BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 56 (1981), pp. 415-440

20. *Tsunami Energy in Relation to Parameters of the Earthquake Fault Model.*

By Kinjiro KAJIURA,

Earthquake Research Institute.

(Received July 30, 1981)

Abstract

Tsunami energy generated by an earthquake is estimated on the basis of a simple fault origin model of the earthquake. Tsunami energy E_t is given by

$$\log E_t(\text{ergs}) = 2M_w + \log F + 5.5$$

where M_w is the moment-magnitude of earthquake and F is a function of fault parameters (maximum F is about 0.1), such as the dip angle δ, slip angle λ and the relative depth $h^*(=H^*/L$; where H^* is the mean depth of the fault plane with the length L and width W). The aspect ratio $(=W/L)$ is assumed to be 1/2.

The variation of F with respect to the full range of δ, λ, or h^* (<1.0) is about a factor of 10. In particular, the difference of tsunami energy between the vertical faults with the dip and strike slips is conspicuous. Since the depth dependence of the tsunami energy is given in terms of the relative depth h^*, the decrease of energy with the increase of the fault depth H^* is more significant for smaller earthquakes.

The results are compared with empirical values of tsunami energy published so far. The general trend of $\log E_t$ with respect to M_w is consistent with the above formula. However, it is noted that the values of tsunami energy derived in the past on the basis of the energy flux method were systematically overestimated by a factor of 10 or more. On the other hand, the maximum tsunami energy (Chilean earthquake of 1960) would be around 10^{23} ergs and somewhat lower than the value expected from the formula.

1. Introduction

It is known that major tsunamis are generated at the time of large shallow earthquakes occurring under the sea. More than 20 years ago IIDA (1958) showed a relationship between the physical sizes of tsunamis and earthquakes by correlating the so-called Imamura-Iida magnitude m of tsunamis with the earthquake magnitude M (JMA magnitude M_j determined by the Japan Meteorological Agency was used) of the corresponding earthquakes, though the correlation was not so good. To appreciate this kind of empirical relation between the physical sizes of tsunamis and earthquakes, we need to understand several different problems: 1) What is the most appropriate measure to represent the overall size of a tsunami? 2) Is the Gutenberg-Richter surface wave magnitude M_s adequate as a measure of the "size" of a very large earthquake? 3) What is the cause of the essential scatter in the relation of the tsunami magnitude (a single parameter representation of the "size" of a tsunami, whatever the definition may be) to the earthquake magnitude (or the total seismic moments M_0)?

The first problem has been discussed by many researchers, each of whom criticized the deficiencies of the Imamura-Iida scale and proposed a new definition of the tsunami magnitude scale (WATANABE, 1964; IIDA *et al.*, 1967; SOLOVIEV, 1970; HATORI, 1973; ADAMS, 1974; ABE, 1979; MURTY and LOOMIS, 1980). After all, it seems to be the consensus from the beginning of the use of a tsunami magnitude scale by IMAMURA (1942, 1949) that the scale should in someway be related to the total physical size (total energy) of the tsunami and not merely an index to represent the intensity of a tsunami at a specific location. MURTY and LOOMIS (1980) proposed to define the magnitude scale directly on the basis of the total tsunami energy. However, to determine the tsunami energy from observed wave data is not an easy task in reality and only a few reliable estimates are available so far. HATORI (1970) summarized the data published before 1970. The second problem has been argued the past 10 years or so on the basis of physical models of the earthquake source and a new definition of the earthquake magnitude scale M_w, namely a moment-magnitude scale based on the total earthquake moment M_0 was proposed by KANAMORI (1977) to overcome the saturation problem of the seismic wave amplitude for great earthquakes ($M_s \geqq 8$) and, at the same time, to guarantee the smooth continuation to the surface wave magnitude M_s for earthquakes of moderate sizes. This continuation was examined later by PURCARU and BERCKHEMER (1978) and SINGH and HAVSKOV (1980). As for the third problem, it has been empirically known that 1) the tsunami generation is influenced by the depth of the earthquake hypocenter: deep-focus earthquakes with the focal depth larger than, say, 60 km hardly ever generate a very large tsunami (IIDA, 1958, 1963a), and 2) tsunamis seem to be generated by earthquakes having predominantly a large dip-slip component of the fault dislocation and earthquakes of the strike slip type ordinarily result in only minor tsunamis (IIDA, 1970; WATANABE, 1970).

