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4. The Coordinate Transformation of the Japan Geodetic Reference System to the Japan Hydrographic Department Geoid.

By Manabu YATABE*,
Earthquake Reasearch Institute.

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Abstract

The geodetic reference system of Japan based on a Bessel ellipsoid does not fit well the geoid in and around Japan, so that the astrogeodetic deflections of the vertical mostly point to the northwest. This unsuitability is caused by the following facts: (a) The geoidal slope around the Tokyo geodetic datum station is steeper than those in other areas. (b) The astrogeodetic deflection of the vertical is taken as zero at the Tokyo geodetic datum station.

During the period from 1947 to 1977, the astrogeodetic deflections of the vertical were newly observed at 324 triangulation points by the Geographical Survey Institute. Meanwhile, the geoidal heights in and around Japan have been obtained by the Hydrographic Department of Japan by combining land- and sea-surface gravity data with the satellite data GEM 10 B.

The present study is devoted to making the coordinate transformation of the geodetic reference system of Japan bestfitting the Japan Hydrographic Department geoid (Ganeko's geoid). The result shows that the correction for the deflection of the vertical at the Tokyo geodetic datum station amounts to $\xi_0=-11''\!.98$ (towards the north) and $\eta_0=+10''\!.24$ (towards the east). This values are well consistent with the results previously obtained by many geodesists.

1. Introduction

Astronomical determinations of the position of a point on the earth's surface depend essentially on the plumbline, *i.e.* the direction of the gravity vector, at the observation point. The astronomical latitude and longitude are independenly determined by astronomical observations at every point. On the other hand, geodetic determinations are composed of purely geometrical operations as distance and angle measurements. The geodetic latitude and longitude are based on the astronomical latitude and longitude of the geodetic datum station.

* Present Address: OYO Corporation. 2-2-19, Daitakubo, Urawa, 336 JAPAN.

In other words, the geodetic positionings are based on a reference ellipsoid independent of the earth's gravity potential, while the astronomical ones are based on the geoid which is one of the equipotential surfaces of the earth's gravity field. The geodetic latitude and longitude, in general, differ from the astronomical latitude and longitude even if the ellipsoid approximates the geoid most closely for the whole earth. If the reference ellipsoid fits the geoid only for a limited area, the positions determined by the geodetic method may largely disagree with the ones determined by astronomical methods in the remaining areas. In such a case, it is necessary for fitting the reference ellipsoid better to the geoid to transform the size and the position of the ellipsoid, otherwise problems of physical geodesy over a more extensive area can not be treated on the basis of the reference ellipsoid.

For geodetic positionings, a reference ellipsoid should be first adopted with some assumptions. In Japan, a Bessel ellipsoid is adopted as the reference ellipsoid, which is set up with the assumption that the deflection of the vertical is zero at the Tokyo geodetic datum station. The astrogeodetic deflections, *i.e.* the inclination of the geoid to the reference ellipsoid, have been observed nationwidely in Japan by the Geographical Survey Institute (GEOGRAPHICAL SURVEY INSTITUTE, 1951, 1953, 1955, 1958, 1972, 1978). The results show that the vectors of the deflections mostly point to the northwest and the magnitudes of most vectors are more than 10″. This tendency is caused by the unsuitability of the Bessel ellipsoid to the geoid excepting for the local area surrounding the Tokyo datum station. The zero-deflection assumpted at the Takyo geodetic datum station may be quite unsuitable for fitting the ellipsoid to the geoid in and around Japan. To eliminate this unsuitability, it is necessary to adopt another reference ellipsoid instead of the Bessel ellipsoid and/or to correct the position of the ellipsoid.

The geodetic coordinate system of the reference ellipsoid is set independently of the geocentric coordinate system of the geoid. The formula to obtain the geometrical relationship between the two systems was first derived by HELMERT (1880). Afterwards, VENING-MEINESZ (1950) and MOLODENSKII *et al.* (1960) improved Helmert's formula. By using these formulas, we can obtain a best suited ellipsoid. Many geodetists (*e.g.* ATUMI, 1933; KAWABATA, 1935; TORAO, 1949; HAGIWARA, 1967; ONO, 1974; KOZAI, 1980) have, so far, investigated the best suited ellipsoids of Japan by various methods. Their results show that the Bessel ellipsoid fits well the geoid in and around Japan if the deflection at the Tokyo geodetic datum station amounts to about 10″ towards the southeast.

Detailed geoidal heights in and around Japan have been calculated by the Hydrographic Department of Japan (GANEKO, 1980, 1982). The gravity data obtained by land and sea surface measurements is combined with the satellite geopotential coefficients GEM 10 B. The geoidal map thus obtained may indicate a short wavelength geoidal undulation, such as the geoidal low on the Japan trench, and can be used for calculating gravimetric deflections of the vertical by numerical differentiations. In the present paper, the author calculates the gravimetric deflections of the vertical from Ganeko's geoidal height to investigate the relationship with the geodetic reference system of Japan. The coordinate transformation is made for fitting the astrogeodetic deflections newly observed at 324 triangulation stations to the gravimetric deflections calculated here, and the corresponding displacement of the center of the Bessel ellipsoid from that of the normal ellipsoid is obtained.

2. Geoidal Height Map around Japan

The topographic surface of the rotating earth is approximately an elliposid of revolution. In other words, the shape of an equipotential surface of the earth's gravity field is nearly an ellipsoid of revolution. Therefore, the difference between magnitudes of the actual gravity potential and that of the ellipsoidal potential, which is usually called the normal gravity potential in physical geodesy, is so small that the difference can be neglected in the first approximation theory. If we split the actual gravity potential into the normal one and the remaining small disturbing one, we can treat mathematically both as mutually independent quantities. This is the starting point of physical geodesy, simplifying the problem of determining the shape of an equipotential surface of the actual gravity field.

Denoting the normal potential by U and the disturbing one by T, the actual gravity potential can be written as $W=U+T$. U is completely determined by four constants, such as the earth's equatorial radius a, the polar one b, the geocentric gravitational constant GM and the angular velocity of the earth's rotation ω. These constants are called Stokes' constants, which can be actually determined by observations of satellite-orbit perturbations. The formulation of the normal potential is made exact by using the spheroidal coordinate system. Meanwhile, T is determined by both satellite observations and gravity measurements on the earth's surface. The mathematical expression of T is made in an expansion series of spherical harmonics for practical convenience.

There exist, around the earth, innumerable equipotential surfaces of the normal gravity field. Their shapes are geometrically ellipsoid of revolution. Among them, there exists only one ellipsoid with the semiaxes of a and b and with the potential $U=U_0$. This is the normal ellipsoid. U_0 is determined by Stokes' contants. On the other hand, there also exist innumerable equipotential surfaces of W=const. due to the actual gravity field. One of them is consistent with a surface having the potential of $W=U_0$. This surface is called the geoid. It is needless to say that the shape of the geoid is very similar to that of the normal ellipsoid. The normal ellipsoid is a standard ellipsoid best suiting the geoid. The center of gravity is located at the coordinate origin of the normal ellipsoid, and the rotation axis coincides with the semiminor axis of the normal ellipsoid.

The distance from the normal ellipsoid to the geoid is called the geoidal height. It can be calculated by Stokes' integral of gravity anomaly (HEISKANEN and MORITZ, 1967), which is

$$\zeta=\frac{R}{4\pi G}\iint_{\sigma} \Delta g S(\psi)\,\mathrm{d}\sigma\,,$$

where ζ is the geoidal height, R the mean radius of the earth, G the mean gravity over the entire surface of the earth, Δg the gravity anomaly defined on the geoid, σ the solid angle, and $S(\psi)$ Stokes' function of ψ, the spherical distance between the computation point and the surface element $\mathrm{d}\sigma$. Stokes' function is written as

$$S(\psi)=\sum_{n=2}^{\infty}\frac{2n+1}{n-1}P_n(\cos\psi)\,,$$

with Legendre's function P_n of degree n. Sum of geoidal heights over the entire surface of the earth becomes zero. This indicates that the total volume of the geoid is almost equal to that of the normal ellipsoid. Stakes' integral requires gravity anomalies distributing over the entire surface of the earth. However, gravity measurements have never been made in some areas nutil now, so we can not expect to use the gravity data in these gaps. For a numerical calculation of Stokes' integral, we use gravity data actually measured in an area nearby the calculation point, and satellite gravity data for the remaining distant area.

GANEKO (1982) calculated the geoidal height (Fig. 1) from the satellite-determined spherical harmonic coefficients and the gravity anomalies obtained by land- and sea-surface gravity data. According to RAPP's method (1980), Ganeko performs Stokes' integration over a spherical cap area whose angular radius ψ is 10°, and the satellite gravity data GEM 10 B (LERCH *et al.*, 1981) is used for estimating the contribution of areas beyound 10°. The geoidal heights are computed on the grid points of $10'\times10'$ block over an area of latitude 20°N to 50°N and longitude 120°E to 150°E.

