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# *The Orbit of Surface Waves in Weakly Anisotropic Media*

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## Abstract

Surface waves in the weakly anisotropic media are discussed focusing attention on the relation between the Rayleigh wave orbit and its velocity and stability.

The wave number in weakly anisotropic media involves imaginary terms. Rayleigh waves in an anisotropic medium always face a stability problem due to the imaginary term.

In weakly anisotropic media, the longitudinal and vertical components ($u$, $w$) compose quasi-Rayleigh waves which resemble Rayleigh waves in an isotropic medium. The horizontally transverse components ($v$) behave as quasi-Love waves which are independent of the former quasi-Rayleigh waves with respect to the elastic coefficients. The quasi-Love waves then decrease in amplitude with depth in the material.

The increase (or decrease) of Rayleigh wave velocity due to anisotropy of the media leads to shrinkage (or expansion) of quasi-Rayleigh wave orbit. In this case, the vertical component of the Rayleigh wave is more sensitive than the horizontal one to anisotropy.

The Rayleigh wave orbit is quite sensitive to the velocity change due to anisotropy. For instance, a one percent velocity increase causes about 10 percent increase in the Rayleigh wave orbit size (major and/or minor axis).

When the quasi-Rayleigh waves are stable (or unstable), the Rayleigh wave orbit axis tilts backward (or forward). When a Rayleigh wave increases (decreases) in amplitude by 10 or 20 percent after traveling the distance of the wavelength, the inclination angle of the orbit at the surface is in the range 5∼30 degrees.

## Introduction

Seismic velocity anisotropy is a widespread phenomenon in the Earth's materials. A large variety of mechanisms give rise to anisotropy: crystal alignments, grain alignments, preferential alignments of cracks (including pore closure under pressure), stress induced effects, the interleaving of thin sedimentary beds. Until about 1980, seismic data could be adequately explained by assuming isotropy, so anisotropy could be largely ignored. With the increasing resolution of seismic observations, however, there is a growing awareness that the assumption of isotropy is often violated. Anisotropy has been widely detected in the crust and upper mantle (e.g. FUCHS (1977); STEPHEN (1981); ANDERSON and DZIEWONSKI (1982)) and laboratory measurements imply that the phenomenon must be widespread in both crystalline and sedimentary rocks (CHRISTENSEN and SALISBURY (1979); BACHMAN (1979); BABUSKA (1981)).

There are fundamental differences between wave propagation in isotropic and anisotropic media (CRAMPIN (1977, 1981)). In an isotropic medium, $P$-wave particle motion is normal to a wavefront so the $P$ polarization vector coincides with the phase propagation vector. $S$ motion may be in any direction orthogonal to $P$. In an anisotropic medium, the $P$ polarization vector need not coincide with the phase propagation vector. Two quasi-shear polarizations form a mutually orthogonal set with the $P$. Thus for any particular direction of phase propagation, there are three body waves with fixod orthogonal polarizations.

Even in the case of Rayleigh waves, the effect of anisotropy is very significant. The theory of surface wave propagation in an anisotropic half-space has been discussed by SYNGE (1957) and BUCHWALD (1961), among others. Propagation in a half-space with cubic symmetry has been investigated by STONELY (1955), and BUCHWALD and DAVIS (1963), and with orthorhombic symmetry by STONELY (1963).

In 1975, Crampin calculated the particle motion of surface waves propagating in particular symmetry directions in anisotropic media and showed that propagation in some directions reveals particle motion anomalies diagnostic of the symmetry. In this paper, we will study the effects of the weak but general anisotropy of the media on the Rayleigh waves focusing particular attention on the relation between the Rayleigh wave orbit at the free surface and the velocity increment or the wave stability due to the anisotropy. A homogeneous half-space model is then used.

The theory was developed using *computer algebra* installed on an NEC 9800 computer.

## 1. Expression of Energy

In anisotropic media, the strain energy function $E_n$ can be expressed by use of a strain tensor $U_J$ up to the second order in $U_J$

$$E_n = (1/2!) c_{JP} * U_J * U_P, \tag{1.1}$$

where $*$ indicates the product, summations over repeated indices $\{J, P\}$ being implied (this convention will be used, unless stated otherwise), and $c_{JP}$ the elastic coefficients defined by K. BRUGGER (1964), i.e.,

$$c_{JP} = \partial^2 E_n / \partial U_J \partial U_P. \tag{1.2}$$

In the above, Voigt Notation $\{ij \sim J\}$

$$\{11\sim 1,\ 22\sim 2,\ 33\sim 3,\ 23\sim 4,\ 13\sim 5,\ 12\sim 6\}$$

and the convention

$$U_{ij} = (1/2)*(1+\delta_{ij})*U_J \tag{1.3}$$

with

$$U_{ij} = (u_{ij} + u_{ji})/2 \tag{1.4}$$

are used, where $u_{ij} = \partial u_i / \partial x_j$ and $\{u_1, u_2, u_3\}$ are displacement components in the directions of the coordinate axes $\{x_1, x_2, x_3\}$. In later discussions, $\{u_1, u_2, u_3\}$ and $\{x_1, x_2, x_3\}$ will be alternatively expressed by $\{u, v, w\}$ and $\{x, y, z\}$. $\delta_{ij}$ is the kronecker delta with suffix $i$, $j$.

In the later discussion, the elastic coefficients $C_{JP}$ normalized by $\rho\omega^2$ will be used, i.e.

$$C_{JP} = c_{JP} / (\rho\omega^2), \tag{1.5}$$

where $\rho$ is the density of the medium and $\omega$ the angular frequency of the Rayleigh waves.

In the weakly anisotropic case, the above normalized elastic coefficients are expressed as

$$\begin{array}{llllll}
C_{11} = L_{2m} + d_{11}, & & C_{22} = L_{2m} + d_{22}, & & C_{33} = L_{2m} + d_{33}, & \\
C_{12} = L_a + d_{12}, & & C_{13} = L_a + d_{13}. & & C_{23} = L_a + d_{23}, & \\
C_{44} = m_u + d_{44}, & & C_{55} = m_u + d_{55}, & & C_{66} = m_u + d_{66}, & \\
C_{14} = d_{14}, & C_{15} = d_{15}, & C_{16} = d_{16}, & C_{24} = d_{24}, & C_{25} = d_{25}, & C_{26} = d_{26} \\
C_{34} = d_{34}, & C_{35} = d_{35}, & C_{36} = d_{36}, & C_{45} = d_{45}, & C_{46} = d_{46}, & C_{56} = d_{56},
\end{array} \tag{1.6}$$

where $L_a = \lambda / (\rho\omega^2)$ and $m_u = (\mu / \rho\omega^2)$ are the elastic coefficients normalized by $\rho\omega^2$ in the case of an isotropic medium, $L_{2m} = L_a + 2*m_u$, and $d_{ij}$ ($i, j$: integers) is the weak deviation of the anisotropic elastic coefficients from the isotropic ones.

## 2. Equations

The stress tensor $S_{ij}$ is related to the energy function

$$S_{ij}=\partial E_n/\partial u_{ij}, \tag{2.1}$$

By use of the above relation, the governing equations can be expressed (LANDAU and LIFSHITZ, 1985) by

$$\rho\partial^2 u_i/\partial t^2=\partial S_{ij}/\partial x_j \qquad (i=1, 2, 3), \tag{2.2}$$

where $t$ is the time factor.