With advances in the physical understanding of the earthquake mechanism (see, for example, AKI, 1972; KANAMORI and ANDERSON, 1975) these problems can be discussed from a unified point of view. The generation of tsunamis has been discussed directly on the basis of a simple source model of earthquakes in the solid-liquid coupled system (PODYAPOLSKY, 1970; YAMASHITA and SATO, 1974; ALEXEEV and GUSIAKOV, 1974; WARD, 1980). This is considered to be the most straightforward approach to the tsunami generation problem. However, to understand the gross characteristics of tsunamis, such as the total energy, in relation to an assumed fault model of large earthquakes, it may be advantageous to use rough estimate method. Since the major part of the permanent crustal deformation at the time of an ordinary large earthquake is believed to be completed in a relatively short time interval (within a minute or so) compared with the generated tsunami "period", the total tsunami energy may be roughly estimated from the initial potential energy of the water surface disturbance computed on the basis of the instantaneous deformation of the ocean bottom, which is equivalent to the static deformation caused by the earthquake fault.

If this assumption is accepted, the relation of the tsunami energy to various earthquake parameters can be found by analyzing the dependence of the static deformation of the sea bottom on the fault parameters of an earthquake. It is significant that the recent advance of numerical methods to solve the shallow water hydrodynamical equations enable us to reproduce the coastal height distribution of the tsunami generated by a given large scale bottom deformation with considerable accuracy and, in fact, AIDA (1977, 1979a) confirmed that, in many cases of large tsunamis, the distribution of tsunami heights observed along the coast can be explained to a reasonable degree of accuracy by a suitably assumed earthquake fault model consistent with seismological data. Thus, in the present paper, we focus attention on the dependence of tsunami energy on various parameters of an earthquake fault. At the same time, the reliability of various estimates of tsunami energy published so far are critically reviewed and compared with the estimate based on the fault model.

Up to the present, the operational tsunami warning practice depends heavily on the empirical relation analogous to Iida's relation because the information obtained immediately after the occurrence of a large earthquake is limited to something like the Gutenberg-Richter earthquake magnitude M and the location of the epicenter. For tsunamis propagating from a distant source region, it is possible to supplement waterwave data at various places later to improve the prediction, but for tsunamis generated in the vicinity of the coastal area of interest, or where no waterwave data is available, it is advantageous to use the earthquake moment M_0 and possibly the kinematic parameters of the fault model, if they can be estimated quickly after the earthquake occurs. It appears that the use of these parameters in operational tsunami warning is possible (KANAMORI, personal communication, 1981). In this respect, the present study may be regarded as a kind of sensitivity study of various fault parameters influencing the total tsunami energy. Of course, there are other dynamical problems in connecting the total tsunami energy to the wave heights realized along the coast. However, these problems are outside the scope of the present study.

2. Relation between tsunami energy and earthquake magnitude

It is instructive to first review some relations concerning the earthquake fault model. Take a simple fault plane of a rectangular shape as shown in Fig. 1, where L is the length, W the width, S the area $(=LW)$, and D the mean dislocation on the fault. The dip angle of the fault plane is δ, and λ is the slip angle of the dislocation on the fault plane measured from the horizontal axis. Then, the dislocation theory in elasticity applied to the earthquake fault model predicts the following approximate relations for physical quantities such as the total seismic moment M_0, the mean stress drop $\Delta\sigma$, and the change of total strain energy W (see AKI, 1972):

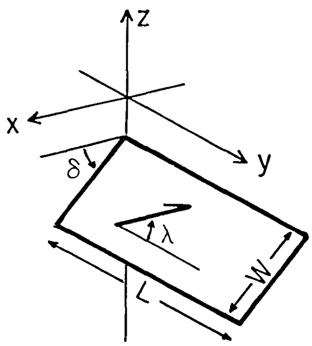

Fig. 1. Geometry of a fault model: (x, y) are rectangular coordinates on the free surface of a semi-infinite solid with the z-axis vertically upward.