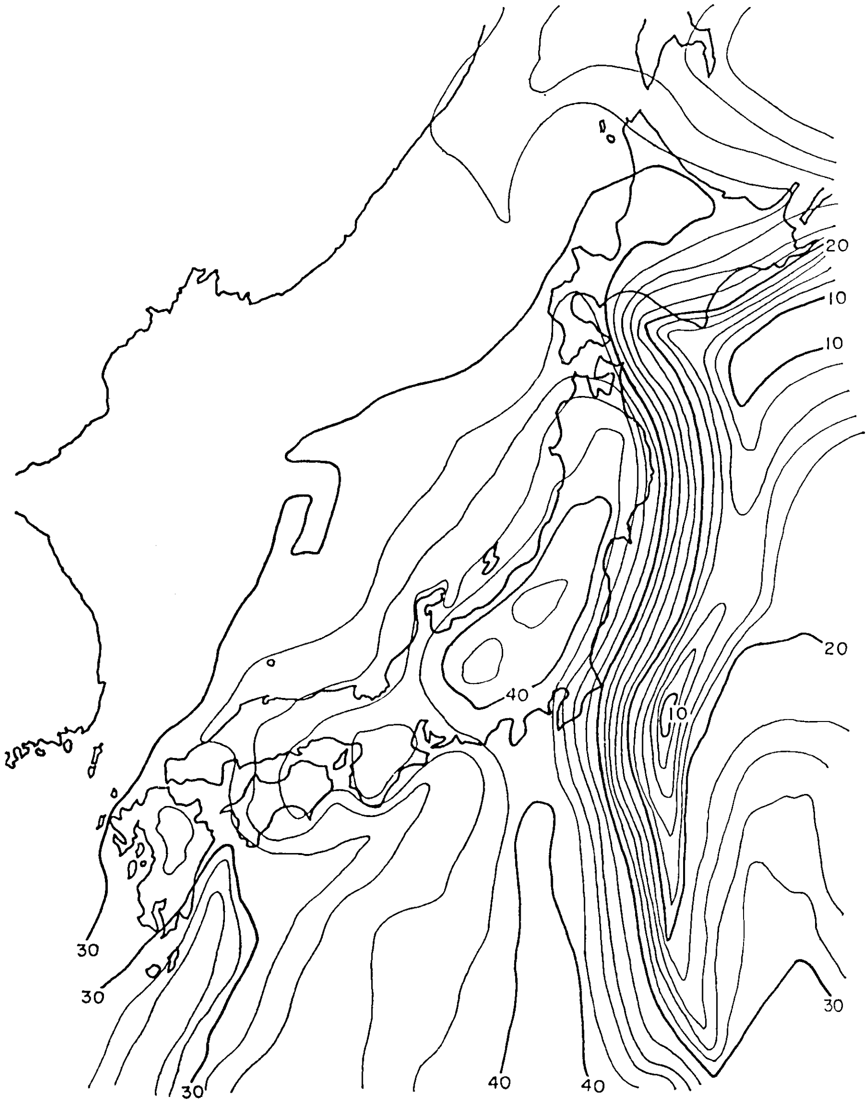

Fig. 1. Japan Hydrographic Department geoidal height in and around Japan (after GANEKO, 1982). Contour interval: 2 m.

As seen in Fig. 1, the geoidal height amounts to about 40 m from Central Japan to the Tohoku District, but it decreases to 10 m on the Japan trench and to about 30 m on the Japan Sea. In general, the geoidal height is higher on the Japan island-arcs than on the surrounding ocean areas. This fact indicates that the geoid heaves up above the excess mass of the crust forming the island-arcs. It should be noticed that the geoidal height is conspiciously low along the trench. The geoid depression along the trench is caused by mass deficiencies

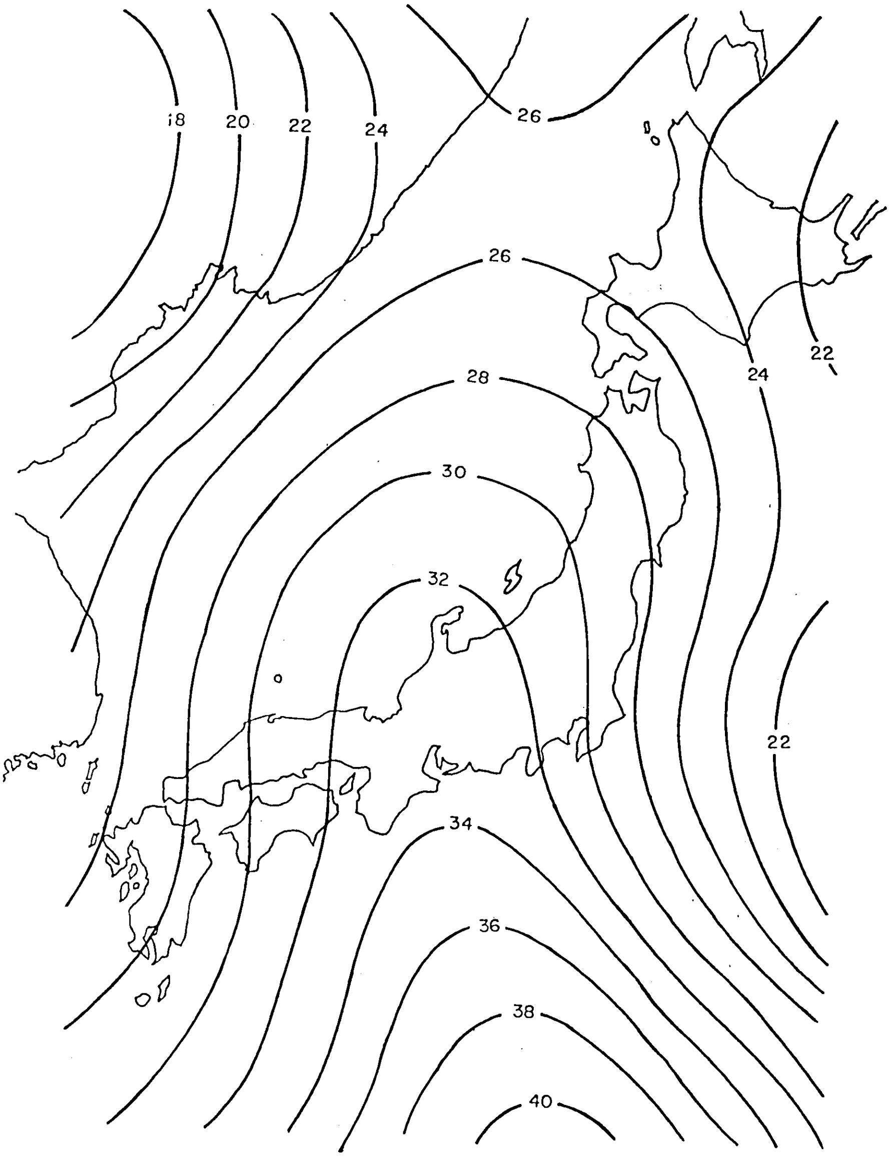

Fig. 2. Geoidal height calculated from the satellite gravity data GEM 10B with the geopotential coefficient set of 36×36. Contour interval: 2 m.

due to the subduction of the Pacific lithospheric plate and by relatively low density convecting materials underlying the plate. The thick sedimentary layers developed along the trench may intensify this tendency. The geoidal height map calculated only from satellite-determined spherical harmonic expansion coefficients of gravity potential can not reveal such a short wavelength geoidal height anomaly over the Japan tranch (Fig. 2). In contrast, Ganeko's geoidal height map (Fig. 1) combined with the surface gravity data well shows the detailed undulation of the geoidal surface. In this sense, the present study chooses Ganeko's geoidal height for calculating deflections of the vertical at triangulation points on Japanese islands.

3. Gravimetric Deflections of the Vertical

As previouly mentioned about the normal ellipsoid, the geoid is defined as an equipotential surface whose potential is equal to the normal potential. The shaps of the geoid is well approximated by that of the normal ellipsoid. Consequently, the geoidal height, which is defined as a discrepancy between both surfaces, is small compared to the topographic relief.

The gravimetric deflection of the vertical is defined as the gradient of the geoid to the normal ellipsoid. For example, consider the excess mass located beneath the ground surface (Fig. 3). Since the gravitational attraction of this mass is added to the normal gravity, the resultant gravity vector slightly inclines in the direction of the excess mass. As a result, an equipotential surface, to which the resultant gravity vector is normal, rises up slightly above the excess mass. Inversely, the geoid becomes low on a mass deficiency.

On the other hand, the normal ellipsoid is independent of the actual gravity field. The surface of the normal ellipsoid is not affected by the gravitational attraction of a mass deficiency. This fundamental importance can be applied to the definition of the deflection of the vertical, that is to say, an angle of intersection between a normal to

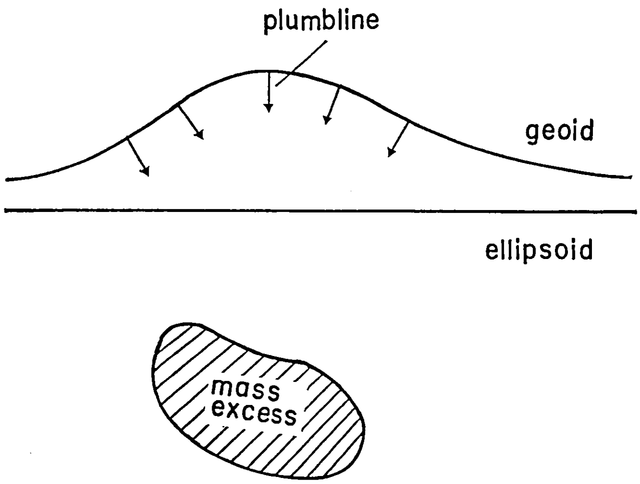

Fig. 3. Geoidal undulation due to an excess mass located beneath the ground surface.

the geoid (this is called "plumbline") and that to the normal ellipsoid (this is called "vertical"). Since the geoidal height is measured from the normal ellipsoid, the deflection component can be obtained by differentiating the geoidal height.

The mathematical procedure for obtaining the deflection of the vertical from the geoidal height is as given below. By using the spherical approximation in which the earth is a sphere with radius R, the meridian and prime-vertical components of the deflections are calculated by

$$\left.\begin{aligned} \xi^g &= -\frac{1}{R}\frac{\partial \zeta}{\partial \phi}, \\ \eta^g &= -\frac{1}{R\cos\phi}\frac{\partial \zeta}{\partial \lambda}, \end{aligned}\right\} \qquad (1)$$

where ζ denotes the geoidal height, ϕ the latitude and λ the longitude (HEISKANEN and MORITZ, 1967). The deflections of the vertical such defined are based on the actual gravity field of the earth, so that they are called gravimetric deflections of the vertical to discriminate from the astrogeodetic deflections as will be mentioned in a forthcoming section. The suffix "g" of the notations ξ and η in the above formulas means gravimetric deflections.