By use of (2, 1) and the expressions for energy in the foregoing section, the above two equations are reduced to the following.

$$\rho*u_{t2}=q_1, \quad \rho*v_{t2}=q_2, \quad \rho*w_{t2}=q_3, \tag{2.3}$$

where

$$\begin{aligned}q_1=&u_{x2}*c_{11}+v_{xy}*c_{12}+v_{y2}*c_{26}+v_{yz}*c_{25}+w_{xz}*c_{13}\\&+w_{yz}*c_{36}+w_{z2}*c_{35}+c_{16}*P_{a2}+c_{15}*P_{a3}+c_{66}*P_{a4}\\&+c_{56}*P_{a6}+c_{55}*P_{a8}+c_{14}*P_{a7}+c_{46}*P_{a11}+c_{45}*P_{a12},\end{aligned}$$

$$\begin{aligned}q_2=&u_{x2}*c_{16}+u_{xy}*c_{12}+u_{xz}*c_{14}+v_{y2}*c_{22}+w_{xz}*c_{36}\\&+w_{yz}*c_{23}+w_{z2}*c_{34}+c_{26}*P_{b6}+c_{25}*P_{b8}+c_{66}*P_{b2}\\&+c_{56}*P_{b4}+c_{46}*P_{b7}+c_{45}*P_{b9}+c_{24}*P_{b12}+c_{44}*P_{b13},\end{aligned}$$

$$\begin{aligned}q_3=&u_{x2}*c_{15}+u_{xy}*c_{14}+u_{xz}*c_{13}+v_{xy}*c_{25}+v_{y2}*c_{24}\\&+v_{yz}*c_{23}+w_{z2}*c_{33}+c_{36}*P_{c9}+c_{35}*P_{c10}+c_{56}*P_{c2}\\&+c_{55}*P_{c4}+c_{46}*P_{c6}+c_{45}*P_{c8}+c_{34}*P_{c14}+c_{44}*P_{c12},\end{aligned}$$

with

$$P_{a2}=2*u_{xy}+v_{x2}, \quad P_{a3}=2*u_{xz}+w_{x2}, \quad P_{a4}=u_{y2}+v_{xy},$$
$$P_{a6}=2*u_{yz}+v_{xz}+w_{xy}, \quad P_{a7}=v_{xz}+w_{xy}, \quad P_{a8}=u_{z2}+w_{xz},$$
$$P_{a11}=v_{yz}+w_{y2}, \quad P_{a13}=v_{z2}+w_{yz},$$
$$P_{b2}=u_{xy}+v_{x2}, \quad P_{b4}=u_{xz}+w_{x2}, \quad P_{b6}=u_{y2}+2*v_{xy},$$
$$P_{b7}=u_{yz}+2*v_{xz}+w_{xy}, \quad P_{b8}=u_{yz}+w_{xy}, \quad P_{b9}=u_{z2}+w_{xz},$$
$$P_{b12}=2*v_{yz}+w_{y2}, \quad P_{b13}=v_{z2}+w_{yz},$$
$$P_{c2}=u_{yz}+v_{x2}, \quad P_{c4}=u_{xz}+w_{x2}, \quad P_{c6}=u_{y2}+v_{xy},$$
$$P_{c8}=u_{yz}+v_{xz}+2*w_{xy}, \quad P_{c9}=u_{yz}+v_{xz},$$
$$P_{c10}=u_{z2}+2*w_{xz}, \quad P_{c12}=v_{yz}+w_{y2}, \quad P_{c14}=v_{z2}+2*w_{yz},$$

and

$$\begin{aligned}
&u_{t2}=\partial^2u/\partial t^2, && n_{x2}=\partial^2u/\partial x^2, && u_{xy}=\partial^2u/\partial x\partial y,\\
&u_{xz}=\partial^2u/\partial x\partial z, && u_{y2}=\partial^2u/\partial y^2, && u_{yz}=\partial^2u/\partial y\partial z,\\
&u_{z2}=\partial^2u/\partial z^2,\\
&v_{t2}=\partial^2v/\partial t^2, && v_{x2}=\partial^2v/\partial x^2, && v_{xy}=\partial^2v/\partial x\partial y,\\
&v_{xz}=\partial^2v/\partial x\partial z, && v_{y2}=\partial^2v/\partial y^2, && v_{yz}=\partial^2v/\partial y\partial z,\\
&v_{z2}=\partial^2v/\partial z^2,
\end{aligned}$$

$w_{t2}=\partial^2 w/\partial t^2$, $w_{x2}=\partial^2 w/\partial x^2$, $w_{xy}=\partial^2 w/\partial x\partial y$,
$w_{xz}=\partial^2 w/\partial x\partial z$, $w_{y2}=\partial^2 w/\partial y^2$, $w_{yz}=\partial^2 w/\partial y\partial z$,
$w_{z2}=\partial^2 w/\partial z^2$,

## 3. Surface Conditions

By use of relation (2.1), the surface conditions are expressed as

$$\begin{aligned} S_{31}&=u_{11}*C_{15}+u_{21}*C_{56}+u_{22}*C_{25}+u_{12}*C_{56}+u_{23}*C_{45}\\ &+u_{31}*C_{55}+u_{13}*C_{55}+u_{32}*C_{45}+u_{33}*C_{35}=0,\\ S_{32}&=u_{11}*C_{14}+u_{21}*C_{46}+u_{22}*C_{24}+u_{12}*C_{46}+u_{23}*C_{44}\\ &+u_{31}*C_{45}+u_{13}*C_{45}+u_{32}*C_{44}+u_{33}*C_{34}=0,\\ S_{33}&=u_{11}*C_{13}+u_{21}*C_{36}+u_{22}*C_{23}+u_{12}*C_{36}+u_{23}*C_{34}\\ &+u_{31}*C_{35}+u_{13}*C_{35}+u_{32}*C_{34}+u_{33}*C_{33}=0,\\ &\text{at } z=0. \end{aligned} \tag{3.1}$$

## 4. Splitting of Lobe of the Quasi-Rayleigh Waves

In order to obtain the expression for the Rayleigh waves, we assume the following for the displacements:

$$u=A_u*E_p,\quad v=A_v*E_p,\quad w=A_w*E_p, \tag{4.1}$$

where

$$E_p=\exp(-i*hk*x-ab*z)$$

with $hk$ and $ab$, in general complex, being the x- and z- wave numbers. In the above expression, the time factor $\exp(i*\omega t)$ is omitted. This convention will be followed in the subsequent discussion.

Since weakly anisotropic media are considered in this paper, $hk$ and $ab$ are expressed as

$$hk=k_r+dk_r,\quad ab=ab_o+dab, \tag{4.2}$$

where $k_r$ is the wave number in the isotropic medium, $ab_o=a_o$ $(=\alpha)$ or $b_o(=\beta)$ with

$$a_o^2=k_r^2-1/(L_a+2*m_u) \text{ and } b_o^2=k_r^2-1/m_u, \tag{4.3}$$

$dk_r$ and $dab$ indicate the deviations of the wave number and $\alpha$ (or $\beta$), respectively, in the weakly anisotropic medium from an isotropic one.

Substituting (4.1) into Eqs. (2.3) and taking the first order of $d_{ij}$, we have the deviations $dab=\{da,\ db_1$ and $db_2\}$ of $ab$ from $a_o$ and $b_o$ in the isotropic case, i.e.,

$$da=ab-a_o, \quad db_1 \text{ or } db_2=ab-b_o,$$

as follows, where it will be found later that the deviation of $b_o$ splits in the anisotropic case.

$$da=d_{11}*dL_1+d_{33}*dL_2+d_{55}*dL_3+d_{13}*dL_4 \\ +i*d_{15}*dL_5+i*d_{35}*dL_6+dk_r*dL_7 \tag{4.4}$$

with

$$dL_1=k_r^4/(2*a_o), \quad dL_2=a_o^3/2, \quad dL_3=-2*a_o*k_r^2,$$
$$dL_4=-a_o*k_r^2, \quad dL_5=-2*k_r^3, \quad dL_6=2*a_o^2*k_r,$$
$$dL_7=k_r/a_o.$$

$$db_1=dN+dS, \quad db_2=dN-dS, \tag{4.5}$$

where

$$dN=dN_1*d_{15}-dN_1*d_{35}+dN_5*d_{11}+dN_5*d_{33}-2*dN_5*d_{13} \\ +d_{46}*dN_3+d_{44}*dN_{10}+dk_r*dN_4+d_{66}*dN_8+d_{55}*dN_9 \tag{4.5.1}$$

with

$$dN_1=i*k_r*C_f/(2*m_u), \quad dN_3=-i*k_r/(2*m_u),$$
$$dN_4=k_r/b_o, \quad dN_5=-k_r^2*b_o/4,$$
$$dN_8=k_r^2/(4*m_u*b_o), \quad dN_9=C_f^2/(4*m_u^2*b_o),$$
$$dN_{10}=-b_o/(4*m_u), \quad C_f=2*k_r^2*m_u-1,$$

and

$$dS=(S_{re}+i*S_{im})^{(1/2)}, \tag{4.5.2}$$

with $S_{re}$ and $S_{im}$ given in Appendix A1.

