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34. *On the Cross-correlational Determination of Phase Velocity.*

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Abstract

The cross-correlation method employs a pair of seismograms at two adjacent stations and determines phase velocity of dispersive waves via a cross-correlogram of the two records. Its principle is already known, but it has been seldom used in seismology. We have re-examined this method a little more in detail.

Sets of experimental signals have been used to test the technique. Tests using synthesized seismograms have proven the usefulness of this method if appropriate corrections are taken into account. However, as for the actual seismograms recorded at our array stations, the calculated data tend to scatter considerably as the period increases or decreases. With respect to accuracy, our tests suggested no superiority of this method over the conventional phase comparison method. This seems to be inconsistent with the conclusions of former authors (DZIEWONSKI *et al.*, 1968; LANDISMAN *et al.*, 1969).

1. Introduction

Seismologists have worked out extensive analyses of dispersive surface waves in order to explore the underground structures. Compared to the travel time analysis of body waves, especially with those using controlled explosion sources, this approach to the objectives may be less advantageous regarding the rigidity and precision of the results. Yet, the use of relatively long period signals in this technique allows us to get information about the deep structure of medium because of high energy compared with signals from explosion sources. This point would be advantageous in urban areas if we take the cost of explosions into consideration. This is the case of the Monitoring Chain on Crustal Activity in the South Kanto district (KASAHARA *et al.*, 1980), for example. This system, which was constructed in 1980 under the earthquake prediction project, is equipped with broad-band seismographs at each of its stations. Thus a minute re-examination of a proper technique for surface wave analysis became necessary in order to study the crustal structures of this populated area using surface wave dispersion.

A dispersion curve of surface waves may be drawn from seismograms with respect to either of the two quantities, *i. e.* group or phase velocities. The group velocity analysis has produced a lot of valuable information about the earth's deep structures on the basis of the existing world-wide seismic network. For our purpose of studying the structures on a local scale, however, the phase velocity analysis seems more convenient than that of the group velocity, since a system with such a close installation of seismographs as ours (station separation: ca. 15 km) may permit even direct reading of phase velocity from seismograms.

The principle of phase velocity determination was given by SATÔ (1955). Let the wave forms at a pair of stations be subject to the Fourier analysis, then the phase velocity there is given as a function of several variables, *i. e.* difference of epicentral distance of the stations, phase difference of the corresponding spectral components between them. In practice, the phase difference after the Fourier analysis is associated with the uncertainty for $2n\pi$ (n: integer), therefore this term must be adjusted properly after some seismological consideration (SATÔ, 1955). SATÔ's method has been popularly used for the succeeded phase velocity analyses since then. It must be mentioned, however, that this original method brings about some difficulties in practical use. That is, in addition to the above-stated uncertainty in phase differences, it needs the Fourier transformation separately of the respective record, at the expense of the machine time and at the risk of increasing phase angle errors in the results.

These difficulties can be reduced, to some extent, by the use of an alternative technique of cross-correlation. This possibility has been suggested by DZIEWONSKI *et al.* (1968), LANDISMAN *et al.* (1969), together with a small number of examples.

In spite of its advantages as stated above, however, it has not been used much for productive work since then. This paper purposes to re-evaluate its usefulness and to consider practical procedures necessary for its use in our routine observations.

2. Theory and procedure

Theory—SATÔ (1955) has shown that the phase velocity of surface waves between stations 1 and 2 can be determined using the following formula.

$$V(\omega)=\frac{\omega(x_2-x_1)}{\phi_1(\omega)-\phi_2(\omega)+2n\pi} \tag{1}$$

where V, ω, x_k, $\phi_k(\omega)$ denote the phase velocity, angular frequency, epicentral distance of the k-th station, and phase angle of the Fourier transform of the seismogram at the k-th station, respectively. In eq. (1), n is an integer which cannot be determined uniquely. If the distance difference x_2-x_1 and the phase difference $\phi_1(\omega)-\phi_2(\omega)$ of the stations are given, the phase velocity may be calculated by this formula. Usually $\phi_1(\omega)-\phi_2(\omega)$ has been calculated from the difference of the phase angles of the Fourier transforms of the respective seismograms.

