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The Stress Field Induced by Antiplane Shear Cracks —Application to Earthquake Study—

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Abstract

The problem of two interacting antiplane shear cracks in the elastic half-space consisting of two vertical slabs of different rigidity is analyzed. The problem is solved by modeling the slip along the cracks as a continuous distribution of screw dislocations. The unknown dislocation density distribution is found by solving numerically the system of integral equations. The stress field induced by a single antiplane crack and two colinear cracks is studied and compared with that due to inplane shear cracks. The results are applied to some problems of geophysical interest. It has been found that both the free surface and lateral inhomogenities of the medium should have an effect on seismic and aseismic activity following major faulting in the medium. Also, the breaking of a barrier or seismic gap should result in a stress increase in the regions outside the fracture zone.

1. Introduction

Insight into the mechanism of earthquake generation is of paramount importance in the analysis of seismicity, of medium deformation, and of other earthquake related phenomena. It is generally accepted that an earthquake is a consequence of the inability of a medium to withstand increasing shear stress. When some critical value of the stress locally equals the strength of the material, fracture takes place resulting in a shear stress drop on the fracture surface. This basic concept of a tectonic earthquake, which was first formulated by REID (1910) from the study of surface deformations accom-

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panying the San Francisco earthquake of 1906, is still accepted today.

Starting from the above assumptions it is natural that dislocation (VOLTERRA, 1907; STEKETEE, 1958; MARUYAMA, 1963, 1964, 1966; BURRIDGE and KNOPOFF, 1964) and crack (GRIFFITH, 1921; IRWIN, 1958; KOSTROV, 1964, 1966; KEILIS-BOROK, 1959) theories have been applied as a main tool in mathematical and physical descriptions of mechanics of earthquakes. There is no essential conceptual difference between a dislocation and a crack. In both cases it is assumed that a displacement discontinuity (in general variable in time and in space) is introduced in the medium along some surfaces. In the case of a dislocation, however, the displacement discontinuity is arbitrarily prescribed as a boundary condition on the dislocation surface, while in the case of a crack model, which is based on more realistic physical foundations, the stress drop is used as a boundary condition on the crack surface.

Both static and dynamic (kinematic) crack and dislocation models are widely applied in the study of various problems related to seismology (CHINNERY, 1961, 1963; DAS and AKI, 1977; HASKELL, 1964, 1969; HASTIE and SAVAGE, 1970; KOSTROV, 1970; MADARIAGA, 1976; MATSU'URA *et al.*, 1981; MIKUMO, 1973; MIKUMO and MIYATAKE, 1979; SATO, 1969, 1979; SATO *et al.*, 1979; SAVAGE, 1966; TEISSEYRE, 1964, 1969, 1970; VVEDENSKAYA, 1956; among others). Static models are used for the description of processes involved in the pre-seismic and post-seismic stages of an earthquake (KASAHARA, 1969), while dynamic models are applied to study the time and space development of rupture and wave field generated by an earthquake.

With an increasing flow of more accurate seismological data and with the development of computation techniques, the crack and dislocation models also become more complex and more realistic in their description of observed phenomena. These models when applied to seismology serve a double purpose. First they are used in the interpretation of observational data, and second they attempt to gain further insight into the physics of earthquake generation.

It should be emphasized, however, that the natural conditions are still far from the idealized assumptions taken in the present dislocation and crack models. The medium structure in which earthquakes occur is extremely heteregoneous and often unknow in detail. Also, there is a great degree of mathematical complexity in constructing the models, especially the crack models. For this reason usually two-dimensional crack models are considered, which are much easier to deal with than the three-dimensional models.

The purpose of this paper is to examine the problem of antiplane shear cracks (mode III cracks) which interact with each other in an external shear field. This is a two-dimensional problem for which the relative slip along the cracks may be imagined to be the result of a continuous distribution of screw dislocations. Since the expressions for the elastic field due to a screw dislocation are relatively easy to derive (SAVAGE, 1974), it is possible to choose fairly realistic medium models. We assume here that the cracks are located in an elastic inhomogeneous half-space consisting of two slabs of different rigidity separated by a vertical plane. Choosing such a model for the medium it is possible to find out simultaneously the effect of a free surface (also the effect of a surface low rigidity layer) as well as that of a laterally inhomogeneous medium.

First a brief description of the theory is given. Then some results are presented and discussed in relation to some problems of seismological interest.

