression)}. \tag{7.1.7}$$

For amplitude and phase of v-component in (7.1.1), we have then

$$A_v = \pm (R_v^2 + I_v^2)^{(1/2)} \quad (+ \text{ for } R_v > 0 \text{ and } - \text{ for } R_v < 0),$$
$$P_v = \tan^{-1}(I_v/R_v) \quad \text{(principal value assumed)}, \tag{7.1.8}$$

where

$$R_v = V_d \theta_d + V_{14} d\theta_{14} + V_{34} d\theta_{34} + V_{56} d\theta_{56},$$
$$I_v = V_d \theta_y + V_{16} d\theta_{16} + V_{36} d\theta_{36} + V_{45} d\theta_{45}, \tag{7.1.9}$$

$$V_d = k_{rm}^{(1/2)}(4b_m^2 + 1) a_m R_{Lam}/(2b_m^2),$$
$$V_{14} = -2k_{rm}^{(1/2)}(2b_m^2 R_{L2m} + 1) a_m^3 R_{Lam}/(b_m^2 C_f R_{L2m}),$$
$$V_{34} = -2k_{rm}^{(1/2)}(2b_m^2 R_{L2m} + 2R_{Lam} + 1) a_m R_{Lam}/(C_f R_{L2m}),$$
$$V_{56} = -k_{rm}^{(1/2)}(2b_m^2 R_{L2m} + R_{Lam}) a_m R_{Lam}/(b_m^2 R_{L2m}), \tag{7.1.10}$$
$$V_{16} = 2k_{rm}^{(1/2)}(a_m R_{Lam} + b_m C_f R_{L2m}) a_m R_{Lam}/(b_m C_f R_{L2m}),$$
$$V_{36} = 2k_{rm}^{(1/2)}(a_m b_m C_f R_{L2m} + k_{rm} R_{Lam}) R_{Lam}/(b_m C_f R_{L2m}),$$
$$V_{45} = k_{rm}^{(1/2)}(2a_m b_m R_{L2m} - R_{Lam}) a_m^2 R_{Lam}/(b_m^3 R_{L2m}), \tag{7.1.11}$$

7.2. *Surface wave orbit at the free surface on the x–z plane*

In this section, we will obtain the expressions for surface wave orbit at the free surface projected onto the x–z plane (the absolute coordinates).

From the first two expressions in (7.1.1), the orbit equation projected onto the x–z plane is obtained as follows.

$$-2wu(P_u - P_w)/(a_u a_w) + w^2/a_w^2 + u^2/a_u^2 = 1 \tag{7.2.1}$$

with

$$a_u = A_u E_{\text{van}}, \quad a_w = A_w E_{\text{van}}.$$

In order to examine the inclination of the orbit, the coordinates of displacements (u, w) are rotated into new ones (u_{xz}, w_{xz}) on the x–z plane by

$$u = u_{xz} C_{xz} - w_{xz} S_{xz}, \quad w = u_{xz} S_{xz} + w_{xz} C_{xz},$$
$$C_{xz} = \cos \Theta_{xz}, \quad S_{xz} = \sin \Theta_{xz}, \tag{7.2.2}$$

where Θ_{xz} is a rotation angle.

$$u_{xz}w_{xz}\Theta_{xz1}+u_{xz}^2\Theta_{xz2}+w_{xz}^2\Theta_{xz3}=1\,, \tag{7.2.3}$$

where

$$\Theta_{xz1}=-2C_{2xz}(P_u-P_w)/(a_u a_w)-S_{2xz}/a_u^2+S_{2xz}/a_w^2\,,$$
$$\Theta_{xz2}=-S_{2xz}(P_u-P_w)/(a_u a_w)+C_{xz}^2/a_u^2+S_{xz}^2/a_w^2\,,$$
$$\Theta_{xz3}=S_{2xz}(P_u-P_w)/(a_u a_w)+S_{xz}^2/a_u^2+C_{xz}^2/a_w^2\,,$$

with

$$C_{2xz}=\cos(2\Theta_{xz})\,,\quad S_{2xz}=\sin(2\Theta_{xz})\,.$$

If the rotation angle is determined so as to make $\Theta_{xz1}=0$ in (7.2.3), such an angle indicates the inclination of the orbit axis of surface waves.