The cross-correlational determination will conveniently replace this original technique by taking the equivalence of the two quantities, *i.e.* the phase of the Fourier transformed cross-correlogram of the two records and the phase difference between the corresponding spectral components in the respective record (DZIEWONSKI *et al.*, 1968 ; LANDISMAN *et al.*, 1969).

The Fourier transform of the seismogram $f_1(t)$, at station 1 is expressed as

$$F_1(\omega)=\int_{-\infty}^{\infty} f_1(t)\exp(-i\omega t)dt \tag{2}$$

where i denotes $\sqrt{-1}$. A similar relation holds on the seismogram $f_2(t)$ at station 2. The cross-correlation of the two seismograms is expressed as follows.

$$R(\tau)=\int_{-\infty}^{\infty} f_1(t+\tau)f_2(t)dt \tag{3}$$

where τ denotes the time delay. Let us take the Fourier transform of $R(\tau)$. Then the next formula is derived by the well-known equivalence of convolution in the time domain and multiplication in the frequency domain.

$$S(\omega)=\int_{-\infty}^{\infty} R(\tau)\exp(-i\omega\tau)d\tau=F_1(\omega)F_2^*(\omega) \tag{4}$$

where the asterisk denotes the complex conjugate. If we take its arguments we obtain the phase difference as,

$$\mathrm{Arg}\{S(\omega)\}=\mathrm{Arg}\{F_1(\omega)\}-\mathrm{Arg}\{F_2(\omega)\}=\phi_1(\omega)-\phi_2(\omega) \tag{5}$$

This means that we can determine the phase difference $\phi_1(\omega)-\phi_2(\omega)$ from the cross-correlation.

From the physical viewpoint, this may be compared to a study of frequency response of a filter. In this analogy, the crustal segment under

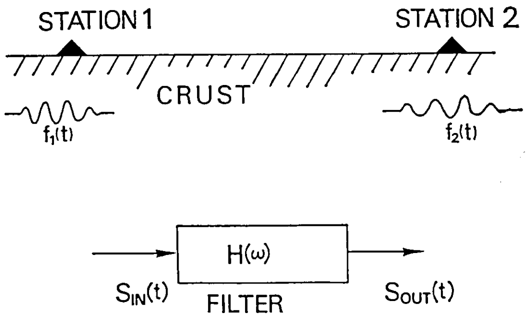

Fig. 1. A schematic illustration of a section of the crust supposed as a linear filter. $f_1(t), f_2(t), S_{\mathrm{IN}}(t), S_{\mathrm{OUT}}(t), H(\omega)$ denote seismograms at stations 1 and 2, input and output signals and the frequency response, respectively.

consideration plays the role of a filter which receives the seismogram f_1 at station 1 (closer to the epicenter) as input outputs f_2 (See Fig. 1). Suppose the stations are equipped with seismographs having identical characteristics, then we assume linearity of the system and obtain the next formula,

$$S_{io}(\omega)=H(\omega)S_{ii}(\omega) \tag{6}$$

where $S_{ii}(\omega)$, $S_{io}(\omega)$, $H(\omega)$ represent the auto-correlation of the input signal, the cross-correlation of the input and output signals and the frequency response of the filter, respectively, with respect to the frequency charcteristics. It must be that the auto-correlation is an even function about the coordinates origin of time-delay and the argument of $S_{ii}(\omega)$ is zero in the all frequency range. Consequently the necessary information about the phase of $H(\omega)$ is known immediately from that of $S_{io}(\omega)$.

Instrumental corrections—In the above discussion we assumed the identity of instrumental characteristics at the two stations. To satisfy this condition, the record must be corrected, or equalized, for the respective instrumental responses for the necessary period range. The anti-aliasing filtering may also be necessary, if it is not sufficient in the original records.

