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22. *A New Technique for the Group Velocity Analysis of Dispersive Seismic Waves.*

By Seiji TSUBOI,
Geophysical Institute, Faculty of Science, University of Tokyo,
and
Shuhei OKUBO,
Earthquake Research Institute.

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Abstract

A new technique of high resolution group velocity analysis for the study of dispersive seismic surface waves is proposed. This method comprises the development of the Instantaneous Frequency Analysis which has a basis in the Maximum Entropy Spectral Analysis. The feasibility of this method is tested using synthetic seismograms. Particular attention is paid from the practical point of view to the optimum length of the linear prediction filter and the resolution to the interference of the fundamental mode and the higher mode. We compare this method with a conventional bandpass filtration technique using the same synthetic seismograms. Our results show that this method has the advantage of retrieving the higher mode group velocity even for shallow earthquakes.

1. Introduction

The measurments of surface wave dispersion are now a necessary element in the quantification of mechanical properties under a propagation path. With the development of the theory of surface waves, we can calculate theoretical dispersion curves of higher modes without difficulty. As the higher modes have their maximum amplitude in the upper portion of the mantle, the determination of dispersion curves of the higher modes from an observation is seriously important to estimate the upper mantle structure. To date, however, the fundamental mode has been primarily used for these studies. This is because the coventional technique so far has difficulty in the determination of the higher modes dispersion curves.

In the present paper, we apply the Maximum Entropy Spectral Analysis (BURG, 1967) and the Instantaneous Frequency Analysis (GRIFFITHS, 1975) to determine group velocities from observational surface wave trains. The application of the Instantaneous Frequency Analysis to the estimation of group velocities was first discussed by MCCOWAN (1978). To demonstrate the feasibility of this method, we shall show some examples in the following. Test calculations are performed using the synthetic Rayleigh waves in which both the fundamental and the first higher mode are contained. Particular attention is paid to the resolution to retrieve group velocities of the first higher mode. From a practical point of view further attention is paid to the determination of the optimum filtering parameters such as the length of the linear prediction filter.

We compare this method with the conventional bandpass filtration technique (DZIEWONSKI and HALES, 1972) using the same synthetic Rayleigh wave seismograms. For the best selection of parameters our new method is superior to the band pass filtration technique in retrieving the first higher mode group velocity from seismograms of shallow earthquakes.

2. Numerical examples

It is well known that the Maximum Entropy Spectral Analysis gives a high resolution spectrum to a stationary time series from the linear prediction error filter. If we allow the time variable prediction error filter, we can get an algorithm to calculate the time-varying spectrum with high resolution. In fact, GRIFFITHS (1975) proposed a completely different method to determine the filter coefficient for a non-stationary time series. It is the Instantaneous Frequency Analysis and its geophysical application is discussed by OKUBO (1982). We shall first review this procedure and then show its application to group velocity analysis.

From the definition of the liner prediction filter, we get the prediction $\hat{x}(k)$ of the current data at a time k as

$$\begin{aligned}\hat{x}(k)&=\sum_{l} g_l(k)x(k-l)\\&=\boldsymbol{G}^T(k)\boldsymbol{X}(k-1)\end{aligned}\tag{1}$$

where $g_l(k)$ is the linear prediction filter coefficient and $x(k)$ is the current data. $\boldsymbol{G}(k)$ and $\boldsymbol{X}(k-1)$ are defined as follows

$$\boldsymbol{G}^T(k)=[g_1(k), \cdots, g_L(k)]$$
$$\boldsymbol{X}^T(k-1)=[x(k-1), \cdots, x(k-L)]$$

where T denotes transpose and L is the filter length. Starting with an initial guess $g_l(0)$, the filter coefficients are updated successively by the steepest descent procedure and we will get the filter coefficients which minimize the mean-square prediction error $E[\varepsilon^2(k)]=E\{[x(k)-\hat{x}(k)]^2\}$, where E means the expectation. The algorithm is written as

$$G(k+1)=G(k)+\mu[x(k)-\hat{x}(k)]X(k-1) \tag{2}$$

$$\mu=\alpha/Lr_x(0)\ , \qquad 0<\alpha<2 \tag{3}$$

where $r_x(l)$ is the autocorrelation of $x(k)$. The upper limit of α is determined from the convergence properties (GRIFFITHS, 1975). Once the filter coefficients are obtained, the instantaneous power spectrum at a time k is calculated from

$$P(f;k)=1/|\sum_l g_l(k)\exp(i2\pi f l\Delta t)|^2 \tag{4}$$

where Δt is the sampling interval.