Solving $\Theta_{xz1}=0$ associated with the coupled term $u_{xz}w_{xz}\Theta_{xz1}$, we have a dip angle Θ_{xz} of the orbit and the orbit equation on the x–z plane as follows.

$$\Theta_{xz}=dP_{uw}A_uA_w/(A_u^2-A_w^2) \tag{7.2.4}$$

with

$$dP_{uw}=P_u-P_w\,,$$

and

$$u_{xz}^2/(A_u^2 f_{au}^2 E_{\mathrm{van}}^2)+w_{xz}^2/(A_w^2 f_{aw}^2 E_{\mathrm{van}}^2)=1\,, \tag{7.2.5}$$

with

$$f_{au}=1+dP_{uw}^2A_u^2/(2A_u^2-2A_w^2)\,,$$
$$f_{aw}=1-dP_{uw}^2A_w^2/(2A_u^2-2A_w^2)\,.$$

In the above expression, Θ_{xz} is of the order of d_{ij}, since P_u and P_w are of the order of d_{ij}. On the other hand, the lengths of major and minor axes of the orbit (7.2.5) are affected only by the terms of second order of d_{ij}, i.e., dP_{uw}^2. This indicates that the sensitivity of the orbit dip angle due to anisotropy of the medium is higher than that of the orbit axis.

8. Surface Waves on the Drift Coordinates

In general, surface waves in anisotropic media drift due to the existence of anisotropy of the media. In this section, we will obtain the expressions for two kinds of orbits at the free surface in the drift coordinates, i.e., the *sagittal* (x_d–z_d) plane and the (y_d–z_d) plane perpendicular to it, respectively. Here (x_d, y_d, z_d) indicate the drift coordinates. The latter (y_d–z_d) plane is named the *transverse* plane for the sake of later discussion.

8.1. Displacements at the free surface in the drift coordinates

The absolute coordinates are converted to the drift ones (x_d-, y_d- and z_d-axes) by rotation around the z-axis:

$$x = x_d C_d - y_d S_d, \quad y = x_d S_d + y_d C_d,$$
$$u_d = uC_d + vS_d, \quad v_d = -uS_d + vC_d, \quad w_d = w,$$
$$C_d = \cos\theta_d, \quad S_d = \sin\theta_d, \tag{8.1.1}$$

where (u_d, v_d, w_d) are the (x_d, y_d, z_d)-components of displacement in the drift coordinates, respectively, and θ_d is the drift angle (azimuth) of the absolute coordinates to the drift ones, where the sagittal plane coincides with the x_d–z_d plane.

By substituting (7.1.1) into (8.1.1) and taking the first order of d_{ij}, we have the expressions for new displacements, i.e.,

$$u_d = u,$$
$$v_d = v - \theta_d R_{Lam} a_m / (2k_{rm}{}^{(1/2)}) E_{\mathrm{van}} \sin\Theta_{xn},$$
$$w_d = w, \tag{8.1.2}$$

with

$$E_{\mathrm{van}} = \exp(\theta_x k_r x_d + \theta_y k_r y_d),$$
$$\Theta_{xn} = \omega t - (1+\theta_r) k_r x_d, \tag{8.1.3}$$

where u, v and w are displacements on the absolute coordinates given by (7.1.1) with substitution of $x = x_d$ and $y = y_d$.

Since coordinates are rotated around the z-axis, the displacement w is of course invariant. In a weakly anisotropic medium, the longitudinal component of displacement u is also invariant to first order in d_{ij}, as shown in (8.1.2).

After some reduction, we find

$$\theta_y = (\partial u_{di} / \partial \eta) / u_{di} \quad (i = 1, 2, 3) \tag{8.1.4}$$

where $u_{d1} = u_d$, $u_{d2} = v_d$, $u_{d3} = w_d$ and $\eta = k_r y_d$. This expression indicates that θ_y is expressed by the normalized gradient of each wave component in the transverse direction in the drift coordinates.

By use of (6.2.3) and (8.1.4), we have the expressions

$$d\theta_{14} = (\partial u_{di} / \partial \eta) / u_{di} |_{d14},$$
$$d\theta_{34} = (\partial u_{di} / \partial \eta) / u_{di} |_{d34},$$
$$d\theta_{56} = (\partial u_{di} / \partial \eta) / u_{di} |_{d56}, \tag{8.1.5}$$

i.e., $\{d\theta_{14}, d\theta_{34}, d\theta_{56}\}$ stand for the normalized gradients in the transverse direction associated with the elastic coefficients $\{d_{14}, d_{34}, d_{56}\}$, respectively.

As found above, θ_y in (8.1.3) is closely related to the distribution of waves in the y_d-direction (transverse).

