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Efficient Active Suppression of Vibration Based upon Explicit Treatment of Actuator Characteristics

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Abstract

A new approach to efficient active suppression of seismic vibration of structures is developed. The key concept of the new method is explicit treatment of the characteristics of the actuators: power limit and delay of response. Actuators with the newly developed control criterion can work even under extremely large seismic loading.

In order to achieve the objectives, the classical theory of optimal control is applied and optimality conditions of operation of the actuators are obtained. A solution procedure is proposed for one-degree-of-freedom system, and a concise control criterion is derived. The controllability of the proposed procedure is investigated by a synthesis procedure. Next, the case of a delayed actuator is studied by modeling it with a first-order differential equation, and a similar concise criterion is obtained.

Finally, the efficiency of the present method is examined by means of numerical simulation. The result shows high efficiency of the proposed method in suppressing the excessive vibration responses, in comparison to the classical regulator method.

1. Introduction

This paper presents a new approach to suppression of vibration of structures under seismic ground motions.

Considering the technical background which has been established in these several decades, we employ the concept of feedback as the basic strategy of the present study (YAO, 1972). But conventional methods, which apply the optimal regulator theory, have several disadvantages if one tries to directly apply them to suppress seismic vibrations.

The present approach starts with precise application of classical variational theory to remove the disadvantages of the conventional methods. In order to discuss the aim of the present study and the functions which the new method must possess, we first review briefly the conventional regulator theory.

Even before the regulator method, the feedback concept was widely recognized. For convenience of description, let us take an example of one-degree-of-freedom system excited by an external disturbance. If we pose a force which is always proportional to the velocity or the displacement of the system, only the sign being inverted, the vibration is in some sense moderated. The velocity or displacement feedback is equivalent to an augmentation of the damping force or resistant force of the system, respectively.

This problem has been investigated in detail by the regulator theory which formulates the problem as minimization of an appropriately introduced objective function. The objective function is an integral over a prescribed time interval of a simple sum of two quadratic forms: one consisting of the variables representing the state of vibration such as displacement and/or velocity and the other including the control force.

By assuming the solution in a feedback type form or, equivalently, by assuming the optimal control force to be expressible as a linear combination of the state variables, one obtains as the new unknown variable the gain matrix which assigns feedback weights to the input signals.

The gain matrix is governed by a Riccati equation with respect to time, which is to be solved simultaneously with the equation of motion of the system. The boundary condition of the gain matrix is not given at the initial moment but at the terminal moment, while the equation of motion has the usual initial condition.

This complexity of the boundary conditions is a reflection of the mathematical globalness of the problem; i.e., the optimal solution at every time point cannot be specified by information available at that time only, but is determined after having compared all possible subsequent processes.

In order to eliminate this obstacle, most practical studies have assumed the gain matrix to be constant over the time interval (YANG, 1975; CHUNG *et al.*, 1989), or restricted the time interval of control to be very short (YANG, 1987). This theory provides theoretical background to the classical, essentially empirical, feedback technique; recent practical studies of vibration control have been founded on this basis (YOSHIDA *et al.*, 1991, SETO *et al.*, 1991).

So long as the power requested by the commanding signal remains within the power limit of the actuator, this method meets no troubles. But in such a case, the actuator would rarely exhibit its maximum ability, and reasonably economical design of the actuators would never allow such an extra margin. And yet the seismic load to the structures and therefore compensating control force needed are extremely unstationary; the actuator cannot avoid the request for power output greatly exceeding its ability.

In such situations, of course, the output of the actuator would behave in a nonlinear manner around its peak ability, and, as a result, it deviates from the expected optimal output. The actuator thus fails to work normally just at the critical moments when the reduction of vibration is most desirable. Simple application of the classical feedback technique or the simplified regulator method is therefore inadequate for our problem.

It is true that the second term of the objective function including the control force has a so-called penalty effect; it restrains the control force from excessive growth. But this effect is indirect and cannot completely prevent the request for excessive control force; consequently, explicit treatment of the constraints of the power of the actuators is therefore of primary importance.

A mathematical model must be supported by assurance of the existence of a physical model which it represents. In the present study, the new physical concept is direct force control. This concept is convenient because, if it is properly designed, it works normally even under excessively large deformation (UDWADIA and TABAIE, 1981). On the contrary, the direct objective of control in conventional methods has been the displacement. But this way of control cannot respond properly to such a strong external disturbance as seismic force.

One way of realizing direct force control is to introduce linear induction motors as actuators (FUJITA *et al.*, 1990; TAKAHASHI *et al.*, 1990). Detailed and systematic description of this model is not presented in this paper, but the result of the present paper will afford a mathematical basis for such actuators.

In Chapter 2, the basic mathematical model which is suitable for formulation of the problem is given. The control device is constructed as follows: The time interval is divided into very short subintervals, and the optimal output, or, more precisely, the optimal input signal of the actuator is determined at the beginning of every subinterval. The derivation follows the general treatment of optimal control directly based upon variational calculus. This approach is necessary so as to explicitly incorporate the constraint of the power limit of the actuator.

The present approach also allows consideration of the effect of delayed actuator response. The general results of this theory is the well established switching theorem (OLDENBURGER, 1966): the optimal operation is a successive switching between the upper and the lower bounds of actuator power. Accordingly, higher efficiency of the new method than the conventional regulator method is expected because the simple feedback criterion accompanying a constant gain matrix, which usually requests intermediate values of actuator power, cannot be optimal.

In Chapter 3, the general theory established in the preceding chapter is applied to a one-degree-of-freedom system in which the actuator requires no rise time. The optimality condition which has been stated in an abstract form in Chapter 2 is expressible in analytical form. Taking the general switching property of the optimal control of linear systems into account, we seek the optimal solution in the form of constant, peak value operation. The equations are then completely solvable, and a concise control criterion is obtained which selects one of the peak values of the actuator with reference to the value of displacement and/or velocity of the system.

The obtained result demonstrates the existence of an uncontrollable domain in the ensemble of initial conditions in the sense that if the initial condition of a subinterval belongs to this domain, no optimal operation is found by the above procedure. The origin of such an uncontrollable domain is made clear in Chapter 4. This is accomplished by a kind of synthesis procedure: construction of a globally optimal solution by assembling local solutions obtained by the above solution procedure.

The synthesis procedure, an analytical treatment of the globalness of the control problem, is generally difficult and no universal method has been established. Only few classical problems, such as the shortest time problem of one-degree-of-freedom system have been published (BELLMAN *et al.* 1956; LASALLE, 1960; WONHAM and JOHNSON, 1964). In this paper, this approach is successfully extended to a class of vibration suppression problem of one-degree-of-freedom system.

It is concluded that, for such subintervals, truly optimal operation requires at least one extra switching at an appropriate intermediate moment in the subintervals. In practice, however, the subintervals are very short; accordingly, the uncontrollable domain is spatially very narrow, and only few subintervals are uncontrollable. As a result, the influence of such uncontrollability is negligible.

In Chapter 5, we reanalyze the one-degree-of-freedom system, taking the effect of actuator response delay into consideration. With reference to the actual linear motors, the behavior of the actuator is formulated in terms of a first order linear differential equation which contains a term representing the reaction of the structure to the actuator. A similar solution procedure to that of the preceding chapter is constructed and an extended form of control criterion is obtained.

In Chapter 6, results of numerical examination of the presented method are shown in comparison to those of the corresponding regulator method.

2. General Formulation

Let us consider a finite-degree-of-freedom system. A finite number of variables which completely specify the behavior of the system, the so-called state variables, are denoted as $\{x(t)\}=\{x_1(t), x_2(t), \cdots, x_N(t)\}^t$ in which N is the dimension of the problem and $\{a\}^t$ denotes the transposed vector of $\{a\}$.

(1) Equation of Motion

We assume that the law of dynamics governing the process of change of the state variables is expressed in terms of the equation of motion which is a system of ordinary differential equations. Following conventional usage (PONTRYAGIN *et al.*, 1962), we adopt a system of first order differential equations as the fundamental equation:

$$\dot{x}_j(t)=h_j(t;\ x_1, x_2, \cdots, x_N;\ u_1, u_2, \cdots, u_M) \qquad j=1, 2, \cdots, N \tag{1}$$

In order to express the equation of motion in terms of first order ordinary differential equations, one must assign the degree of freedom to the first order time derivatives of the original variables; accordingly, the dimension N of the problem is ordinarily equal to double the number of physical degrees of freedom of the system.

The functions on the right hand sides depend upon the control variables $u(t)=\{u_1(t), u_2(t), \cdots, u_M(t)\}$ as well as the state variables. The capital letter M is the number of control variables. In ordinary problems of vibration control, the control variables represent input signals to the actuators. Let the control variables be limited with some prescribed upper and lower bounds:

$$u_{j,\min} \leqq u_j \leqq u_{j,\max} \qquad j=1, 2, \cdots, M \tag{2}$$

These parameters represent the maximum acceptable input signal of the actuators.

The initial values of the state variables are prescribed as follows:

$$x_1(0)=X_1,\ x_2(0)=X_2, \cdots, x_N(0)=X_N \tag{3}$$

(2) Objective Function

Let us divide the time interval into sufficiently short subintervals, and let us define the problem as follows: Determine at the beginning of each subinterval the values of the control variables which minimize an appropriately prescribed objective function defined as an indicator of the state of vibration over the subinterval. Let us denote this subinterval as $(0, T)$ without loss of generality, and our problem is how to construct reasonable control criteria over the interval $(0, T)$ at the initial time $t=0$.

Quadratic forms of the state variables are conventionally employed as the measure of the level of vibration. The present study follows this approach. One representative form is the integral of the quadratic form over the whole subinterval: i.e.,

$$J'=\int_0^T \frac{1}{2}\{x(t)\}^t[Q]\{x(t)\}dx \tag{4}$$

in which $[Q]$ is a positive definite matrix whose components represent the weights of the contributions of the state variables to the measure of the vibration magnitude.

Another convenient alternative is the value of the quadratic form at the end of the subinterval:

$$J=\{x(T)\}^t[Q]\{x(T)\} \tag{5}$$

So long as the time interval is sufficiently short, there is no significant difference between the solutions obtained under the above two objective functions. But Eq. (5) is used in the subsequent discussions because it makes theoretical considerations easier. Our problem is to minimize J (or J') subject to equality constraint (1) associated with initial condition (3) and inequality constraint (2).

Following the established methodology, let us introduce the Lagrange multipliers $p_j(t)$ $(j=1, 2, \cdots, N)$ in order to incorporate the equations of motion (1) into the objective function (MAREC, 1979, MORTON, 1969). The extended objective function takes the following form:

$$\hat{J}=\{x(T)\}^t[Q]\{x(T)\}+\int_0^T \{p(t)\}^t(\{\dot{x}\}-\{h\})dt \tag{6}$$

in which $\{p\}=(p_1, p_2, \cdots, p_N)$; this is called the costate variable hereafter. The modified problem includes two groups of unknown variables $\{x\}$ and $\{p\}$. The minimization condition of the modified objective function (6) is as follows:

1) Equation of motion (1).
2) Linear and homogeneous differential equations of the costate variables:

$$\{\dot{p}(t)\}=-[H]\{p(t)\}, \tag{7}$$

in which H is the Jacobian matrix of the transformation $\{x\}\to\{h\}$; i.e., $H_{ij}=\partial h_i/\partial x_j$. This equation is referred to as the covariant equation in this paper. If the equation of motion is linear, these equations do not contain any state variables.