In general, the group velocity of the surface wave of a frequency f is

$$U(f)=D/t_g(f) \tag{5}$$

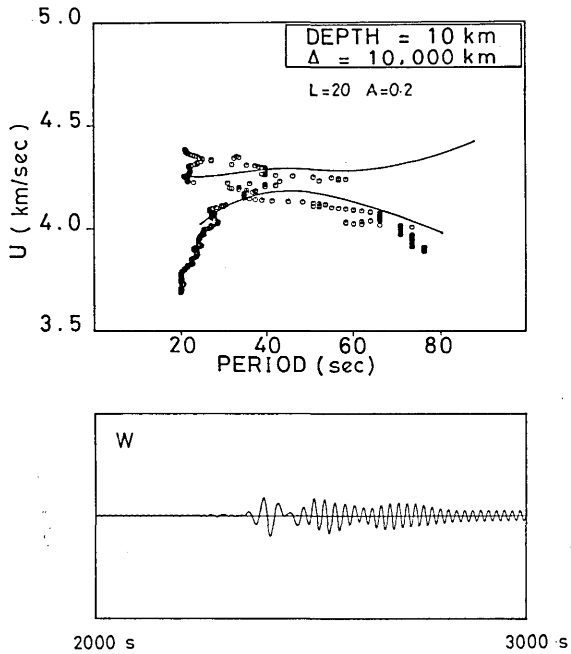

Fig. 1. (Top) Dispersion curves of the synthetic Rayleigh wave. Solid lines are the theoretical group velocity for the fundamental mode and the first higher mode. Plotted symbols are the estimated group velocity by the Instantaneous Frequency Analysis. Filter length $L=20$ and $\alpha=0.2$. (bottom) The wavetrain of the synthetic Rayleigh wave seismogram. Focal depth is 10 km.

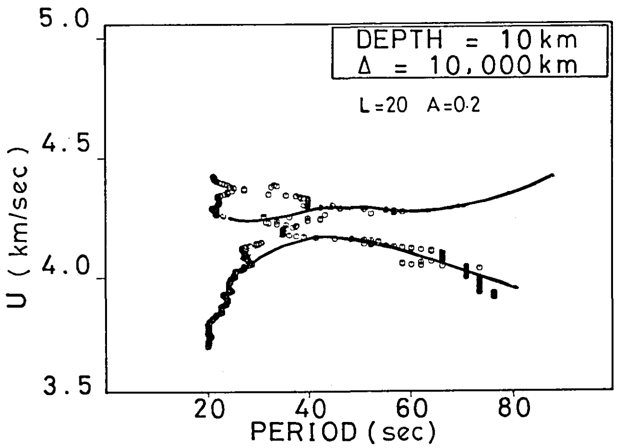

Fig. 2. Dispersion curves of synthetic Rayleigh wave. Parametars are the same as used in Fig. 1. Group arrival time is defined as $(k-L/2)\Delta t$.

where D is the epicentral distance and $t_g(f)$ is the group arrival time. If we find a spectral peak f at $t=k\Delta t$, we can say that the group arrival time t_g of the wave which has a frequency f is $k\Delta t$. Hence we can get the dispersion relation of group velocity by applying Eqs. (2) and (4) to the observational wavetrain successively to plot the position of the spectral peak of $P(f;k)$. In practice, we define the group arrival time as $(k-L/2)\Delta t$. This is because we use $x(k-l)$ for $l=1,\cdots,L$ to predict $x(k)$ as shown in Eq. (1). This indicates that the present $\hat{x}(k)$ is affected by the past L data and we had better define the group arrival time at about a half length of the filter.