8.2. *Surface wave orbits at the free surface in the drift coordinates*

Since $(u_d, w_d) = (u, w)$ to first order of d_{ij} as found from (8.1.2), the expression for the orbit projected onto the x_d–z_d plane (drift coordinates) is also given by those in *subsection 7.2* (absolute coordinates). Let θ_{dxz} be the dip angle of the surface wave orbit at the free surface on the sagittal plane (drift coordinates). θ_{dxz} is given by

$$\theta_{dxz}=\Theta_{xz},$$

where Θ_{xz} is given by (7.2.4).

By using (v_d, w_d) in (8.1.2) and taking the first order of d_{ij}, the expression for the orbit projected onto the transverse y_d–z_d plane becomes:

$$w_d v_d \Theta_{d1}+w_d^2\Theta_{d2}+v_d^2\Theta_{d3}=A_w^2 d_{vvv}^2 E_{\mathrm{van}}^2, \tag{8.2.1}$$

where

$$d_{vvv}=R_{Lam}m_u^{(1/2)}a_m dk_{yR}+2I_v k_{rm},$$
$$\Theta_{d1}=-4d_{vvv}k_{rm}A_w P_w-8k_{rm}^2 A_w R_v,$$
$$\Theta_{d2}=4k_{rm}^2 R_v^2+d_{vvv}^2,$$
$$\Theta_{d3}=4k_{rm}^2 A_w^2+4k_{rm}^2 A_w^2 P_w^2.$$

By using

$$v_d=v_{dyz}C_{dyz}-w_{dyz}S_{dyz}, \quad w_d=v_{dyz}S_{dyz}+w_{dyz}C_{dyz},$$
$$C_{dyz}=\cos\theta_{dyz}, \quad S_{dyz}=\sin\theta_{dyz}, \tag{8.2.2}$$

the coordinates are transformed into new ones, where θ_{dyz} is the rotation angle of the new coordinates on the y_d–z_d plane and (v_{dyz}, w_{dyz}) are the new variables after the rotation.

In the same way as that in the foregoing *subsection 7.2*, eliminating the coupled term of the rotated equation, we have the dip angle θ_{dyz} and the rotated new orbit equation on the y_d–z_d plane.

$$\theta_{dyz}=-R_v/A_w \tag{8.2.3}$$

and

$$v_{dyz}^2/(I_{dv}^2 E_{\mathrm{van}}^2)+w_{dyz}^2/(A_w^2 E_{\mathrm{van}}^2)=1 \tag{8.2.4}$$

with

$$I_{dv}=I_v+\theta_d a_m R_{Lam}/(2k_{rm}^{(1/2)}). \tag{8.2.5}$$

9. Implications of the Inclination of Surface Wave Orbit on the Sagittal Plane with the Stability of Waves

In this section, the orbit inclination of surface waves on the sagittal $(x_d$–$z_d)$ plane will be considered.

9.1. *Expression for the dip angle of surface wave orbit on the sagittal plane*

As discussed in *subsection 8.2*, the dip angle θ_{dxy} of the surface wave orbit at the free surface on the sagittal plane is expressed as $\theta_{dxz}=\Theta_{xz}$, where Θ_{xz} is given by (7.2.4). By use of (7.1.5) and (7.1.6), this expression becomes

$$\theta_{dxz}=S_{g15}d\theta_{15}+S_{g35}d\theta_{35} \tag{9.1.1}$$

with

$$S_{g15}=(2048b_m^{14}+8192b_m^{12}+13568b_m^{10}+12224b_m^{8}+6640b_m^{6}+2232b_m^{4}+416b_m^{2}+25)R_{L2m}^{2}uuu^{2}/(128k_{rm}^{(1/2)}(32b_m^{6}+64b_m^{4}+40b_m^{2}+7)b_m^{3}C_f^2k_{rm}^{3}R_{Lam}^{2}),$$

$$S_{g35}=(4096b_m^{20}+22528b_m^{18}+54784b_m^{16}+77184b_m^{14}+69536b_m^{12}+41872b_m^{10}+17152b_m^{8}+4734b_m^{6}+820b_m^{4}+70b_m^{2}+1)R_{L2m}^{2}uuu^{2}/(128k_{rm}^{(1/2)}(64b_m^{10}+160b_m^{8}+144b_m^{6}+54b_m^{4}+8b_m^{2}+1)b_m^{3}C_f^2k_{rm}^{4}R_{Lam}^{2}), \quad (9.1.2)$$

where a characteristic equation $a_m=C_f^2/(4k_{rm}b_m)$ in an isotropic medium is used in order to eliminate a_m.