$$M_0 = \mu DS \tag{1}$$

$$\Delta\sigma = C\mu DS^{-1/2} \tag{2}$$

and

$$W = \bar{\sigma} DS \tag{3}$$

where μ is the rigidity with the value between $3\sim 7\times 10^{11}$ dyne cm^{-2} depending on the crustal condition in the earth, and C (2.4$\sim$5) is the shape factor depending on the fault geometry and slip direction. For a buried dip-slip faulting on a rectangular fault, $C=(16/3\pi)\sqrt{L/W}$. $\bar{\sigma}$ is the average stress given by $\bar{\sigma}=(\sigma_1+\sigma_0)/2$ with σ_0 and σ_1 the average stresses on the fault plane immediately before and after the earthquake occurrence and the stress drop $\Delta\sigma$ is equal to $(\sigma_0-\sigma_1)$. The total seismic energy E_s may be formally related to the change of strain energy W due to dislocation in the form $E=\eta W$ where η is the efficiency related to σ_f. From (1) and (3), the minimum estimate of strain energy release is $W_{\min}=(\Delta\sigma/2\mu)M_0$ and it seems reasonable to assume $E_s\sim W_{\min}$.

One basic empirical relation in the simple fault model of shallow earthquakes is that the fault area S and the earthquake moment M_0 are closely related (ABE, 1975; KANAMORI and ANDERSON, 1975); namely,

$$M_0 = \alpha S^{3/2} \tag{4}$$

The constant of proportionality α appears to vary by a factor of 3 depending on the tectonic setting of the region in which the earthquake takes place: $\alpha\sim 1.23\times 10^7$ dyne cm^{-2} for interplate earthquakes and about 3×10^7 dyne cm^{-2} for intraplate earthquakes (S and M_0 are expressed in *c. g. s.* units). Substitution of (4) into (1) yields $D/S^{1/2}=\alpha/\mu$ and (2) becomes $\Delta\sigma = C\alpha$. Roughly speaking, the length to width ratio of the fault plane L/W is about 2 (ABE, 1975; GELLER, 1976), so that the former relation suggests the geometrical similarity of the earthquake fault, and the latter relation indicates that the difference of α is the reflection of the variation of the stress drop $\Delta\sigma$. It is very often said that the stress drop $\Delta\sigma$ is about 30 bars for earthquakes on interplate boundaries. ABE (1975) actually estimated α from $\Delta\sigma/C$ with $\Delta\sigma=30$ bars and $C=2.44$.

Now extending the earthquake energy-magnitude relationship

$$\log E_s = 1.5\, M_s + 11.8 \tag{5}$$

to the range where the surface wave magnitude M_s suffers a saturation problem, a new magnitude scale M_w may be defined by the substitution of the minimum strain energy release $W_{\min}$ for E_s (KANAMORI, 1977):

$$\log E_s = \log M_0 + \log (\Delta\sigma/2\mu) = 1.5\, M_w + 11.8 \tag{6}$$

Kanamori took a constant strain drop $\Delta\sigma/\mu \sim 10^{-4}$ for major earthquakes occurring along interplate boundaries and gave

$$M_w = (\log M_0 - 16.1)/1.5 \tag{7}$$

The usefulness of this magnitude scale for earthquakes of moderate sizes was examined by PURCARU and BERCKHEMER (1978) and SINGH and HAVSKOV (1980). Since the strain drop $\Delta\sigma/\mu$ is fixed in (7), this magnitude scale is a function of the total moment M_0 only and is called the moment-magnitude.

In the fault model of earthquakes there is one important dynamic parameter called the rise time τ_s which is related to the movement of the fault dislocation. The mean velocity of dislocation D/τ_s is proportional to the initial effective stress σ_e and if σ_e is approximated by $\Delta\sigma$, the rise time τ_s is given by $\tau_s = \mu D/(\beta\Delta\sigma)$ (GELLER, 1976) where β is the shear wave velocity. Substituting relevant numerical values, the velocity of dislocation is found to be around 1 m sec^{-1}. Invoking the geometrical similarity of the fault and the constancy of $\Delta\sigma$, it follows that

$$\tau_s\beta/S^{1/2} = 1/C < 1 \tag{8}$$

On the other hand, the time scale corresponding to the rupture propagation with the rupture velocity $v\,(v \sim 0.7\beta)$ over the whole length L of a fault is $L/v \sim 2S^{1/2}/\beta$ and about 5 to 10 times greater than the rise time τ_s. Therefore, we assume the representative time scale related to the permanent deformation of the crust to be $\tau^* = S^{1/2}/\beta$.