Next we apply this method to synthetic Rayleigh wave seismograms. We used the vertical component of the synthetic Rayleigh wave computed for the model PC-MAX (YOSHIDA, 1978). The assumed source is a pure dip-slip motion along a vertical fault plane (YOSHIDA, 1982). This synthetic seismograms contain both the fundamental mode and the first higher mode. The focal depths are 10 km, 100 km and 200 km. The epicentral distance D is 10,000 km. The sampling interval Δt is 2 sec. An example of the calculation is shown in Fig. 1 for $L=20$ and $\alpha=0.2$. The initial value of the filter coefficient is set at $G^T(k)=[1,0,\cdots]$. We also tried the solution of the normal equation as the initial value of the filter coefficient and got no significant difference with the former. As the sampling interval Δt is 2 sec, this filter length corresponds to 40 sec. In this case, we plot the estimation so that the group arrival time is defined at the top of the filter. The estimated group velocity is obtained both for the fundamental and the first higher mode with a slight difference from the theoretical value. We find that the group arrival time should be defined at about $L/2$.

Then we shifted the definition of the group arrival time by $L\Delta t/2$ and the result is shown in Fig. 2. For the fundamental mode, the agreement between the estimated and the theoretical group velocity becomes excellent. Note that for the first higher mode our new method yields group velocity around 40 sec. We take the group arrival time as $(k-L/2)\Delta t$ hereafter.

From a practical point of view, we should determine the optimum choice of the filter length L and the convergence factor α in Eq. (3). First we determine the optimum length of the prediction filter. Figs. 3 and 4 show the results for $L=10$ and 30 respectively. For $L=10$ (Fig. 3), we cannot get the group velocity for a period greater than about 60 sec. In the case of $L=30$ (Fig. 4) we can get the group

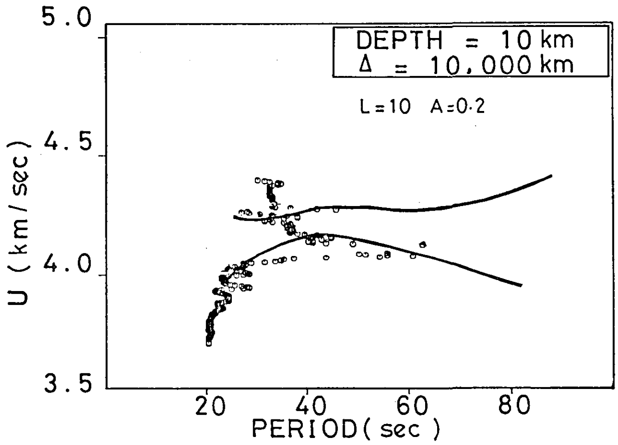

Fig. 3. Dispersion curves of synthetic Rayleigh wave. Filter length $L=10$ and $\alpha=0.2$. For further details, see captions on Figs. 1 and 2,

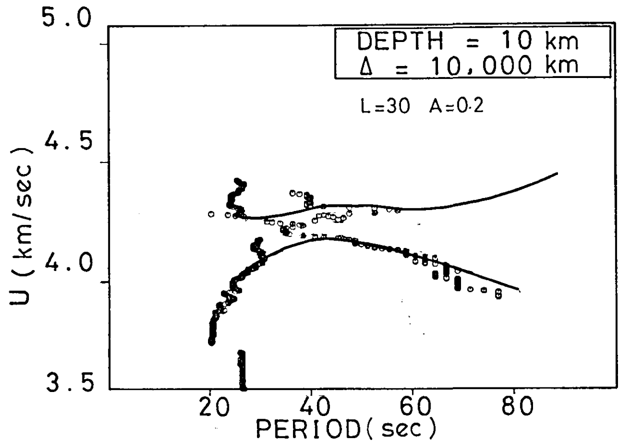

Fig. 4. Dispersion curves of synthetic Rayleigh wave. Filter length $L=30$ and $\alpha=0.2$. For further details, see captions on Figs. 1 and 2.

velocity as long as 80 sec. These results show that we should take the filter length as long as a characteristic period of surface waves. Next we alter the convergence factor α. GRIFFITHS (1975) suggested that $\alpha=0.75$ gave good results in his examples. Following GRIFFITHS we fist examined $\alpha=0.75$ and got unsatisfactory results. We tried several values of α by trial and error and concluded that α should be as small as 0.5 in the present case. Examples of calculations are shown in Figs. 5 and 6 for $\alpha=0.1$ and $\alpha=0.4$. The estimation is not as good for $\alpha=0.1$ (Fig. 5) as with $\alpha=0.2$ (Fig. 2). For $\alpha=0.4$ the result is competitive with that shown in Fig. 2. From these calculation we should conclude at present that there seems to be no criterion for the