In the expression (9.1.1), the coefficients S_{g15} and S_{g35} are always positive (see (9.1.2)). Therefore, when $d\theta_{15}$ or $d\theta_{35}$ is positive (negative), the inclination θ_{dxz} of the wave orbit associated with $(d\theta_{15}, d\theta_{35})$ is also positive (negative). On the other hand, θ_x is expressed as $\theta_x=d\theta_{15}+d\theta_{35}$ (see (6.1.3)) and θ_x is the factor indicating the stability of surface waves at the free surface (see (7.1.1) and (7.1.2)). Therefore, we obtain the following important result for surface waves at the free surface. The illustration is given in Fig. 2.

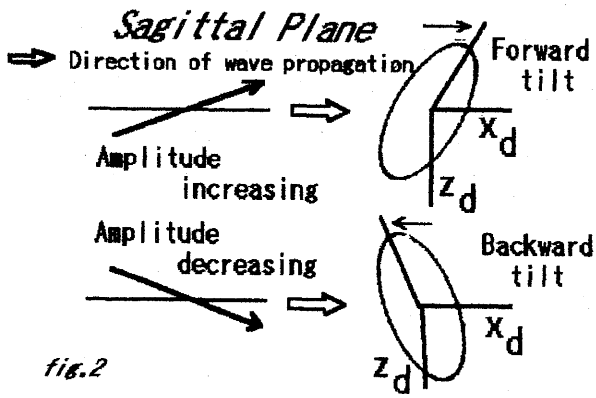

Fig. 2. Implication of the stability of waves with the inclination of surface wave orbit at the free surface on the sagittal plane.

(i) When the advancing wave increases (then $d\theta_{15}$ or $d\theta_{35}$ positive) in the direction of wave propagation, the orbit tilts forward.

(ii) When the advancing wave decreases (then $d\theta_{15}$ or $d\theta_{35}$ negative) in the direction of wave propagation, the orbit tilts backward.

The same feature has been also found in *paper M*, where drift is not taken into consideration. The results of the numerical computation in the present paper are very similar to those in *paper M*, so that numerical results concerning the orbit on the sagittal plane are omitted in order to avoid redundancy.

10. Implications of the Inclination of Surface Wave Orbit on the Transverse Plane with the Drift Angle and Azimuthal Distribution of Waves

In this section, we will discuss the dependences of the inclination of surface wave orbit on the drift angle of waves and the distribution of waves in the azimuthal (transverse) direction.

As for uneven distribution of waves in the azimuthal direction, three causes are considered. The first cause is that due to the existence of anisotropy of an elastic medium, the second is the directivity of the wave source and the third is the refraction of waves due to the discontinuity of the medium. In this paper, we will consider the effect of uneven distribution of waves on the inclination of wave orbit due to anisotropy of the medium.

10.1. Expression for surface wave orbit dip angle on the transverse plane

Substituting (7.1.5) and (7.1.9) into (8.2.3) and taking the terms up to first order in d_{ij}, we have the expression for dip angle θ_{dyz} of the wave orbit on the transverse plane.

$$\theta_{dyz}=\theta_{Ev}+\theta_{Dr}\,, \tag{10.1.1}$$

$$\theta_{Ev}=D_{14}d\theta_{14}+D_{34}d\theta_{34}+D_{56}d\theta_{56}\,, \tag{10.1.2}$$

$$\theta_{Dr}=D_{Dr}(d\theta_{16}+d\theta_{36}+d\theta_{45})\,, \tag{10.1.3}$$

with

$$\begin{aligned}
D_{14}&=C_f^2(1+2b_m{}^2R_{L2m})/(2k_{rm}{}^{(1/2)}b_m{}^3R_{L2m})\,,\\
D_{34}&=8k_{rm}{}^{(3/2)}b_m(3+2(L_m+b_m{}^2R_{L2m}))/(C_f^2R_{L2m})\,,\\
D_{56}&=4k_{rm}{}^{(3/2)}(R_{Lam}+2b_m{}^2R_{L2m})/(C_fb_mR_{L2m})\,,\\
D_{Dr}&=-2k_{rm}{}^{(3/2)}(1+4b_m{}^2)/(C_fb_m)\,,
\end{aligned} \tag{10.1.4}$$

where the characteristic equation $a_m=C_f^2/(4k_{rm}b_m)$ in an isotropic medium is used in order to eliminate a_m. In the above expression, θ_{Ev} is a term associated with the azimuthal distribution of waves through $\{d\theta_{14}, d\theta_{34}, d\theta_{56}\}$ (see (6.2.3), (7.1.1) and (7.1.2)). On the other hand, θ_{Dr} is a term associated with drift angle $\theta_{\rm drift}$ through $\{d\theta_{16}, d\theta_{36}, d\theta_{45}\}$ (see (7.1.4)).