Now the representative time scale τ of the tsunami generated by the crustal deformation of the ocean bottom may be expressed by $\tau = S^{1/2}/c$ where $c(=(gd)^{1/2}$: d the depth of water) is the velocity of a shallow water gravity wave. Thus, the ratio $\tau^*/\tau (= c/\beta)$ plays an important role in the efficiency of tsunami generation due to the bottom deformation. If $\tau^*/\tau \ll 1$, it is known that the generated tsunami energy E_t is almost independent of τ^* and the same as the energy E_{t0} in the case of the instantaneous deformation of the bottom (for a model of uniform uplift, E_t/E_{t0} is about 0.9 for $\tau^*/\tau \sim 0.2$: KAJIURA, 1970). In realistic situations, $c/\beta \leqq 1/20$ and the condition of $\tau^*/\tau \ll 1$ always holds. If $\tau^*/\tau > 1$, the ratio E_t/E_{t0} is generally proportional to $(\tau^*/\tau)^{-n}$ (n is unity for the case of one-dimensional propagation and n is somewhat larger for the case of two-dimensional propagation). It is known that in the case of an instantaneous deformation of a very large portion of the sea bottom ($S^{1/2}/d \gg 1$), the initial disturbance at the sea surface is approximately equal to the bottom displacement at least where the gravity wave part is concerned. Of course, smaller scale components of the bottom displacement are attenuated by the hydrodynamic effect in water, so that the surface disturbance is a smoothed version of the displacement with a filter function of 1/cosh (kd) where k is a wave number. That is, the bottom deformation on a scale less than about $2d$ is very strongly attenuated in the water surface disturbance. The oscillatory time dependence of the bottom displacement does not contribute significantly to the final gravity wave generation, if the angular frequency of oscillation ω is in the range of $\omega^2 d/g > 1$.

Thus, we may obtain a rough estimate of the total tsunami energy E_t from the initial potential energy corresponding to a static mound of water generated at the sea surface by the sudden displacement z of the sea bottom:

$$E_t = \frac{1}{2}\rho g \int_{S'} z^2 dS \tag{9}$$

where ρ is the density of water and S' is the surface area in which the vertical static displacement z takes place at the time of the earthquake occurrence. Now, scaling the bottom displacement in terms of the dimensions of a plane fault, we get

$$z_* = z/D, \quad dS_* = dS/S, \quad h = H/L, \quad r = W/L$$

where H is the depth of the upper rim (parallel to the free surface) of a fault plane, and (9) can be formally written as

$$E_t = \frac{1}{2}\rho g D^2 S F(\delta, \lambda, h, r) \tag{10}$$

where

$$F(\delta, \lambda, h, r) = \int_{S'/S} z_*(\delta, \lambda, h, r)^2 dS_* \tag{11}$$

Note that $F(\delta, \lambda, h, r)$ is a non-dimensional function apparently independent of the absolute values of D and S (namely independent of M_w).

Apart from the factor F, (10) shows that E_t is proportional to $D^2 S$ and, taking the relations (1) and (4), we have

$$E_t = \frac{1}{2}\rho g(\alpha^{2/3}/\mu^2) M_0^{4/3} F(\delta, \lambda, h, r) \tag{12}$$

The proportionality of E_t to $M_0^{4/3}$ was mentioned by AIDA (1977) but WARD (1980) suggested M_0^2 dependency by assuming a moment tensor point source.

Now, adopting the relation (7) and using the value $\alpha = 1.23 \times 10^7$ dyne

cm^{-2}, $\mu=5\times10^{11}$ dyne cm^{-2}, and $\rho g=10^3$ *c. g. s.* units it follows that

$$\log E_t=2M_w+5.50+\log F \tag{13}$$

Because of the range of variation of α and μ, the value of the constant term in (13) is somewhat uncertain and the value of E_t may vary by a factor of about 2. Since the maximum value of F is about 0.11 as will be introduced later, the gross upper limit of the total tsunami energy may be formally shown as

$$\log E_t=2M_w+4.54 \tag{14}$$

Comparing (14) with (6), we have

$$\log (E_t/E_s)=0.5\,M_w-7.26 \tag{15}$$

the ratio E_t/E_s increases with the increase of earthquake magnitude, but even for maximum earthquake ($M_w\sim9.5$), the ratio is less than 10^{-2}. The relation suggested by (13) will be checked later by the examination of an empirical relationship between the estimated tsunami energy by various methods and the moment-magnitude M_w of the corresponding earthquake.

The difference between the tsunami energy and seismic energy in their dependence to the earthquake moment M_0 or magnitude M_w may be interpreted as the effect of the rise time τ_s of the fault dislocation. For seismic wave energy the rise time τ_s plays an essential role (HASKELL, 1964) but for tsunami energy, the time scale of tsunami generation is much larger than the time scale of the crustal deformation ($\tau^*/\tau\ll1$) so that the total energy of gravity waves is almost independent of the time scale τ^* related to the permanent deformation of the crust (a kind of saturation problem).