2. Outline of theory

Consider a semi-infinite, elastic medium consisting of two vertical slabs of rigidity μ_1 and μ_2, as shown in Fig. 1. The surface $y=0$ is the free surface and the slabs are in rigid contact. We assume here that the system under study is invariant in the z-direction and the mode of deformation is two-dimensional antiplane shear. This means that the only nonzero components of displacement and stress are w (displacement component in the z-direction), τ_{xz}, and τ_{yz}, respectively.

Suppose that as a result of external stress τ^e_{xz} and τ^e_{yz}, acting in the medium, two antiplane cracks A and B interacting with each other are formed in the medium (Fig. 1). Each crack is defined by the coordinates $(x_i, y_i)(i=A, B)$ of one of its tips, its length L_i, and the angle α_i that the crack plane makes with the x-axis.

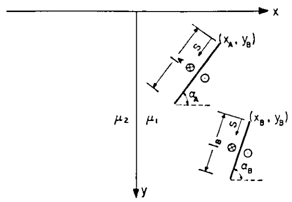

Fig. 1. Geometry of the model analysed. An elastic half-space consisting of two vertical slabs of rigidity μ_1 and μ_2 and containing two antiplane cracks A and B is loaded by shear stress τ^e_{xz} and τ^e_{yz}. Each crack is defined by the coordinates of one of its tips (x_i, y_i) $(i=A, B)$, length L_i, angle α_i that the crack makes with the x-axis, and magnitude of the stress drop along the crack. The distance along the crack is measured by parameter s.

Let us introduce a parameter s which denotes the distance along a crack from its tip to any point on the crack. The slip distribution $\Delta w_i(s)$ along each crack may be considered to be a result of a continuous distribution of screw dislocations of density $g_i(s)$, so that the Burger's vector for the distribution between neighboring points s and $s+ds$ is $b=[d\Delta w_i(s)/ds]ds=g_i(s)ds$. The expressions for displacement and stress fields induced by a single screw dislocation in the medium under consideration are given in the appendix. The functions $g_A(s)$ and $g_B(s)$ are calculated on the assumption that the tangential stress at each point s of the cracks due to dislocations is equal to the prescribed stress drop at that point, $\sigma_i(s)$ (The stress drop is the friction stress, $\tau_i^f(s)$ minus the external tangential stress, $\tau^e_{xz}(s)\sin\alpha_i+\tau^e_{yz}(s)\cos\alpha_i$). The following system of integral equations for $g_A(s)$ and $g_B(s)$ is obtained

$$\frac{1}{\pi}\int_0^{L_A} g_A(s')\left[\frac{1}{s-s'}+K_A(s,\,s')\right]ds'+\frac{1}{\pi}\int_0^{L_B} g_B(s')K_{AB}(s,\,s')ds' =\sigma_A(s),\ s\in[0,\,L_A]\ , \tag{1}$$

$$\frac{1}{\pi}\int_0^{L_A} g_A(s')K_{BA}(s,\,s')ds'+\frac{1}{\pi}\int_0^{L_B} g_B(s')\left[\frac{1}{s-s'}+K_B(s,\,s')\right]ds' =\sigma_B(s),\ s\in[0,\,L_B]\ . \tag{2}$$

The left hand sides of equations (1) and (2) give the value of tangential stress at the point s of crack A (eq. (1)) and of crack B (eq. (2)) induced by screw dislocations located along crack A (first integrals) and along crack B (second integrals). The kernels K_A, K_{AB}, K_{BA}, and K_B are regular, the only singularities are set apart. Kernel $K_{AB}(K_{BA})$ describes the effect of crack $B(A)$ on the dislocations of the crack A (B). The explicit expressions for the kernels for the geometry of cracks as shown in Fig. 1 can be easily found using the formulas (A3) and (A4). Nevertheless they are rather lengthy and they are not reproduced here.

The solution of equations (1) and (2) can be found in the class of unbounded functions on the segments $[0, L_A]$ and $[0, L_B]$ (MUSKHELISHVILI, 1953). Such solutions, however, are non-unique and additional conditions are required to make them so. In the case considered here, the uniqueness of the solution is obtained from the condition that while moving along contour C containing crack A (B) we have

$$\int_C \frac{\partial w(t)}{\partial t}dt=0\ .$$

This condition, which means that a displacement discontinuity occurs only on the cracks, yields two additional integral equations

$$\int_0^{L_A} g_A(s')ds'=0\ , \tag{3}$$

$$\int_0^{L_B} g_B(s')ds' = 0 \ . \tag{4}$$

Equations (1)-(4) form a complete system of integral equations for calculating $g_A(s)$ and $g_B(s)$.