To calculate gravimetric deflections in Japan, we differentiate the geoidal height obtained by GANEKO (1982) according to (1). The data of the geoidal height is given on the grid points of $10' \times 10'$ blocks and stored in a magnetic tape for computation use (Fig. 4). For numerically computing a geoidal height at a point, we interpolate the geoidal height between the point and the neighboring grid points in the formula

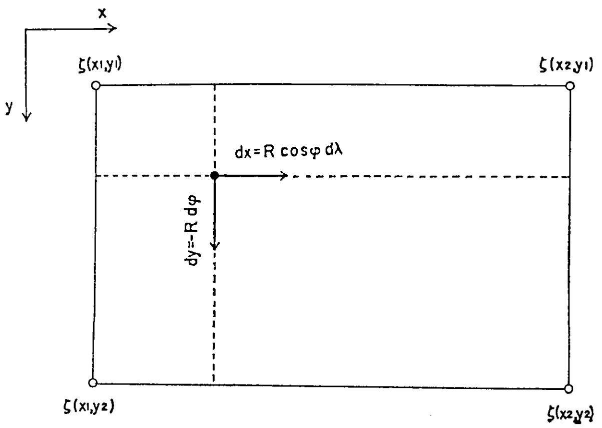

Fig. 4. Schematic illustration of numerical differentiation. Geoidal height (solid circle) is calculated by interpolating from four neighboring geoidal heights (open circles).

$$\zeta(x, y)=c_1x+c_2y+c_3xy+c_4 . \tag{2}$$

Then we differentiate (2) with respect to x and y directions and we get

$$\left.\begin{aligned} \xi^g &= \frac{\partial\zeta}{\partial y}=c_2+c_3x ,\\ \eta^g &= -\frac{\partial\zeta}{\partial x}=-(c_1+c_3y) . \end{aligned}\right\} \tag{3}$$

If the four coefficients c_1, c_2, c_3 and c_4 are determined from the data of geoidal height at every $10'\times10'$ block, the gravimetric deflections ξ^g and η^g are calculated from the above formulas (3).

Fig. 5 shows the gravimetric deflections at triangulation points calculated from Ganeko's geoidal data. We see that most deflection

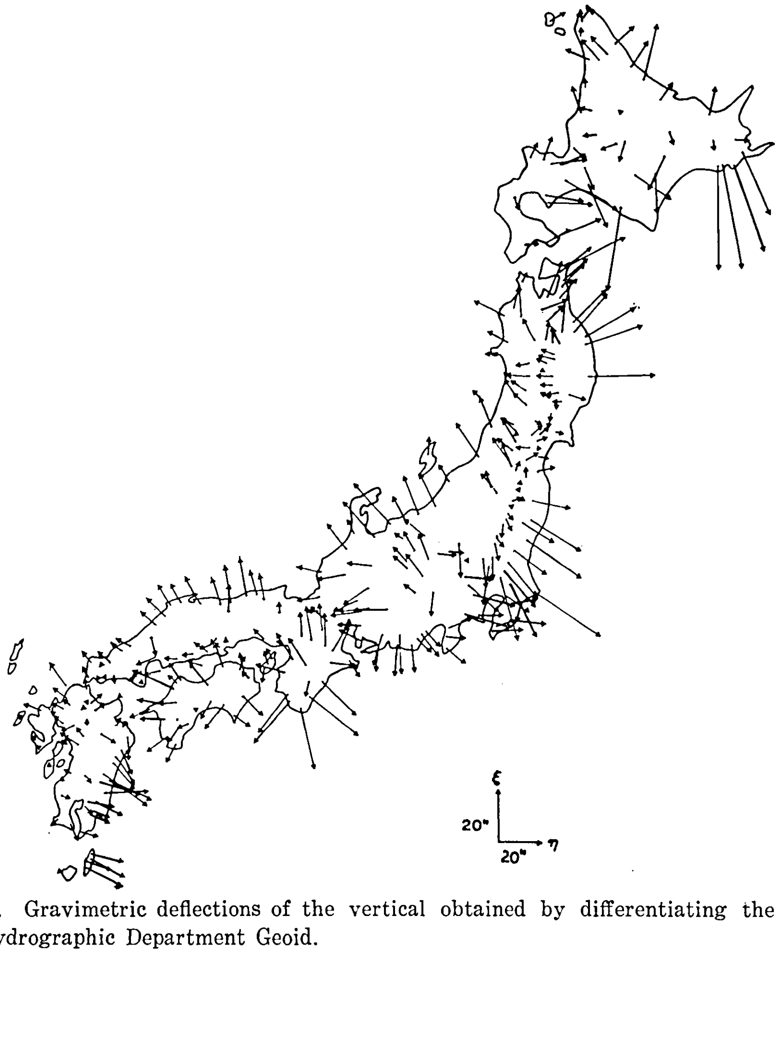

Fig. 5. Gravimetric deflections of the vertical obtained by differentiating the Japan Hydrographic Department Geoid.

vectors are directed towards the sea. Their magnitudes are small in the inland and large near the seashore. In the Pacific coast, they amount to about 35″ at maximum. The magnitudes of the vectors on the Pacific coast are generally larger than those on the Japan Sea coast. We can see that this tendency is due to the geoidal depression in the Japan trench. Generally speaking, it is reasonable to calculate deflections by Vening-Meinez' integral (see HEISKANEN and MORITZ, 1967) rather than by numerically differentiating the geoidal heights, because values calculated by numerical differentiation may possibly contain many errors. However, the deflections calculated from Ganeko's geoid is considered to be rather plausible as shown in Fig. 5., since the deflection vectors are well consistent with the slopes of the geoid.

4. Astrogeodetic Deflection of the Vertical

For the purpose of determining the position of a star, an astronomical instrument is usually placed on a horizontal plane by adjusting the highly precise levels. In this case, the plumbline, *i.e.* the direction of the earth's gravity fields, is perpendicular to the horizontal plane. This fact implies that an astronomical positioning is based on the geoid, *i.e.* the astronomical determinations of longitude and latitude are made on the basis of plumblines. This means the astronomical longitude and latitude depend on the geoidal height undulation at the observation point.

On the contrary, geodetic positioning is based on a reference ellipsoid. The geodetic latitude and longitude are determined on the basis of verticals normal to the reference ellipsoid. The reference ellipsoid is not always equal to the normal ellipsoid, but an ellipsoid is adopted by law as the standard base surface by which geodetic positions are regulated.

Before the position of a triangulation station is determined, it is necessary to know the dimensions of the reference ellipsoid and to fix definitely the latitude and longitude of a specific station as well as the azimuth of the direction to another triangulation station. This selected position is the geodetic datum determing the relative positions of all triangulation points on land with respect to the adopted reference ellipsoid.

The reference ellipsoid of Japan is a Bessel ellipsoid (the equatorial radius $a=6,377,397$ m and the flattening $f=1/299.2$) assigned to the Tokyo datum station. It is established as follow:

(a) The plumbline passing through the Tokyo geodetic datum is consistent with the normal-line to the Bessel ellipsoid at latitude 35°39′17.″5148 N and longitude 139°44′40.″5020 E.

(b) The azimuth reckoned from the Tokyo geodetic datum towards the Kanozan triangulation point amounts to 156°25′28.″442 on the Bessel ellipsoid.

(c) The surface of the Bessel ellipsoid passes 24.4140 m under the leveling datum at Tokyo.

(a) assumes that the astrogeodetic deflection of the vertical at the Tokyo geodetic datum station is zero. Therefore, the astronomical latitude and longitue are equal to the geodetic ones at the datum station. The geoidal surface has an undulation due to the mass distribution of the earth. On the contrary, the reference ellipsoid does not depend on the mass distributions because it is geometrically established. The astronomical latitude and longitude are not generally equal to the geodetic ones at any stations excepting for the geodetic datum stations.

Fig. 6. Astrogeodetic deflections of the vertical in Japan with respect to the Bessel ellipsoid (after the Geographical Survey Institute). Most vectors point to the northwest due to the unsuitability of a reference ellipsoid.

The astrogeodetic deflections of the vertical at 324 triangulation points in Japan were observed by the Geographical Survey Institute during the period from 1947 to February 1977. The result is shown in Fig. 6. We can see that almost all vectors are directed towards the northwest. This indicates that the geoid slopes towards the northwest to the reference ellipsoid. We can consider the causes as follows; the zero deflection components at the Tokyo geodetic datum station is assumed but the geoidal slope is steep around there. Therefore, the ellipsoid does not always fit the geoid all over Japan even if the elliposid best fits the geoid around the Tokyo geodetic datum station. If we adopt any other deflection values at the Tokyo geodetic datum station, more plausible distribution of the astrogeodetic deflections fitting gravimetric deflections as shown in Fig. 5 can be obtained. The real existence of deviations between both kinds of deflections implies that astronomically determined positions are different from geodetically determined ones. Accordingly, the latitude and longitude determined by astronomical observations are not always equal to those determined by geodetic observations. Then the deflection of the vertical can be newly defined by the difference between two kinds of latitude and longitude. Its meridian and prime-verical components are respectively

$$\left.\begin{aligned} \xi^a &= \phi^a - \phi^g , \\ \eta^a &= (\lambda^a - \lambda^g) \cos \phi^g , \end{aligned}\right\} \qquad (4)$$

where ϕ^a and λ^a are the astronomical, and ϕ^g and λ^g the geodetic latitudes and longitudes.

Fig. 7 illustrates schematically the difference in the definitions of the deflection of the vertical. We suppose an ellipsoid passing through a point P, where the ellipsoidal normal is not generally consistent with the plumbline. The angle between the plumbline and the normal to

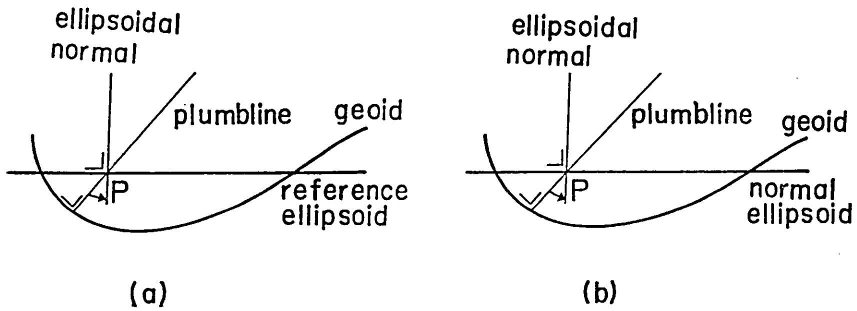

Fig. 7. Definition of the deflection of the vertical. (a) Astrogeodetic deflection of the vertical referring to a reference ellipsoid. (b) Gravimetric deflection of the vertical referring to the normal ellipsoid.

the reference ellipsoid is the astrogeodetic deflection of the vertical at the point P (Fig. 7 (a)). On the other hand, in the case of gravimetric deflection, the standard surface is the normal ellipsoid instead of the reference ellipsoid (Fig. 7 (b)). This fundamental importance may suggest the coordinate transformation of the reference ellipsoid to the normal ellipsoid.

5. Transformation of the Geodetic Coordinate System

The astrogeodetic geoid related to the astrogeodetic deflections is clearly different from the gravimetric geoid. The astrogeodetic geoid is based on the reference ellipsoid established independently from the world geodetic system, so that the local astrogeodetic datum should be transformed in such a way as deviations between both geoids diminish. Such a condition may also be realized by minimizing the sum of the squares of differences between astrogeodetic deflections and gravimetric ones at all triangulation points.