3. Dependence of tsunami energy on earthquake fault parameters

For a given earthquake magnitude, F defined by (11) shows the dependence of tsunami energy on geometric parameters of the fault. To find the factor F, we assume a uniform rectangular plane fault with given parameters δ, λ, h and r in a semi-infinite elastic solid with equal Lamé constants. Only cases of $r=1/2$ are considered, though the aspect ratio varies between $2\geqq r\geqq1/4$ (ABE, 1975; GELLER, 1976). The static displacement z_* of the free surface of a solid space is computed within an area $2L\times2L$ by the analytical formula given by MANSHINHA and SMYLIE (1971). And z^2_* is integrated in a limited region delineated by $|z_*/z_{*\max}|\geqq0.1$ with $z_{*\max}$ representing the absolute value of the maximum computed displacement. Actual computations are carried out for $0° \leqq \delta$, $\lambda \leqq 90°$, and $0.04 \leqq h \leqq 0.44$ with the intervals 15°, 30°, and 0.1 respectively. In addition, the computation for $h^* = 1.0$ is also made. It should be noticed that the relative depth h of the upper rim of the fault is related to the relative mean depth h^* of the fault by

$$h^* = h + (1/4) \sin \delta \tag{16}$$

so that the apparent conclusion as to the dependence of F on δ and λ is affected by the choice of the representative depth.

Figure 2 shows the dependence of F on δ and λ for $h = 0.04$ which is the case when the upper rim of the fault plane is close to the surface. The maximum value of $F (\sim 0.11)$ is found when $\lambda = 90°$ and δ is $45° \sim 60°$ and F decreases sharply for small values of δ. In Fig. 3a, b, c, the mean depth h^* is fixed ($h^* = 0.29$, 0.44 and 1.0) instead of the depth of the upper rim. The variation of F with respect to δ is not so conspicuous and within a factor of 2, except for cases of strike slip faults ($\lambda = 0°$) in which the values of F vary by a factor of 10 between $\delta = 0°$ and 90°. That is, F is smallest in the case of a vertical strike slip fault. It is seen in Fig. 3a that the variations of F have an opposite trend with respect to δ for λ larger or smaller than about 45°. The variation of F with respect to λ depends on δ and the largest change occurs for a vertical fault

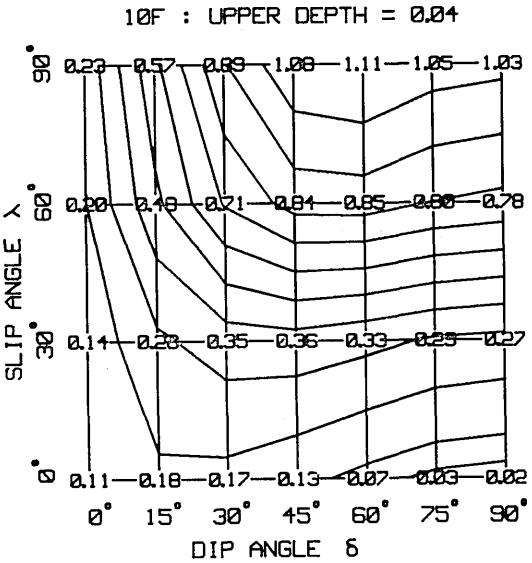

Fig. 2. Dependence of F on the dip angle δ and the slip angle λ when the relative depth h of the upper rim of the fault is fixed at $h = 0.04$. Inserted numerical values are *10F*.