3) Initial conditions (3) for the state variables.
4) Terminal conditions for the costate variables:

$$Q_j x_j(T) + p_j(T) = 0 \qquad j = 1, 2, \cdots, N \tag{8}$$

where, for simplicity, $[Q]$ is regarded as diagonal.

5) Optimality condition: The terms $\{p(t)\}^t[H']\{u(t)\}$ must be maximum for all components of the control variables at every instant t, in which the component H'_{ij} is defined as the derivative of h_i with respect to u_j ($i=1, 2, \cdots, N$; $j=1, 2, \cdots, M$). In the subsequent discussion, we say that the costate variables and the control variables are consistent if they satisfy the optimality condition.

Because of the optimality condition, u_j must take its upper bound or its lower bound whenever the term

$$\sum_{i=1}^{N} p_i(t) H'_{ij}$$

takes positive or negative sign, respectively. It may take an intermediate value at the instant when this term vanishes. Consequently, the optimal control process is expected to be constructed as an appropriately arranged sequence of alternating switching of the peak values of actuator power.

If we employ the integral form of the objective function (4), a small change of the result occurs. The terminal conditions for the costate variables are simply as follows:

$$p_j(T) = 0 \qquad j = 1, 2, \cdots, N \tag{8$'$}$$

But at the same time several new terms representing the contribution of the state variables are added to the covariant equations (7). This disadvantage is serious if the equation of motion is linear; coupling between the state variables and the costate variables occurs. They are independent of each other if the instantaneous form (5) is employed.

Although the optimality condition of control is stated in a formal manner, its solution is not easy, and an iterative approach must be introduced. As a preliminary study, let us start the analysis by assuming some control criterion, or, in other words, let us assume the control variables. Then the equation of motion (1) is solvable. The obtained solution of the state variables specifies the terminal value of the costate variables through Eq. (8). Then the covariant equation (7) is solvable. We thus obtain a solution.

The obtained state variables and costate variables must be consistent with the prescribed control variables everywhere in the interval. Of course, except for accidental rare cases, the above obtained solution does not satisfy this condition, and it is necessary to modify the solution.

We must therefore seek an appropriate rule how to modify the first assumption so as to improve the solution. Unfortunately, this is difficult because the degree of freedom of the indefiniteness of the control variables is very large; i.e., even if a local change of the control variables yields some improvement of the objective function, there is no assurance that repetition of such procedures eventually leads to the truly optimal solution.

This difficulty is removed if we only consider the suppression of vibration over a very short time interval relative to the characteristic period of the structure. According to the switching property of the optimal operation, it is reasonable to expect that no change of the control variables is needed in most of the subintervals if the subintervals are sufficiently short. In other words, switching of the control variables would only be necessary for exceptionally few subintervals. Along this line of thought, we first seek solution for the state and costate variables for constant control variables. If the obtained costate variables and the assumed control variables are consistent, then we may accept the solution as truly optimal. Otherwise we must abandon the solution as false and seek another type of solution.

3. One-Degree-of-Freedom System

Imagine a simplified example of one-degree-of-freedom system which is demonstrated in Fig. 1. We try to reduce the vibration of the mass point by applying a control force $f(t)$.

Initial conditions are prescribed as follows in which $x_1(t)=x(t)$ is the displacement and $x_2(t)$ is its derivative:

$$x_1(0)=X_1 \quad \text{and} \quad x_2(0)=X_2 \tag{9}$$

Let the matrix Q of the objective function be diagonal and let

$$Q_{jj}=Q_j. \qquad j=1,2 \tag{10}$$

The equation of motion takes the following form:

$$\dot{x}_1=x_2, \qquad \dot{x}_2=-(K/m)(x_1-u)-2cx_2 \tag{11}$$

in which m is the mass of the structure; K is the stiffness of the spring; $2c$ is the damping coefficient divided by m; $u=[2cm\dot{z}(t)-f(t)]/K+z(t)$.

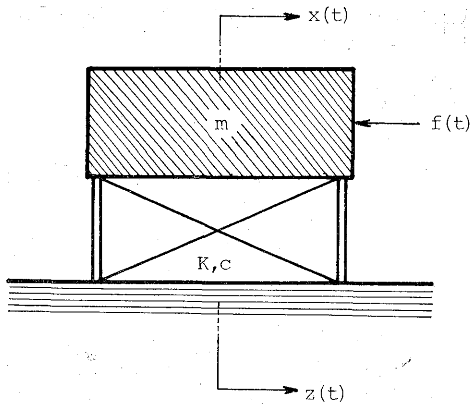

Fig. 1. Concept of one-degree-of-freedom system.

Since the subinterval is very short, the displacement $z(t)$ of the foundation, representing the external disturbance, is regarded as constant; accordingly, the control variable u remains constant in the subinterval.

The covariant equation (7) is as follows:

$$\dot{p}_1=(K/m)p_2, \qquad \dot{p}_2=-p_1+2cp_2 \tag{12}$$

The state variables are solved as follows:

$$x_1(t)=e^{-ct}\left\{(X_1-u)\left[\cos(\omega t)+\frac{c}{\omega}\sin(\omega t)\right]+\frac{X_2}{\omega}\sin(\omega t)\right\}+u \tag{13}$$

$$x_2(t)=e^{-ct}\left\{-(X_1-u)\left(\frac{c^2}{\omega}+\omega\right)\sin(\omega t)+X_2\left[\cos(\omega t)-\frac{c}{\omega}\sin(\omega t)\right]\right\} \tag{14}$$

in which $\omega=\sqrt{(K/m-c^2)}$.

The costate variables are solved as follows:

$$p_2(t)=\frac{e^{-c(T-t)}}{\omega}[C_1\sin(\omega t)+C_2\cos(\omega t)] \tag{15}$$

in which

$$C_1=-(X_1-u)e^{-cT}\left\{Q_1\cos^2\omega T+\left[\frac{c}{\omega}Q_1+c\left(\frac{c^2}{\omega}+\omega\right)Q_2\right]\sin\omega T\cdot\cos\omega T\right.$$
$$\left.+\omega\left(\frac{c^2}{\omega}+\omega\right)Q_2\sin^2\omega T\right\}+X_2e^{-cT}\left\{cQ_2\cos^2\omega T-\left[\frac{Q_1}{\omega}+\left(\frac{c^2}{\omega}-\omega\right)Q_2\right]\right.$$
$$\left.\times\sin\omega T\cdot\cos\omega T-cQ_2\sin^2\omega T-uQ_1\cos\omega T\right\} \tag{16}$$

and

$$C_2=(X_1-u)e^{-cT}\left\{[Q_1-(c^2+\omega^2)Q_2]\sin\omega T\cdot\cos\omega T+\left[\frac{c}{\omega}Q_1+c\left(\frac{c^2}{\omega}+\omega\right)Q_2\right]\right.$$

$$\times \sin^2 \omega T\Big\} + X_2 e^{-cT}\Big[\omega Q_2 \cos^2 \omega T - 2cQ_2 \sin \omega T \cdot \cos \omega T$$

$$+\left(\frac{Q_1}{\omega} + \frac{c^2}{\omega} Q_2\right) \sin^2 \omega T\Big] + uQ_1 \tag{17}$$

Specifically, the boundary values of p_2 are as follows:

$$p_2(0) = \frac{e^{-cT}}{\omega} C_2 \tag{18}$$

and

$$p_2(T) = e^{-2cT} Q_2 \Big[-(X_1 - u)\left(\frac{c^2}{\omega} + \omega\right) \sin \omega T + X_2\left(\cos \omega T - \frac{c}{\omega} \sin \omega T\right)\Big] \tag{19}$$

If the value $u = u_{\max}$ makes these boundary values of p_2 negative, then the operation $u = u_{\max}$ is truly optimal. Similarly if the value $u = u_{\min}$ makes them potitive, then the operation $u = u_{\min}$ is acceptable as optimal. Otherwise, the two operations are not optimal or, in other words, the above procedure has failed to find the optimal solution.

The validity of the above procedure depends upon the combination of the initial values of X_1 and X_2 of the state variables. The critical condition for the initial value and the terminal value of the second costate variable is expressible by two parallel lines on the $X_1 - X_2$ plane, as shown in Fig. 2-(a) and -(b), respectively. In this example, the damping factor is zero, $\omega T = 0.1$ [rad], and $Q_2 = 0.5 Q_1/\omega^2$.

The strip areas between the pairs of parallel lines are the uncontrollable domain in the sense that, if the initial condition drops into this domain, the constant peak value control cannot be optimal in the subinterval. This is because the values of the control variable and the second costate variable do not satisfy the optimality condition. If, on the contrary, the point which corresponds to the initial conditions of the subinterval is situated above or below these uncontrollable domains, the constant peak value control, $u = u_{\min}$ or $u = u_{\max}$, is optimal.

Overlaying these figures, one obtains the total uncontrollable domain as is shown in Fig. 3. The meaning of the points of intersection R_+ and R_- of the border lines will be discussed in the next chapter. The shaded narrow apertures lying between the two strips are also uncontrollable domain, because the consistency conditions of the control variable demanded by the initial and terminal values of the second costate variable contradict each other.

In the actual control operation, the nondimensional length of the subinterval ωT is kept very small; the strips are very narrow and uncontrollability scarcely occurs. All the more, the uncontrollable domain

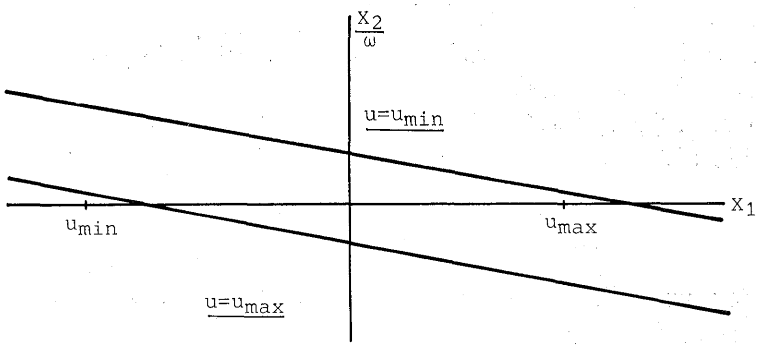

Fig. 2-(a). Optimal operation at the initial time point.

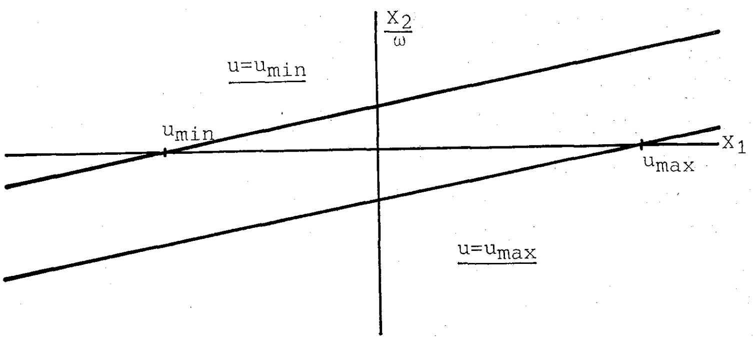

Fig. 2-(b). Optimal operation at the final time point.