The transformed astrogeodetic deflections are written as

$$\left.\begin{aligned} \tilde{\xi}^a &= \xi^a + \Delta\xi^a , \\ \tilde{\eta}^a &= \eta^a + \Delta\eta^a , \end{aligned}\right\} \qquad (5)$$

where $\Delta\xi^a$ and $\Delta\eta^a$ are correction values for the astrogeodetic deflection components ξ^a and η^a. After transforming the coordinate, $\tilde{\xi}^a$ and $\tilde{\eta}^a$ should coincide with the gravimetric deflections ξ^g and η^g. Because of inaccuracies in measurements and computations, there remain small nonzero residuals $\tilde{\xi}^a - \xi^g$ and $\tilde{\eta}^a - \eta^g$, so that $\Delta\xi^a$ and $\Delta\eta^g$ can be determined by the method of least squares, *i.e.*

$$\sum [(\tilde{\xi}^a - \xi^g)^2 + (\tilde{\eta}^a - \eta^g)^2] \rightarrow \text{minimum} . \qquad (6)$$

The sum is extended over all triangulation points.

For actually determining these residuals, they should be expressed in forms available for numerical computations. Denoting the origin of the reference system in the geocentric coordinate by $(\Delta X, \Delta Y, \Delta Z)$, they can expressed as follows (MOLODENSKII *et al.*, 1962):

$$\left.\begin{aligned} \Delta\xi^a &= A\Delta X + B\Delta Y + C\Delta Z + D\frac{\Delta a}{a} + E\frac{\Delta b}{b} , \\ \Delta\eta^a &= A'\Delta X + B'\Delta Y . \end{aligned}\right\} \qquad (7)$$

In the above formulas, the coefficients are given by

$$A = \frac{\sin\phi^g \cos\lambda^g}{M+H} , \quad B = \frac{\sin\phi^g \sin\lambda^g}{M+H} , \quad C = -\frac{\cos\phi^g}{M+H} ,$$

$$D=-\frac{\sin\phi^g\cos\phi^g}{M+H}(N+M)\,,\quad E=\frac{\sin\phi^g\cos\phi^g}{M+H}\left(\frac{b^2}{a^2}N+M\right),$$

$$A'=\frac{\sin\lambda^g}{N+H}\,,\quad B'=-\frac{\cos\lambda^g}{N+H}\,,$$

$$\Delta a=a^\dagger-a\,,\quad \Delta b=b^\dagger-b\,,$$

where

a: equatorial radius of the Bessel ellipsoid,
b: polar radius of the Bessel ellipsoid,
$a^\dagger$: equatorial radius of the normal ellipsoid,
$b^\dagger$: polar radius of the normal ellipsoid,
H: height of a triangulation point above the Bessel ellipsoid,
M: radius of curvature in meridian,
N: radius of curvature in prime-vertical,
ϕ^g: geodetic latitude,
λ^g: geodetic longitude.

In (7), we assume that the rotation axis of the Bessel ellipsoid is parallel to the one of the earth itself. In this case, the Cartesian coordinates are taken as the Greenwich meridian, the 90°E longitude and the rotation axis as the X, Y and Z axes, respectively. How to derive (7) with the coefficients will be given in the Appendix. In such a way, we determine $\Delta\xi^a$ and $\Delta\eta^a$, and transform ξ^a and η^a into $\tilde{\xi}^a$ and $\tilde{\eta}^a$ by the minimum condition (6). Fig. 8 shows the astrogeodetic deflections of the vertical transformed as best-fitting the gravimetric deflections of the vertical which are calculated by numerically differentiating Ganeko's geoidal height. Fig. 8 and Fig. 5 are so similar that we can not discriminate each other. This resemblance suggests that these two systems of deflections of the vertical coincide almost perfectly, despite the fact that these systems are obtained by essentially different kinds of observations. The results of computation are summarized in Table 1.

As $\Delta a/a$ and $\Delta b/b$ are known quantities in (7), the minimum condition (6) yields only ΔX, ΔY and ΔZ. By using recently determined geodetic parameters, these values are calculated as

$$\left.\begin{aligned}\Delta X&=-149.57\text{ m}\pm 62.28\text{ m},\\ \Delta Y&=+541.82\text{ m}\pm 58.29\text{ m},\\ \Delta Z&=+706.07\text{ m}\pm 63.83\text{ m}.\end{aligned}\right\}\qquad (8)$$

These results indicate that the center of the Bessel ellipsoid is located about 900 m towards the point (51°.5 N, 105°.4 E) in the geocentric coordinates.

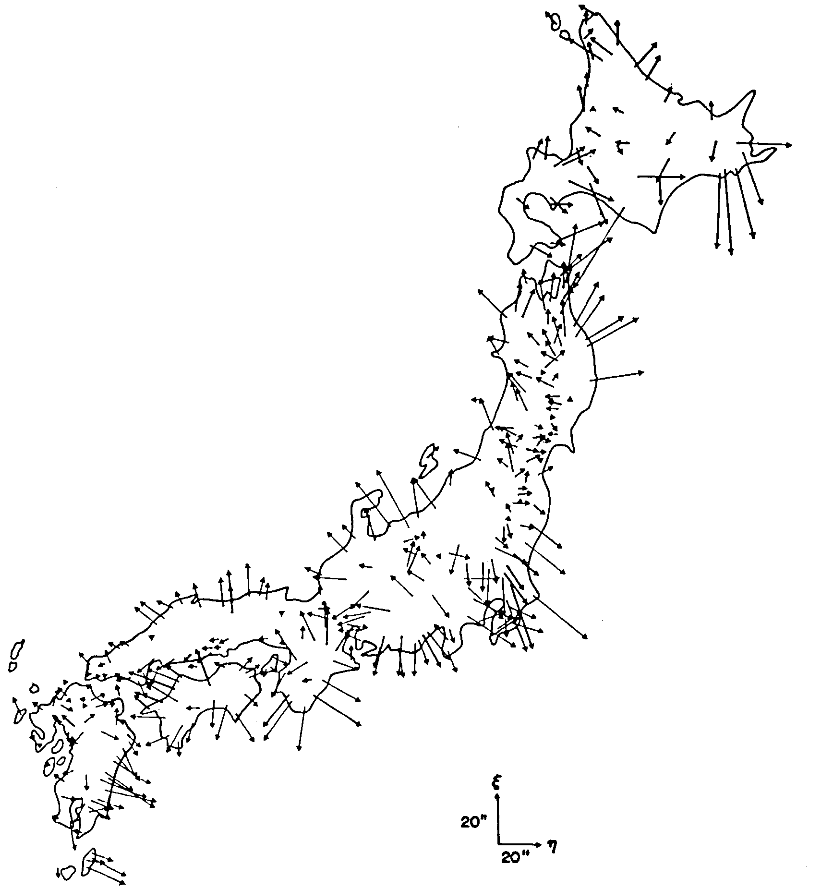

Fig. 8. Astrogeodetic deflections of the vertical with respect to the best suited ellipsoid.

The displacement of the Bessel ellipsoid can be converted into ξ_0 and η_0, the deflections of the vertical at the Tokyo datum station. The geodetic values for the Tokyo datum station

$$\left.\begin{aligned} \phi^g &= 35°39'17''.5148\text{ N}, \\ \lambda^g &= 139°44'40''.5020\text{ E}, \\ H &= 24.414\text{ m}, \\ M &= 6{,}356{,}445.256\text{ m}, \\ N &= 6{,}384{,}638.497\text{ m}, \end{aligned}\right\}$$

and the values (8) are substituted into the equation (7), obtaining the following results.

$$\left.\begin{aligned} \xi_0 &= -11''.98 \pm 2''.04, \\ \eta_0 &= +10''.24 \pm 1''.94. \end{aligned}\right\} \tag{9}$$

Table 1. Deflection of the vertical in Japan.
No.: station number,
ϕ: geodetic latitude (for example, 3565 indicates 35°65N),
λ: geodetic longitude (for example, 13974 indicates 139°74E),
H: height above the Bessel ellipsoid (unit m),
ξ^a: ξ-component of the astrogeodetic deflection referring to the Bessel ellipsoid (unit ″),
η^a: η-component of the astrogeodetic deflection referring to the Bessel ellipsoid (unit ″),
ξ^g: ξ-component of the gravimetric deflection calculated from Ganeko's geoid (unit ″),
η^g: η-component of the gravimetric deflection calculated from Ganeko's geoid (unit ″),
$\bar{\xi}^a$: ξ-component of the astrogeodetic deflection referring to the normal ellipsoid (unit ″),
$\bar{\eta}^a$: η-component of the astrogeodetic deflection referring to the normal ellipsoid (unit ″).

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\bar{\xi}^a$	$\bar{\eta}^a$
0	3565	13974	24.41	0.00	0.00	−4.81	10.64	−11.98	10.24
1	3406	13454	279.50	14.29	−3.15	2.19	−1.70	2.13	5.68
2	3366	13441	125.10	9.28	−4.31	−7.51	8.13	−3.04	4.49
3	3424	13433	36.70	16.13	−3.89	4.20	1.19	4.06	4.88
4	3406	13299	39.70	17.15	−15.31	5.75	−6.03	5.12	−6.91
5	3429	13389	462.45	13.19	−12.55	7.78	−3.36	1.18	−3.90
6	3398	13340	244.70	24.79	−14.37	6.85	−8.13	12.69	−5.85
7	3351	13256	319.90	15.72	−19.58	2.02	−11.02	3.49	−11.30
8	3361	13299	713.60	19.99	−22.80	−0.72	−8.14	7.77	−14.40
9	3381	13279	75.40	18.43	−28.44	6.45	−17.01	6.31	−20.09
10	3321	13255	113.70	13.01	−22.72	−0.63	−9.65	0.65	−14.44
11	3295	13273	253.70	9.29	−18.10	−5.19	−8.52	−3.19	−9.77
12	3339	13329	143.21	8.57	−8.51	−2.13	0.52	−3.77	−0.02
13	3304	13306	47.50	4.72	−14.21	−4.39	−3.68	−7.75	−5.79
14	3355	13351	27.10	4.02	−10.67	−5.38	−4.27	−8.27	−2.12
15	3434	13569	109.60	20.61	−15.81	9.84	−7.56	8.47	−6.66
16	3372	13539	34.67	2.95	−24.12	−14.38	−13.96	−9.43	−15.05
17	3345	13576	74.49	−3.60	−11.95	−21.30	5.63	−16.13	−2.78
18	3371	13600	41.30	1.90	11.07	−14.01	21.66	−10.54	20.31
19	3446	13677	478.00	16.99	−9.04	−5.00	4.18	4.80	0.41
20	3504	13695	55.80	11.42	−19.93	0.06	−11.37	−0.54	−10.44
21	3420	13518	45.30	17.46	−19.86	2.86	−12.28	5.30	−10.85
22	3472	13784	33.30	0.14	−8.93	−11.63	−1.86	−12.04	0.80
23	3511	13895	111.20	7.80	−8.02	2.76	9.42	−4.33	2.01
24	3436	13486	137.40	10.25	−6.62	3.66	−2.08	−1.81	2.30
25	3532	13932	181.30	1.55	10.66	−4.57	19.38	−10.53	20.78
26	3499	13580	221.10	14.48	−8.38	7.58	−0.55	2.60	0.80
27	3444	13633	419.80	9.37	0.61	−0.80	10.35	−2.79	9.93
28	3477	13614	166.70	23.33	−9.34	11.18	−0.34	11.33	−0.07
29	3404	13579	375.10	11.12	−13.28	−5.05	−0.60	−1.16	−4.10
30	3407	13620	59.70	4.38	6.43	−11.91	14.58	−7.93	15.72
31	3477	13527	931.30	11.72	−8.20	1.36	0.49	−0.20	0.83
32	3202	13149	14.80	2.34	12.25	−6.02	17.57	−10.44	20.23