It is noted here that the coefficients in the expressions (10.1.2) and (10.1.3) have constant signs, i.e., $\{D_{14}, D_{34}, D_{56}\}$ are always positive and D_{Dr} is always negative. This fact leads to the following very important results. The term *rightward* or *leftward* in the following discussions will be used in the positive direction of the advancing wave ($x>0$).

(i) When $d\theta_{16}$, $d\theta_{36}$ or $d\theta_{45}$ is positive (negative), the associated drift angle $\theta_{\rm drift}$ (7.1.4) is positive (negative) and the dip angle of the orbit θ_{Dr} ($D_{Dr}<0$) is negative (positive), so that, when the wave drifts rightward (leftward), the orbit axis of the wave on the transverse plane tilts leftward (rightward). The illustration is given in Fig. 3.

(ii) When $d\theta_{14}$, $d\theta_{34}$ or $d\theta_{56}$ is positive (negative), the associated increment of θ_{Ev} is also positive (negative), so that, when the amplitude of the wave increases (decreases) rightward from (8.1.5), the orbit axis of the wave on the transverse plane tilts rightward (leftward). The illustration is given in Fig. 4.

Feature (i) is closely associated with the deviation of the wave propagation

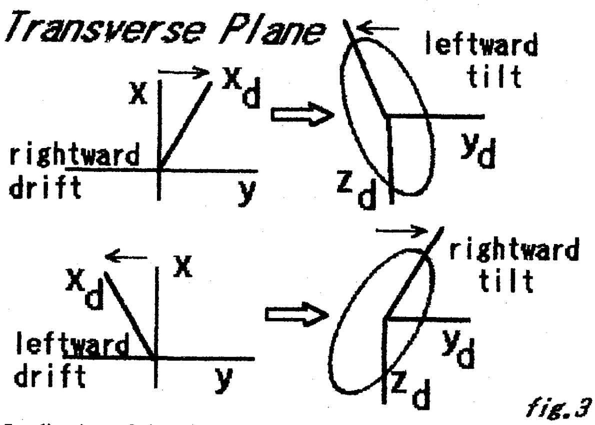

Fig. 3. Implication of the drift of surface waves with the inclination of the wave orbit at the free surface on the transverse plane. The terms of *rightward* and *leftward* indicate the transverse direction in the direction of wave propagation.

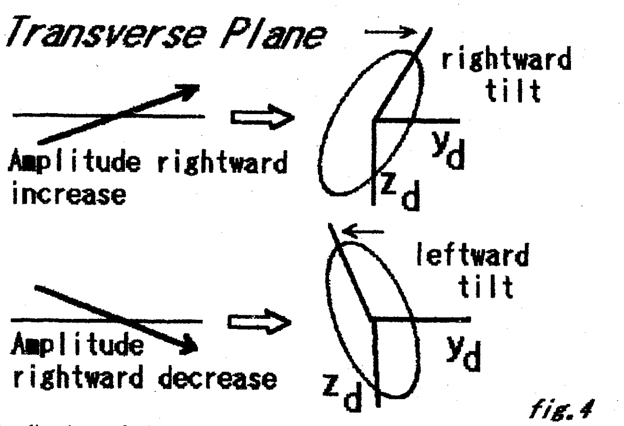

Fig. 4. Implication of the azimuthal distribution of waves with the inclination of surface wave orbit at the free surface on the transverse plane.

from the great circle path in practical problems. Feature (ii) is produced by the uneven distribution of waves in the azimuthal direction due to anisotropy of the medium.

The factors $\{D_{14}, D_{34}, D_{56}\}$ in (10.1.2) and D_{Dr} in (10.1.3) indicate the extent of the response of the dip angle θ_{dyz} of the orbit, on the transverse plane, versus the drift angle θ_{drift} (7.1.4) and the wave gradients (8.1.5).