The seismographs at our stations (vertical component only) are of the PELS-73 type (PROJECT TEAM FOR THE DEVELOPMENT OF SMALL-SIZE LONG-PERIOD SEISMOMETER, 1974). They are used with natural period and damping constant at ca. 8 sec and ca. 3.1, respectively, so that their response to the ground velocity may possibly be uniform around the period of free oscillation. Yet periodic maintenance of instrument and instrumantal correction prior to the correlation analysis are useful for better results.

Azimuthal correction—If the observed seismic path is not parallel to the station alignment, the distance difference x_2-x_1 in formula (1) must be substituted by the normal projection along the seismic path. As we are concerned with local structures by use of a closely located station pair, this correction must be done accurately. For practical use of the present technique, therefore, tripartite observation is generally recommended. This system allows us to derive both velocity and azimuth of a seismic path by vectorial combination of wave slowness (MARUYAMA and KAYANO, 1969).

3. Test and examples

Figs. 2a and 2b illustrate the first test of the technique. Here we used theoretical surface wave synthesized by YOSHIDA (1978, 1982) using upper mantle model PC-MAX. Here abscissa t denotes time elapsed after the shock ocurred. The sampling interval of these time series is 2 seconds,

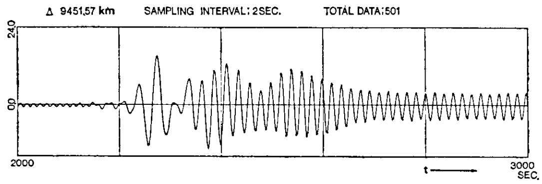

Fig. 2a. Synthesized Rayleigh waves by Yoshida's model PC-MAX at a hypothetical station. The epicentral distance is 9451.57 kilometers.

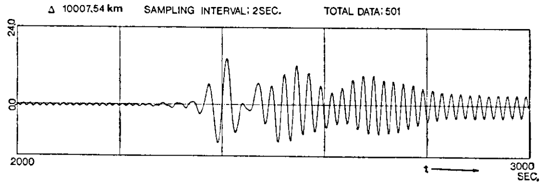

Fig. 2b. Synthesized Rayleigh waves by the same model as that of Fig. 2a at another hypothetical station. The epicentral distance is 10007.54 kilometers.

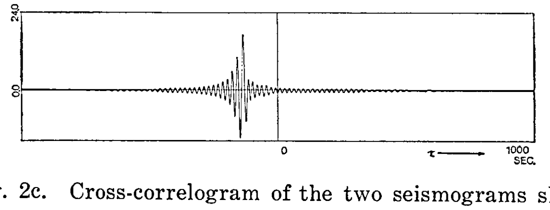

Fig. 2c. Cross-correlogram of the two seismograms shown in Figs. 2a and 2b. The abscissa τ represents correlational time delay.

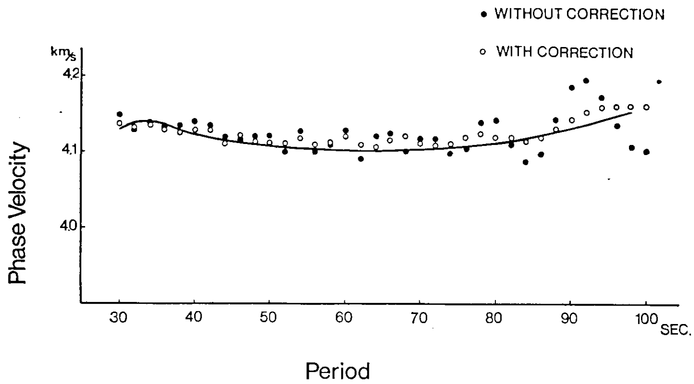

Fig. 3. Phase velocities of synthetic Rayleigh waves calculated by the cross-correlation method. Solid and empty circles represent the calculated values without and with correction, respectively. The curve crossing circles represent the theoretical phase velocities.

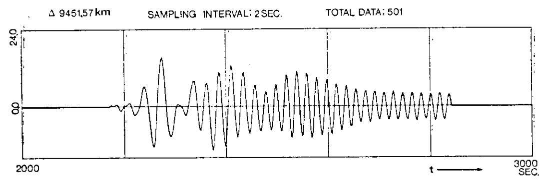

Fig. 4a. Synthetic waves with the same wave form as those of Fig. 2a with both ends replaced with zeros.