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\bar{\xi}^a$	$\bar{\eta}^a$
33	3191	13142	5.00	8.32	13.67	0.52	18.48	−4.50	21.63
34	3237	13164	22.80	3.05	6.94	−9.88	9.60	−9.59	14.96
35	3395	13093	64.10	13.06	−9.47	3.12	−2.44	1.15	−1.65
36	3450	13351	68.80	8.25	−12.98	2.49	−5.33	−3.63	−4.43
37	3525	13996	352.40	19.36	0.71	2.86	10.44	7.19	11.00
38	3496	13978	193.60	19.55	0.54	0.48	12.73	7.27	10.79
39	3325	13418	184.87	3.89	−1.80	−3.94	6.48	−8.59	6.93
40	3353	13379	144.81	0.83	−14.72	−7.51	−7.17	−11.50	−6.09
41	3317	13315	658.00	6.68	−12.33	−2.64	−2.23	−5.74	−3.88
42	3547	13380	177.00	20.93	−9.94	9.12	−3.68	9.44	−1.31
43	3490	13371	697.31	10.90	−11.73	−2.61	−1.93	−0.83	−3.13
44	3367	13535	131.06	2.59	−24.85	−15.16	−15.53	−9.81	−15.79
45	3469	13803	220.58	1.35	−10.01	−9.29	0.37	−10.86	−0.23
46	4054	14158	52.50	25.29	−0.09	13.95	15.30	15.22	10.62
47	4024	14179	185.66	18.67	5.12	10.62	21.23	8.44	15.88
48	4129	14129	132.60	17.80	−2.67	8.77	12.88	8.10	7.96
49	4095	14092	323.40	19.76	−8.48	12.52	6.09	9.95	2.06
50	4018	14013	211.10	11.90	−19.57	4.24	−9.27	1.85	−9.24
51	4049	14094	707.50	14.35	−10.43	5.61	5.54	4.34	0.11
52	4060	14038	233.80	19.53	−4.76	10.59	−1.27	9.63	5.64
53	4061	13990	208.00	19.26	−22.50	5.45	−13.61	9.42	−12.23
54	3876	14033	184.10	10.49	−19.18	1.73	−5.66	−0.22	−8.79
55	3990	13976	716.30	16.45	−14.76	−0.54	−4.20	6.31	−4.52
56	3933	14119	319.90	10.26	−16.81	−0.81	−5.12	−0.29	−6.20
57	3988	14059	933.30	12.48	−15.55	−0.36	−4.84	2.24	−5.10
58	3819	14083	236.40	11.76	−8.58	−0.67	3.35	0.75	1.94
59	3937	14055	121.40	17.64	−17.69	4.25	−7.54	7.17	−7.25
60	3692	14002	298.43	6.19	−9.73	−4.29	−0.97	−5.28	0.58
61	3654	13918	1673.90	0.75	−13.15	−13.18	0.20	−10.79	−3.07
62	3648	13802	1447.90	13.48	−14.12	6.71	−4.67	2.08	−4.34
63	3574	13806	1680.70	18.51	−19.09	3.17	−4.82	6.74	−9.30
64	3729	13849	992.90	21.20	−18.94	12.10	−7.83	10.05	−9.04
65	3812	13952	438.20	13.81	−22.12	11.81	−10.13	2.92	−11.95
66	3533	13698	293.00	8.64	−20.78	−1.90	−13.26	−3.20	−11.28
67	3758	13971	580.60	12.53	−8.99	6.19	−2.66	1.38	1.24
68	3706	14075	751.10	2.03	10.03	−11.25	22.52	−9.46	20.53
69	3535	13607	87.00	12.04	−5.19	0.68	−4.04	0.30	4.06
70	3632	14020	518.20	3.31	−3.36	−13.38	14.26	−8.44	7.00
71	3570	14086	73.60	−4.67	13.49	−16.53	28.95	−16.75	24.02
72	3678	13682	637.20	15.24	−14.62	4.87	−2.54	4.04	−5.16
73	3608	13609	656.14	13.87	−16.14	1.82	−9.56	2.44	−6.88
74	3535	13402	1252.00	21.52	−9.14	7.79	−0.56	9.96	−0.45
75	3525	13260	253.00	17.38	−18.64	5.94	−8.46	5.90	−10.35
76	3355	13137	26.40	13.00	−4.92	2.19	1.80	0.88	3.02
77	3559	13603	913.50	15.67	−9.25	−1.46	−9.39	4.03	−0.01
78	3294	13188	259.50	6.94	−3.57	−7.01	−0.72	−5.48	4.52
79	3483	13486	271.60	9.44	−9.12	2.66	−4.09	−2.42	−0.20
80	3132	13097	280.80	14.35	−1.18	−3.24	8.33	1.31	6.65
81	3436	13249	212.20	6.16	−14.73	−3.28	−6.78	−5.70	−6.47

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\tilde{\xi}^a$	$\tilde{\eta}^a$
82	3173	13045	676.80	11.46	−3.54	−1.76	3.46	−1.37	4.14
83	3401	13151	101.40	10.34	−8.98	−2.77	−0.96	−1.59	−1.00
84	3277	13076	10.30	13.42	−11.00	3.94	−6.09	1.01	−3.23
85	3274	12992	421.40	12.87	−5.94	−1.81	−0.38	0.51	1.59
86	4393	14445	23.70	15.49	−10.69	9.75	1.83	6.57	0.74
87	3918	13955	57.80	10.57	−14.00	3.09	−3.32	0.14	−3.82
88	4452	14306	49.90	16.38	−4.82	9.12	6.08	7.92	6.26
89	4341	14242	308.60	6.69	−9.45	−2.63	−6.60	−2.19	1.47
90	4300	14175	272.80	2.81	−6.28	−7.74	3.80	−6.17	4.47
91	4238	14255	718.70	−20.67	−31.04	−30.56	−6.14	−30.04	−20.09
92	4540	14165	210.80	11.92	−10.63	3.30	−0.75	4.06	0.09
93	4328	14357	370.52	0.57	−16.27	−12.25	−5.86	−8.53	−5.06
94	4259	14039	498.80	5.04	−5.65	−5.34	2.27	−3.96	4.75
95	4295	14447	95.58	−18.26	−13.20	−37.39	0.92	−27.63	−1.77
96	4174	14055	482.30	5.66	0.83	0.47	4.55	−3.74	11.27
97	3343	12992	268.80	14.04	−11.14	1.94	−7.85	1.98	−3.61
98	3497	13847	307.20	2.68	0.23	−5.36	6.31	−9.46	10.13
99	3469	13893	307.80	5.61	−6.18	−5.50	6.75	−6.70	3.84
100	4350	14455	420.80	1.78	−13.84	−3.24	0.93	−7.35	−2.39
101	4482	14207	30.31	16.36	−26.16	7.34	−8.04	8.17	−15.33
102	4307	14502	68.10	−16.48	−3.08	−31.81	13.00	−25.87	8.49
103	4398	14249	740.60	9.55	−15.04	0.28	−0.49	0.92	−4.10
104	4355	14368	449.49	5.57	−13.83	−4.05	1.56	−3.42	−2.59
105	4399	14166	64.02	17.52	−13.26	9.37	−3.82	9.00	−2.53
106	4321	14087	87.79	17.86	−9.13	8.54	3.02	9.08	1.39
107	4293	14292	134.62	9.81	9.49	−8.82	15.47	0.64	20.55
108	4082	14121	124.20	18.52	−3.98	8.80	6.08	8.62	6.63
109	3856	14118	222.30	11.37	−14.07	−0.32	2.46	0.48	−3.46
110	3737	14021	968.20	11.60	−3.19	0.92	2.93	0.30	7.16
111	3929	14036	412.10	15.80	−12.86	7.05	−6.05	5.32	−2.47
112	3657	13989	158.74	6.63	−8.00	−5.51	1.36	−4.98	2.27
113	3876	13975	273.80	21.10	−15.27	11.88	−6.18	10.46	−5.03
114	3600	13919	875.79	11.87	3.74	−0.87	13.65	0.10	13.83
115	3477	13821	150.90	4.62	−6.51	−4.92	5.83	−7.58	3.32
116	3772	13882	633.79	15.37	−9.24	5.30	−4.00	4.37	0.75
117	3669	13717	145.28	21.40	−14.14	12.42	−14.14	10.13	−4.59
118	3578	13704	1034.56	4.66	−17.10	−5.94	−9.12	−6.99	−7.58
119	3491	13742	789.19	−1.66	−14.48	−11.17	−3.72	−13.72	−4.86
120	3711	13826	19.46	25.33	−12.74	14.43	−7.24	14.13	−2.90
121	3693	13745	17.65	26.47	−24.32	14.62	−16.92	15.27	−14.69
122	3623	13719	494.32	12.17	−14.60	1.16	−10.06	0.70	−5.04
123	3658	13659	141.23	17.38	−16.85	9.71	−8.77	6.12	−7.46
124	3598	13655	227.02	12.75	−23.14	−1.42	−10.77	1.23	−13.76
125	3525	13626	137.31	13.66	−18.49	4.17	−6.59	1.85	−9.19
126	3623	14019	67.20	1.16	−0.63	−13.38	14.18	−10.62	9.72
127	4005	14176	813.90	20.66	11.81	6.75	23.39	10.35	22.56
128	3469	13354	504.70	9.18	−9.30	−3.28	−2.49	−2.62	−0.75
129	3662	14059	623.70	−0.45	7.11	−13.21	24.29	−12.11	17.57
130	3692	13792	482.70	33.14	−24.74	10.18	−6.80	21.89	−14.99