The lobe of the Rayleigh waves associated with the distortional part splits in an anisotropic medium. Expressions (4.4) and (4.5) involve the imaginary terms. This implies the existence of the generalized Rayleigh waves discussed by SYNGE (1957) which are propagated at an inclination to the free surface.

## 5. Expressions for Surface Waves

Since we have three components of $ab$ as discussed in the foregoing section, the Rayleigh waves in the anisotropic media are expressed as

$$u=E_{aL}*A_u+E_{b1}*B_{u1}+E_{b2}*B_{u2},$$
$$v=E_{aL}*A_v+E_{b1}*B_{v1}+E_{b2}*B_{v2},$$
$$w=E_{aL}*A_w+E_{b1}*B_{w1}+E_{b2}*B_{w2}, \tag{5.1}$$

where

$$a=a_0+da,\ b_j=b_0+db_j \qquad (j=1,2)$$
$$E_{aL}=\exp(-i*x*hk-z*a),$$
$$E_{b1}=\exp(-i*x*hk-z*b_1),$$
$$E_{b2}=\exp(-i*x*hk-z*b_2),$$

and $A_u, A_v, A_w, B_{u1}, B_{v1}, B_{w1}, B_{u2}, B_{v2}, B_{w2}$ are the amplitudes determined by the surface conditions.

## 6. Characteristic Equation

In this section, the deviation, $dk_r$, of the wave number in (4.2) will be obtained.

Substituting (5.1) into (3.1), we have, taking the terms up to the first order of $d_{ij}$,

$$dk_r=R_{dkr}+i*I_{dkr}, \tag{6.1}$$
$$R_{dkr}=dk_{r11}+dk_{r33}+dk_{r55}+dk_{r13}, \tag{6.2}$$
$$I_{dkr}=dk_{r15}+dk_{r35}, \tag{6.3}$$

where

$$dk_{r11}=d_{11}*Dr_1,\quad dk_{r33}=d_{33}*Dr_2,\quad dk_{r55}=d_{55}*Dr_3,\quad dk_{r13}=d_{13}*Dr_4,$$
$$dk_{r15}=d_{15}*Dr_5,\quad dk_{r35}=d_{35}*Dr_6,$$
$$Dr_0=8*\mathrm{SQRT}(m_u)*(16*b_{0m}{}^4+28*b_{0m}{}^3+18*b_{0m}{}^2+7*b_{0m}+2)$$
$$*(4*b_{om}+3)*b_{om}*m_u*uuu,$$
$$Dr_1=-(2*(64*b_{0m}{}^5+64*b_{0m}{}^4-104*b_{0m}{}^3-164*b_{0m}{}^2$$
$$-62*b_{0m}-3)*(4*b_{0m}+3)*b_{0m}{}^2*k_{rm}{}^2)/(\mathrm{SQST}(k_{rm})*Dr_0),$$
$$Dr_2=-((512*b_{0m}{}^8+1152*b_{0m}{}^7+128*b_{0m}{}^6-1712*b_{0m}{}^5-1856*b_{0m}{}^4$$
$$-784*b_{0m}{}^3-136*b_{0m}{}^2-10*b_{0m}-1)*C_f{}^3)/(8*\mathrm{SQRT}(k_{rm})*Dr_0*k_{rm}),$$
$$Dr_3=-(2*(8*b_{0m}{}^2+12*b_{0m}+5)*(4*b_{0m}+3)*b_{0m}*C_f{}^5*k_{rm})/\mathrm{SQRT}(k_{rm})*Dr_0),$$
$$Dr_4=-(4*\mathrm{SQRT}(k_{rm})*(96*b_{0m}{}^5+232*b_{0m}{}^4+192*b_{0m}{}^3+59*b_{0m}{}^2$$
$$+5*b_{0m}+1)*b_{0m}*C_f{}^3)/Dr_0,$$
$$Dr_5=-(8*\mathrm{SQRT}(b_{0m})*(32*b_{0m}{}^3+64*b_{0m}{}^2+40*b_{0m}+7)*b_{0m}C_f{}^3*k_{rm}{}^2)/Dr_0,$$
$$Dr_6=4*\mathrm{SQRT}(b_{0m})*(64*b_{0m}{}^5+160*b_{0m}{}^4+144*b_{0m}{}^3+54*b_{0m}{}^2$$
$$+8*b_{0m}+1)*C_f{}^3*k_{rm}/Dr_0,$$

with $b_{0m}=b_0^2*m_u$ (for $b_0$, refer to (4.3)),

$$C_f=2*b_{0m}+1,\quad k_{rm}=b_{0m}+1,\quad uuu=8*b_{0m}{}^2+8*b_{0m}+1.$$

In the derivation of the above, the characteristic equation in the case of isotropic media

$$C_f{}^4-16*k_{rm}{}^2*b_{0m}*a_{0m}=0$$

with

$$a_{0m}=a_0^2*m_u=k_{rm}-1/(2+L_m)$$

was used in order to eliminate the ratio $L_m$.

As found from (6.1), the deviation term $dk_r$ includes the imaginary terms, $I_{dkr}$, associated with $d_{15}$ and $d_{35}$. This implies that the Rayleigh waves in an anisotropic medium always face a stability problem depending on the sign of the imaginary terms. As known from (4.4) and (4.5), $da$, $db_1$ and $db_2$ also include imaginary terms, so the energy profile of the quasi-Rayleigh waves along the vertical line is always moving in response to the stable or unstable state of the waves. It must be noted here that the stability of the Rayleigh waves depends on only two elastic coefficients, $d_{15}$ and $d_{35}$, out of the 21 coefficients in a weakly anisotropic medium.

## 7. Surface Wave Orbit at the Free Surface

Substituting (5.1) into (3.1), we can obtain the following expressions for the Rayleigh waves at the free surface.

$$\begin{aligned} u&=A_{ur}*\cos(O_{kr}+P_{ur}),\\ w&=A_{wr}*\cos(O_{kr}+P_{wr}),\\ v&=A_{vr}*\cos(O_{kr}+P_{vr}), \end{aligned} \tag{7.1}$$

where

$$\begin{aligned} &A_{ur}=a_{ur}*E_{im},\quad A_{wr}=a_{wr}*E_{im},\quad A_{vr}=a_{vr}*E_{im},\\ &E_{im}=\exp(I_{dkr}*x),\quad O_{kr}=\omega*t-x*(k_r+R_{dkr}). \end{aligned} \tag{7.2}$$

For the amplitudes in (7.1), these are expressed as

$$a_{ur}=u_0+u_1*dk_{r11}+u_2*dk_{r33}+u_3*dk_{r55}+u_4*dk_{r13}, \tag{7.3}$$

$$a_{wr}=a_{w0}-w_1*dk_{r11}-w_2*dk_{r33}-w_3*dk_{r55}-w_4*dk_{r13}, \tag{7.4}$$

$$a_{vr}=\mathrm{SQRT}((d_{14}*v_1+d_{34}*v_2+d_{56}*v_3)^2+(d_{16}*v_4+d_{36}*v_5+d_{45}*v_6)^2), \tag{7.5}$$

where

$$u_0=2*\mathrm{SQRT}(k_{rm})*\mathrm{SQRT}(b_{0m})/C_f^2, \tag{7.3.1}$$

$$a_{w0}=1/C_f, \tag{7.4.1}$$

and the coefficients

$$u_1,\ u_2,\ u_3,\ u_4,\ w_1,\ w_2,\ w_3,\ w_4,\ v_1,\ v_2,\ v_3,\ v_4,\ v_5,\ v_6$$

are given in Appendix A2.