These response factors will be evaluated numerically for some specified values of $L_m(=\lambda/\mu)$. In the computation of (10.1.4), the characteristic equation

$$a_m = C_f^2/(4k_{rm}b_m)$$

in an isotropic medium and the following expressions are used.

$$R_{Lam} = 1 + L_m, \quad R_{L2m} = 2 + L_m,$$
$$C_f = 2b_m^{\ 2} + 1, \quad k_{rm} = b_m^{\ 2} + 1.$$

For $L_m(=\lambda/\mu)=0.5$, 1.0, 2.0, 3.0, 4.0, 5.0, 10.0 and 50.0, the computed values of D_{14}, D_{34}, D_{56} and D_{Dr} are arranged here.

$L_m=0.5$	$D_{14}=7.63382$	$D_{34}=4.97841$	$D_{56}=8.29767$	$D_{Dr}=-7.50053$
$L_m=1$	$D_{14}=7.66250$	$D_{34}=4.79698$	$D_{56}=9.09526$	$D_{Dr}=-7.62737$
$L_m=2$	$D_{14}=7.46135$	$D_{34}=4.63438$	$D_{56}=10.28832$	$D_{Dr}=-7.83848$
$L_m=3$	$D_{14}=7.17333$	$D_{34}=4.56124$	$D_{56}=11.09147$	$D_{Dr}=-7.98157$
$L_m=4$	$D_{14}=6.89985$	$D_{34}=4.52031$	$D_{56}=11.65893$	$D_{Dr}=-8.08125$
$L_m=5$	$D_{14}=6.66135$	$D_{34}=4.49435$	$D_{56}=12.07887$	$D_{Dr}=-8.15398$
$L_m=10$	$D_{14}=5.89614$	$D_{34}=4.43968$	$D_{56}=13.17846$	$D_{Dr}=-8.33976$
$L_m=50$	$D_{14}=4.78053$	$D_{34}=4.39430$	$D_{56}=14.43625$	$D_{Dr}=-8.54366$

As found in the above values, each value of D_{14}, D_{34}, D_{56} and D_{Dr} is very flat over a wide range of L_m from 0.5 to 50. By taking the values $D_{14}=7.0$, $D_{34}=4.5$, $D_{56}=11.0$ and $D_{Dr}=-8.0$ as the typical values for each D_{ij}, we can affirm that the response of the orbit inclination on the transverse plane to the anisotropy of the medium is much higher than that of the drift angle and transverse gradient.

11. Conclusion

In weakly anisotropic media, the axis of the surface wave orbit at the free surface tilts forward (backward) when the amplitude of waves increases (decreases) in the direction of wave propagation. This feature has also been found in the previous paper where drift is not taken into account. Even in the theory considering drift (present paper), the above-mentioned feature holds valid.

In anisotropic media, drift and uneven distribution of waves in the azimuthal direction occur. These properties are derived from the characteristic equation for surface waves as the solutions. Such a drift and uneven distribution yield the inclination of surface wave orbit at the free surface in the *transverse* direction (perpendicular to the advancing direction of waves). The obtained result is as follows. The terms *rightward* and *leftward* used here indicate azimuthal directions in the direction of wave propagation.

(i) When waves drift rightward (leftward), the orbit tilts leftward (rightward).

(ii) When the amplitude of waves increases (decreases) rightward, the orbit tilts rightward (leftward).

Thus, drift of surface waves in anisotropic media can be explained by use of the fundamental solutions to the characteristic equation of surface waves.

References

BRUGGER, K., 1964, Thermodynamic Definition of Higher Order Elastic Coefficients, *Phys. Rev.*, **133**, 6A, A1611–A1612.

BUCHWALD, V. T., 1961, Rayleigh waves in anisotropic media, *Q. Jl. Mechs. appl. Math.*, **14**, 461–469.

BUCHWALD, V. T. and DAVIS, A., 1963, Surface waves in elastic media with cubic symmetry, *Q. Jl. Mechs. appl. Math.*, **16**, 283–294.
CRAMPIN, S., 1975, Distinctive particle motion of surface waves as a diagnostic of anisotropic layering, *Geophys. J. R. astr. Soc.*, **40**, 177–186.
LANDAU, L. D. and E. M. LIFSHITZ, 1985, (佐藤常三訳), 弾性理論, 東京図書, 149–150.
MASON, W. P., 1964, *Physical Acoustics*, Vol. 1A, p. 1, Academic Press Inc., New York.
MOMOI, T., 1991, The Orbit of Surface Waves in Weakly Anisotropic Media, *Bull. Earthq. Res. Inst.*, **66**, 475–495.
STONELEY, R., 1955, The propagation of surface elastic waves in a cubic crystal, *Proc. R. Soc. A.*, **232**, 447–458.
STONELEY, R., 1963, The propagation of surface waves in an elastic medium with orthorhombic symmetry, *Geophys. J. R. astr. Soc.*, **8**, 176–186.
SYNGE, J. L., 1957, Elastic waves in anisotropic media, *J. Math. Phys.*, **35**, 323–334.
THURSTON, R. N. and K. BRUGGER, 1964, Third-Order Elastic Constants and the Velocity of Small Amplitude Elastic Waves in Homogeneously Stressed Media, *Phys. Rev.*, **133**, A1604.
VOIT, W., 1928, *Lehrbuch der Kristallphysik*, Teubner, Leipzig.