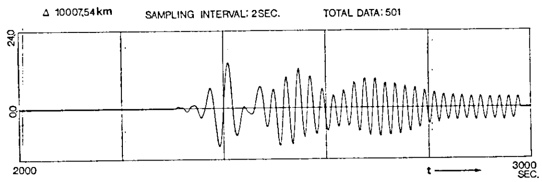

Fig. 4b. Synthetic waves with the same wave form as those of Fig. 2b with both ends replaced with zeros.

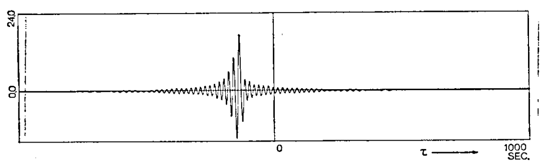

Fig. 4c. Cross-correlogram of the seismograms shown in Figs. 4a and 4b.

and they are synthesized from $t=2000$ second to $t=3000$ second (total data number$=$501). Figs. 2a and 2b are seismograms to be recorded at hypothetical stations, epicentral distances of which are 9451.57 kilometers and 10007.54 kilometers, respectively. Fig. 2c is the cross-correlogram of the two seismograms, by use of eq. (3), where abscissa τ is the correlational time delay. Solid circles in Fig. 3 represent the phase velocities calculated by the cross-correlation method. The curve in this figure represents the theoretical values. Calculated data explain the theoretical curve satisfactorily up to a period of about 70 seconds. Beyond that, however, the experimental data are predominated by periodic disturbances about the theoretical curve.

Next we tested these seismograms with some corrections at both ends of the seismograms. Seismograms illustrated in Figs. 4a and 4b have the same wave forms as those shown in Figs. 2a and 2b, respectively, except that their data at the both ends are replaced with zeros. Fig. 4c is the cross-correlogram of the two seismograms. Empty circles in Fig. 3 show the resultant phase velocities which show the better accuracy of the method by this correction.

To investigate the effect of data truncation at the both ends of the seismogram (Hereafter we call it the truncation effect) more minutely, we used the theoretical waves with no dispersion as to the phase velocity.

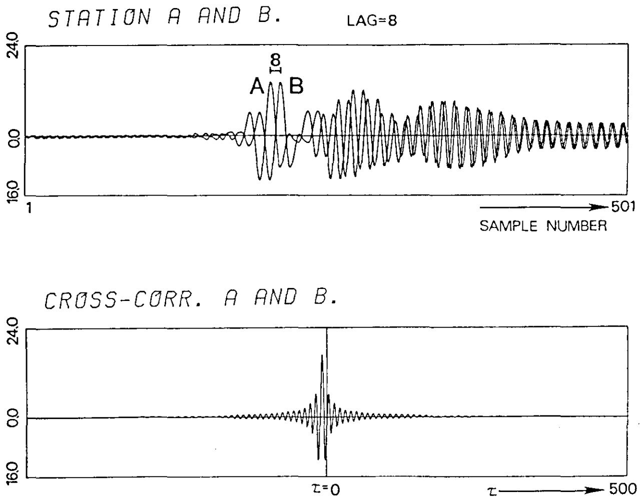

Fig. 5a (upper). Time relation of seismograms of stations A and B. The seismogram B is delayed from A for 8 time units.

Fig. 5b (lower). Cross-correlogram of the two seismograms shown in Fig. 5a. The abscissa τ represents correlational time delay.