Since the kernals in integral equations (1) and (2) have a Cauchy-type singularity, it follows from the theory of singular integral equations that $g_A(s)$ and $g_B(s)$ should have square-root singularities at the crack tips, that is

$$g_A(s) = \frac{f_A(s)}{\sqrt{(L_A - s)s}} \ , \quad s \in [0, L_A] \ ,$$

$$g_B(s) = \frac{f_B(s)}{\sqrt{(L_B - s)s}} \ , \quad s \in [0, L_B] \ ,$$

where $f_A(s)$ and $f_B(s)$ are continuous bounded functions.

The system of integral equations (1)-(4) can not be, in general, solved analitically. In the present numerical solution, quadrature formulas given by ERDOGAN and GUPTA (1972) have been used.

Once the densities of dislocations $g_A(s)$, $g_B(s)$ are known, the value of stress at any point of the medium (x, y) caused by cracks A and B can easily be found by the superposition principle. We have

$$\tau_{xz}(x, y) = \int_0^{L_A} \tau_{xz}(s', x, y) g_A(s')ds' + \int_0^{L_B} \tau_{xz}(s', x, y) g_B(s')ds' \ ,$$

$$\tau_{yz}(x, y) = \int_0^{L_A} \tau_{yz}(s', x, y) g_A(s')ds' + \int_0^{L_B} \tau_{yz}(s', x, y) g_B(s')ds' \ ,$$

where $\tau_{xz}(s', x, y)$ and $\tau_{yz}(s', x, y)$ can be immediately obtained from formulas (A3) and (A4) with

$$\left.\begin{aligned} \xi &= x_A - s' \cos \alpha_A \ , \\ \eta &= y_A + s' \sin \alpha_A \ , \end{aligned}\right\} s' \in [0, L_A] \ ,$$

and

$$\left.\begin{aligned} \xi &= x_B - s' \cos \alpha_B \ , \\ \eta &= y_B + s' \sin \alpha_B \ , \end{aligned}\right\} s' \in [0, L_B] \ ,$$

respectively.

3. Results

The model described above was used to study the stress field induced by faulting. It was assumed that the rigidity contrast μ_2/μ_1 between the vertical slabs was equal to 2. This value corresponds approximately to the laterally asymmetric P-wave velocity pattern found for the segment of the San Andreas fault near Bear Valley, California (HEALY and PEAKE, 1975). As for the fault itself, a model similar to that associated with the 1966 Parkfield California earthquake was chosen (SCHOLZ *et al.* 1969; ARCHULETA and DAY, 1980), i.e., it is a buried vertical strike-slip fault whose upper edge is located at a depth of 3 km. Two cases were considered. In the first case the fault was modeled as a single antiplane crack with a vertical extension of 6 km. In the second case the fault was assumed to consist of two vertical cracks whose upper and lower tips were located at depths of 3 and 6 km, and 7 and 10 km, respectively. Three positions of the fault in relation to the vertical boundary dividing the slabs were chosen. The horizontal distance between the boundary and the fault were taken as 3, 1, and 0 km. Both a constant stress drop and a stress drop linearly decreasing with depth were considered. Since the results obtained for these two cases are basically similar, only a few results for the constant stress drop are reproduced here. Also, only the τ_{xz} component of stress induced by the fault is presented below. In the model considered, this component of stress corresponds to tectonic shear stress acting in the medium which causes the fracture, so the τ_{xz} pattern is of great importance in quantitative analyses of stress redistributions as a result of fracture formation.