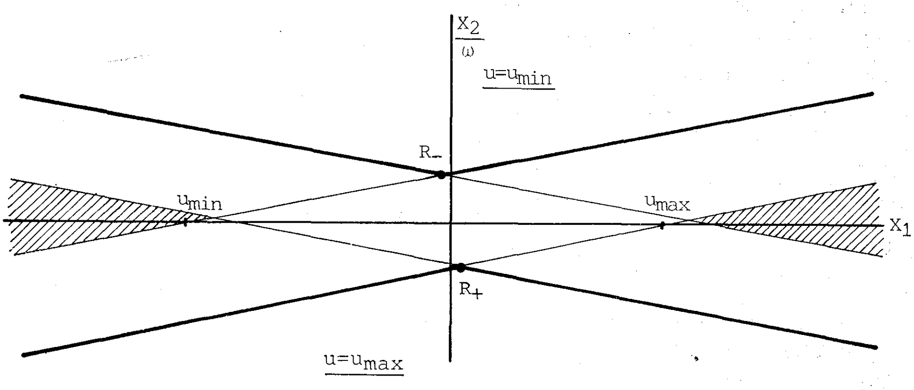

Fig. 3. Optimal operation and the uncontrollable domain.

is confined to the neighborhood of X_1-axis; i.e., it occurs only when the velocity of the system remains very small. The choice of control variable value has therefore little effect.

The existence of the uncontrollable domain thus makes no practical trouble. Nevertheless, it is desirable from the theoretical viewpoint to make clear the nature of the uncontrollability. The origin of the uncontrollable domain is investigated in the next chapter through a synthesis procedure. Appropriate operations for the uncontrollable domain will also be considered.

4. Origin and Nature of Uncontrollability

In the preceding chapter, a control criterion was derived for one-degree-of-freedom system by seeking a procedure assigning a constant value to the control variable during each subinterval. It was also found that a certain domain of initial conditions existed such that if the initial condition of the subinterval belonged to this domain, no optimal operation was found. It is therefore necessary to inquire into the nature of this uncontrollability.

The above uncontrollability was detected by examination of only the values of the second costate variable at only two time points, the initial and the terminal points. Strictly speaking, this examination must be applied to every time point, and, if the subinterval is not sufficiently small, it is possible that the consistency condition between the signs of the second costate variable and the control variable is violated at some intermediate times even though they are consistent at the initial and terminal points. It is therefore necessary to pick up all of the uncontrollable domain.

This is performed by the so-called synthesis procedure of the control process, which is a standard procedure to approach the global properties which are intrinsic to control problems. The analysis of global properties in the case of subintervals whose length is not infinitesimally short is generally very difficult, and it is therefore necessary to simplify the model as much as possible. Fortunately, one-degree-of-freedom system without damping controlled by non-delayed control force accepts the synthesis.

In order to simplify the description of the procedure, the variables are normalized and nondimensionalized as follows: We employ ωt as the independent time variable and denote it as t. Differentiation with respect to this new time variable is denoted by primes. The quantities x_2/ω and ωT are denoted as x_2 and θ, respectively. The nondimensionalized time interval θ need not be infinitesimally small; i.e., the analysis is not necessarily mathematically local, so long as θ does not exceed $\pi/2$. (The case of more than $\pi/2$ does not have theoretical or practical meaning.)

We may assign unity to the diagonal elements of the matrix of the objective function and neglect the contribution of external disturbance without losing the generality.

The equation of motion and the covariant equation takes the following form in which u is the normalized peak value of the actuator power:

$$x_1' = x_2, \qquad x_2' = -x_1 + u \tag{20}$$

$$p_1' = p_2, \qquad p_2' = -p_1 \tag{21}$$

If the control variable remains constant, the second costate variable is solvable as follows:

$$p_2(t) = [X_2 + u \sin\theta] \cos(t) - [(X_1 - u) + u \cos\theta] \sin(t) \tag{22}$$

Specifically,

$$p_2(0) = X_2 + u \sin\theta \tag{23}$$

and

$$p_2(\theta) = -(X_1 - u) \sin\theta + X_2 \cos\theta \tag{24}$$

The first condition requires that the absolute value of the second state variable, the normalized velocity, be not more than $u \sin\theta$. The feasible domain defined by the second condition is the declined stripe:

$$(X_1 - u) \tan\theta < X_2 < (X_1 + u) \tan\theta \tag{25}$$

Let ϕ denote the argument of the point $((X_1-u)+u \cos\theta,\ X_2+u \sin\theta)$, and

$$p_2(t) = -(\text{positive constant}) \sin(t-\phi) \ldots\ldots 0 \leqq t \leqq \theta \tag{62}$$

This representation enables a synthesis to be done as follows: Consider the plane which is spanned by the pair of initial values of state variable components and draw circles of radius u whose centers are located at the two points $(u, 0)$ and $(-u, 0)$, as is shown in Fig. 4. Draw two parallel lines which pass the centers of the circles and whose gradients are $\tan\theta$. Let us denote each one of the intersections between the circles and the lines which are nearer to the origin as R_+ and R_-, respectively. Let P denote the point (X_1, X_2), and the angles between the X_1-axis and the two lines R_+P and R_-P give the value of ϕ in Eq. (26). We donote them ϕ_+ and ϕ_-, respectively.

Let X_2 be nonnegative because the situation is quite similar for negative X_2. We divide the half plane into subdomains as is shown in Fig. 5 with the above introduced two lines $X_2 = (X_1 \pm u) \tan\theta$ together with the horizontal line $X_2 = u \sin\theta$.

The subdomains I, V and VI have already been recognized as uncontrollable by means of the initial sign of the second costate variable

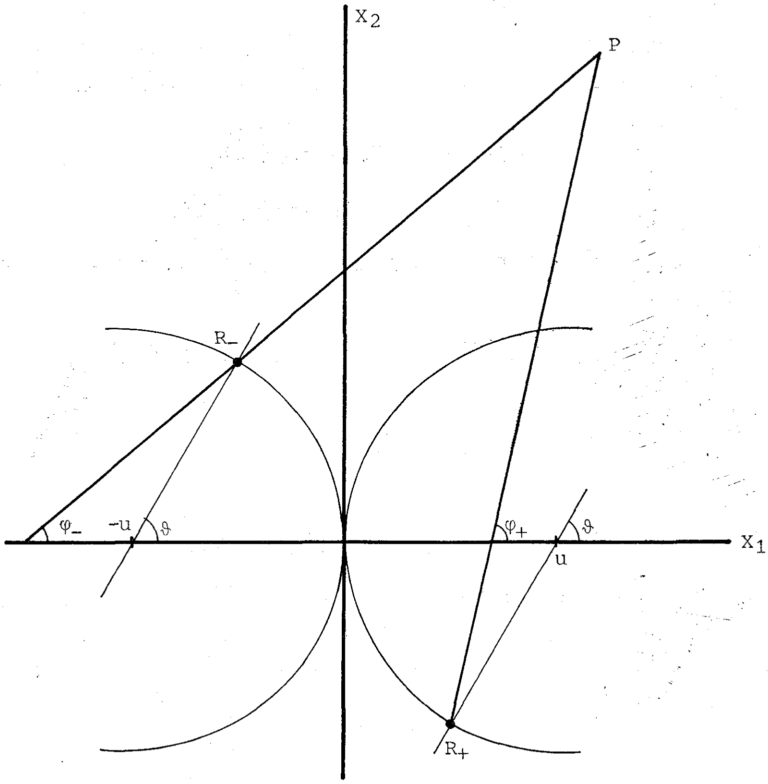

Fig. 4. Change of the sign of the costate variable.

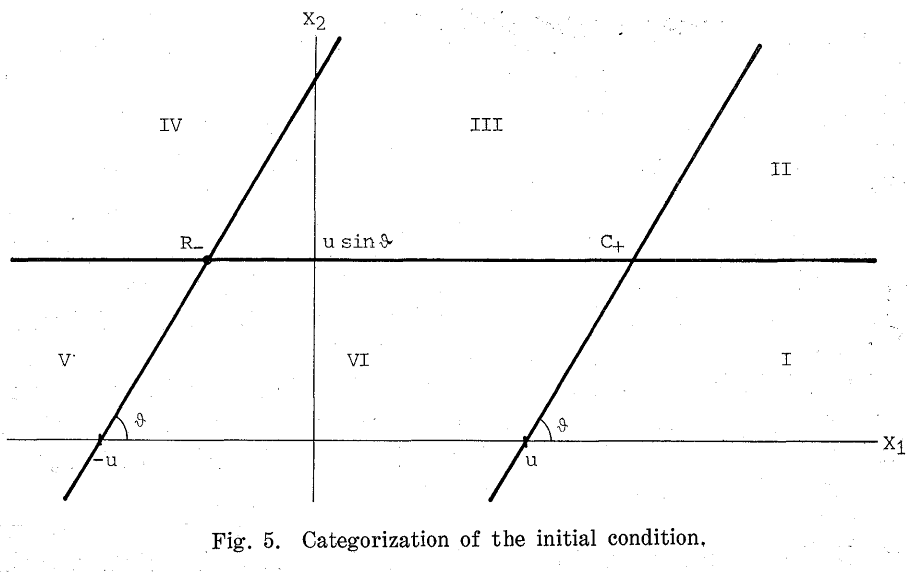

Fig. 5. Categorization of the initial condition.

p_2. Similarly, subdomains III and VI are judged as uncontrollable by the terminal sign of p_2. All the subdomains are classified systematically as follows by Eq. (26).

I:	$-\theta<\phi_-<0<\phi_+<\theta$.	Therefore no feasible operation exists.
II:	$0<\phi_-<\phi_+<\theta$.	No feasible operation exists because p_2 changes the sign in the interval $(0, \theta)$.
III:	$0<\phi_-<\theta<\phi_+$.	No feasible solution exists.
IV:	$\theta<\phi_-<\phi_+$.	The variable p_2 is always positive and, therefore, the negative peak value of the control variable is optimal.
V:	$\pi/2<\phi_+<\pi<\phi_-$.	No feasible operation exists.
VI:	$\phi_-<0,\ \theta<\phi_+$.	No feasible operation exists.

It is thus concluded that optimal operation can be constructed by means of the constant operation only in subdomain IV.

The next question is specification of the origin of the uncontrollable domain. In order to answer this question, let us solve the equation of motion (20) without fixing the value of the control variable $u(t)$. The objective function becomes

$$J=\left[X_1-\int_0^\theta u(t)\sin(t)\,dt\right]^2+\left[X_2+\int_0^\theta u(t)\cos(t)\,dt\right]^2 \tag{27}$$

Consequently, the objective function is identifiable as the distance between the initial point $P=(X_1, X_2)$ and the newly defined point R:

$$R=\left(\int_0^\theta u(t)\sin(t)\,dt,\ -\int_0^\theta u(t)\cos(t)\,dt\right). \tag{28}$$

Let $\sum$ denote the domain to which the point R can reach by some feasible control $u(t)$. We simply call this domain the feasible domain. Again referring to Fig. 4, the domain $\sum$ can be drawn as follows: Let R_+ and R_- denote the two points $\{u(1-\cos\theta), -u\sin\theta\}$ and $\{-u(1-\cos\theta), u\sin\theta\}$, respectively. These are reachable by means of the constant peak value operations $+u$ and $-u$, respectively. Draw two circles of radius $2u$ with centers at $C_+=(1+\cos\theta, \sin\theta)$ and $C_-=(-1-\cos\theta, -\sin\theta)$. The shaded intersection of the two circles is the domain reachable within one intermediate switching; the domain $\sum$ must contain this domain.

This figure makes it possible to completely interpret the results shown in Fig. 5 as follows: The terminal point of the shortest path which connects the initial points situated in domain I, II, or III to the feasible domain does not coincide with R_+ or R_-, but is situated at an intermediate point on the bordering circular arc S_- of the domain.