Appendix A

The coefficients in (7.1.5) and (7.1.6) are expressed as

$$a_{u11}=-k_{rm}{}^{(1/2)}(2048b_m{}^{16}+8192b_m{}^{14}+13568b_m{}^{12}+12672b_m{}^{10}+8416b_m{}^{8}+4896b_m{}^{6}+2296b_m{}^{4}+636b_m{}^{2}+69)C_fR_{L2m}\,uuu/(128(64b_m{}^{10}+64b_m{}^{8}-104b_m{}^{6}-164b_m{}^{4}-62b_m{}^{2}-3)b_m{}^{3}k_{rm}{}^{4}),$$

$$a_{u33}=-k_{rm}{}^{(1/2)}(131072b_m{}^{28}+884736b_m{}^{26}+2736128b_m{}^{24}+5165056b_m{}^{22}+6717440b_m{}^{20}+6474240b_m{}^{18}+4867584b_m{}^{16}+2942592b_m{}^{14}+1431040b_m{}^{12}+542704b_m{}^{10}+152208b_m{}^{8}+29520b_m{}^{6}+3600b_m{}^{4}+224b_m{}^{2}+3)R_{L2m}\,uuu/(128(512b_m{}^{16}+1152b_m{}^{14}+128b_m{}^{12}-1712b_m{}^{10}-1856b_m{}^{8}-784b_m{}^{6}-136b_m{}^{4}-10b_m{}^{2}-1)b_m{}^{3}C_f^2k_{rm}{}^{4}),$$

$$a_{u55}=k_{rm}{}^{(1/2)}(2048b_m{}^{16}+7168b_m{}^{14}+10496b_m{}^{12}+8960b_m{}^{10}+5600b_m{}^{8}+2880b_m{}^{6}+1088b_m{}^{4}+248b_m{}^{2}+27)R_{L2m}\,uuu/(128(8b_m{}^{4}+12b_m{}^{2}+5)b_m{}^{3}C_f^2k_{rm}{}^{4}),$$

$$a_{u13}=k_{rm}{}^{(1/2)}(8192b_m{}^{20}+34816b_m{}^{18}+64512b_m{}^{16}+71424b_m{}^{14}+55936b_m{}^{12}+33632b_m{}^{10}+14880b_m{}^{8}+4384b_m{}^{6}+888b_m{}^{4}+167b_m{}^{2}+21)R_{L2m}\,uuu/(128(96b_m{}^{10}+232b_m{}^{8}+192b_m{}^{6}+59b_m{}^{4}+5b_m{}^{2}+1)b_m{}^{3}C_f^2k_{rm}{}^{3}),$$

$$a_{w11}=-(1024b_m{}^{14}+3584b_m{}^{12}+5504b_m{}^{10}+5376b_m{}^{8}+3920b_m{}^{6}+2000b_m{}^{4}+576b_m{}^{2}+65)C_f^2R_{L2m}\,uuu/(128(64b_m{}^{10}+64b_m{}^{8}-104b_m{}^{6}-164b_m{}^{4}-62b_m{}^{2}-3)b_m{}^{4}k_{rm}{}^{3}),$$

$$a_{w33}=-(65536b_m{}^{28}+475136b_m{}^{26}+1605632b_m{}^{24}+3352576b_m{}^{22}+4840448b_m{}^{20}+5133056b_m{}^{18}+4145408b_m{}^{16}+2596800b_m{}^{14}+1260528b_m{}^{12}+464032b_m{}^{10}+124880b_m{}^{8}+23680b_m{}^{6}+3112b_m{}^{4}+273b_m{}^{2}+11)R_{L2m}\,uuu/(128(512b_m{}^{16}+1152b_m{}^{14}+128b_m{}^{12}-1712b_m{}^{10}-1856b_m{}^{8}-784b_m{}^{6}-136b_m{}^{4}-10b_m{}^{2}-1)b_m{}^{4}C_fk_{rm}{}^{4}),$$