Figures 2 and 3 show the changes in the field of τ_{xz} normalized by the stress drop for a single crack model, for distances of 3 and 1 km between the fault and the boundary dividing the slabs. Figure 4 shows the change in the τ_{xz} component of stress (hereafter called simply stress) for the fault modeled by two

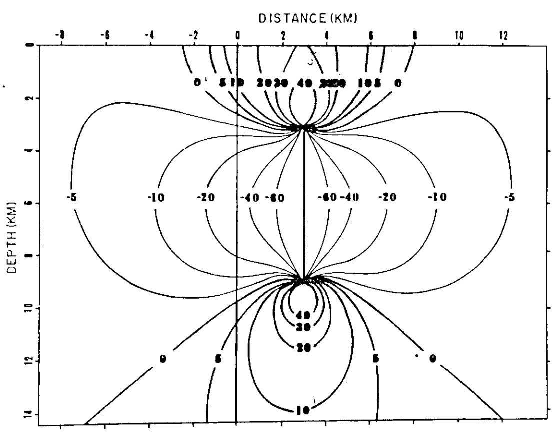

Fig. 2. Contour lines for the $\tau_{xz}(x, y)$ component of stress change normalized by the stress drop for a single antiplane vertical crack located at a distance of 3 km from the boundary dividing slabs of different rigidity. The rigidity contrast μ_2/μ_1 between the slabs is equal to 2. The numbers indicate the change in τ_{xz} (positive numbers mean increase, negative decrease), as the percentage of the stress drop on the crack. Stippled regions indicate regions where τ_{xz} increases due to the presence of the crack. The vertical and horizontal scales are only approximately equal.

colinear cracks located at a distance of 1 km from the boundary. The results obtained can be summarized as follows:

1) There is a general decrease of the stress in the regions adjacent to a crack except in the zones near the crack tips where there is an increase of stress.

2) The presence of a free surface (or of a surface low rigidity layer) influences the stress in such a way that the area of increased stress near the upper tip of a crack is visibly smaller but of much greater magnitude than that near the lower tip.

3) The presence of a slab of higher rigidity introduces a distinct asymmetry in the stress pattern. The magnitude of stress changes introduced by a crack (both positive and negative) are visibly higher near the side of the crack that is closer to the slab of higher rigidity. The lines of maximum stress near the crack tips, which lie in the crack plane in the case of a homogeneous medium, are inclined towards the high rigidity slab.

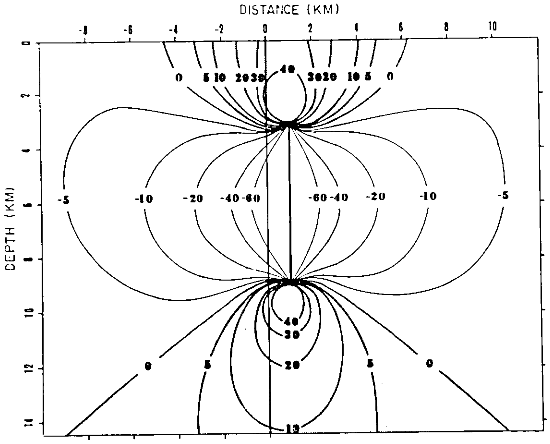

Fig. 3. Same as Fig. 2 for a single antiplane vertical crack at a distance of 1 km from the vertical boundary.

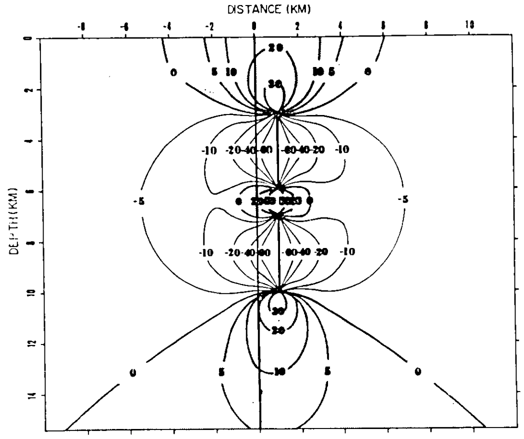

Fig. 4. The model of a fault consisting of two colinear antiplane cracks at a distance of 1 km from the vertical boundary. See the caption for Fig. 2 for other details.

4) The effect introduced by a second colinear crack is similar to that of a free surface, i.e., the increase of stress between the inner tips of the cracks is more localized spatially and of greater magnitude than those near the outside tips.

5) The stress near the outer tips of a single crack is much higher than that in the corresponding case of two colinear cracks with a combined extent equal to the single crack.