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\bar{\xi}^a$	$\bar{\eta}^a$
131	3670	13786	709.80	12.68	−2.97	1.24	−2.72	1.34	6.77
132	3610	13791	953.60	21.36	−5.11	4.44	−5.28	9.76	4.64
133	3613	13791	721.90	20.03	−6.23	4.49	−5.60	8.44	3.52
134	3537	13745	554.70	14.38	−25.14	−0.65	−15.52	2.51	−15.51
135	3534	13713	159.50	9.33	−21.43	−1.39	−13.28	−2.52	−11.89
136	3234	13055	567.30	10.86	−10.11	0.74	−2.87	−1.72	−2.40
137	3167	13116	918.00	9.35	3.29	−4.89	11.35	−3.56	11.17
138	3290	13197	581.50	7.36	−5.37	−6.76	−0.84	−5.08	2.74
139	3291	13051	15.30	8.66	−7.40	−2.85	−1.77	−3.67	0.30
140	3321	13044	4.50	13.60	−11.99	2.20	−4.30	1.40	−4.31
141	3331	13095	164.30	13.72	−8.72	1.95	−0.70	1.53	−0.90
142	3360	13068	112.50	14.02	−7.18	1.94	−2.73	1.97	0.57
143	3400	13113	114.60	12.72	−7.94	0.70	−0.57	0.81	−0.07
144	3395	13198	298.70	7.10	−8.71	−1.58	0.0	−4.90	−0.59
145	3418	13223	189.50	5.38	−4.48	−2.52	−2.46	−6.53	3.71
146	3431	13285	138.10	5.81	−12.71	0.46	−4.71	−6.10	−4.35
147	3444	13322	180.80	9.15	−10.67	−0.23	−4.36	−2.73	−2.20
148	3462	13383	96.90	11.90	−10.90	1.86	−2.76	0.04	−2.26
149	2777	12899	644.80	7.64	7.47	−5.03	11.60	−6.72	14.73
150	3552	13481	124.40	16.08	−8.25	8.08	−2.03	4.52	0.66
151	3553	13401	80.80	23.42	−10.90	13.21	−2.65	11.94	−2.21
152	3545	13336	4.40	19.69	−12.19	9.81	−7.52	8.23	−3.69
153	3537	13272	41.00	12.66	−14.82	6.40	−5.56	1.22	−6.50
154	3508	13233	89.00	17.21	−16.58	3.99	−6.08	5.68	−8.36
155	3467	13182	56.10	12.31	−11.82	2.17	−5.34	0.64	−3.75
156	3440	13140	5.20	15.07	−10.57	2.94	−4.94	3.31	−2.62
157	3821	13845	947.10	13.59	−6.98	5.32	0.71	2.85	2.91
158	3672	13825	744.40	14.49	−9.98	6.62	−9.59	3.12	−0.14
159	3567	13851	349.60	3.91	−4.48	−9.01	−1.19	−7.94	5.43
160	3045	13022	600.10	11.15	−6.39	−1.37	−1.85	−2.20	1.22
161	3483	13592	681.20	21.44	−14.20	11.07	−2.78	9.49	−4.99
162	3457	13496	475.80	9.82	−3.24	2.72	−2.32	−2.16	5.71
163	3416	13440	691.30	14.66	−3.62	7.37	−1.64	2.56	5.17
164	3076	13102	177.30	10.74	1.45	−3.81	14.06	−2.54	9.29
165	3125	13049	43.40	3.59	−4.93	−5.71	3.35	−9.45	2.76
166	3146	13073	123.20	5.86	−11.41	−3.43	4.52	−7.10	−3.65
167	3227	13098	200.10	6.91	−7.76	−2.81	2.32	−5.73	0.07
168	3537	14036	43.70	9.05	2.82	−2.01	17.24	−3.12	13.22
169	3559	13445	536.20	21.58	−10.44	13.01	−3.02	10.08	−1.63
170	3551	13298	341.80	15.10	−11.10	6.14	−4.69	3.70	−2.70
171	3465	13162	532.80	13.29	−12.74	2.02	−3.86	1.63	−4.73
172	3299	13085	199.90	6.75	−12.66	2.81	−3.25	−5.57	−4.87
173	3297	13149	210.40	10.27	−0.91	−3.79	6.48	−2.11	7.07
174	3323	13165	94.50	18.43	−7.34	2.22	1.33	6.15	0.68
175	3359	13116	55.30	14.07	−3.99	2.27	4.47	1.98	3.89
176	3379	13045	114.50	14.46	−10.59	3.31	−5.11	2.51	−2.91
177	3439	13095	149.40	15.34	−11.96	3.01	−3.75	3.62	−4.14
178	3423	13128	425.40	9.54	−8.47	−1.62	−1.28	−2.28	−0.55
179	2914	12921	289.00	8.87	−3.76	−2.23	6.41	−4.95	3.56

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\tilde{\xi}^a$	$\tilde{\eta}^a$
180	3822	14017	986.40	17.12	−12.93	3.45	−5.77	6.20	−2.59
181	3735	14071	1192.60	8.26	−6.21	−2.83	16.18	−3.10	4.27
182	3780	14080	672.10	14.08	−6.00	1.58	7.13	2.90	4.51
183	3901	14119	595.70	10.57	−15.07	−0.82	−0.06	−0.13	−4.46
184	3920	14141	870.60	10.15	−10.68	−1.94	6.17	−0.49	−0.02
185	3954	14198	731.40	13.54	12.39	0.71	28.15	2.98	23.20
186	3528	13961	207.90	15.92	1.23	0.66	11.16	3.79	11.43
187	4065	14031	1625.50	18.72	−8.90	8.31	2.54	8.86	1.48
188	4024	14095	1024.20	15.29	−12.82	6.18	1.15	5.17	−2.28
189	3985	14100	2040.50	15.45	−3.32	1.47	−4.93	5.15	7.24
190	4066	14088	1584.60	19.99	−12.38	3.68	10.77	10.06	−1.85
191	4039	14131	615.40	23.50	−12.46	12.74	1.30	13.40	−1.82
192	4040	14159	740.10	25.24	1.07	13.98	14.08	15.11	11.78
193	4116	14132	338.50	19.81	−6.21	8.51	10.08	10.05	4.43
194	4138	14144	400.00	22.13	8.29	12.64	24.16	12.45	18.96
195	4112	14040	669.60	13.35	−9.76	2.67	1.84	3.68	0.64
196	4185	14110	691.10	17.58	10.22	6.95	20.15	8.16	20.80
197	4318	14066	872.20	15.69	−5.00	5.46	3.09	6.93	5.47
198	4307	14120	1023.70	16.50	2.02	1.15	13.40	7.62	12.63
199	4331	14152	410.00	3.34	−8.78	−5.31	4.12	−5.48	1.91
200	4243	14101	911.00	4.35	−3.38	−2.24	15.73	−4.80	7.18
201	4484	14183	181.80	13.96	−12.83	4.37	−10.79	5.81	−2.06
202	4537	14102	490.00	11.42	−13.59	3.51	5.95	3.63	−3.03
203	4270	14196	252.40	−5.22	−3.35	−15.94	8.27	−14.37	7.45
204	4431	14165	49.90	13.45	−10.07	3.79	−0.40	5.08	0.66
205	4396	14184	730.50	10.70	−9.56	1.17	−4.78	2.14	1.21
206	4364	14209	795.60	12.56	−16.88	−0.37	−4.05	3.83	−6.05
207	4501	14240	761.00	17.35	−11.50	5.80	7.10	9.20	−0.59
208	4547	14190	172.10	11.01	−19.50	2.29	−2.59	3.14	−8.71
209	4485	14215	631.80	14.58	−19.91	6.11	−7.06	6.40	−9.06
210	4523	14160	96.10	12.95	−6.97	3.17	−0.10	5.01	3.74
211	4464	14275	525.20	17.56	−1.66	9.19	7.39	9.20	0.34
212	4339	14264	1912.20	8.48	−15.89	−6.97	−3.17	−0.44	−4.92
213	4297	14337	250.00	−1.24	−11.14	−14.63	0.36	−10.45	0.02
214	4416	14349	437.20	13.70	−8.01	6.00	5.11	5.01	3.18
215	4319	14527	75.40	−10.24	−2.42	−23.11	12.55	−19.61	9.21
216	4336	14506	50.60	9.62	11.75	−0.29	6.48	0.36	23.33
217	4294	14472	205.20	−20.62	−7.02	−38.76	10.10	−30.02	4.47
218	3320	13184	484.00	14.00	−6.57	−2.09	−1.01	1.70	1.51
219	3315	13054	350.50	15.32	−12.58	−2.18	−4.03	3.09	−4.88
220	3344	13051	404.40	10.00	−3.70	1.48	−4.02	−2.10	4.00
221	3230	13143	1405.20	5.42	4.04	−12.96	18.77	−7.24	12.00
222	3140	13016	591.10	11.73	−8.54	−2.21	−2.27	−1.22	−0.94
223	3284	13062	685.40	10.60	−9.79	−1.22	−2.79	−1.77	−2.06
224	3113	13077	897.30	12.37	−12.46	−2.10	7.83	−0.73	−4.69
225	3068	13097	161.40	12.39	−0.42	−3.70	10.56	−0.92	7.41
226	3212	13040	687.30	13.83	−12.01	−1.41	−0.87	1.17	−4.34
227	3240	13010	525.90	11.46	−9.29	−1.65	−1.56	−1.06	−1.71
228	3291	12975	560.80	12.13	−6.31	−0.14	−1.01	−0.14	1.17