The situation is quite similar to the points in domain V; the terminal

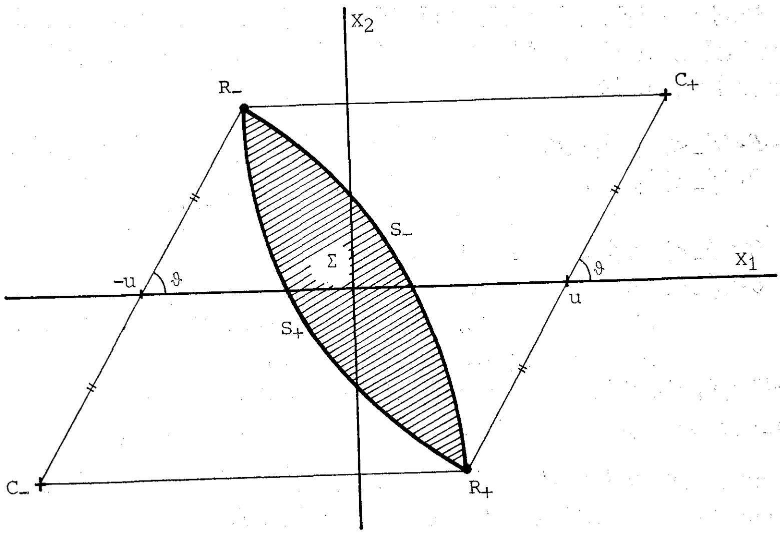

Fig. 6. Reachable domain of feasible operations.

point of the shortest path is on the bordering arc S_+. The shortest paths starting from the initial points in domain VI are totally included in $\sum$ or reach $\sum$ at points on S_1 or S_2. Only the shortest path from the points in domain IV reaches $\sum$ at point R_-.

The above analysis demonstrates that the uncontrollable domain of the initial conditions is the ensemble of the initial conditions under which the control process containing one intermediate switching is better than the constant value operations.

The most practical countermeasure against this sort of uncontrollability would be shortening of the subintervals. In practice, the non-dimensional interval is very short; therefore, the uncontrollable domain is expected to be a very narrow area around the abscissa or, in other words, the optimal procedure is violated only exceptionally when the velocity of the structure approximately vanishes.

The loss of optimality because of this type of uncontrollability is also expected to remain small if $\sum$ is small. Considering the uniformness of the distribution of initial points over the uncontrollable domain, it is reasonable to make the control variable zero for the subinterval.

5. First Order Delay System

If the control force $f(t)$ in Fig. 1 is supplied, for example, by an ordinary linear induction motor, it is expressible in terms of the input current I to the motor; i.e., if the current remains within some prescribed range, $f(t)$ is proportional to I:

$$f(t)=F\cdot I(t), \tag{29}$$

in which F is constant.

The electromagnetic motion is represented in terms of a first order ordinary differential equation:

$$L\dot{I}(t)+RI(t)-K_e x_2(t)=e(t), \tag{30}$$

in which the left hand side includes the inductance L, resistance R and the counterelectromotive force $K_e x_2$, respectively. The parameters L, R and K_e are the motor constants. The right hand side term is the input voltage.

Introducing the current as the third state variable, one obtains the equation of motion as follows:

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=-\omega_0^2x_1-2cx_2-\kappa x_3+Z\\ \dot{x}_3&=kx_2-\gamma x_3+u\end{aligned} \tag{31}$$

in which $\omega_0^2=K/m$; $\kappa=F/m$; $\gamma=R/L$; $k=K_e/L$; $u=u(t)=e(t)/L$ and $Z=Z(t)=[2mc\dot{z}(t)+Kz(t)]/m$. The remaining parameters have been introduced in Chapter 3.

Let the objective function be identical to Eq. (5) associated with Eq. (10):

$$J=\frac{1}{2}\{Q_1[x_1(T)]^2+Q_2[x_2(T)]^2)\} \tag{32}$$

Introduce the costate variables p_1, p_2, and p_3, and the Hamiltonian of the problem is given as follows:

$$H=p_1x_2-p_2(\omega_0^2x_1+2cx_2+\kappa x_3-Z)+p_3(kx_2-\gamma x_3+u) \tag{33}$$

Let capital X's express the initial conditions for the subinterval under consideration:

$$x_j(0)=X_j \qquad j=1, 2, 3 \tag{34}$$

Similar to Eq. (8), the terminal condition is as follows:

$$\begin{aligned}&p_j(T)=-Q_jx_j(T) \qquad j=1, 2\\ &p_3(T)=0\end{aligned} \tag{35}$$

The consistency condition between the costate variable and the control variable is as follows: Let u_0 denote the maximum value of u, and for any t belonging to the subinterval, $u(t)$ must be equal to $+u_0$

or $-u_0$, while at the same time the product $p_3(u)\cdot u(t)$ must be nonpositive.

(1) Solution Procedure

Let Ω denote the matrix composed of the coefficients of the equation of motion (31):

$$\{\dot{x}(t)\}=[\Omega]\{x(t)\} \tag{36}$$

Then the covariant equation is expressible as follows:

$$\{\dot{p}(t)\}=-\{\Omega\}^t\{p(t)\}. \tag{37}$$

Let ρ_1, ρ_2 and ρ_3 denote the eigenvalues of $[\Omega]$. The vector $\{a_j\}$ defined as follows is the eigenvector of $[\Omega]$ corresponding to ρ_j:

$$\{a_j\}=(\rho_j+\gamma,\ \rho_j(\rho_j+\gamma),\ \kappa\rho_j)^t \qquad j=1,2,3 \tag{38}$$

The ρ_j's are also the eigenvalues of $[\Omega]^t$, and its eigenvector $\{b_j\}$ corresponding to ρ_j is as follows:

$$\{b_j\}=(-\omega_0^2(\rho_j-\gamma),\ \rho_j(\rho_j-\gamma),\ \kappa\rho_j)^t \qquad j=1,2,3 \tag{39}$$

Let us define the 3×3 matrices $[A]$ and $[B]$ by arranging the column vectors $\{a_j\}$ and $\{b_j\}$, respectively. Then the costate variable is expressed as follows:

$$\{p(t)\}=[B][D(T-t)][C]\{x(0)\}, \tag{40}$$

in which

$$[C]=-[B]^{-1}[Q][A][D(-T)][A]^{-1}; \tag{41}$$

$[D]$ and $[Q]$ are 3×3 diagonal matrices such that $D_{jj}(t)=e^{-\rho_j t}$, $Q_{jj}=Q_j$ $(j=1,2)$ and $Q_3=0$. Particularly, the third costate variable $p_3(t)$ takes the following form:

$$p_3(t)=\sum_{j,k} b_{3j}e^{-\rho_j(T-t)}C_{jk}x_k(0) \tag{42}$$

Let $u=u_0$ and specify the initial values of the state variables. Because Eq. (42) is linear and homogeneous with respect to the initial values, the condition that $p_3(t)$ is nonpositive for every t in $(0, T)$ defines as the controllable domain a certain convex domain in the space spanned by the initial values. The situation is quite similar to the case $u=-u_0$, and another convex domain is defined as the controllable domain. The null operation $u=0$ is assigned to the initial conditions which do not belong to the controllable domains. If T is sufficiently small, it is enough to examine the sign of p_3 only at $t=0$ and $t=T$.

(2) Short Delay Approximation

The inductance L in Eq. (30) is usually very small if we employ ordinary linear induction motors. If this is the case, the coefficient matrix $[\Omega]$ is nearly singular and the above mentioned general treatment is not applicable. In this case, a perturbation method can bring a regularized algorithm by identifying the decreasing order of the large quantity γ $(=R/L)$ as follows.

If L is small, one of the eigenvalues of the coefficient matrix $[\Omega]$ is a negative real number of very large absolute value. Denote it as ρ_3, and

$$\rho_3=-\gamma+2\sigma+4\sigma\eta\gamma^{-1}+O(\gamma^{-2}) \tag{43}$$

in which $\sigma=K_e\kappa/(2R)$ and $\eta=\sigma+c$. The parameter σ is an indicator of the stiffness and the damping ability of the actuator and η is the damping coefficient of the total system. The principal first two terms are functions of the intrinsic constants of the actuator only and independent of the properties of the structure.

The remaining two eigenvalues describe the mechanical behavior of the system and are specified by the following quadratic equation to accuracy of order γ^{-1}:

$$\rho^2+2\eta(1+2\sigma\gamma^{-1})+\omega_0^2+8\sigma\eta(\sigma+\eta)\gamma^{-1}=0 \tag{44}$$

It is obvious that the real part of these roots is negative. By considering the principal terms which do not contain γ^{-1}, we obtain the following classification:

1) If η is larger than ω_0, the system is overdamping:

$$\rho=-\eta\left(1+\frac{2\sigma}{\gamma}\right)\pm\left[\sqrt{\eta^2-\omega_0^2}-\frac{2\sigma\eta(\eta+2\sigma)}{\sqrt{\eta^2-\omega_0^2}}\frac{1}{\gamma}\right] \tag{45}$$

2) If η is smaller than ω_0,

$$\rho=-\eta\left(1+\frac{2\sigma}{\gamma}\right)\pm i\left[\sqrt{\omega_0^2-\eta^2}-\frac{2\sigma\eta(\eta+2\sigma)}{\sqrt{\omega_0^2-\eta^2}}\frac{1}{\gamma}\right] \tag{46}$$

The latter should be the ordinary case because the stiffness of the available actuators is supposed to be very much weaker than that of the structures.

Comparison of Eqs. (13) and (46) immediately shows that the behavior of both solutions is similar except for the replacement of the original damping constant c by $\eta=\sigma+c$. Accordingly, the response of the control system is qualitatively similar to that of the case of a nondelayed control force so long as the rise time or, equivalently, the ratio of the inductance of the actuator to its resistance, is sufficiently small. The influence of the delay is analyzed by observing the change of the right hand side of Eq. (46) when the characteristic rise time γ gradually changes. This requires an extensive numerical investigation; detailed discussion of the results will be discussed elsewhere.

6. Numerical Examinations

To illustrate the efficiency and the advantages of the present method compared to the classical linear regulator method, two examples of simulation of a one-degree-of-freedom lumped mass system are presented in this chapter: one system with a non-delayed control force and another with a delayed control force supplied by a first order time-lag actuator.

The objective function takes the following form:

$$J=\frac{1}{2}[x^2(T)+q\dot{x}^2(T)] \tag{47}$$

where x and $\dot{x}$ denote the displacement and velocity of the mass point,

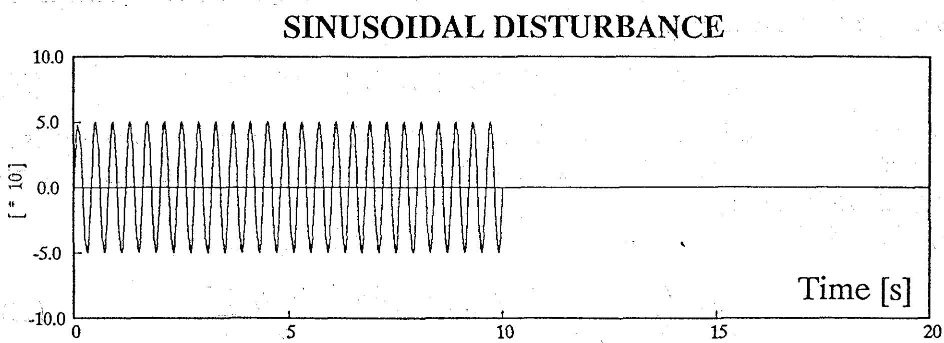

Fig. 7. Slowly-varying sinusoidal disturbance.