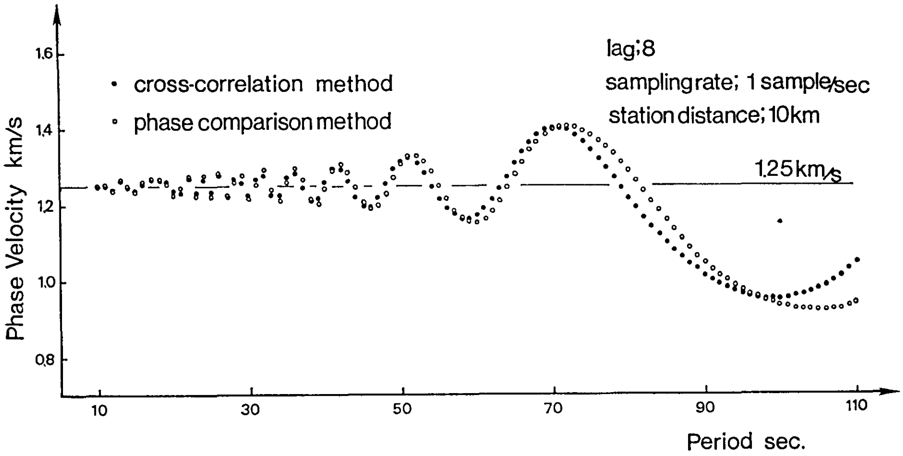

Fig. 6. Calculated phase velocities between stations A and B by two different methods. Empty and solid circles represent the values by the phase comparison and the cross-correlation methods, respectively. The horizontal line represents the theoretical values.

Seismogram **A** shown in Fig. 5a has the same form as the one shown in Fig. 2b, and seismogram **B** is of the same wave form but with uniform delay for 8 sample numbers compared with seismogram **A**. Here we assume that the sampling rate of each seismogram is one sample per second and that the distance between the two stations is 10 km. The theoretical phase velocity must then be 10 km/8 sec = 1.25 km/sec. Fig. 5b is the cross-correlogram of the two seismograms. Fig. 6 compares the phase velocities calculated by two different methods. Solid and empty circles represent, respectively, the phase velocities by the cross-correlation method and by the phase comparison method, that is, the conventional technique of finding the phase differences between the Fourier transformed seismograms. The horizontal line in this figure represents the theoretical value, 1.25 km/sec, in the present case. Also in this case, the data of the two groups are predominated by periodic disturbances as the period increases. Theoretically the cross-correlation of two seismograms with the same wave forms is equivalent to auto-correlation of any one of the two seismograms with some time lag if the two seismograms are composed of an infinite number of data. So, calculated phase velocities must agree exactly with theoretical ones. Then we may say that the amount of disturbance shown in Fig. 6 directly represents the magnitude of the truncation effect at both ends of the seismograms. In fact, we could get the exact phase velocities by replacing data at the both sides of the two seismograms in Fig. 5a with zeros.

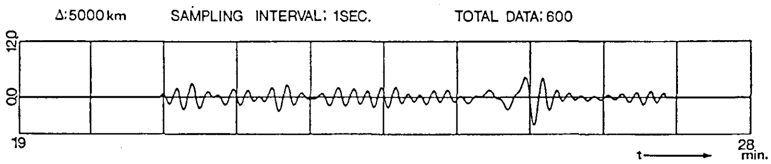

Fig. 7a. Synthesized surface waves by the method of Aki. The epicentral distance is 5000 kilometers.

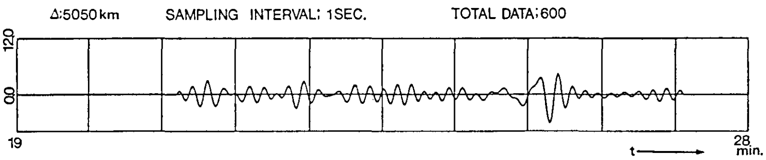

Fig. 7b. Synthesized surface waves by the method of Aki. The epicentral distance is 5050 kilometers.

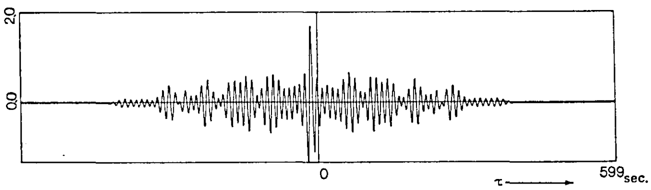

Fig. 7c. Cross-correlogram of the two seismograms shown in Figs. 7a and 7b. The abscissa τ represents correlational time delay.