For the phases in (7.1), these are expressed as

$$P_{ur}=\pi/2-(dk_{r15}*U_5+dk_{r35}*U_6), \tag{7.6}$$

$$P_{wr}=\pi-(dk_{r15}*W_5+dk_{r35}*W_6), \tag{7.7}$$

$$P_{vr}=\tan^{-1}((d_{16}*v_4+d_{36}*v_5+d_{45}*v_6)/(d_{14}*v_1+d_{34}*v_2+d_{56}*v_3)), \tag{7.8}$$

where the coefficients

$$U_5,\ U_6,\ W_5,\ W_6$$

are given in Appendix A3.

By comparing the expressions in (7.3), (7.4) and (7.5) in the case of amplitude and (7.6), (7.7) and (7.8) in the case of phase, it is found that the elastic coefficients $\{d_{11}, d_{33}, d_{55}, d_{13}$ for amplitude and $d_{15}, d_{35}$ for phase$\}$ in $A_{ur}, A_{wr}, P_{ur}, P_{wr}$ associated with the longitudinal and vertical components are completely different from those $\{d_{14}, d_{34}, d_{56}, d_{16}, d_{36}, d_{45}\}$ in $A_{vr}$ and $P_{vr}$ associated with transverse component. This result implies that, in a weakly anisotropic medium, the $u$ and $w$ components compose a quasi-Rayleigh wave which resembles the Rayleigh wave in an isotropic medium. The $v$ component is a quasi-Love wave which decreases with depth. These quasi-Love waves are independent of the former quasi-Rayleigh waves with regard to the elastic coefficients. The typical difference between quasi-Love waves and pure Love waves in an isotropic medium is that quasi-Love waves are propagated with the velocity of the quasi-Rayleigh instead of so-called $S$ waves. Similar features have been also found by STONELEY (1955, 1963) in the particular crystals and by CRAMPIN (1970) in multilayered anisotropic media.

## 8. Dependence of quasi-Rayleigh wave orbit on the velocity

In order to examine the relation between velocity and quasi-Rayleigh wave amplitude, the coefficients in expressions (7.3) and (7.4) are evaluated for the value of the ratio $L_m=\lambda/\mu$ in the range 0.5 to 50. Only four numerical instances of the evaluated results are given here. For intervening values of $L_m$, the numerical values are approximately those that would be obtained by interpolation.

The quantities $u_0$ and $a_{w0}$ are always positive for any value of $L_m$ (see (7.3.1) and (7.4.1)):

when $L_m=0.5$,

$u_1=4.4$, $u_2=9.4$, $u_3=2.4$, $u_4=5.9$,

$w_1=-13.7$, $w_2=-19.6$, $w_3=-6.5$, $w_4=-13.7$,

when $L_m=1$,

$u_1=4.3$, $u_2=8.5$, $u_3=2.1$, $u_4=5.9$,

$w_1=-14.3$, $w_2=-19.6$, $w_3=-6.2$, $w_4=-15.1$,

when $L_m=10$,

$u_1=4.4$, $u_2=5.4$, $u_3=1.4$, $u_4=4.9$,

$w_1=-17.3$, $w_2=-18.8$, $w_3=-5.6$, $w_4=-18$;

when $L_m=50$,

$u_1=4.5$, $u_2=4.3$, $u_3=1.3$, $u_4=4.6$,

$w_1=-18.1$, $w_2=-18.5$, $w_3=-5.4$, $w_4=-18.3$,

where the factor $\sqrt{m_u}$ is eliminated.

By use of the above evaluated results, the signs of the coefficients in the expressions for the amplitude are illustrated below.

$$a_{ur}=\underset{>0}{u_0}+\underset{>0}{u_1}*dk_{r11}+\underset{>0}{u_2}*dk_{r33}+\underset{>0}{u_3}*dk_{r55}+\underset{>0}{u_4}*dk_{r13},$$

$$a_{wr}=\underset{>0}{a_{w0}}-\underset{>0}{w_1}*dk_{r11}-\underset{>0}{w_2}*dk_{r33}-\underset{>0}{w_3}*dk_{r55}-\underset{>0}{w_4}dk_{r13},$$

where the evaluation took the preceding sign (+ or −) into account.

The above example indicate that when the velocity increases (or decreases), $dk_{rij}$ (components of real part of $dk_r$) decreases (or increases) and, as a result, the amplitudes $a_{ur}$ and $a_{wr}$ decrease (or increase). That is to say, the velocity increase (or decrease) due to anisotropy causes shrinkage (or expansion) of the orbit. From the physical point of view, the above result is reasonal, since when the velocity decreases (or increases) the wave energy is concentrated (or dispersed).

As found from the numerical values of $\{u_j, w_j\}$ ($j$: suffix), the absolute values of $w_j$ are larger (about 2∼4 times) than those of $u_j$. This implies that the vertical component of the Rayleigh waves is more sensitive than the horizontal component to anisotropy.

Here, the sensitivity of Rayleigh wave amplitude to velocity change are evaluated. For this purpose, the following two quantities are defined.

(a) Rate of Increase of Velocity:

$$dV_r=dv_r/v_r,$$

where there exists the relation $\omega = k_r * v_r = (k_r + R_{dkr}) * (v_r + dv_r)$. Here $k_r$ and $v_r$ are the Rayleigh wave number and velocity in an isotropic medium. $R_{dkr}$ and $dv_r$ are the real part of $dk_r$ (see (6.1)) and increase of the velocity due to anisotropy. By taking the first order of $R_{dkr}$, the above is reduced to

$$dV_r = -R_{dkr}/k_r, \tag{8.1}$$

(b) Rate of Increase of Amplitudes:

$$A_{uR} = (a_{ur} - u_0)/u_0, \quad A_{wR} = (a_{wr} - a_{w0})/a_{w0}. \tag{8.2}$$

By using (6.2), (8.1) is reduced to

$$dV_r = dV_{r11} + dV_{r33} + dV_{r55} + dV_{r13}, \tag{8.3}$$

where $dV_{rij}$ indicates the velocity change due to the anisotropy associated with $d_{ij}$, i.e.,

$$dV_{rij} = -dk_{rij}/k_r,$$

with $ij$ (suffices) = 11, 33, 55 and 13.

After some manipulation, (8.2) are also reduced to

$$\begin{aligned} A_{uR} &= A_{uR11} + A_{uR33} + A_{uR55} + A_{uR13}, \\ A_{wR} &= A_{wR11} + A_{wR33} + A_{wR55} + A_{wR13}, \end{aligned} \tag{8.4}$$

where

$$A_{uRij} = dV_{rij} * S_{uij} \text{ and } A_{wRij} = dV_{rij} * S_{wij} \tag{8.5}$$

with $ij$ = 11, 33, 55, 13 and

$$\begin{aligned} S_{u11} &= -k_r * u_1/u_0, & S_{u33} &= -k_r * u_2/u_0, \\ S_{u55} &= -k_r * u_3/u_0, & S_{u13} &= -k_r * u_4/u_0, \\ S_{w11} &= k_r * w_1/a_{w0}, & S_{w33} &= k_r * w_2/a_{w0}, \\ S_{w55} &= k_r * w_3/a_{w0}, & S_{w13} &= k_r * w_4/a_{w0}. \end{aligned} \tag{8.6}$$

From the above expressions (8.3) to (8.5), it is found that $S_{uij}$ and $S_{wij}$ ($ij$ = 11, 33, 55, 13) determine the sensitivity of Rayleigh wave amplitude for velocity change due to anisotropy $d_{ij}$. In order to see the order of the sensitivity, the numerical evaluation of (8.6) are carried out for the following specified $L_m$.