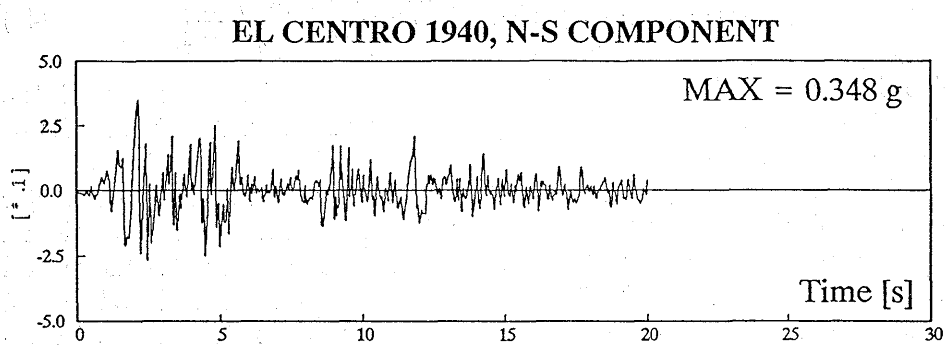

Fig. 8. NS component of El Centro seismic record of 1940.

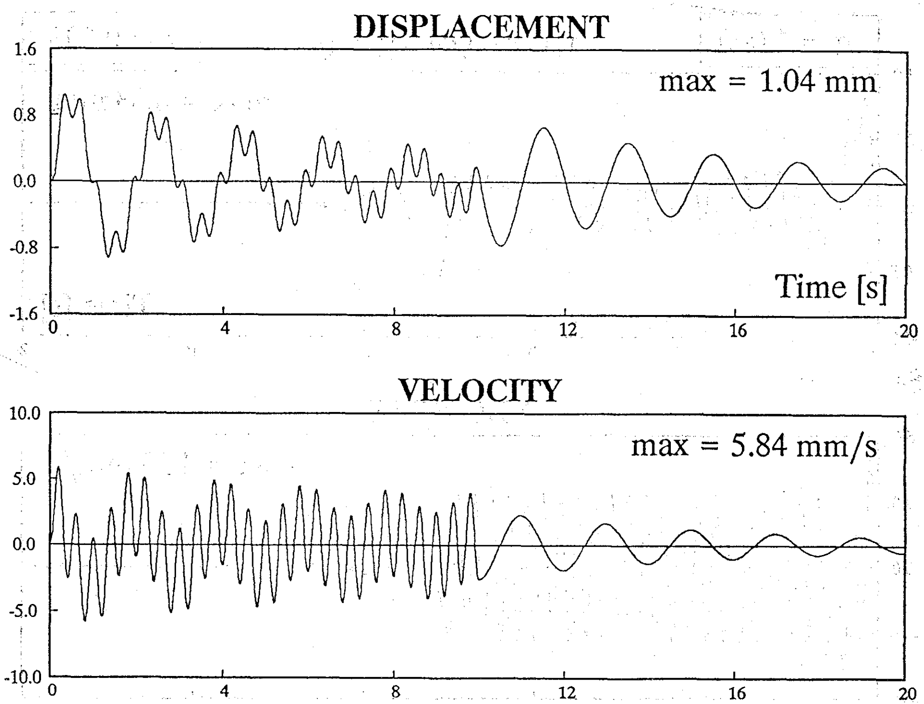

Fig. 9. Time histories of displacement and velocity under sinusoidal disturbance without control force.

respectively, and q is a selective positive parameter. The time interval of each operation is denoted as T.

Let the frequency and the damping ratio of the system be 3.16[rad/sec] and 0.05, respectively. The mass is 1[Ns²/mm] and 0.02[Ns²/mm] for the non-delayed and the delayed case, respectively. The time interval T is 0.02[sec]. The parameter q is increased from 0.1 to 10.

Two types of input disturbance are investigated:

1) Slowly varying sinusoidal disturbance of 10 sec duration with amplitude and period of 50[mm/sec²] and 0.4[sec], respectively, as shown in Fig. 7.
2) Seismic disturbance defined as 2 percent scale of NS component of the El Centro seismic record of 1940, as shown in Fig. 8.

(1) Case of Non-delayed Control Force

1) Sinusoidal Disturbance

Fig. 9 shows time histories of the displacement and velocity of the one-degree-of-freedom model without control force subjected to a sinusoidal disturbance, while Figs. 10 and 11 show controlled response

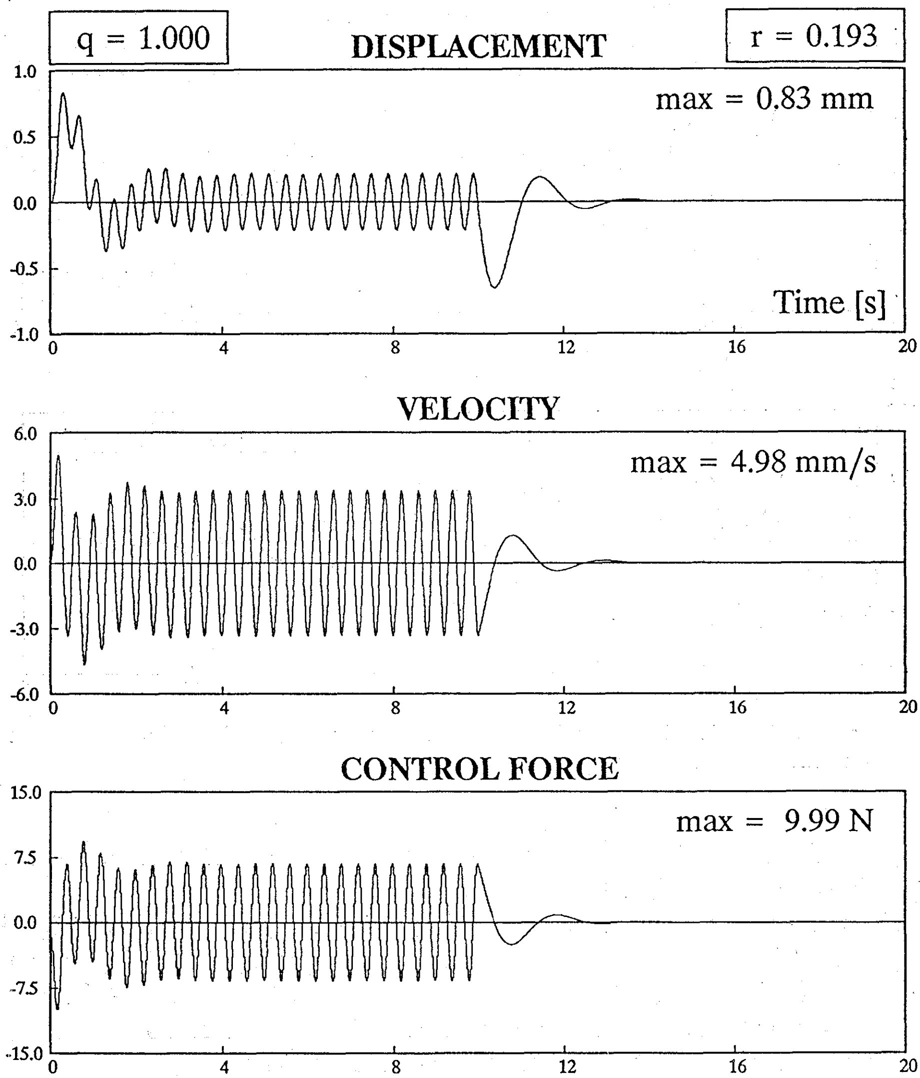

Fig. 10. Time histories of displacement, velocity and control force under sinusoidal disturbance obtained by the regulator method.

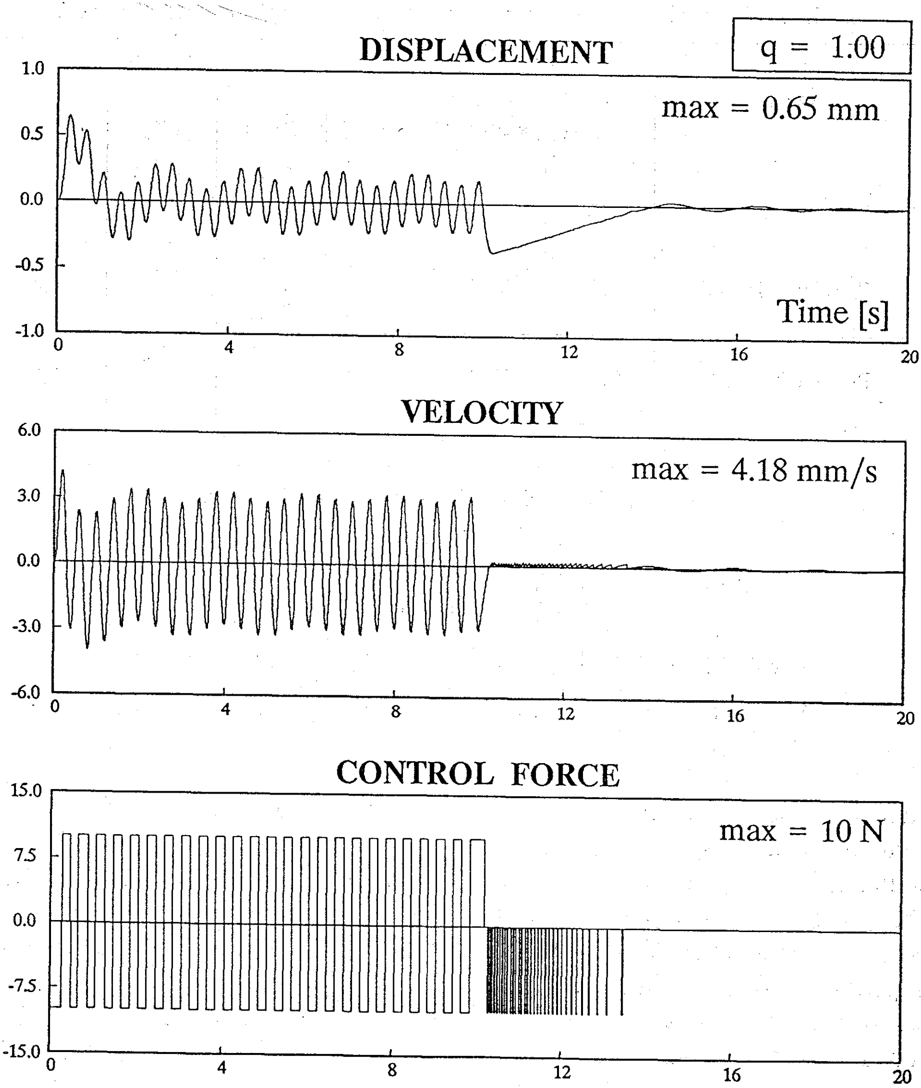

Fig. 11. Time histories of displacement, velocity and control force under sinusoidal disturbance obtained by the present method ($q=1$).

Table 1. Comparison of displacement reduction and control force efficiency.

Case:	Sinusoidal			Seismic		
	Disp. [mm]	Red. [%]	η [%]	Disp. [mm]	Red. [%]	η [%]
Uncontrolled	1.04			3.47	(max)	
				1.16	(rms)	
Classical Method	0.83	20.2	12.3	1.99	42.7	6.4
				0.44	62.1	
Present Method	0.65	38.0	53.9	1.11	67.9	41.1
				0.21	81.9	

Table 2. Comparison of control force efficiency.