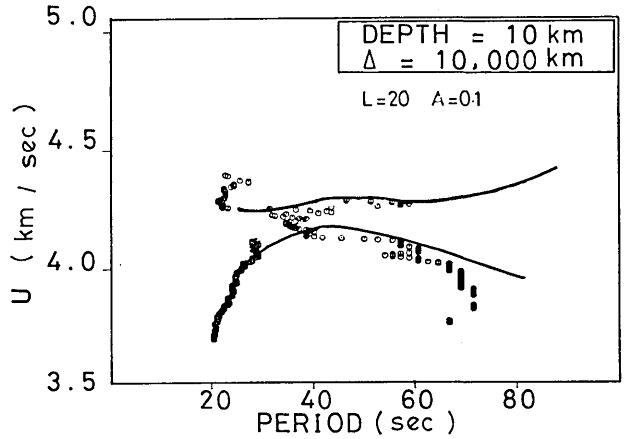

Fig. 5. Dispersion curves of synthetic Rayleigh wave. Filter length $L=20$ and $\alpha=0.1$. For further details, see captions on Figs. 1 and 2.

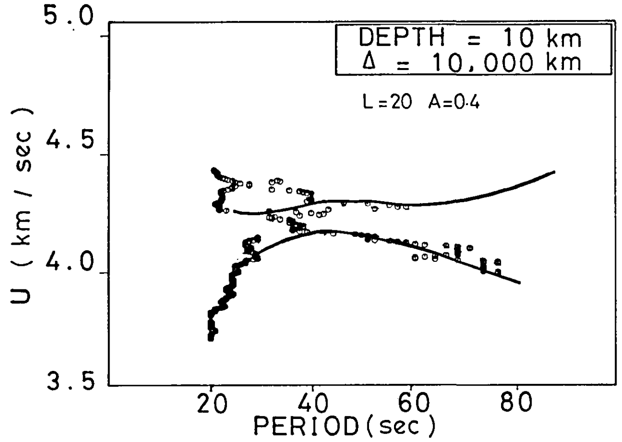

Fig. 6. Dispersion curves of synthetic Rayleigh wave. Filter length $L=20$ and $\alpha=0.4$. For further details, see captions on Figs. 1 and 2.

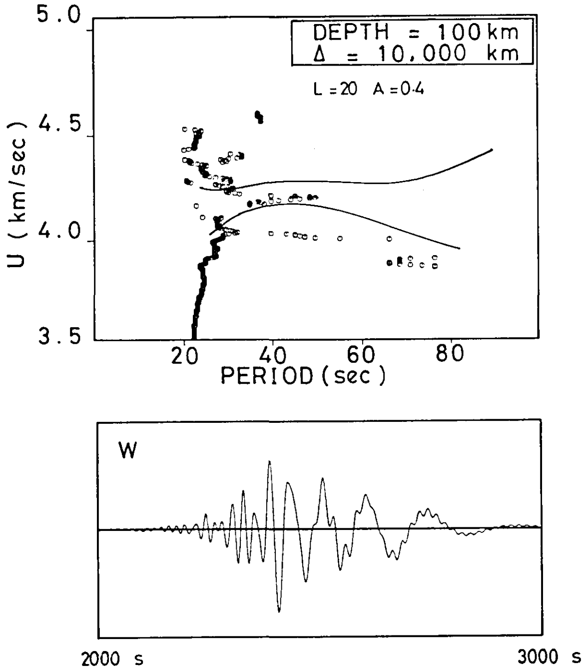

Fig. 7. Dispersion curves of synthetic Rayleigh wave. Focal depth is 100 km. $L=20$, $\alpha=0.4$. For further details, see captions on Figs. 1 and 2.