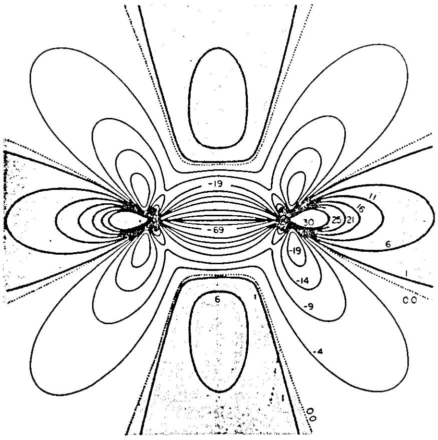

Fig. 5. Contour lines for normalized stress changes for inplane crack. Stippled regions indicate regions of increased stress due to the presence of the crack (after KOSTROV and DAS, 1982)

The results given by 1) and 2) are the same as those found in the antiplane case of a two-dimensional Volterra dislocation (RYBICKI, 1971).

It is interesting to compare the main features of the stress field induced by an antiplane crack with that of a plane crack. Figure 5 (after KOSTROV and DAS, 1982) shows the changes in the τ_{12} component of stress as a percentage of the stress drop on the crack located in an infinite homogeneous medium. (In the chosen coordinate system the x_1-axis is in the plane of the crack and the x_2-axis is perpendicular to it. The displacement discontinuity vector is directed along the x_1-axes and the stress component τ_{12} corresponds to the stress component τ_{xz} analysed in this paper).

The most essential difference between the stress patterns induced by inplane and antiplane cracks is that in the case of antiplane cracks there is no increase in stress off the crack plane in the normal direction.

4. Application and Discussion

In applying the results to some seismological problems, we should note the simplifying assumptions taken in the present model. This means that the model can be useful only in explaining and drawing conclusions related to some general features of mechanics of faulting. Nevertheless such conclusions are of great importance for practical purposes, in particular for earthquake prediction.

The first conclusion based on results 1) and 2) is that when fracture takes place on a buried vertical strike-slip fault, a further extension of failure towards the Earth's surface should be expected in the form of aftershocks (seismic activity) and/or creep (aseismic activity). This conclusion is in agreement with the data related to the 1966 Parkfield earthquake. There exists exceptionally detailed data for the aftershocks sequence of this earthquake (EATON *et al.*, 1970). Comparing their histograms, which shows the number of aftershocks for various magnitude thresholds as a function of depth (Fig. 6), with the stress pattern shown in Fig. 2 (or Fig. 3) one can see the good correlation of spatial distribution of aftershocks with the theortical results. There is a visible concentration of aftershocks close to the fault edges and the number of aftershocks near the upper edge is greater than those close to the lower edge. The stress in the upper region may have been released in an aseismic way as well, as evidenced by substantial creep which accompanied the aftershock sequence (SCHOLZ *et al.*, 1969).

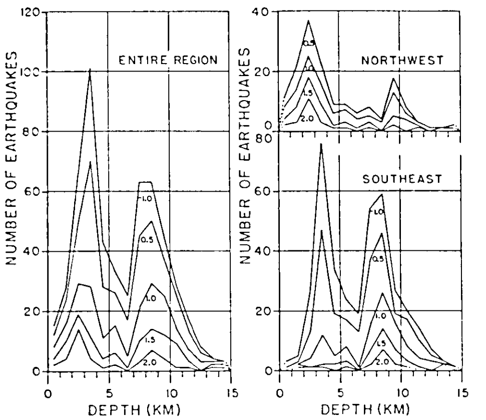

Fig. 6. Histograms showing the number of aftershocks following the 1966 Parkfield earthquake for various magnitude thresholds as a function of focal depth for the entire hypocentral region, and for its northwestern and sotheastern sections (after EATON *et al.*, 1970)

New results connected with this study indicate that in the presence of a high rigidity slab near an earthquake fault, the development of seismic and/or aseismic activities may have an asymmetric character with a general inclination of the zone of aftershocks and creep towards the slab. This result also suggests the possibility of a similar trait for a dynamically developing rupture, although further study of this problem is obviously necessary. Such results would also suggest the possibility of a departure from purely vertical surfaces for strike-slip faults located in a laterally inhomogeneous medium.

The stress pattern induced by an inplane shear crack (see Fig. 5) was used by NIEWIADOMSKI and RITSEMA (1980), as well as by DAS and SCHOLZ (1981), to explain the existence of aftershocks off the fault plane for a number of earthquakes, and in all these cases the location of off fault aftershocks is in good agreement with the pattern of increasing stress predicted by the theoretical solutions.

Using a Volterra rectangular dislocation model (three-dimensional case) in a homogeneous half-space, the same effect was found and used to explain off fault aftershock distributions by YAMASHINA (1978, 1980) and STEIN and LISOWSKI (1983).