$$a_{w55}=(1024b_m{}^{16}+4096b_m{}^{14}+7296b_m{}^{12}+7872b_m{}^{10}+5872b_m{}^{8}+3136b_m{}^{6}$$
$$+1128b_m{}^{4}+233b_m{}^{2}+19)R_{L2m}uuu/(128(8b_m{}^{4}+12b_m{}^{2}+5)b_m{}^{4}C_f k_{rm}{}^{4}),$$

$$a_{w13}=(4096b_m{}^{20}+19456b_m{}^{18}+41984b_m{}^{16}+55680b_m{}^{14}+51072b_m{}^{12}+33392b_m{}^{10}$$
$$+15216b_m{}^{8}+4736b_m{}^{6}+1069b_m{}^{4}+188b_m{}^{2}+17)R_{L2m}uuu/(128(96b_m{}^{10}$$
$$+232b_m{}^{8}+192b_m{}^{6}+59b_m{}^{4}+5b_m{}^{2}+1)b_m{}^{4}C_f k_{rm}{}^{3}),$$

and

$$a_{u15}=k_{rm}{}^{(1/2)}(8192b_m{}^{18}+32768b_m{}^{16}+56320b_m{}^{14}+57088b_m{}^{12}+41280b_m{}^{10}$$
$$+24064b_m{}^{8}+11008b_m{}^{6}+3504b_m{}^{4}+712b_m{}^{2}+81)R_{L2m}uuu/(128(32b_m{}^{6}$$
$$+64b_m{}^{4}+40b_m{}^{2}+7)b_m{}^{3}C_f^2 k_{rm}{}^{4}),$$

$$a_{u35}=k_{rm}{}^{(1/2)}(16384b_m{}^{24}+90112b_m{}^{22}+223232b_m{}^{20}+333312b_m{}^{18}+341888b_m{}^{16}$$
$$+262016b_m{}^{14}+157632b_m{}^{12}+74672b_m{}^{10}+26816b_m{}^{8}+6958b_m{}^{6}+1266b_m{}^{4}$$
$$+154b_m{}^{2}+9)R_{L2m}uuu/(128(64b_m{}^{10}+160b_m{}^{8}+144b_m{}^{6}+54b_m{}^{4}$$
$$+8b_m{}^{2}+1)b_m{}^{3}C_f^2 k_{rm}{}^{5}),$$

$$a_{w15}=(4096b_m{}^{18}+18432b_m{}^{16}+37376b_m{}^{14}+46208b_m{}^{12}+39648b_m{}^{10}+24784b_m{}^{8}$$
$$+11056b_m{}^{6}+3284b_m{}^{4}+589b_m{}^{2}+53)R_{L2m}uuu/(128(32b_m{}^{6}+64b_m{}^{4}$$
$$+40b_m{}^{2}+7)b_m{}^{4}C_f k_{rm}{}^{4}),$$

$$a_{w35}=(8192b_m{}^{24}+49152b_m{}^{22}+136192b_m{}^{20}+232704b_m{}^{18}+275520b_m{}^{16}+239136b_m{}^{14}$$
$$+155456b_m{}^{12}+75424b_m{}^{10}+26718b_m{}^{8}+6666b_m{}^{6}+1113b_m{}^{4}+113b_m{}^{2}$$
$$+5)R_{L2m}uuu/(128(64b_m{}^{10}+160b_m{}^{8}+144b_m{}^{6}+54b_m{}^{4}+8b_m{}^{2}+1)b_m{}^{4}C_f k_{rm}{}^{5}).$$

弱い異方性媒質における表面波の軌道とその漂流現象および方位角方向への波の空間分布との関係

桃　井　高　夫

一般に，異方性媒質において表面波は媒質の異方性のために，漂流現象やその進行波の横（方位角）方向に振幅の不均質な空間分布を引き起こす．

本論文では弱い異方性媒質において，波の進行方向と垂直な平面に投影された表面波の軌道の傾きと波の漂流現象および方位角方向の波の空間分布との間の関係が論じられている．つぎのような結論が得られている．

「左右」の記述は波の進行方向に向かって方位角方向への左右を意味する．

(i)　進行波が右（左）に漂流するとき波の軌道は左（右）に傾く．

(ii)　右方向に波の振幅が増加（減少）するとき波の軌道は右（左）に傾く．

特記すべきことは，上述の現象が異方性媒質における表面波の特性方程式の解として基本的に存在することである．