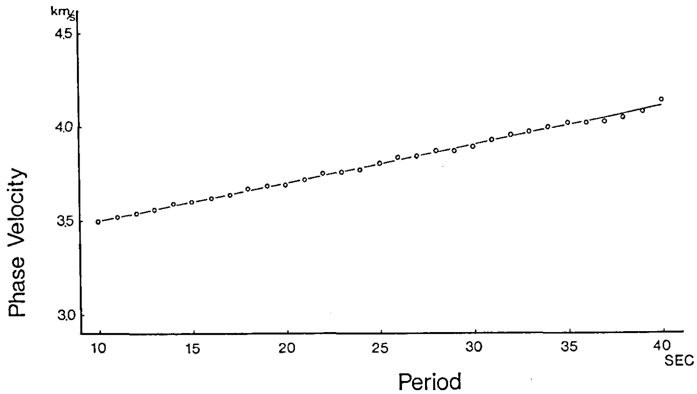

Fig. 8. Calculated phase velocities of the surface waves shown in Figs. 7a and 7b. The straight line crossing the data represents the theoretical values.

Figs. 7a and 7b represent another theoretical seismogram synthesized by the method of AKI (1960) assuming the phase velocity dispersion of the medium shown by the straight line in Fig. 8, the epicentral distances of which are 5000 and 5050 kilometers respectively. The sampling rate of these seismograms is one piece of data per second, and each seismogram is composed of 600 pieces of data. Both ends of the data series are replaced with zeros to avoid the truncation effect. Fig. 7c represents cross-correlogram of these two seismograms. Empty circles shown in Fig. 8

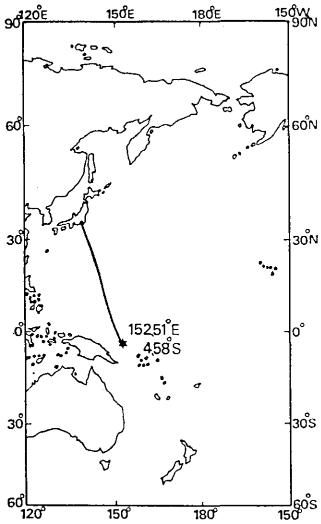

Fig. 9. Location of the epicenter of the New Britain Is. earthquake, May 10, 1983 (Ms: 6.7) and the great circle path from the epicenter to the stations.

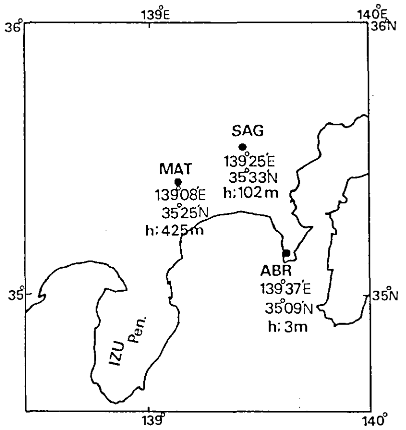

Fig. 10. Local map of the stations, Matsuda (MAT), Sagamihara (SAG) and Aburatsubo (ABR).

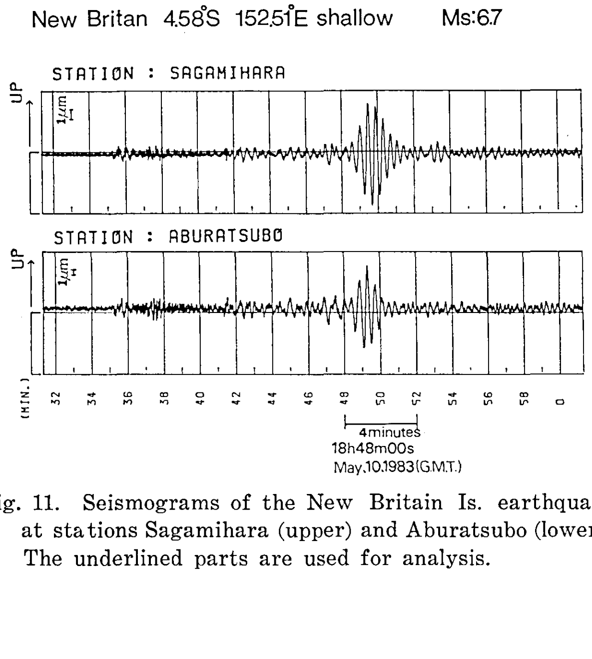

Fig. 11. Seismograms of the New Britain Is. earthquake at stations Sagamihara (upper) and Aburatsubo (lower). The underlined parts are used for analysis.