When $L_m = 0.5$,

$S_{u11} = -9.8$, $S_{u33} = -20.9$, $S_{u55} = -5.4$, $S_{u13} = -13.0$,
$S_{w11} = -21.9$, $S_{w33} = -31.3$, $S_{w55} = -10.3$, $S_{w13} = -21.8$,

when $L_m=1$,

| | | | |
|---|---|---|---|
| $S_{u11}=-9.4$, | $S_{u33}=-18.8$, | $S_{u55}=-4.7$, | $S_{u13}=-12.9$, |
| $S_{w11}=-21.3$, | $S_{w33}=-29.1$, | $S_{w55}=-9.3$, | $S_{w13}=-22.5$, |

when $L_m=10$,

| | | | |
|---|---|---|---|
| $S_{u11}=-10.0$, | $S_{u33}=-12.3$, | $S_{u55}=-3.3$, | $S_{u13}=-11.2$, |
| $S_{w11}=-22.2$, | $S_{w33}=-24.1$, | $S_{w55}=-7.1$, | $S_{w13}=-23.1$, |

when $L_m=50$,

| | | | |
|---|---|---|---|
| $S_{u11}=-10.3$, | $S_{u33}=-10.6$, | $S_{u55}=-3.1$, | $S_{u13}=-10.6$, |
| $S_{w11}=-22.7$, | $S_{w33}=-22.9$, | $S_{w55}=-6.8$, | $S_{w13}=-22.9$, |

For other intervening values of $L_m$, the similar order including minus sign holds.

From the above values, the amplitude sensitivity is found to be very high, or 10 to 20 times the velocity change. For instance, a one percent change of velocity due to anisotropy leads to a 10 to 20 percent change of the Rayleigh wave amplitude. For a 10 percent velocity change, the size of the orbit is approximatedly doubled (see Fig. 1a).

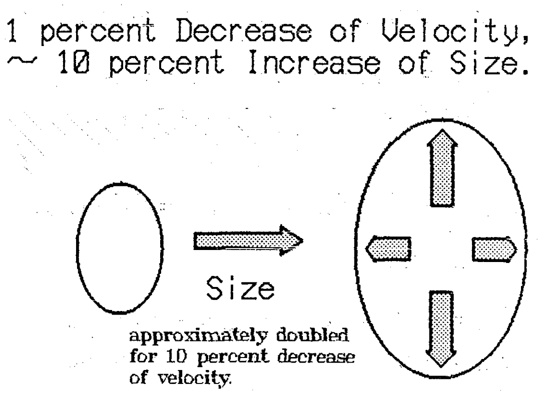


Fig. 1a. Expansion of the Rayleigh orbit due to the velocity change caused by anisotropy of the medium.

The above result implies that the analysis of the medium anisotropy by the Rayleigh wave orbit is more convenient than that by the usual velocity method because of high sensitivity of the Rayleigh wave orbit.

## 9. Dependence of inclination of Rayleigh wave orbit on the stability

From the first two exprәssions of (7.1), the orbit equation projected on the $x-z$ plane (sagittal plane) is obtained as follows.

$$-2*u*w*\cos(\Delta P)/(A_{ur}*A_{wr})+u^2/A_{ur}^2+w^2/A_{wr}^2=(\sin \Delta P)^2, \tag{9.1}$$

where

$$A_{ur}=a_{ur}*\exp(I_{dkr}*x),\quad A_{wr}=a_{wr}*\exp(I_{dkr}*x),\quad \Delta P=P_{wr}-P_{ur}.$$

In order to eliminate the coupled term, the first term of (9.1), the coordinates of the displacements $(u, w)$ are rotated into new ones $(u_n, w_n)$ by

$$u_n=w*S_t+u*C_t,\qquad w_n=w*C_t-u*S_t,$$
$$C_t=\cos(\Theta_{rot}),\qquad S_t=\sin(\Theta_{rot}),$$

where $\Theta_{rot}$ is the rotation angle.

The rotated equation becomes

$$u_n*w_n*C_{uw}+w_n^2*C_w+u_n^2*C_u=(\sin \Delta P)^2, \tag{9.2}$$

where

$$C_{uw}=\sin(2*\Theta_{rot})*(-1/A_{ur}^2+1/A_{wr}^2)$$
$$-2*\cos(2*\Theta_{rot})*\cos(\Delta P)/(A_{ur}*A_{wr}),$$
$$C_w=(\sin \Theta_{rot})^2/A_{ur}^2+(\cos \Theta_{rot})^2/A_{wr}^2$$
$$+2*\cos \Theta_{rot}*\cos \Delta P*\sin \Theta_{rot}/(A_{ur}*A_{wr}),$$
$$C_u=(\cos \Theta_{rot})^2/A_{ur}^2+(\sin \Theta_{rot})^2/A_{wr}^2$$
$$-2*\cos \Theta_{rot}*\cos \Delta P*\sin \Theta_{rot}/(A_{ur}*A_{wr}).$$

Putting the first expression $C_{uw}$ equal to zero and taking the terms to first order of $d_{ij}$, we have the rotation angle $\Theta_{rot}$ as follows.

$$\Theta_{rot}=d\Theta_{15}+d\Theta_{35}, \tag{9.3}$$

with

$$d\Theta_{15}=K_{15}*dk_{r15},\quad d\Theta_{35}=K_{35}*dk_{r35}, \tag{9.4}$$

where

$$K_{15}=2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(b_{0m})*(2048*b_{0m}^7+8192*b_{0m}^6+13568*b_{0m}^5$$
$$+1224*b_{0m}^4+6640*b_{0m}^3+2232*b_{0m}^2+416*b_{0m}+25)$$
$$/((32*b_{0m}^3+64*b_{0m}^2+40*b_{0m}+7)*C_f^2),$$
$$K_{35}=2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(b_{0m})*(4096*b_{0m}^{10}+22528*b_{0m}^9+54784*b_{0m}^8$$
$$+77184*b_{0m}^7+69536*b_{0m}^6+41872*b_{0m}^5+17152*b_{0m}^4+4734*b_{0m}^3$$
$$+820*b_{0m}^2+70*b_{0m}+1)/((64*b_{0m}^5+160*b_{0m}^4+144*b_{0m}^3+54*b_{0m}^2$$
$$+8*b_{0m}+1)*C_f^2*k_{rm}). \tag{9.5}$$

In the above expressions (9.4), it is noted that $K_{15}$ and $K_{35}$ are always positive (constant sign). This implies that the sign of the rotation angle

$\Theta_{rot}$ depends on only the signs of $dk_{r15}$ and $dk_{r35}$, which are the increments of imaginary terms of $dk_r$ (see (6.3)). This feature leads to the results that, for the orbit of the Rayleigh waves at the free surface (see Fig. 1b),

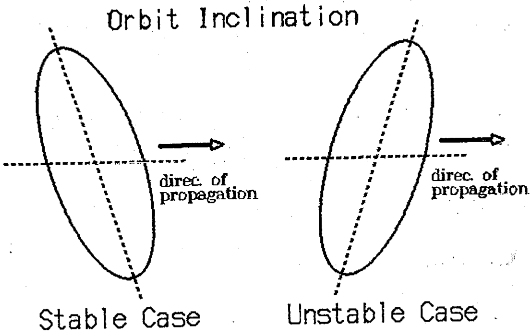


Fig. 1b. Inclination of the Rayleigh wave orbit axis due to the stability.

(i) when the Rayleigh waves are stable ($dk_{r5}, dk_{r35}<0$), the orbit axis is rotated counterclockwise ($\Theta_{rot}<0$), i.e., tilts backward, while

(ii) when the Rayleigh waves are unstable ($dk_{r15}, dk_{r35}>0$), the orbit axis is rotated clockwise ($\Theta_{rot}>0$), i.e., tilts forward.

In order to examine the dependence of rotation angle $\Theta_{rot}$ on the increase (decrease) of the amplitude, we introduce the following rate of increase (decrease) of amplitude, $R_{amp}$, in the distance of Rayleigh wavelength $L_r$.

$$\begin{aligned} R_{amp} &= (A_{ur}-a_{ur})/a_{ur} \\ &= (A_{vr}-a_{vr})/a_{vr} \\ &= (A_{wr}-a_{wr})/a_{wr}, \\ &= -1+\exp(L_r * I_{dkr}). \end{aligned} \tag{9.6}$$

After some reduction by use of (6.3), the above becomes

$$Dk_{r15}+Dk_{r35}=ln(1+R_{amp}), \tag{9.7}$$

where

$$Dk_{r15}=dk_{r15}*L_r, \quad Dk_{r35}=dk_{r35}*L_r,$$

and $ln$ is the natural logarithm.