Case:	Sinusoidal		Seismic	
	Max. [N]	η [%]	Max. [N]	η [%]
Classical Method	20.4	15.7	22.4	3.9
Present Method	10.0	53.9	10.0	41.1

time histories by employing the classical regulator method and the present method, respectively. In both cases, the maximum control force is limited to 10[N]. From these figures and Table 1 it can be seen that the present method performs better than the classical one. The reason why the present method performs better than the classical one can be explained by observing the time histories of the control force. In the classical method, the maximum control force is produced only at certain times. On the other hand, in the present method, at every time the control force reaches its maximum value.

To measure the efficiency in utilizing the available control energy, let us introduce an efficiency factor as follows:

$$\int_{t_0}^{t_1} u(t)^2 dt \Big/ \int_{t_0}^{t_1} u_{\max}^2 dt \tag{48}$$

where t_0 and t_1 are the time limits of the simulation. The efficiency factors in Table 1 show that the present method is much more efficient in utilizing the maximum control energy. To achieve the same amount of vibration suppression as shown in Table 2, the maximum control force required by the classical method is double that of the present method.

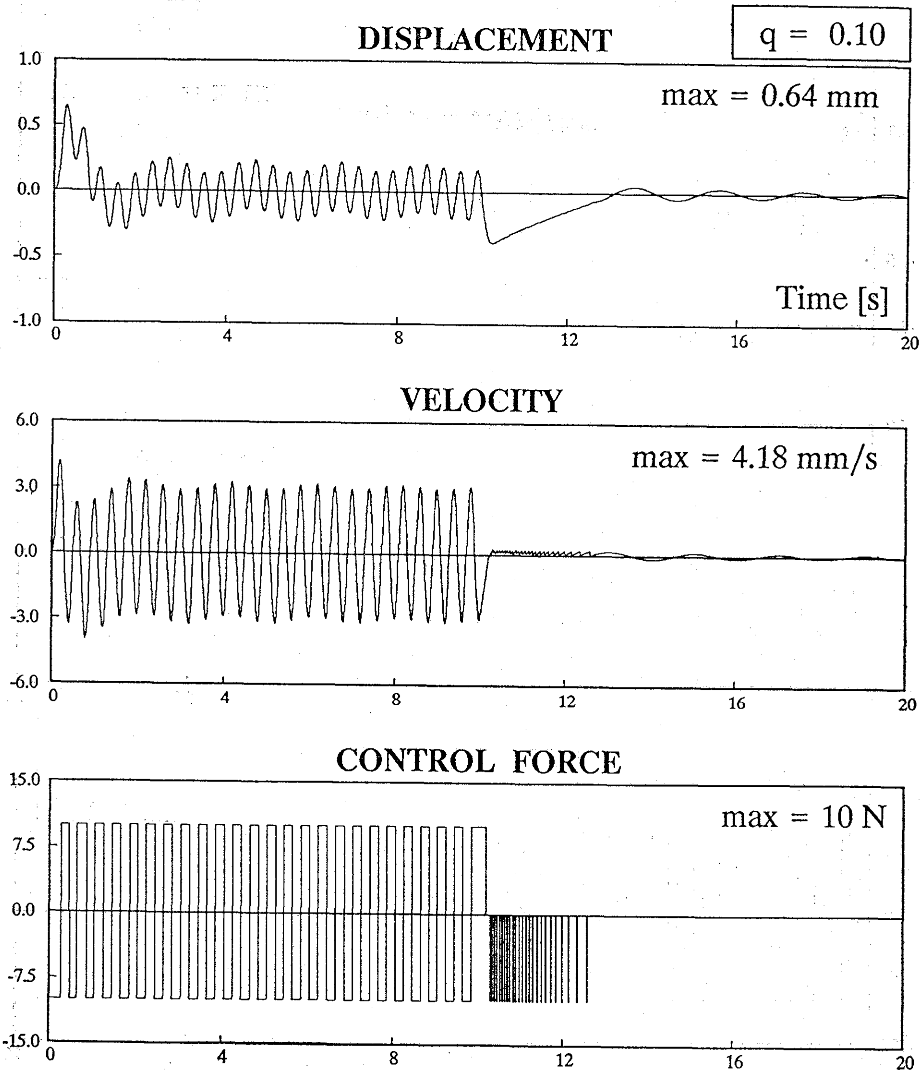

Fig. 12. Time histories of displacement, velocity and control force under sinusoidal disturbance obtained by the present method ($q=0.1$).

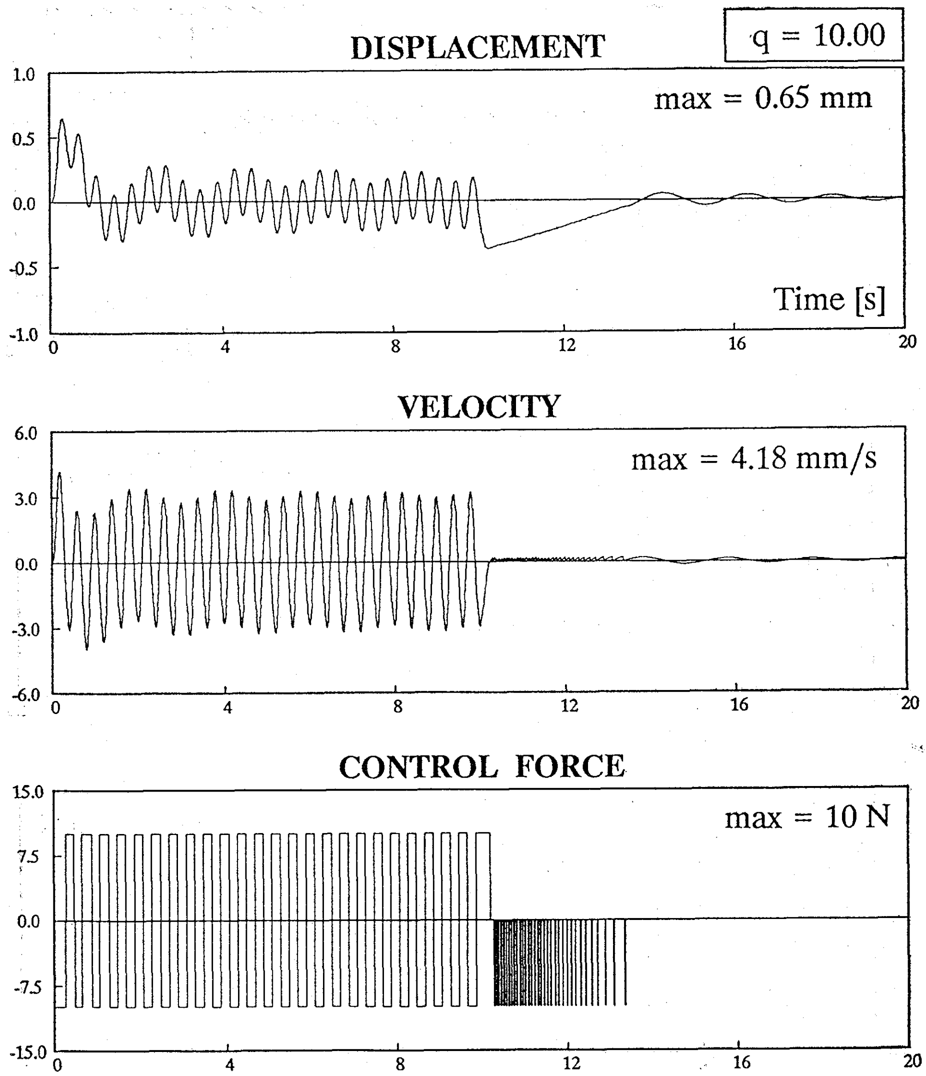

Fig. 13. Time histories of displacement, velocity and control force under sinusoidal disturbance obtained by the present method ($q=10$).

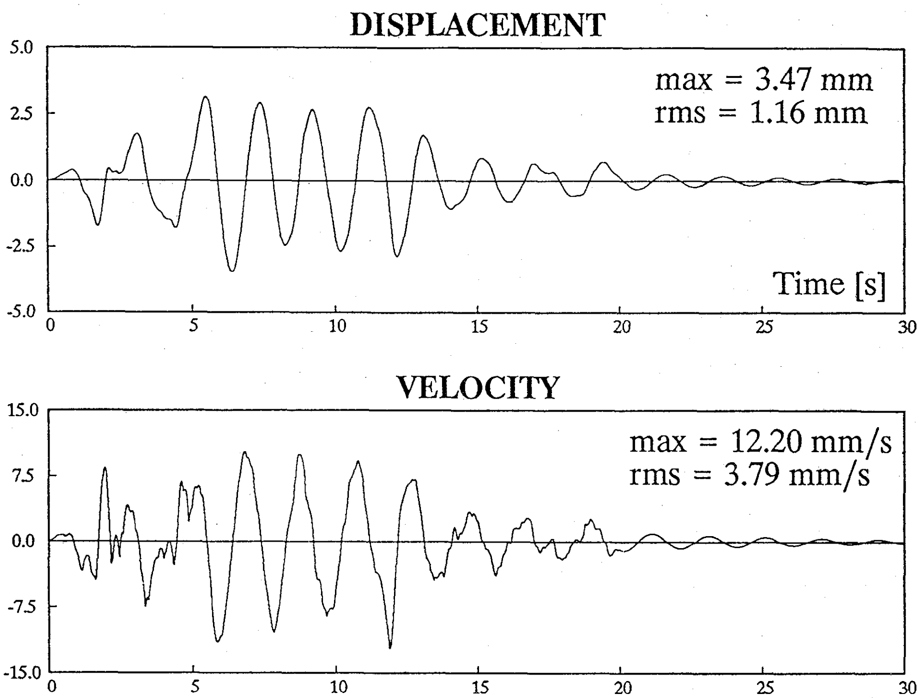

Fig. 14. Time histories of displacement and velocity under seismic disturbance motion without control force.

In the previous simulation, the weighting factor q was taken to be 1. If q is assigned a value different from 1, for example $q=0.1$ or $q=10$, the response time histories of the system are almost the same as for $q=1$, as is seen from Figs. 12 and 13, respectively. From this observation, it can be concluded that the new method is insensitive to the weighting factor q.

2) Seismic Disturbance

Fig. 14 shows the time histories of uncontrolled responses of the model due to a seismic disturbance as shown in Fig. 8. The time histories of controlled responses by employing the classical method and the new one are presented in Figs. 15 and 16, respectively. As in the case of a sinusoidal disturbance, the maximum control force is limited to 10[N]. In Table 1, the root mean square values of the displacement response are also given to show that the new method is not only efficient in suppressing the maximum vibration response which occurs only at certain times, but also efficient in suppressing the root mean square values which represent the average energy content of the system.