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\bar{\xi}^a$	$\bar{\eta}^a$
229	3347	13093	1199.60	12.51	−10.57	0.69	0.98	0.39	−2.75
230	3386	13092	597.80	13.17	−8.74	3.12	−1.28	1.22	−0.93
231	3372	13167	266.60	11.21	−3.37	−0.57	−0.69	−0.86	4.66
232	3346	13031	967.20	18.14	−7.52	−3.43	0.59	6.06	0.12
233	3401	13126	250.20	12.45	−6.91	−0.49	0.82	0.54	1.00
234	3333	12947	514.30	16.26	−10.11	−2.13	−0.28	4.19	−2.71
235	3343	13126	758.10	13.59	−2.40	2.62	3.48	1.43	5.51
236	3391	13044	94.60	13.65	−10.02	7.74	−6.86	1.75	−2.34
237	3318	13089	1230.80	14.89	−4.67	2.22	0.21	2.65	3.14
238	3417	13332	169.80	12.99	−13.53	1.93	−6.35	0.98	−5.04
239	3343	13221	395.10	15.97	−14.58	−2.51	−7.89	3.73	−6.40
240	3421	13242	542.00	5.78	−11.33	−1.63	−6.47	−6.14	−3.09
241	3385	13214	526.70	6.60	−4.24	−0.61	−1.77	−5.45	3.92
242	4378	13238	1218.20	10.93	−8.05	−6.24	1.18	−0.74	0.18
243	3705	14053	674.90	10.96	−4.29	−8.59	14.53	−0.51	6.15
244	3771	14057	599.40	14.28	−9.87	3.67	0.07	3.09	0.58
245	3582	14028	42.40	−10.40	−0.96	−19.31	11.18	−22.37	9.42
246	3544	14021	173.20	7.18	1.78	2.37	12.08	−4.94	12.14
247	3751	14036	575.60	11.53	−5.62	0.90	−0.24	0.28	4.77
248	3645	14029	255.40	3.22	1.47	−9.36	14.06	−8.48	11.85
249	3566	14017	32.30	−7.86	−1.12	−6.26	9.12	−19.88	9.23
250	3275	13300	450.80	8.07	−8.10	−5.85	−3.82	−4.52	0.30
251	3394	13285	986.00	17.03	−24.17	1.67	−2.68	4.96	−15.81
252	3474	13394	499.50	9.36	−10.44	−0.80	−2.47	−2.46	−1.77
253	4279	14140	865.80	1.80	8.49	−10.36	22.44	−7.23	19.15
254	4172	14096	290.80	8.76	−3.84	4.58	9.61	−0.71	6.71
255	4104	14125	24.00	19.28	−10.30	8.16	8.41	9.47	0.32
256	4010	14128	509.20	21.07	−14.34	6.58	−3.81	10.84	−3.71
257	4308	14135	15.20	14.89	2.45	3.21	9.37	5.99	13.10
258	4232	14101	187.70	9.27	−0.75	−1.01	18.87	0.07	9.81
259	4064	14127	76.40	18.95	−4.90	11.21	4.59	8.96	5.73
260	3953	14109	552.30	12.44	−8.95	−0.99	−6.01	1.99	1.63
261	3614	13974	13.40	−2.08	−10.73	−13.59	−2.24	−13.85	−0.50
262	3599	14003	21.60	−9.34	−7.51	−23.25	8.32	−21.21	2.80
263	3837	14033	90.70	13.23	−17.11	8.20	−5.58	2.35	−6.72
264	3800	14012	215.20	13.97	−14.14	8.30	−5.99	2.95	−3.81
265	3753	13986	188.50	16.69	−13.86	6.80	−6.41	5.50	−3.60
266	3736	14036	258.10	12.19	−10.50	1.64	−0.18	0.87	−0.10
267	3777	14044	89.20	13.17	−6.69	2.58	1.74	2.02	3.37
268	3800	14062	76.00	13.46	−4.39	2.35	4.63	2.39	6.07
269	3561	13479	566.60	16.54	−7.24	8.54	−3.21	5.02	1.66
270	3533	13515	536.50	9.90	−8.95	2.07	−0.49	−1.77	0.05
271	3423	13482	608.30	10.33	−10.11	−3.68	−0.33	−1.78	−1.20
272	3490	13832	501.40	0.09	−1.80	−7.83	8.80	−12.06	8.06
273	3468	13742	62.30	5.46	−12.05	−8.48	−1.52	−6.70	−2.43
274	3447	13554	165.30	22.92	−18.91	9.05	−6.79	10.85	−9.80
275	3485	13788	259.90	−3.04	−11.00	−14.80	0.13	−15.17	−1.26
276	3495	13884	981.90	17.34	−14.97	−0.23	7.46	5.16	−4.97
277	3469	13628	756.40	19.00	−3.61	9.36	6.61	6.95	5.70

Table 1. (Continued)

No.	ϕ	λ	H	ξ^a	η^a	ξ^g	η^g	$\bar{\xi}^a$	$\bar{\eta}^a$
278	3833	14079	145.20	11.18	−7.47	−0.30	2.51	0.23	3.04
279	3883	14101	24.20	8.50	−9.21	−2.66	−0.09	−2.26	1.35
280	3913	14049	98.90	17.94	−14.83	4.37	−4.80	7.38	−4.40
281	3941	14049	28.20	14.48	−14.49	4.46	−6.30	4.04	−4.06
282	3865	14086	108.10	9.47	−6.50	2.94	2.79	−1.35	4.03
283	3844	14089	24.20	14.41	−7.94	1.26	2.86	3.50	2.59
284	3819	14063	228.80	10.65	−3.36	0.11	4.93	−0.34	7.11
285	3142	13088	102.70	7.85	1.84	−3.61	9.26	−5.14	9.64
286	3167	13104	174.90	10.37	2.17	−2.36	10.53	−2.52	10.02
287	3201	13137	123.30	5.43	13.32	−5.20	17.69	−7.35	21.27
288	3242	13167	191.60	2.77	3.24	−10.33	8.56	−9.86	11.27
289	3270	13176	645.40	1.66	−1.67	−8.86	4.48	−10.85	6.38
290	3040	13097	46.00	6.16	8.57	−6.91	15.06	−7.26	16.40
291	3060	13105	173.50	6.20	9.14	−7.79	14.88	−7.14	16.99
292	3647	13888	1390.70	9.59	−1.14	−0.93	6.21	−1.95	8.87
293	3628	13838	827.00	14.58	−12.46	9.57	−3.68	3.01	−2.59
294	4027	14079	170.40	13.50	−13.82	7.55	−4.55	3.14	−3.32
295	3961	14060	72.60	11.98	−16.98	1.22	−10.30	1.61	−6.52
296	3986	14117	207.60	13.13	−17.09	1.47	−7.30	2.81	−6.49
297	3946	14116	81.90	13.18	−17.33	−0.13	−5.63	2.68	−6.73
298	3632	13925	66.60	3.51	−8.71	0.21	−0.76	−8.14	1.40
299	3563	13947	161.70	3.42	8.24	−5.01	18.47	−8.54	18.40
300	3587	13939	73.70	0.57	6.90	−9.82	13.16	−11.28	17.04
301	3916	14115	50.60	12.01	−17.20	−0.17	−3.57	1.38	−6.60
302	3969	14097	207.50	10.59	−11.02	0.69	0.10	0.22	−0.47
303	3893	14108	135.50	10.33	−11.03	−2.56	−0.85	−0.39	−0.45
304	4021	14111	371.40	16.99	−7.24	7.23	0.67	6.83	3.35
305	3851	14033	136.30	12.92	−9.00	2.15	−0.73	2.10	1.39
306	3860	14046	146.60	13.54	−17.53	3.74	−5.56	2.75	−7.11
307	3923	14108	117.20	10.53	−11.54	1.22	−3.23	−0.06	−0.96
308	3998	14114	324.50	14.98	−14.22	1.80	−4.38	4.71	−3.62
309	3875	14080	349.30	9.47	−7.01	−1.23	3.98	−1.29	3.50
310	3861	14071	191.10	11.65	−6.34	4.16	4.34	0.83	4.15
311	3480	13690	81.60	14.26	−15.04	3.20	−7.71	2.20	−5.56
312	3549	13688	167.30	6.25	−18.07	−4.18	−9.66	−5.51	−8.60
313	3787	14013	337.40	21.59	−13.12	7.67	−4.02	10.52	−2.78
314	3760	14043	345.70	11.14	−7.29	1.03	0.05	−0.08	3.12
315	3597	13822	1011.10	14.84	−17.05	8.38	−7.82	3.15	−7.22
316	3634	13859	933.80	11.26	−9.74	−0.54	7.14	−0.31	0.19
317	3722	14035	312.30	13.42	−12.04	1.17	1.33	2.04	−1.65
318	3712	14028	345.70	10.81	−11.50	−2.11	2.16	−0.60	−1.12
319	3705	14009	394.50	6.27	−8.00	−4.78	2.15	−5.15	2.32
320	3679	13994	209.40	7.63	−4.81	−3.29	2.89	−3.89	5.48
321	3633	13987	46.70	2.84	−8.13	−8.23	−0.14	−8.87	2.14
322	3630	13960	25.00	0.85	−9.96	−10.91	1.05	−10.84	0.24
323	3502	13611	405.10	21.74	−9.81	9.18	−1.75	9.85	−0.55
324	3514	13663	403.30	10.93	−0.98	1.40	−0.30	−0.96	8.43

These are the deflection components measured from the ellipsoidal normal at the Tokyo datum station. The correction values for the geodetic latitude and longitude are $\Delta\phi^g=+11''.98(=-\xi_0)$ and $\Delta\lambda^g=-12''.60(=-\eta_0/\cos\phi_0)$, respectively, where ϕ_0 denotes the latitude at the Tokyo geodetic datum station. As compared with the world geodetic system, the geodetic positions of all the triangulation points in Japan are shifted by these values $\Delta\phi^g$ and $\Delta\lambda^g$, because the geodetic latitudes and longitudes in Japan are based on the astronomical ones of the Tokyo geodetic datum station.

6. Discussion and Conclusions

The correction values for the geodetic system of Japan obtained by some geodesists are summarized in Table 2. ATUMI (1933), KAWABATA (1935) and TORAO (1949) used the transformation formula derived by HELMERT (1880). They estimated the deflections of the vertical at the Tokyo datum statum station referring to the normal ellipsoid by minimizing the sum of squares of transformed astrogeodetic deflections. Such a method, however, has no physical meaning, because the improved reference ellipsoid does not always fit the geoid. On the other hand, HAGIWARA (1967) and ONO (1974) used the transformation formula derived by VENING-MEINESZ (1950). They minimized the squared sum of differences between the astrogeodetic and geodetic deflections. This method has physical meaning and is thought to be better than the method by previous researchers. The method used here is essentially similar to this method, determining only ΔX, ΔY and ΔZ, the displacements between the centers of the Japan reference ellipsoid and of the normal ellipsoid bestfitted to the geoid.