Let us here assume the expression

$$R_{amp}=dR_{amp15} \text{ and/or } dR_{amp35}, \tag{9.8}$$

where $dR_{amp15}$ and $dR_{amp35}$ are the variation of $R_{amp}$ associated with $Dk_{r15}$

and/or $Dk_{r35}$, respectively. The expression (9.7) becomes

$$Dk_{r15}=ln(1+dR_{amp15}),\quad Dk_{r35}=ln(1+dR_{amp35}). \tag{9.9}$$

By use of the above notations, the expressions (9.4) are transcribed as follows.

$$d\Theta_{15}=D\Theta_{15}*ln(1+dR_{amp15}),\quad d\Theta_{35}=D\Theta_{35}*ln(1+dR_{amp35}) \tag{9.10}$$

with

$$D\Theta_{15}=k_r*K_{15}/(2\pi),\quad D\Theta_{35}=k_r*K_{35}/(2\pi). \tag{9.11}$$

The above (9.11) can be evaluated numerically by use of (9.5). The evaluated results for some $L_m(=\lambda/\mu)$ are arranged below (unit: radian).

When $L_m=0$,
$D\Theta_{15}=2.10766,\quad D\Theta_{35}=2.44763;$
when $L_m=0.5$,
$D\Theta_{15}=1.37527,\quad D\Theta_{35}=1.59366;$
when $L_m=1$,
$D\Theta_{15}=1.11201,\quad D\Theta_{35}=1.27053;$
when $L_m=10$,
$D\Theta_{15}=0.65916,\quad D\Theta_{35}=0.68145;$
when $L_m=100$,
$D\Theta_{15}=0.59309,\quad D\Theta_{35}=0.59431.$ (9.12)

In the case of particular rate of increase (decrease) of amplitude, say $dR_{amp15}$ or $dR_{amp35}\sim +10$ or $+20$ ($-10$ or $-20$) percent, the inclination angles are evaluated by use of (9.10) and (9.12) as follows (unit: degree), where the values in parentheese are for decrease of the amplitude.

[a] Case of $dR_{amp15}$ or $dR_{amp35}=0.1$ $(-0.1)$,
i.e., 10 percent increase (decrease) of amplitude:

when $L_m=0$,
$d\Theta_{15}=11.51°\ (-12.72°),\quad d\Theta_{35}=13.37°\ (-14.48°);$
when $L_m=0.5$,
$d\Theta_{15}=7.51°\ (-8.30°),\quad d\Theta_{35}=8.70°\ (-9.62°);$
when $L_m=1$,
$d\Theta_{15}=6.07°\ (-6.71°),\quad d\Theta_{35}=6.94°\ (-7.67°);$
when $L_m=10$,
$d\Theta_{15}=3.60°\ (-3.98°),\quad d\Theta_{35}=3.72°\ (-4.11°);$
when $L_m=100$,
$d\Theta_{15}=3.24°\ (-3.58°),\quad d\Theta_{35}=3.25°\ (-3.59°).$

[b] Case of $dR_{amp15}$ or $dR_{amp35}=0.2$ $(-0.2)$,
i.e., 20 percent increase (decrease) of amplitude:

when $L_m=0$,
$d\Theta_{15}=22.02°$ $(-26.95°)$, $d\Theta_{35}=25.57°$ $(-31.29°)$;

when $L_m=0.5$,
$d\Theta_{15}=14.37°$ $(-17.58°)$, $d\Theta_{35}=16.65°$ $(-20.37°)$;

when $L_m=1$,
$d\Theta_{15}=11.62°$ $(-14.22°)$, $d\Theta_{35}=13.27°$ $(-16.24°)$;

when $L_m=10$,
$d\Theta_{15}=6.88°$ $(-8.43°)$, $d\Theta_{35}=7.12°$ $(-8.71°)$;

when $L_m=100$,
$d\Theta_{15}=6.20°$ $(-7.58°)$, $d\Theta_{35}=6.21°$ $(-7.60°)$.

As shown above, when the Rayleigh waves increase (decrease) in amplitude by 10 or 20 percent after traveling the distance of the wavelength, the inclination angles of the orbit at the free surface are in the range $5\sim30$ degrees, and the inclination angles in the case of decreasing amplitude are slightly larger than in the case of increasing amplitude.

As already mentioned before, the Rayleigh wave stability depends on only the elastic coefficients $d_{15}$ and $d_{35}$, so knowing the inclination angles leads to a possibility of analysis on medium structure particularly associated with $d_{15}$ and $d_{35}$.

## 10. Conclusion

Important conclusions are illustrated in Figs. 1a and 1b.

The wave number in a weakly anisotropic medium involves imaginary terms. As a result, quasi-Rayleigh waves in anisotropic media always face a stability problem.

In a weakly anisotropic medium, the longitudinal and vertical components $(u, w)$ compose quasi-Rayleigh waves which resemble Rayleigh waves in an isotropic medium. The horizontally transverse component $(v)$ is a quasi-Love wave which is independent of the former quasi-Rayleigh wave with respect to the elastic coefficients and decrease in amplitude with depth in the medium.

The increase (or decrease) of the quasi-Rayleigh wave velocity due to anisotropy of the medium leads to shrinkage (or expansion) of the quasi-Rayleigh wave orbit. In this case, the vertical component of the Rayleigh wave is more sensitive than the horizontal component.

The quasi-Rayleigh wave orbit is very sensitive to the velocity change due to anisotropy. For instance, a one percent increase of velocity causes about 10 percent increase in the Rayleigh wave orbit size. For larger increase of the velocity, say 10 percent, the orbit size is practically doubled. This consequence seems important, since it implies that information on the anisotropy of the Earth can be obtained from study of the quasi-Rayleigh wave orbit because of the high sensitivity of the orbit in comparison with velocity.

When the quasi-Rayleigh waves are stable (or unstable), the Rayleigh wave orbit axis at the free surface tilts backward (or forward). And the inclination of the orbit depends on only two elastic coefficients $d_{15}$ and $d_{35}$ out of the 21 coefficients in the weakly anisotropic medium. This fact makes it possible to investigate the elastic medium associated with $d_{15}$ and $d_{35}$.

As for the inclination angle, when the Rayleigh waves increase (decrease) in amplitude by 10 or 20 percent after traveling the distance of the wavelength, the inclination angles of the orbit at the surface are in the range 5∼30 degrees.

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## Appendix A1

The real and imaginary parts inside the root of the expression (4.5.2) are given here.

$$\begin{aligned} S_{re}= & d_{55}*d_{66}*S_{re1}+d_{55}*d_{44}*S_{re4}+d_{55}*d_{11}*S_{re5}+d_{55}*d_{33}*S_{re5}-2*d_{55}*d_{13}*S_{re5} \\ & +d_{66}*d_{44}*S_{re2}+d_{66}*d_{11}*S_{re3}+d_{66}*d_{33}*S_{re3}-2*d_{66}*d_{13}*S_{re3}+d_{56}*d_{14}*S_{re10} \\ & -d_{56}*d_{34}*S_{re10}+d_{44}*d_{11}*S_{re8}+d_{44}*d_{33}*S_{re8}-2*d_{44}*d_{13}*S_{re8}-d_{46}*d_{15}*S_{re10} \\ & +d_{46}*d_{35}*S_{re10}+d_{11}*d_{33}*S_{re9}-2*d_{11}*d_{13}*S_{re9}-2*d_{33}*d_{13}*S_{re9} \\ & +4*d_{36}*d_{16}*S_{re3}-d_{36}*d_{45}*S_{re10}+d_{16}*d_{45}*S_{re10}-4*d_{15}*d_{35}*S_{re5} \\ & 4*d_{14}*d_{34}*S_{re8}+d_{55}{}^2*S_{re7}+d_{66}{}^2*S_{re6}-2*d_{56}{}^2*S_{re1}+d_{44}{}^2*S_{re11}+2*d_{46}{}^2*S_{re2} \\ & +d_{11}{}^2*S_{re9}/2+d_{33}{}^2*S_{re9}/2+2*d_{13}{}^2*S_{re9}-2*d_{36}{}^2*S_{re3}-2*d_{16}{}^2*S_{re3} \\ & -2*d_{45}{}^2*S_{re4}+2*d_{15}{}^2*S_{re5}+2*d_{34}{}^2*S_{re5}-2*d_{14}{}^2*S_{re8}-2*d_{34}{}^2*S_{re8}, \end{aligned}$$