represent the calculated phase veloctities by the cross-correlation method. Also in this case, the agreement is markedly good as well as the case of YOSHIDA's model PC-MAX.

The foregoing examples have proven the usefulness of the present method. Let us apply it to a set of seismograms at our array stations, taking an earthquake in the New Britain Islands area (Ms : 6.7 ; May 10, 1983) as an example. Figs. 9 and 10 illustrate the seismic path to the stations and the location of the stations (Sagamihara, Aburatsubo and Matsuda), respectively. The records at the first two stations are given in Fig. 11. The underlined portions of these records, which are interpreted as the fundamental Rayleigh waves and reproduced in Fig. 12, are subject to the analysis to draw the cross-correlogram in Fig. 13. The power spectra of the respective record traces are illustrated in Fig. 14. Fig. 15 represents the final results as interpreted from the present three stations. The phase velocity curve is associated with disturbances toward short and long period ranges, as the signal power tends to drop there. Though we may read out a likely phase velocity curve from it by smoothing out

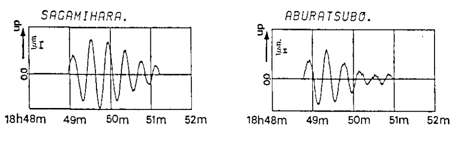

Fig. 12. Details of the used parts of the seismograms in Fig. 11 after a correction for the DC-components.

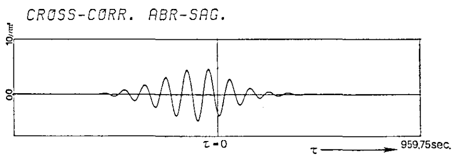

Fig. 13. Cross-correlogram of the seismograms in Fig. 12. The abscissa τ represents the correlational time delay.

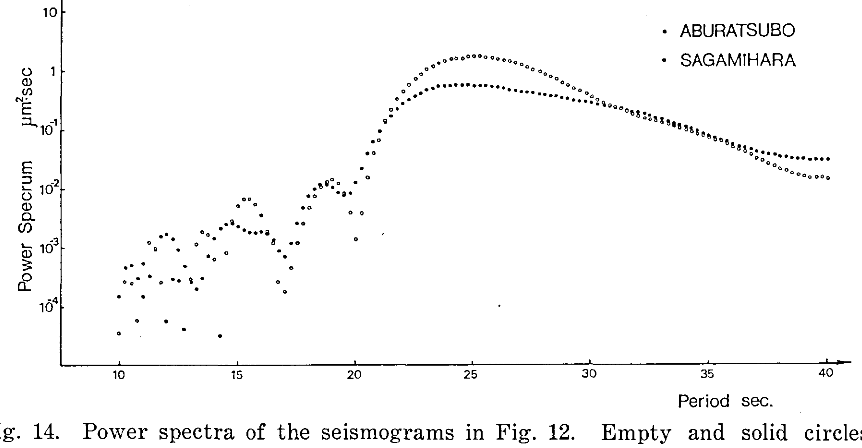

Fig. 14. Power spectra of the seismograms in Fig. 12. Empty and solid circles denote the values at the stations Sagamihara and Aburatsubo, respectively.