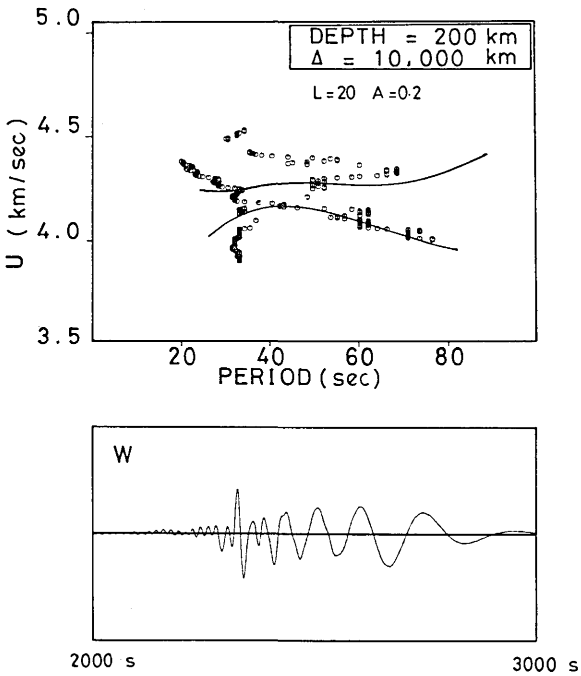

Fig. 8. Dispersion curves of synthetic Rayleigh wave. Focal depth is 200 km. $L=20$, and $\alpha=0.2$. For further details, see captions on Figs. 1 and 2.

determination of α. We must determine α case by case.

We did not add noise to the synthetic seismograms used in these calculations. The effect of noise to this method should be considered from a practical point of view. However, this method has a basis on the Maximum Entropy Spectral Analysis and the presence of the white noise is supposed to have a tendency of stabilizing the result.

The results of the calculation for the focal depth 100 km and 200 km are shown in Figs. 7 and 8. As the focal depth becomes deeper, the higher modes are more effectively excited and the estimated group velocity becomes clearer for the higher mode than for the fundamental mode.

3. Discussion

We might apply this method to observed seismograms to show its feasibility. However, we tried another tests in the present paper. We applied the conventional bandpass filtration technique to the same synthetic seismograms as used in the last section. Our attention was paid to a comparison of the resolution of both methods to the interference of the fundamental mode and the higher mode. Remember that our new method succeeded in giving the group velocity of the higher mode.

The determination of group velocity by the bandpass filtration is based upon the concepts of the instantaneous amplitude and phase (DZIEWONSKI and HALES, 1972). The instantaneous amplitude $a(t)$ and phase $\Phi(t)$ of a time series $f(t)$ are defined as

$$a(t) \exp[i\Phi(t)] = f(t) + iq(t) \tag{6}$$

where $q(t)$ is the Hilbert transform of $f(t)$. Then the instantaneous amplitude (envelope) is

$$a(t) = [f^2(t) + q^2(t)]^{1/2} \tag{7}$$

The algorithm of the bandpass filtration technique is as follows. First we calculate the Fourier transform $F(\omega)$ of seismogram $f(t)$. Then we define the Gaussian shape bandpass filter

$$H(\omega, \omega_n) = \exp\{-\alpha[(\omega - \omega_n)/\omega_n]^2\} \tag{8}$$

where ω_n is the center frequency and α indicates the sharpness of the peak. We apply the bandpass filter (8) to $F(\omega)$ and calculate the Hilbert transform of the windowed spectrum. Next we return to the time domain by the inverse Fourier transform and calculate the envelope from Eq. (7). The peak of the envelope gives us the group arrival time of the wave of frequency ω_n. By applying this procedure successively with ω_n, we can get the dispersion curves of the group velocity from Eq. (5).

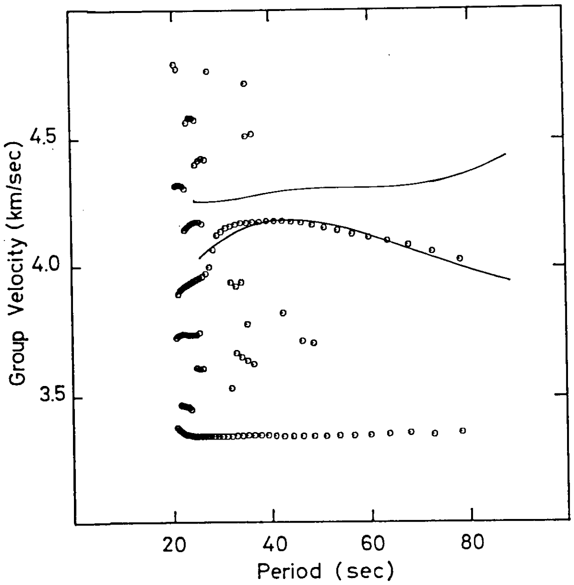

Fig. 9. Dispersion curves of synthetic Rayleigh wave. Plotted symbols are the estimated group velocity by the bandpass filtration technique. Solid lines are theoretical dispersion curves of group velocity for fundamental mode and first higher mode. Focal depth is 10 km and α in Eq. (8) is 50.