$$\begin{aligned} S_{im}= & d_{66}*d_{15}*S_{im2}-d_{66}*d_{35}*S_{im2}+d_{66}*d_{46}*S_{im1}+d_{15}*d_{55}*S_{im4}+d_{15}*d_{44}*S_{im6} \\ & +d_{15}*d_{11}*S_{im8}+d_{15}*d_{33}*S_{im8}-2*d_{15}*d_{13}*S_{im8}-d_{35}*d_{55}*S_{im4}-d_{35}*d_{44}*S_{im6} \\ & -d_{35}*d_{11}*S_{im8}-d_{35}*d_{33}*S_{im8}+2*d_{35}*d_{13}*S_{im8}+2*d_{36}*d_{56}*S_{im2} \\ & +2*d_{36}*d_{14}*S_{im7}-2*d_{36}*d_{34}*S_{im7}-2*d_{56}*d_{16}*S_{im2}-2*d_{56}*d_{45}*S_{im3} \\ & -2*d_{16}*d_{14}*S_{im7}+2*d_{16}*d_{34}*S_{im7}+d_{55}*d_{46}*S_{im3}+d_{46}*d_{44}*S_{im5} \\ & +d_{46}*d_{11}*S_{im7}+d_{46}*d_{33}*S_{im7}-2*d_{46}*d_{13}*S_{im7}-2*d_{45}*d_{14}*S_{im6} \\ & +2*d_{45}*d_{34}*S_{im6}, \end{aligned}$$

$S_{re1}=-k_r{}^2*C_f{}^2/(8*b_0{}^2*m_u{}^3)$, $S_{re2}=-k_r{}^2/(8*m_u{}^2)$,

$S_{re3}=k_r{}^4/(8*m_u)$, $S_{re4}=C_f{}^2/(8*m_u{}^3)$,

$S_{re5}=-k_r{}^2*C_f{}^2/(8*m_u{}^2)$, $S_{re6}=k_r{}^4/(16*b_0{}^2*m_u{}^2)$,

$S_{re7}=C_f{}^4/(16*b_0{}^2*m_u{}^4)$, $S_{re8}=-b_0{}^2*k_r{}^2/(8*m_u)$,

$S_{re9}=b_0{}^2*k_r{}^4/8$, $S_{re10}=k_r{}^2*C_f/(2*m_u{}^2)$,

$S_{re11}=b_0{}^2/(16*m_u{}^2)$,

$S_{im1}=-k_r{}^3/(4*b_0*m_u{}^2)$, $S_{im2}=-k_r{}^3*C_f/(4*b_0*m_u{}^2)$,

$S_{im3}=k_r*C_f{}^2/(4*b_0*m_u{}^3)$, $S_{im4}=k_r*C_f{}^3/(4*b_0*m_u{}^3)$,

$S_{im5}=b_0*k_r/(4*m_u{}^2)$, $S_{im6}=b_0*k_r*C_f/(4*m_u{}^2)$,

$S_{im7}=-b_0*k_r{}^3/(4*m_u)$, $S_{im8}=-b_0*k_r{}^3*C_f/(4*m_u)$.

## Appendix A2

The coefficients in the expressions (7.3), (7.4) and (7.5) are given here.

$$\begin{aligned} u_1= & -(2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(b_{0m})*(2048*b_{0m}{}^8+10240*b_{0m}{}^7+21504*b_{0m}{}^6 \\ & +24320*b_{0m}{}^5+15936*b_{0m}{}^4+6272*b_{0m}{}^3+1688*b_{0m}{}^2+412*b_{0m}+69))/ \\ & ((64*b_{0m}{}^5+64*b_{0m}{}^4-104*b_{0m}{}^3-164*b_{0m}{}^2-62*b_{0m}-3)*C_f{}^3), \end{aligned}$$

$$u_2 = -(2*\mathrm{SQRT}(m_u)*(131072*b_{0m}^{14}+1015808*b_{0m}^{13}+3604480*b_{0m}^{12}$$
$$+7766016*b_{0m}^{11}+11350016*b_{0m}^{10}+11904000*b_{0m}^{9}+9255424*b_{0m}^{8}$$
$$+5450368*b_{0m}^{7}+2469888*b_{0m}^{6}+866672*b_{0m}^{5}+230032*b_{0m}^{4}$$
$$+42160*b_{0m}^{3}+4160*b_{0m}^{2}+28*b_{0m}-21)*b_{0m})/(\mathrm{SQRT}(b_{0m})*(512*b_{0m}^{8}$$
$$+1152*b_{0m}^{7}+128*b_{0m}^{6}-1712*b_{0m}^{5}-1856*b_{0m}^{4}-784*b_{0m}^{3}-136*b_{0m}^{2}$$
$$-10*b_{0m}-1)*C_f^{6}),$$

$$u_3 = 2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(b_{0m})*(4096*b_{0m}^{9}+20480*b_{0m}^{8}+44032*b_{0m}^{7}$$
$$+53248*b_{0m}^{6}+39936*b_{0m}^{5}+19584*b_{0m}^{4}+6608*b_{0m}^{3}+1576*b_{0m}^{2}$$
$$+218*b_{0m}+3)/((8*b_{0m}^{2}+12*b_{0m}+5)*C_f^{7}),$$

$$u_4 = 2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(b_{0m})*(8192*b_{0m}^{10}+43008*b_{0m}^{9}+98304*b_{0m}^{8}$$
$$+129024*b_{0m}^{7}+109440*b_{0m}^{6}+65280*b_{0m}^{5}+29216*b_{0m}^{4}+9664*b_{0m}^{3}$$
$$1992*b_{0m}^{2}+163*b_{0m}-3)*k_{rm}/((96*b_{0m}^{5}+232*b_{0m}^{4}+192*b_{0m}^{3}$$
$$+59*b_{0m}^{2}+5*b_{0m}+1)*C_f^{6}),$$

$$u_5 = -(2*\mathrm{SQRT}(m_u)*(8192*b_{0m}^{9}+40960*b_{0m}^{8}+88064*b_{0m}^{7}+106752*b_{0m}^{6}$$
$$+80960*b_{0m}^{5}+40992*b_{0m}^{4}+14784*b_{0m}^{3}+3904*b_{0m}^{2}+640*b_{0m}+33)*b_{0m})/$$
$$(\mathrm{SQRT}(b_{0m})*(32*b_{0m}^{3}+64*b_{0m}^{2}+40*b_{0m}+7)*C_f^{6}),$$

$$u_6 = -(2*\mathrm{SQRT}(m_u)*(16384*b_{0m}^{12}+106496*b_{0m}^{11}+311296*b_{0m}^{10}$$
$$+540160*b_{0m}^{9}+620416*b_{0m}^{8}+500160*b_{0m}^{7}+294400*b_{0m}^{6}+130192*b_{0m}^{5}$$
$$+43536*b_{0m}^{4}+10486*b_{0m}^{3}+1554*b_{0m}^{2}+88*b_{0m}-3)*b_{0m})/(\mathrm{SQRT}(b_{0m})$$
$$*(64*b_{0m}^{5}+160*b_{0m}^{4}+144*b_{0m}^{3}+54*b_{0m}^{2}+8*b_{0m}+1)*C_f^{6}*k_{rm}),$$

$$w_1 = 2*\mathrm{SQRT}(m_u)*(1024*b_{0m}^{7}+4608*b_{0m}^{6}+8448*b_{0m}^{5}+8256*b_{0m}^{4}+4800*b_{0m}^{3}$$
$$+1808*b_{0m}^{2}+464*b_{0m}+65)*k_{rm}/(\mathrm{SQRT}(k_{rm})*(64*b_{0m}^{5}+64*b_{0m}^{4}$$
$$-104*b_{0m}^{3}-164*b_{0m}^{2}-62*b_{0m}-3)*C_f^{2}),$$