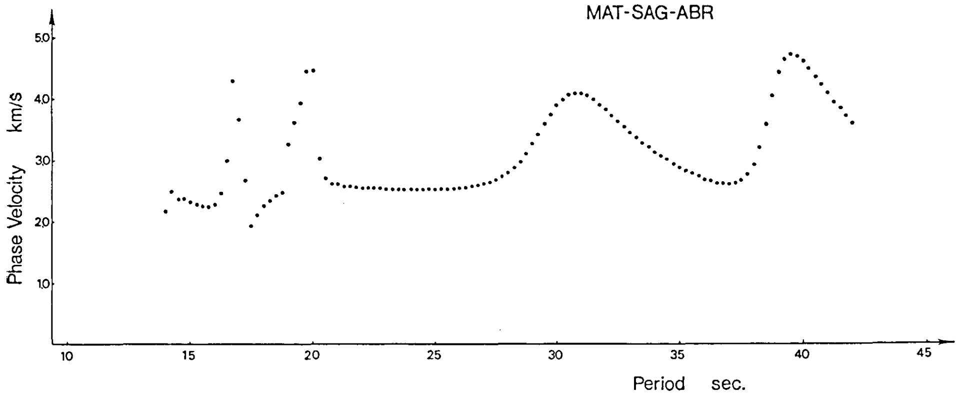

Fig. 15. Phase velocities by the tripartite observation at Matsuda, Sagamihara and Aburatsubo.

these disturbances, the precision of the curve seems to be insufficient, the cause of which is not clear.

4. Discussion and conclusions

The foregoing two chapters have studied the technique of cross-correlational determination of phase velocites with respect to its concept and procedures, together with elementary tests using a few sets of experimental signals. They are surface waves synthesized for two different model media and the Rayleigh waves from a New Britain Island earthquake, as recorded at our array stations in the South Kanto district. The tests using synthesized waves have given the accurate phase velocities as theoretically expected for all the period ranges of our concern if both ends of data series are replaced with zeros. Without this correction the calculated phase velocities are predominated with periodic fluctuations about the theoretical values as the period increases. Irregularity of the phase velocity plots toward both long and short period ranges was also seen in the tests using actual seismograms though the same correction to avoid the truncation effect was made. The cause of this fluctuation may be associated with insufficient signals of corresponding period ranges, since we used the very narrow portions of seismograms for analysis to reduce the contamination by ground noise in spite of the broad ranges of group velocities of the oceanic Rayleigh waves. If we exclude these extreme parts and interpret the principal parts of the plotted data with a smoothed curve, we then obtain a phase velocity diagram. However, its accuracy seems to be a little insufficient the cause of which is not clear.

According to DZIEWONSKI *et al.* (1968) and LANDISMAN *et al.* (1969), greater phase stability is attached to the cross-correlation method because of the improved signal-to-noise ratio which results from the fact that the entire signal is correlated while only a portion of the noise energy enters the cross-correlogram. However, our tests suggested no superiority of cross-correlation metod with respect to accuracy.

Conclusions are as follows. (1), The truncation effect must be taken into account when the cross-correlation method is applied to the determination of phase velocity. (2), The accuracy of the cross-correlation method is not much different from that of the phase comparison method.

The latter conclusion seems to be inconsistent with those of former authors (DZIEWONSKI *et al.*, 1968; LANDISMAN *et al.*, 1969).

Acknowledgement

Discussions with Dr. T. Yoshii were useful for this work. The installation and operation of seismographs at our array stations are due to the efforts of the staff members in our crustal movement study group. The writers are thankful to all their colleagues at the Earthquake Research Institute for their kind cooperation in this work.

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34. 位相速度の相互相関による決定について

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隣接する2観測点の地震の記録の相互相関を用いて波動の位相速度を求める方法の可能性については，二，三の指摘があるが，その利用例は少ない．本報ではこの方法を若干の考察とテストを加えて再評価した．

いくつかの波形を用いてテストしたが，合成波を用いた場合は，データの両端を0にするという補正を加えて満足な結果を得る事ができた．しかし，地震研究所の地殻活動総合観測線（南関東）の地震計の記録を用いたテストでは，長周期及び短周期の帯域でばらつきが見られた．精度に関しては，位相速度を求める際通常用いられている，2つの地震記象のフーリエ変換の位相角の差を取る方法に比べて，大きな差は無かった．この事は，DZIEWONSKI *et al.* (1968) や LANDISMAN *et al.* (1969) の結果とは一致せず，さらに研究が必要であると思われる．