From Table 1, it can be observed that the present method is also

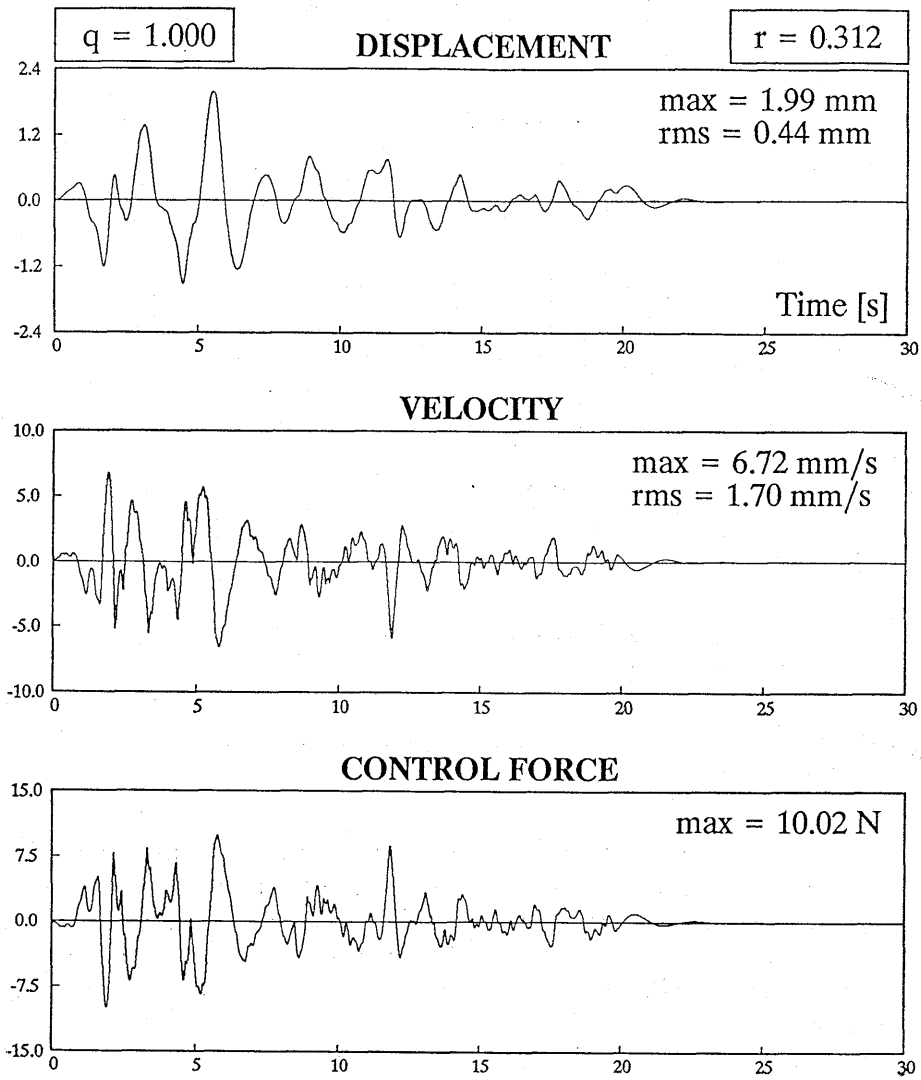

Fig. 15. Time histories of displacement, velocity and control force under seismic disturbance obtained by the regulator method.

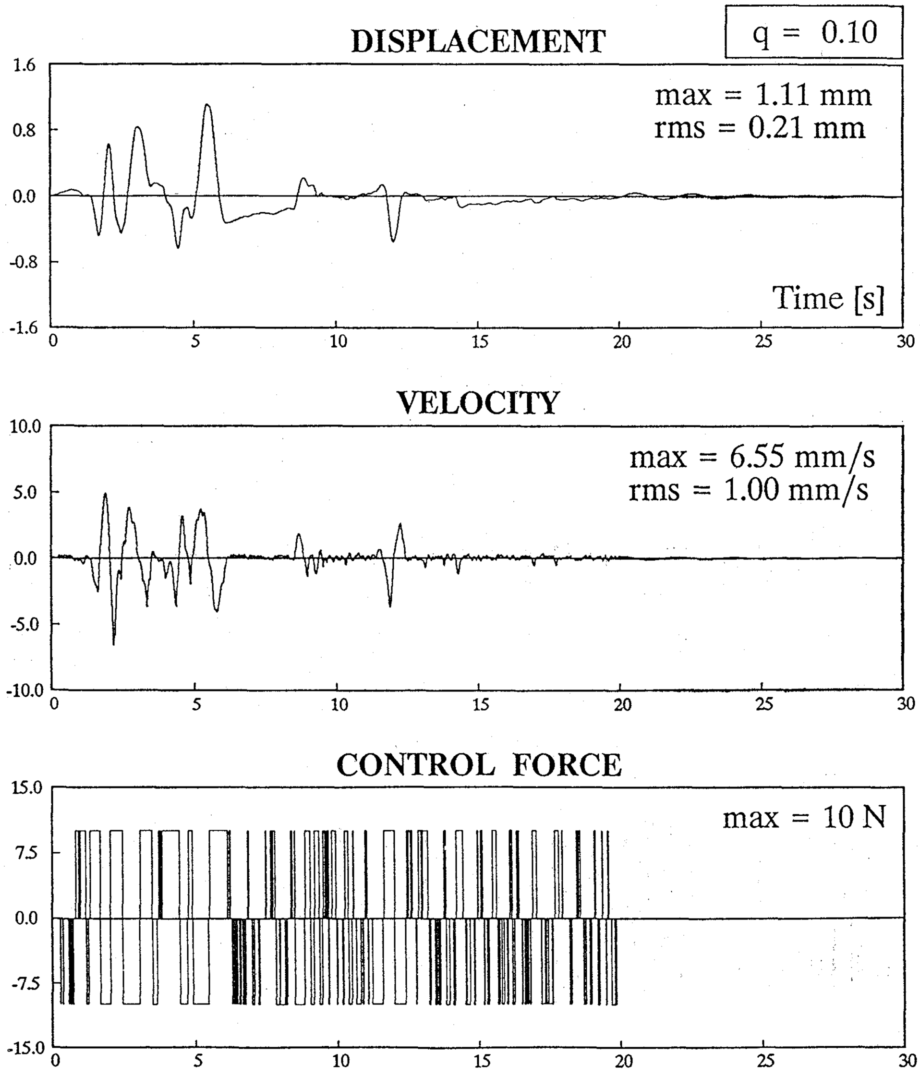

Fig. 16. Time histories of displacement, velocity and control force under seismic disturbance obtained by the present method ($q=1$).

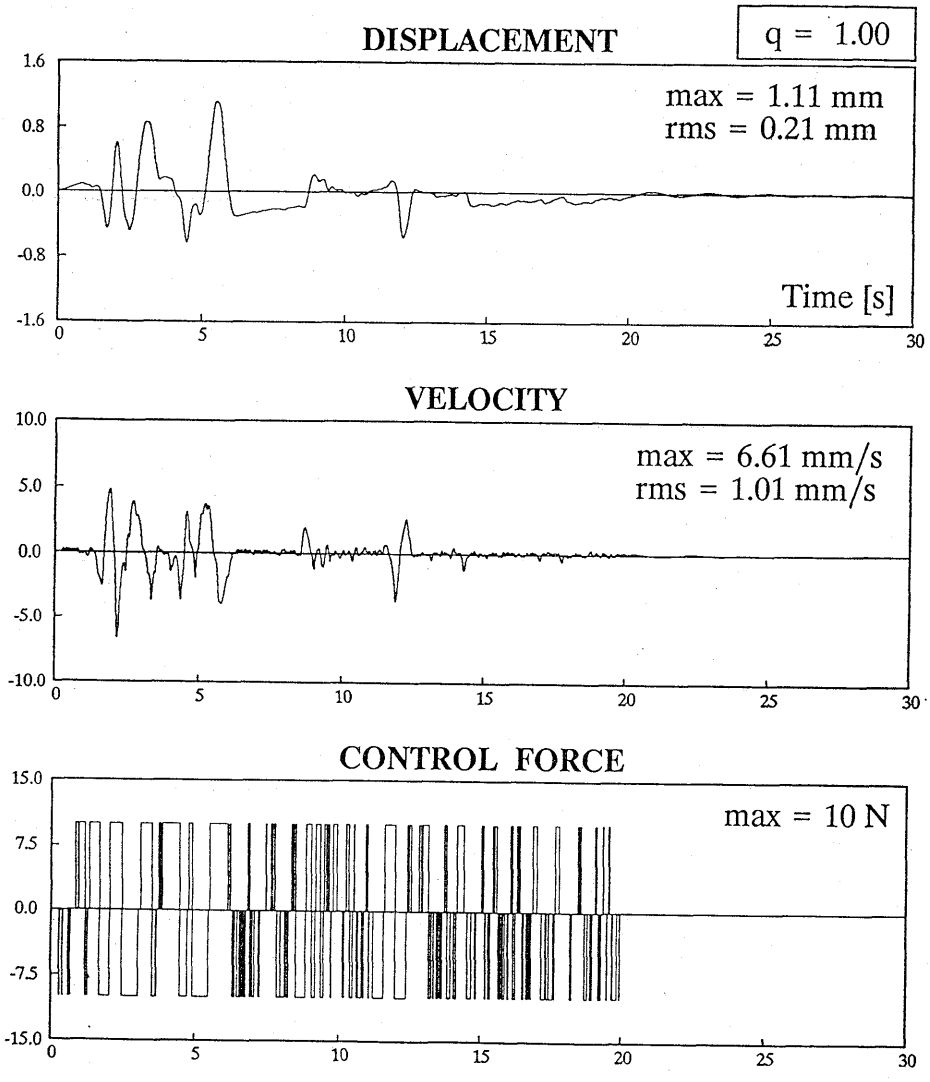

Fig. 17. Time histories of displacement, velocity and control force under seismic disturbance obtained by the present method ($q=0.1$).

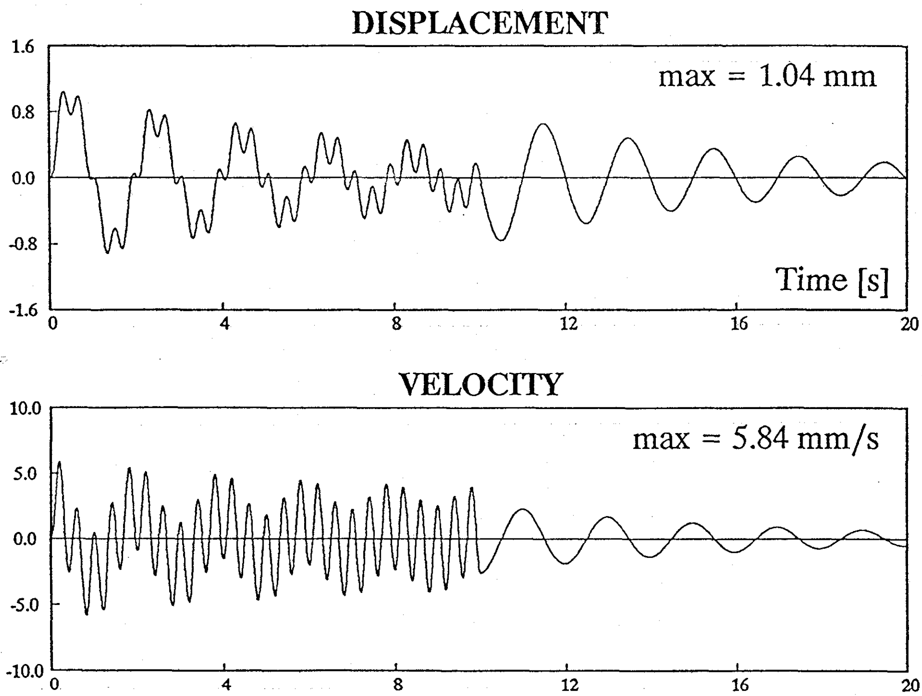

Fig. 18. Time histories of displacement and velocity of delayed system under sinusoidal disturbance without control force.

more efficient than the classical one in suppressing the vibrations due to non-stationary seismic disturbance.

The response time history for $q=0.1$ as presented in Fig. 17 shows only slight difference of vibration from that for $q=1$ in the free vibration period. This fact again shows that the present method is insensitive to the weighting factor q.

(2) Case of Delayed Control Force

In this case, the actuator is modeled as a first-order time-lag system. The characteristics of the actuator are as follows: back-electromotive force constant $K_e=0.046$[Vsec/mm], thrust constant $F=46$[N/A], inductance $L=0.0047$[H] and resistance $R=7.4[\Omega]$.