$$w_2 = 2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(k_{rm})*(65536*b_{0m}^{13}+475136*b_{0m}^{12}+1564672*b_{0m}^{11}$$
$$+3088384*b_{0m}^{10}+4068352*b_{0m}^{9}+3779584*b_{0m}^{8}+2559744*b_{0m}^{7}$$
$$+1290944*b_{0m}^{6}+489008*b_{0m}^{5}+137008*b_{0m}^{4}+26784*b_{0m}^{3}+3216*b_{0m}^{2}$$
$$+176*b_{0m}-1)/((512*b_{0m}^{8}+1152*b_{0m}^{7}+128*b_{0m}^{6}-1712*b_{0m}^{5}-1856*b_{0m}^{4}$$
$$-784*b_{0m}^{3}-136*b_{0m}^{2}-10*b_{0m}-1)*C_f^{5}),$$

$$w_3 = -(2*\mathrm{SQRT}(m_u)*(2048*b_{0m}^{8}+9216*b_{0m}^{7}+17408*b_{0m}^{6}+18048*b_{0m}^{5}$$
$$+11456*b_{0m}^{4}+4800*b_{0m}^{3}+1368*b_{0m}^{2}+222*\mathrm{b}_{0m}+7)*k_{rm})/(\mathrm{SQRT}(k_{rm})$$
$$*(8*b_{0m}^{2}+12*b_{0m}+5)*C_f^{6}),$$

$$w_4 = -(2*\mathrm{SQRT}(m_u)*(4096*b_{0m}^{8}+15360*b_{0m}^{7}+24064*b_{0m}^{6}+20992*b_{0m}^{5}$$
$$+11776*b_{0m}^{4}+4672*b_{0m}^{3}+1264*b_{0m}^{2}+176*b_{0m}+5)*k_{rm}^{3})/(\mathrm{SQRT}(k_{rm})$$
$$*(96*b_{0m}^{5}+232*b_{0m}^{4}+192*b_{0m}^{3}+59*b_{0m}^{2}+5*b_{0m}+1)*C_f^{5}),$$

$$w_5 = -(2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(k_{rm})*(4096*b_{0m}^{8}+18432*b_{0m}^{7}+34816*b_{0m}^{6}$$
$$+36224*b_{0m}^{5}+23264*b_{0m}^{4}+9984*b_{0m}^{3}+2960*b_{0m}^{2}+524*b_{0m}+29))/$$
$$((32*b_{0m}^{3}+64*b_{0m2}+40*b_{0m}+7)*C_f^{5}),$$

$$w_6 = -(2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(k_{rm})*(8192*b_{0m}^{10}+40960*b_{0m}^{9}+90112*b_{0m}^{8}$$
$$+114944*b_{0m}^{7}+94784*b_{0m}^{6}+53696*b_{0m}^{5}+21664*b_{0m}^{4}+6160*b_{0m}^{3}$$
$$+1094*b_{0m}^{2}+82*b_{0m}-1))/((64*b_{0m}^{5}+160*b_{0m}^{4}+144*b_{0m}^{3}+54*b_{0m}^{2}$$

$+8*b_{0m}+1)*C_f^5)$,

and

$$v_1=(8*b_{0m}^2+6*b_{0m}-1)*(k_{rm})/(d_{n2}*C_f^2*m_u),$$
$$v_2=d_{n3}*b_{0m}/(d_{n2}*C_f*m_u),$$
$$v_3=(k_{rm})/(d_{n2}*b_{0m}*m_u),$$
$$v_4=-(\mathrm{SQRT}(k_{rm})*(8*b_{0m}^2+6*b_{0m}-1)*(k_{rm}))/(\mathrm{SQRT}(b_{0m})*d_{n2}*C_f^2*m_u),$$
$$v_5=-(\mathrm{SQRT}(b_{0m})*\mathrm{SQRT}(k_{rm})*d_{n3})/(d_{n2}*C_f*m_u),$$
$$v_6=\mathrm{SQRT}(k_{rm})/(\mathrm{SQRT}(b_{0m})*d_{n2}*m_u),$$

with

$$d_{n1}=16*b_{0m}^4+28*b_{0m}^3+18*b_{0m}^2+7*b_{0m}+2,$$
$$d_{n2}=8*b_{0m}^2+8*b_{0m}+1,\qquad d_{n3}=4*b_{0m}+3.$$

## Appendix A3

The coefficients in the expressions (7.6) and (7.7) are given here.

$$U_5=-(\mathrm{SQRT}(m_u)*(8192*b_{0m}^9+40960*b_{0m}^8+88064*b_{0m}^7+106752*b_{0m}^6$$
$$+80960*b_{0m}^5+40992*b_{0m}^4+14784*b_{0m}^3+3904*b_{0m}^2+640*b_{0m}+33))/$$
$$(\mathrm{SQRT}(k_{rm})*(32*b_{0m}^3+64*b_{0m}^2+40*b_{0m}+7)*C_f^4),$$

$$U_6=-(\mathrm{SQRT}(m_u)*(16384*b_{0m}^{12}+106496*b_{0m}^{11}+311296*b_{0m}^{10}+540160*b_{0m}^9$$
$$+620416*b_{0m}^8+500160*b_{0m}^7+294400*b_{0m}^6+130192*b_{0m}^5+43536*b_{0m}^4$$
$$+10486*b_{0m}^3+1554*b_{0m}^2+88*b_{0m}-3))/(\mathrm{SQRT}(k_{rm})*(64*b_{0m}^5+160*b_{0m}^4$$
$$+144*b_{0m}^3+54*b_{0m}^2+8*b_{0m}+1)*C_f^4*k_{rm}),$$

$$W_5=-2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(k_{rm})*(4096*b_{0m}^8+18432*b_{0m}^7+34816*b_{0m}^6$$
$$+36224*b_{0m}^5+23264*b_{0m}^4+9984*b_{0m}^3+2960*b_{0m}^2+524*b_{0m}+29)/$$
$$((32*b_{0m}^3+64*b_{0m}^2+40*b_{0m}+7)*C_f^4),$$

$$W_6=-2*\mathrm{SQRT}(m_u)*\mathrm{SQRT}(k_{rm})*(8192*b_{0m}^{10}+40960*b_{0m}^9+90112*b_{0m}^8$$
$$+114944*b_{0m}^7+94784*b_{0m}^6+53696*b_{0m}^5+21664*b_{0m}^4+6160*b_{0m}^3$$
$$+1094*b_{0m}^2+82*b_{0m}-1)/((64*b_{0m}^5+160*b_{0m}^4+144*b_{0m}^3+54*b_{0m}^2$$
$$+8*b_{0m}+1)*C_f^4).$$

# 弱い異方性媒質における表面波の軌道

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本論文において弱い異方性媒質における表面波が，特にレーリー波の軌道とその速度または安定性との関係に注意をはらって，論じられている．

弱い異方性媒質における波数は虚数の項を持っている．その結果として異方性媒質におけるレーリー波はつねに安定性の問題に直面している．

弱い異方性媒質においては，等方性媒質におけるのと同様な準レーリー波が発生する，と同時に弾性係数に関してはこれらレーリー波とは全く無関係な準ラブ波が伴走波として発生する．そしてこの準ラブ波の振幅は媒質内部に行くにつれて減少する．

媒質の異方性にもとづくレーリー波の速度の増加（減少）はレーリー波の軌道の収縮（拡張）をおこす．このとき鉛直成分は水平成分よりもより敏感に反応する．

媒質の異方性にもとづくレーリー波の速度変化に対する軌道の変化は非常に大きい．例えば，速度が1パーセント変化するとき軌道の大きさは約 10 パーセント変化する．この事実はきわめて重要と考えられる．感度の悪い速度変化からでなく感度の非常に高いレーリー波の軌道変化から媒質の異方性を調べられる可能性を示唆しているからだ．

レーリー波が安定（不安定）なとき，自由表面におけるレーリー波の軌道の軸は後方（前方）に傾く．レーリー波がその波長の距離を通過するとき振幅で 10～20 パーセント変化するとすると軌道の傾斜角はおよそ 5～30 度位である．