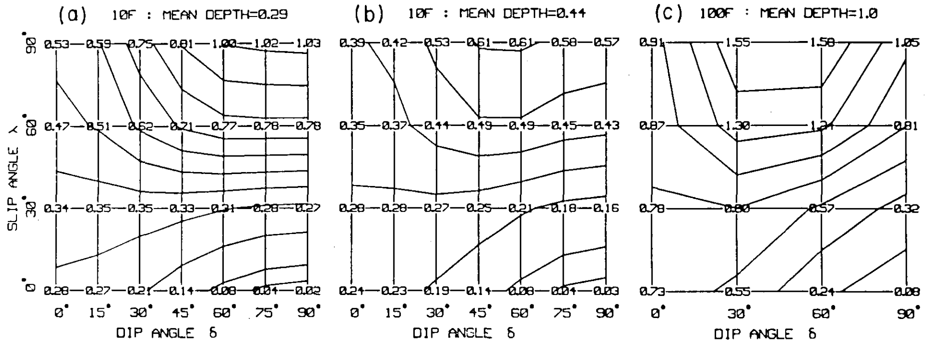

Fig. 3. Dependence of F on the dip angle δ and the slip angle λ when the mean relative depth h^* of the fault is fixed: (a) $h^*=0.29$, (b) $h^*=0.44$, and (c) $h^*=1.0$. Inserted numerical values are *10F* for (a) and (b) and *100F* for (c).

$\delta=90°$ in which the variation amounts to a factor of 10 or more between the dip slip and strike slip faults but for small δ the variation is rather small.

The depth dependence of F can be clearly seen in Fig. 4 which shows the variation of F as a function of the relative depth $h^*(\delta=30°,\ 90°$ and $\lambda=0°,\ 90°)$. Although not shown in the figure, the maximum values for small dip angles ($\delta \leqq 15°$) generally occur at an intermediate depth ($h^* \sim 0.2$). This feature is qualitatively consistent with other studies (PODYAPOLSKY, 1970; YAMASHITA and SATO, 1974). In contrast, for large dip angles ($\delta \geqq 45°$) the maximum values appear at the shallowest depth h^*, except when the slip angle λ is small ($\lambda \leqq 15°$), and the effect of the optimum depth does not show up clearly, probably because the fault ex-

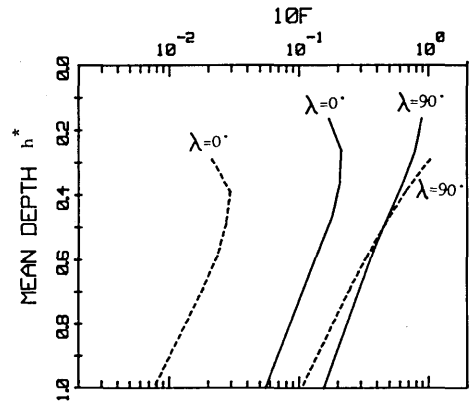

Fig. 4. Variation of F as a function of the relative mean depth h^*: solid lines are for $\delta=30°$ and $\lambda=0°,\ 90°$, and dashed lines are for $\delta=90°$ and $\lambda=0°,\ 90°$.

tends deeper than the optimum depth even in the case when the upper rim of the fault approaches the surface.

For h^* larger than, say, 0.5, the decrease of F with h^* may be regarded as exponential with the decay constant of about 2.4. Since the mean relative depth h^* is given by H^*/L with H^* the mean depth and L the length of the fault respectively, the decrease of F with respect to the increase of the mean depth H^* is relatively faster for smaller earthquake magnitude M_w because L decreases with the decrease of M_w (if we assume $W/L=1/2$, $\alpha=1.23\times10^7$ dyne cm^{-2}, it follows from (4) and (7): $\log L(\mathrm{km})=0.5\,M_w-1.85$). This indicates that, for a fixed mean depth H^* larger than the optimum depth, the relation between E_t and M_w deviates considerably from the relations $\log E_t \sim 2\,M_w$ given by (14). In Fig. 5, the E_t—M_w relation in cases of vertical dip slip faults ($\delta=90°$, $\lambda=90°$) with $h^*=0.29$, $F=0.103$ (the upper rim of the fault very close to the surface) and $H^*=50$ km are shown. For example, at $M_w=7.0$, the tsunami energy when $H^*=50$ km is reduced by a factor of 10 from the value for $h^*=0.29$. In the figure, a case of vertical strike slip ($\delta=90°$, $\lambda=0°$) with $h^*=0.29$, $F=0.002$, is also shown for comparison.