The model of inplane shear cracks can be used to approximate the stress field generated by fracture along short surface strike-slip faults (fault length much less than the depth), or for long buried dipslip faults (deep enough to neglect the effect of the free surface).

The model of antiplane cracks considered in this paper is suitable for long strike-slip faults (length much greater than width). According to the results obtained there should be almost no increase of stress off the plane in such case. This may be one reason for the absence of aftershocks located off the fault plane for the Guatemala earthquake of February 4, 1976 (LANGER *et al.*, 1976).

The results concerning two interacting cracks offer even more important conclusions for a better understanding of mechanics of material failure within the Earth. Such a system of two cracks can be considered as a model of a barrier left in the process of faulting or a model of a seismic gap. It is known that such areas are the most probable places where further material failure may take place in the form of earthquakes (aftershocks or new strong earthquakes) and/or creep (AKI, 1979; DAS and AKI, 1977; KELLEHER *et al.*, 1973; OTSUKA, 1976; SYKES, 1971; a.o.). The method employed in this paper and also in the previous papers dealing with two interacting inplane cracks (NIEWIADOMSKI and RITSEMA, 1980; NIEWIADOMSKI, 1981) makes it possible to estimate quantitatively the stress increase in such failure prone areas. According to the results obtained for both inplane and antiplane cracks (for cracks of equal length) the increase of stress in the region between the inner tips of the cracks may be nearly 100% greater than in the regions close to the outer tips of the cracks. This effect is slightly greater for inplane shear cracks than for antiplane cracks.

Another important conclusion is related to the problem of further stress redistribution in the medium once failure of unfaulted regions takes place, i.e., when aftershocks break barrier or when the earthquake ruptures the seismic gap (or, alternatively, when such areas undergo stress release by creep). Such failure of the material should result in an increase of stress in the regions near the outer tips of already fractured zones. (See result 5 above, the inherent assumption in this reasoning is that there is approximately a uniform stress drop over the whole fractured area after failure of the previously unfaulted regions). Careful re-examination of the results for inplane cracks (NIEWIADOMSKI and RITSEMA, 1980) leads to the same conclusion. The additional increase of stress near the outer tips depends on the ratio of the lengths of the interacting cracks. In the case considered here (two antiplane cracks of equal length) the additional increase of stress may be up to 80% of the increase associated with the initial faulting.

This result means that breaking of a barrier should result in an increased probability for further extention of the zone of failure in the form of aftershocks and/or creep. Geophysical implications of this conclusion in the case of the rupture of the seismic gap are even of greater importance for earthquake prediction, although in this case a more careful and complex analysis is required.

5. Summary and conclusions

By computing the stress field due to two interacting shear cracks in a laterally inhomogeneous half-space it is possible to draw some useful conclusions concerning the mechanics of failure processes in tectonically active areas. The results show that both lateral inhomogenities of the medium and the free surface (or a surface low rigidity layer) influence to a great extent the stress field induced by faulting which should manifest itself in an asymmetric pattern of post-seismic activity.

The stress off the crack plane in the normal direction has completely different characteristics for inplane and antiplane cracks which means that more detailed study of stress field generated by faulting should be undertaken in order to correctly interpret stress patterns existing in the medium.

As for the stress close to the tips of cracks, the results are similar for sets of two colinear inplane and antiplane cracks. In particular, the increase of stress between the inner tips of the cracks is about twice that in the regions close to the outer tips. This result is useful for quantitative estimation of failure probability of a barrier or seismic gap.

The magnitude of the stress increase close to outer tips in the case of two colinear cracks is visibly smaller than in the case of a single crack of the length equal to the combined length of the colinear cracks and the gap between them. The important geophysical conclusion of this result is that once a barrier (or seismic gap) is broken, the further failure of the regions outside the fracture zone becomes more probable.