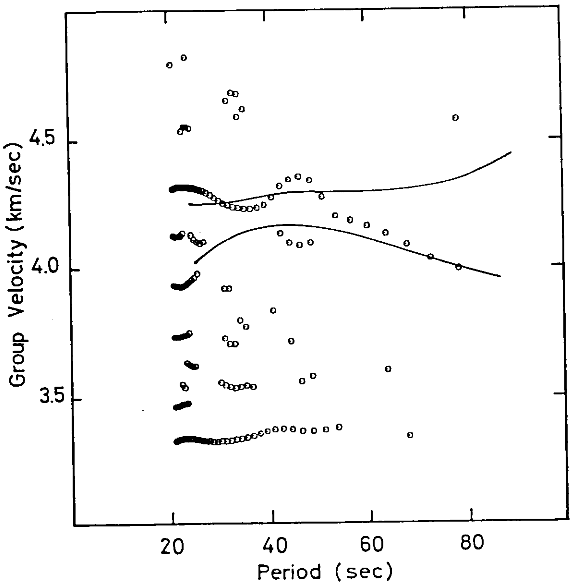

Fig. 10. Dispersion curves of synthetic Rayleigh wave. Focal depth is 200 km and α in Eq. (8) is 50. For further details, see captions on Fig. 9.

The result is shown in Fig. 9 for the synthetic Rayleigh wave seismogram of the focal depth 10 km, which should be compared with Fig. 2. The agreement between the estimation and the theory is excellent for the fundamental mode. However, we cannot estimate group velocity for the first higher mode at all. This shows a marked contrast with the result of the Instantaneous Frequency Analysis, which yielded the estimation of the first higher mode group velocity. Though we admit that the bandpass filtration technique has the advantage of estimating the fundamental mode group velocity, it is not advantageous to use for the estimation of the higher mode group velocity. This situation becomes clearer in case of the focal depth = 200 km. Fig. 10 shows the result for the synthetic seismogram of the focal depth 200 km. This should be compared with Fig. 8. In this case, the interference of the fundamental mode and the higher mode is so severe that the bandpass filtration technique failed to retrieve the separate dispersion curves.

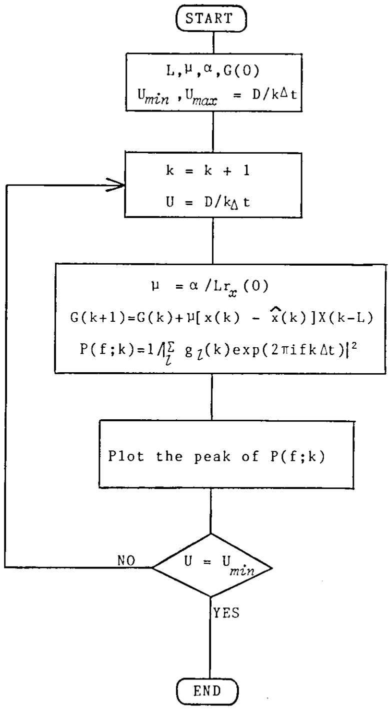

Fig. 11. Flow chart of the Instantaneous Frequency Analysis.

The results in this section show that if we use the Instantaneous Frequency Analysis and the bandpass filtration technique simultaneously for the group velocity analysis, we will be able to determine the dispersion curves of both the fundamental mode and the higher mode effectively. In Fig. 11, we present a flow chart of the Instantaneous Frequency Analysis designed for use as the group velocity analysis.

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22. 分散性地震波のための新しい群速度推定法

東京大学理学部地球物理学教室　坪　井　誠　司
東京大学地震研究所　大久保　修　平

表面波の分散を研究するための，高精度の群速度推定法を考察した．この方法は最大エントロピー法を基礎とする，瞬間周波数解析に基づいている．この方法の有効性を理論記象を用いて調べ，特に実用的観点から，予測フィルターの最適な長さ及び基本モードと高次モードの干渉に対する分解能に注意をはらった．さらにバンドパスフィルターによる方法と比較した．この新しい方法は震源の浅い地震で励起された高次モードの群速度をも推定できる点で従来の方法に優っている．