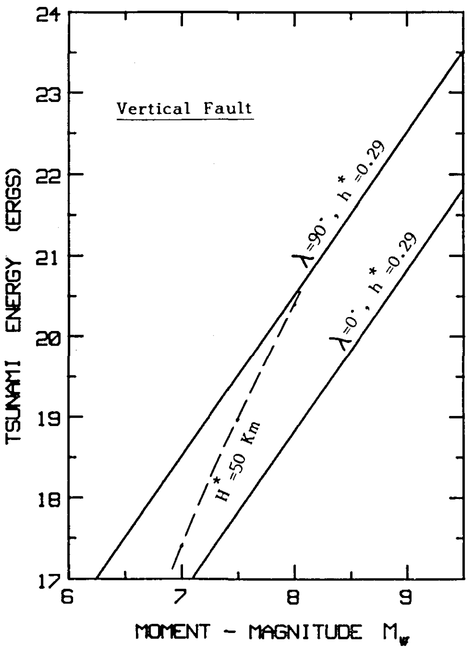

Fig. 5. Relationship between tsunami energy E_t and the moment-magnitude of earthquake M_w in cases of vertical faults ($\delta=90°$) : solid lines are for $h^*=0.29$ corresponding to very shallow faults with the slip angles $\lambda=90°$ and 0°, and a dashed line is for $H^*=50$ km with the slip angle $\lambda=90°$ (dip slip).

4. Empirical methods to estimate tsunami energy

a) Relative energy method

Because of a close relationship between the tsunami magnitude scale m and the total tsunami energy, it may be expedient to explain the definition of the so-called Imamura-Iida magnitude scale. The original definition of the tsunami magnitude scale proposed by IMAMURA (1942) was of a rather qualitative nature, because he intended only to assign the size for old Japanese tsunamis described very briefly in historical documents. An important point in his classification is that the 1896 Sanriku tsunami was assigned grade 4 and the 1933 Sanriku tsunami grade 3. Other tsunamis are classified by comparison with the above two tsunamis. TAKAHASI (1951) translated the qualitative definition of the Imamura's tsunami magnitude in an explicit form:

$$\log_2 (H/H_0) = m - m_0 \tag{17}$$

where H is the "maximum" height and the suffix 0 indicates reference values. (He took $m_0=5$ for $H_0=30$ meters.) Although he stated that H is the maximum height, he apparently used some kind of a mean maximum value for H in practice. This is evident because he assigned the magnitude $m=3$ for the 1933 Sanriku tsunami, which corresponds to $H=7.5$ meters, despite the fact that the maximum observed runup was 28 meters. Later, IIDA (1963) took the reference values in (17) as $m_0=0$ for $H_0=1$ meter. The difference of the magnitude scales defined by Takahasi and Iida is actually small if the definition of H is the same. SOLOVIEV (1970) called this kind of tsunami scale m as an intensity scale i and explicitly defined H as $H=\sqrt{2}\bar{H}$ with $\bar{H}$ the mean of inundation heights over the coast where the tsunami activity was significant (He took $i_0=0$ for $H_0=1$ meter). Originally, the definition of the so-called Imamura-Iida magnitude scale for large tsunamis was stated in terms of the observed maximum height, so that the ambiguity in the definition of H in (17) caused confusion among researchers. The confusion was accelerated when IIDA *et al.* (1967) defined the magnitude scale by (17) in their tsunami catalogue with the statement that H is the actual observed maximum height.

With the accumulation of data on smaller tsunamis, it was felt that a clear distinction between the magnitude scale and the intensity scale was desirable. Various proposals were made to define the magnitude scale based on the wave height estimated at a fixed distance from the tsunami source (WATANABE, 1964; HATORI, 1973; ADAMS, 1974). Discussions of the magnitude scale of the tsunami itself, however, are not the subject of the present paper.

Since the shortest distance between the coast and the tsunami source for most large tsunamis experienced in Japan do not differ significantly, TAKAHASI (1951) considered that the total tsunami energy E_t is proportional only to the square of the representative tsunami height H observed on the coast. Thus, he proposed the formula,

$$E_t/E_{t0}=(H/H_0)^2 \tag{18}$$

where H_0 is the reference tsunami height corresponding to E_{t0}.

Changing (17) to (18), he gave

$$\log(E_t/E_{t0})=0.6020(m-m_0) \tag{19}$$

In this case, he took the 1933 Sanriku tsunami as a standard, for which $m_0=3$ and $E_{t0}=1.6\times10^{23}$ ergs (This value of E_{t0} was estimated by the energy flux method, but was too large by a factor of 50, which will be explained later).

Taking these numerical values into account, (19) becomes

$$\log E_t(\text{erg})=0.6\,m+21.4 \tag{20}$$

IIDA (1963b) extended Takahasi's argument by combining (20) with the formula he derived between m and M (JMA magnitude of the earthquake), namely

$$m=2.61\,M-18.44 \tag{21}$$

to show roughly

$$\log E_t \sim 1.5\,M+10.8 \tag{22}$$

Recalling the Gutenberg-Richter seismic energy-magnitude relation (5), he concluded that $E_t/E_s \sim 1/10$. The numerical value of this ratio depends on the choice of the E_{t0} value. WATANABE (1964) and WILSON (1964) also stated that the ratio of the tsunami energy to the seismic energy is constant.