In the present study, by numerically differentiating Ganeko's geoidal height (Fig. 1) as the model of the world geodetic system, the author obtained the gravimetric deflections and compared them with the astrogeodetic ones based on the geodetic reference system of Japan. The result shows that latitudes and longitudes determined by the geodetic reference system of Japan shifts towards the southeast, relating

Table 2. Astrogeodetic deflection of the vertical at the Tokyo datum station.

Authors		ξ_0	η_0	ΔX	ΔY	ΔZ	Data
ATUMI	(1933)	$-9''.5$	$+9''.4$				68 deflections
KAWABATA	(1935)	$-9''.4$	$+6''.7$				61 deflections
TORAO	(1949)	$-9''.8$	$+7''.3$				166 deflections
HAGIWARA	(1967)	$-11''.1$	$+7''.8$				125 deflections
ONO	(1974)	$-12''.4$	$+9''.6$				192 deflections
KOZAI	(1980)	$-11''.8$	$+8''.8$	-153.96 m	$+493.42$ m	$+691.31$ m	laser ranging
AUTHOR	(1983)	$-12''.0$	$+10''.2$	-149.57 m	$+541.82$ m	$+706.07$ m	324 deflections

to the ones determined by the world geodetic system. The corresponding shift is given by $\Delta X = -149.57$ m, $\Delta Y = +541.82$ m and $\Delta Z = +706.07$ m. Converting these values into the astrogeodetic deflection on the vertical at the Tokyo geodetic datum station, the meridian and primevertical components amount to $\xi_0 = -11''.98$ and $\eta_0 = +10''.24$, respectively. These values are well consistent with the results previously obtained by many investigators as shown in Table 2. Very recently KOZAI (1980) obtained results by using satellite laser ranging data observed at Dodaira Station of the Tokyo Astronomical Observatory. The consistency also matches well with the author's results.

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Appendix

A geodetic reference system is determined by the dimension of a reference ellipsoid (semiaxes a and b) and its position with respect to the earth itself. This relative position can be given by the displacement of the ellipsoidal center from the geocenter and by the inclination of the ellipsoidal axes to the earth-fixed coordinates. However, if we assume that the rotation axis of the earth is consistent with that of the reference ellipsoid, it is enough for a coordinate transformation to consider only the displacement of the ellipsoidal center.

We can express the center of the reference ellipsoid as the displacement vector $\Delta \boldsymbol{r}^*$ in the form

$$\begin{aligned} \Delta \boldsymbol{r}^* &= \boldsymbol{r}^* - \boldsymbol{r} \\ &= \boldsymbol{r}^*(\phi^*, \lambda^*, H^*, a^*, b^*) - \boldsymbol{r}(\phi, \lambda, H, a, b) \\ &= \boldsymbol{r}(\phi + \Delta\phi, \lambda + \Delta\lambda, H + \Delta H, a + \Delta a, b + \Delta b) - \boldsymbol{r}(\phi, \lambda, H, a, b) \\ &\simeq \frac{\partial \boldsymbol{r}}{\partial \phi}\Delta\phi + \frac{\partial \boldsymbol{r}}{\partial \lambda}\Delta\lambda + \frac{\partial \boldsymbol{r}}{\partial H}\Delta H + \frac{\partial \boldsymbol{r}}{\partial a}\Delta a + \frac{\partial \boldsymbol{r}}{\partial b}\Delta b\ . \end{aligned} \tag{A-1}$$

In this equation, the quantities $\boldsymbol{r}$, ϕ, λ, H, a and b denote the vector of a point in the geodetic reference system, the geodetic latitude, the geodetic longitude, the geodetic height, and the semiaxes of the reference ellipsoid, respectively. The asterisked quantities r^*, ϕ^*, λ^*, H^*, a^* and b^* denote the vector of a point in the geocentric system, the astronomical latitude, the astronomical longitude, the astronomical height, and the semiaxes of the normal ellipsoid, respectively. In this appendix, we denote quantities regarded to the geocentric system with the notation "*".

Then the astrogeodetic deflection components ξ and η and the geoidal height ζ are written as follows:

$$\left.\begin{aligned} \xi &= \phi^* - \phi\,, \\ \eta &= (\lambda^* - \lambda)\cos\phi\,, \\ \zeta &= H - H^*\,. \end{aligned}\right\}$$

Since the astronomical quantities ϕ^*, λ^* and H^* are not affected by a datum shift, we can define as

$$\left.\begin{aligned} \Delta\xi &= -\Delta\phi\,, \\ \Delta\eta &= -\Delta\lambda\cos\phi\,, \\ \Delta\zeta &= \Delta H\,. \end{aligned}\right\} \qquad \text{(A-2)}$$

Substituting (A-2) into (A-1), we get

$$\Delta r^* = -\frac{\partial r}{\partial\phi}\Delta\xi - \frac{\partial r}{\partial\lambda}\frac{\Delta\eta}{\cos\phi} + \frac{\partial r}{\partial H}\Delta\zeta + \frac{\partial r}{\partial a}\Delta a + \frac{\partial r}{\partial b}\Delta b\,. \qquad \text{(A-3)}$$

On the contrary, the relation between the curvilinear coordinate (ϕ, λ, H) and Cartesian coordinate (X, Y, Z) are written as

$$\left.\begin{aligned} X &= (N+H)\cos\phi\cos\lambda\,, \\ Y &= (N+H)\cos\phi\sin\lambda\,, \\ Z &= \left(\frac{b^2}{a^2}N + H\right)\sin\phi\,, \end{aligned}\right\} \qquad \text{(A-4)}$$

where N is the radius of curvature in the prime-vertical:

$$N = \frac{a^2}{(a^2\cos^2\phi + b^2\sin^2\phi)^{1/2}}\,.$$

We differentiate (A-4) with respect to ϕ, λ, H, a and b, and substitute the results into the X^*, Y^*, and Z^* components of (A-3). Consequently, we get the following equation system:

$$\begin{aligned} &\Delta\xi(M+H)\sin\phi\cos\lambda + \Delta\eta(N+H)\sin\lambda + \Delta\zeta\cos\phi\cos\lambda \\ &\quad = \Delta X^* - \frac{\Delta a}{a}(N + M\sin^2\phi)\cos\phi\cos\lambda \end{aligned}$$

$$
\left.\begin{aligned}
&+\frac{\Delta b}{b}M\sin^2\phi\cos\phi\cos\lambda\,,\\
&\Delta\xi(M+H)\sin\phi\sin\lambda-\Delta\eta(N+H)\cos\lambda+\Delta\zeta\cos\phi\sin\lambda\\
&\quad=\Delta Y^*-\frac{\Delta a}{a}(N+M\sin^2\phi)\cos\phi\sin\lambda\\
&\quad+\frac{\Delta b}{b}M\sin^2\phi\cos\phi\sin\lambda\,,\\
&\Delta\xi(M+H)\cos\phi-\Delta\zeta\sin\phi\\
&\quad=-\Delta Z^*-\frac{\Delta a}{a}M\sin\phi\cos^2\phi\\
&\quad+\frac{\Delta b}{b}\left(\frac{b^2}{a^2}N+M\cos^2\phi\right)\sin\phi\,,
\end{aligned}\right\}\qquad\text{(A-5)}
$$

where M is the radius of curvature in the meridian:

$$
M=\frac{a^2b^2}{(a^2\cos^2\phi+b^2\sin^2\phi)^{3/2}}\,.
$$

Solving the equation system (A-5) with regard to $\Delta\xi$, $\Delta\eta$ and $\Delta\zeta$, we finally obtain

$$
\left.\begin{aligned}
\Delta\xi&=\frac{\sin\phi\cos\lambda}{M+H}\Delta X^*+\frac{\sin\phi\sin\lambda}{M+H}\Delta Y^*-\frac{\cos\phi}{M+H}\Delta Z^*\\
&\quad-\frac{\sin\phi\cos\phi}{M+H}(N+M)\frac{\Delta a}{a}+\frac{\sin\phi\cos\phi}{M+H}\left(\frac{b^2}{a^2}N+M\right)\frac{\Delta b}{b}\,,\\
\Delta\eta&=\frac{\sin\lambda}{N+H}\Delta X^*-\frac{\cos\lambda}{N+H}\Delta Y^*\,,\\
\Delta\zeta&=-\cos\phi\cos\lambda\Delta X^*-\cos\phi\sin\lambda\Delta Y^*-\sin\phi\Delta Z^*\\
&\quad+\frac{\Delta a}{a}N\cos^2\phi+\frac{\Delta b}{b}\frac{b^2}{a^2}\sin^2\phi\,.
\end{aligned}\right\}
$$

These formulas describe the effect of a shift of a geodetic datum station. The first and second formulas correspond to (7).

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4. 水路部ジオイドに基づく日本測地基準系の変換

地震研究所　矢田部　学

日本ではベッセル楕円体を準拠楕円体として採用し，それを東京測地原点で垂直線偏差0の仮定のもとに設置している．この日本測地基準系は，よく知られているように，地球重心系に対して位置的にずれている．このため，日本測地基準系に基づく位置と地球重心系に基づく位置は一致しない．これでは，グロバールな測地学の問題を扱うために不便である．ベッセル楕円体が実際の地球の重力場に適合するように日本測地基準系を変換することが望ましい．

本論文では，水路部により得られた最新のジオイドのデータを地球重力場のモデルとして採用し，それに適合するように日本測地基準系を変換することを行なった．その結果はベッセル楕円体の中心は地球の重心に対し，$\Delta X=-149.57$ m，$\Delta Y=+541.82$ m，$\Delta Z=706.07$ m ずれていることを示している．これは日本測地基準系での位置は地球重心系での位置に対して南東に約 900 m ずれていることに相当する．そして，東京測地原点で垂直線偏差 $\xi_0=-11''.98$，$\eta_0=+10''.24$ を与えれば，ベッセル楕円体を地球重心系によく適合できるという結果になった．これは，いままでの多くの研究者が，さまざまな方法で得た結果によく一致する．