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Appendix

Consider a single screw dislocation with the Burger's vector b passing through the point $x=\xi$, $y=\eta$ of an infinite, isotropic, homogeous space. The dislocation line is parallel to the z-axis. The only nonvanishing component of displacement due to the dislocation is the z component, $w(x, y)$, which is given by (HIRTH and LOTHE, 1968)

$$w(x, y)=\frac{b}{2\pi}\tan^{-1}\frac{y-\eta}{x-\xi}\,. \tag{A1}$$

The displacement field induced by a single dislocation located at the point $x=\xi$, $y=\eta$, $\xi>0$, in the laterally inhomogeneous medium as shown in Fig. 1 can easily be found using the method of images (ISHII and TAKAGI, 1968; RYBICKI, 1971, 1978; CHINNERY and JOVANOVICH, 1972, RYBICKI and KASAHARA, 1977). We have

$$w(x, y)=\begin{cases}\dfrac{b}{2\pi}\Big[\tan^{-1}\dfrac{y-\eta}{x-\xi}-\tan^{-1}\dfrac{y+\eta}{x-\xi}\\ \qquad -\dfrac{\mu_1-\mu_2}{\mu_1+\mu_2}\Big(\tan^{-1}\dfrac{y-\eta}{x+\xi}-\tan^{-1}\dfrac{y+\eta}{x+\xi}\Big)\Big] & \text{for} \quad x>0\,, \\ \dfrac{b}{\pi}\dfrac{\mu_1}{\mu_1+\mu_2}\Big(\tan^{-1}\dfrac{y-\eta}{x-\xi}-\tan^{-1}\dfrac{y+\eta}{x-\xi}\Big) & \text{for} \quad x<0\,.\end{cases} \tag{A2}$$

The only nonvanishing stress components are

$$\tau_{xz}(x, y)=\begin{cases}-\dfrac{\mu_1 b}{2\pi}\Big[\dfrac{y-\eta}{(x-\xi)^2+(y-\eta)^2}-\dfrac{y+\eta}{(x-\xi)^2+(y+\eta)^2}\\ \qquad -\dfrac{\mu_1-\mu_2}{\mu_1+\mu_2}\Big(\dfrac{y-\eta}{(x+\xi)^2+(y-\eta)^2}-\dfrac{y+\eta}{(x+\xi)^2+(y+\eta)^2}\Big)\Big] \\ \qquad \text{for} \quad x>0\,, \\ -\dfrac{b}{\pi}\dfrac{\mu_1\mu_2}{\mu_1+\mu_2}\Big(\dfrac{y-\eta}{(x-\xi)^2+(y-\eta)^2}-\dfrac{y+\eta}{(x-\xi)^2+(y+\eta)^2}\Big) \quad \text{for} \quad x<0\,,\end{cases} \tag{A3}$$

$$\tau_{yz}(x, y)=\begin{cases}\dfrac{\mu_1 b}{2\pi}\Big[\dfrac{x-\xi}{(x-\xi)^2+(y-\eta)^2}-\dfrac{x-\xi}{(x-\xi)^2+(y+\eta)^2}\\ \qquad -\dfrac{\mu_1-\mu_2}{\mu_1+\mu_2}\Big(\dfrac{x+\xi}{(x+\xi)^2+(y-\eta)^2}-\dfrac{x+\xi}{(x+\xi)^2+(y+\eta)^2}\Big)\Big] \\ \qquad \text{for} \quad x>0 \\ \dfrac{b}{\pi}\dfrac{\mu_1\mu_2}{\mu_1+\mu_2}\Big(\dfrac{x-\xi}{(x-\xi)^2+(y-\eta)^2}-\dfrac{x-\xi}{(x-\xi)^2+(y+\eta)^2}\Big) \quad \text{for} \quad x<0\end{cases} \tag{A4}$$

One can easily verify that the above solution satisfies the boundary conditions, i.e., τ_{yz} vanishes at $y=0$ and w and τ_{xz} are continuous across the boundary $x=0$.

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横ずれ型 shear crack による応力場

——地震学への応用——

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鉛直面により剛性率の異なる領域に2分された半無限媒質中に2つの横ずれ型 shear crack がある場合につき，その相互作用を解析した．解析にあたり，クラックのすべりを模式化し，らせん型くいちがいが連続的に分布しているものとする．一連の積分方程式を数値的に解く事により，くいちがいの分布密度を求める事ができる．横ずれクラックが1個の場合，及び，2つのクラックが一線上に並んでいる場合の応力場を求め，クラックが縦ずれ型の場合と比較した．以上を用いて地球物理学的に重要な問題を考察したところ，地殻内に主断層が生じた後の地震活動（非地震的変形を含む）は，自由表面及び媒質の水平不均質性に影響されるものであることが判った．さらに，既存のバリアあるいは地震空白域が消失した場合には，破砕帯外側の領域に応力の増加をもたらす事が判った．