1) Sinusoidal Disturbance

Figs. 18 and 19 show the uncontrolled and controlled response time histories due to sinusoidal disturbance. From these figures it can be seen that the reduction of displacement response is up to 98 percent which is extremely high. This is due to the fact that the eigenvalues and eigenvectors of the coupled one-degree-of-freedom actuator system are all real-valued, which means that the coupled system is overdamped.

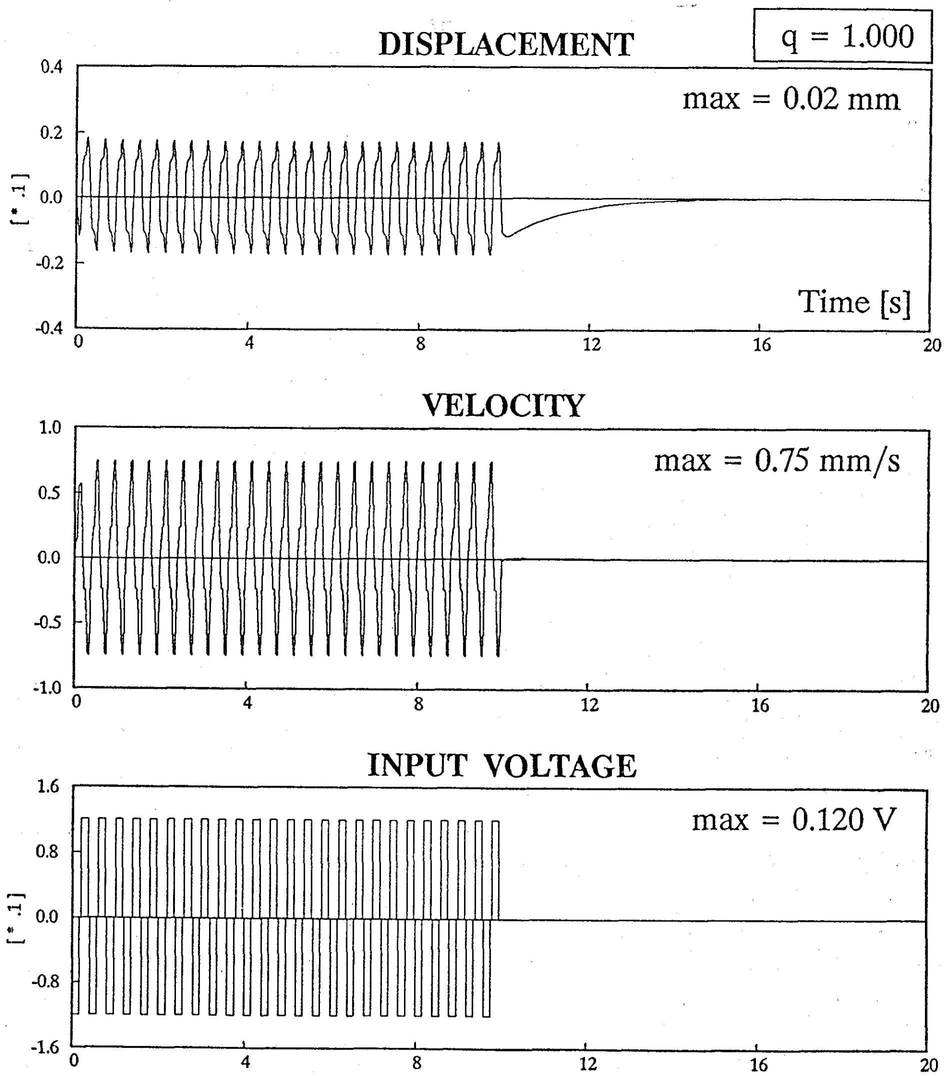

Fig. 19. Time histories of displacement, velocity and input voltage of delayed system under sinusoidal disturbance obtained by the present method.

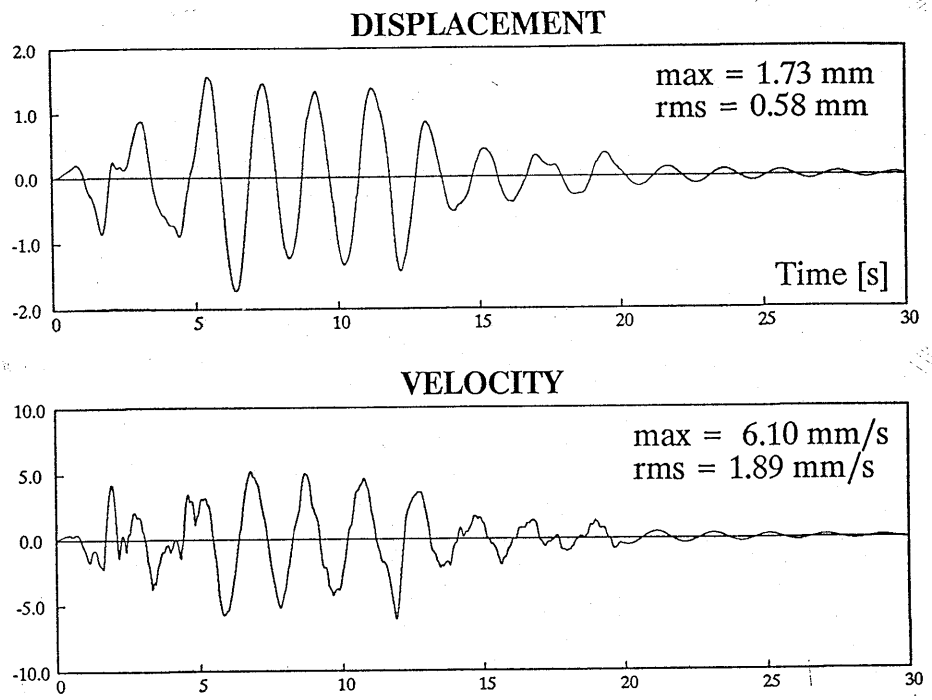

Fig. 20. Time histories of displacement and veolcity of delayed system under seismic disturbance without control force.

The degree of overdamping can be adjusted by varying the parameters of the actuator.

In case of seismic disturbance as shown in Figs. 20 and 21, the reduction of displacement response is also very high, i.e., 94 percent.

7. Conclusions

A new approach to efficient active suppression of seismic vibration of structures is developed.

In order to achieve an explicit treatment of the power limit of actuators, which is the key concept of the study, the classical theory of optimal control is applied, and optimality conditions of operation of the actuators are obtained.

A solution procedure is proposed for a one-degree-of-freedom system, and a concise control criterion is derived. The controllability of the proposed procedure is investigated by developing a synthesis procedure.

Next, the case of delayed actuator is studied by modeling it with a first-order differential equation, and a similar concise criterion is obtained.

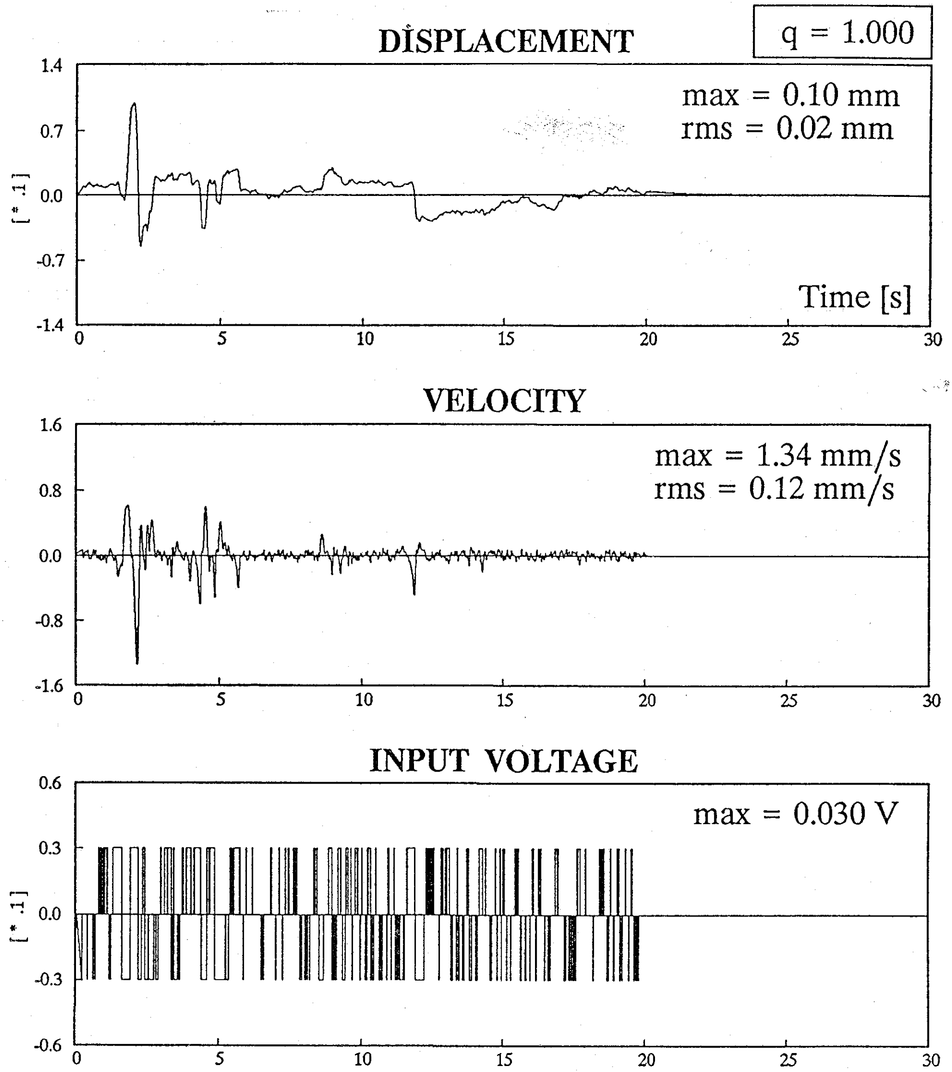

Fig. 21. Time histories of displacement, velocity and input voltage of delayed system under seismic disturbance obtained by the present method.

Finally, the efficiency of the present method is examined by means of numerical simulations, drawing the following conclusions:

(1) The new method is very efficient in suppressing the excessive vibration responses due to both stationary and non-stationary disturbances.

(2) The new method is more efficient than the classical regulator method both in the ability of vibration suppression and in control energy utilization.

(3) The new method is applicable to a one-degree-of-freedom structure with a first order delayed actuator.

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アクチュエータの特性の精密な扱いに基づいた効率的なアクティブ制震

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地震時の構造物の振動をアクティブに抑制する一つの効率的な手法を提案する．その基礎となる考え方は，出力限界や応答遅れなどアクチュエータの能力の制約を精密に扱うことにある．この新しい制御規範によって駆動されるアクチュエータは，巨大地震時にも正常に作動することができるという利点をもつ．

この目的を実現するためには，古典的な最適制御理論を直接適用しなければならない．それは既往の標準的手法である最適レギュレータ理論が問題に適合しないからである．ここで得られた最適性の定式は，まず1自由度系に適用され，その解の構造方法および簡潔な最適操作則が得られる．またこの方法の制御性が，シンセシスの実行によって明らかにされる．次に，一次遅れ系として表現されるアクチュエータを含む系が解析され，遅れのない系と対応した簡潔な最適操作則が求められる．

最後に，本手法の効率が，数値計算によって検証される．その結果，本手法の制震効率が既往の最適レギュレータ理論と比較して高いこと，また遅れ系への適合も良好であることが示される．