It should be noted that Takahasi's conjecture leading to (18), which is followed by several investigators, neglected one important factor in the consideration of the relative tsunami energy, even if the assumption of the equal distance between the coast and the tsunami source is accepted. Namely, the "period" (or the "wave length") of a tsunami which is also a function of the earthquake magnitude (TAKAHASI, 1963). Furthermore, the relation given by (21) greatly lacks certainty. Thus, the formula (22) and its consequences such as the constant energy ratio between the tsunami and earthquake are not well founded.

b) Energy flux method

For a free, progressive, plane, shallow waterwave packet of finite duration, the total flux of energy $E_f(\theta)$ across a section of unit width perpendicular to the direction of the wave propagation θ may be written as

$$E_f(\theta)=\frac{1}{2}\rho g(gd)^{1/2}\sum_{n=1}^{N}A_n^2\Delta\tau_n \tag{23}$$

where A_n is the successive amplitudes (the crest height or trough depth from the mean level), $\Delta\tau_n$ is the duration of the elevation or depression (half the wave "period" of successive waves), and N is the number of half waves radiated from the source. And d is the water depth where A_n is determined, but the variation of d in one wave length should be small.

Assuming the isotropic radiation of energy from a point source, TAKAHASI (1951) proposed to compute the total energy E_t of the tsunami in the form

$$E_t=2\pi RE_f \tag{24}$$

where R is the distance from the epicenter of the earthquake to the location where A_n is estimated.

The refractive effect of the depth variation during the long distance travel of tsunami waves radiated from a point source may be expressed by the refraction coefficient $P(\theta)$ defined by

$$P(\theta)=R\theta_0/S(\theta) \tag{25}$$

where θ is the angle of the observing location from a reference direction at the source, θ_0 is the angle in radian between the neighboring wave rays at the source and $S(\theta)$ is the separation of the same neighboring rays at the distance R from the source. Applying this coefficient, (24) is modified to

$$E_t=(2\pi/\theta_0)S(\theta)E_f(\theta) \tag{26}$$

The geometric optics approximation is valid only if the relative depth change in one wave length is small. Because of the long wave length of ordinary tsunami waves, this approximation is probably not satisfactory near the shelf region and the coast.

In reality, the radiation characteristics of the tsunami energy depend on the characteristics of the earthquake source mechanism and, for an elongated dipole type tsunami source, this effect is very important. The effect may be conveniently expressed by the source directivity coefficient

$Q(\theta)$ defined by

$$Q(\theta)=2\pi RE_f^*(\theta)/E_t \tag{27}$$

where $E_f^*(\theta)$ is the energy flux of radiated waves per unit width at a distance of R when the depth is uniform. For a simple uniform uplift of the rectangular portion $(L\times W)$ of the ocean bottom, $Q(\theta)$ depends on the ratio W/L and if the reference direction is along the major axis of the source, $Q(90°)\sim L/W$ (KAJIURA, 1970).

When the depth variation in the vicinity of the tsunami source is significant, it is impossible to separate the topographic effect from the source directivity defined by (27). However, for a rough approximation, we may replace $E_f^*(\theta)$ by $E_f(\theta)/P(\theta)$ and write

$$E_t=2\pi RE_f(\theta)/(P(\theta)Q(\theta)) \tag{28}$$

The difficulty in applying this method is, apart from the assumptions on $P(\theta)$ and $Q(\theta)$, the estimate of the incoming wave amplitude A_n at the depth d and the duration $\tau=\sum_{n=1}^{N}\Delta\tau_n$ of waves from data taken along the coast where observed wave signatures are strongly modified by such effects as the shoaling, coastal reflection, refraction, diffraction, scattering and wave trapping on the shelf and inside the bay. Despite these difficulties, TAKAHASI (1951), IIDA (1963b) and HATORI (1966) estimated values of the total energy of many tsunamis in Japan on the basis of (24). Although they did not give the details of their computations, it appears that the incoming wave amplitude A_n and the duration of waves τ (they counted 5 half waves) were overestimated roughly 2 to 3 times, respectively. If the effect of the directivity is also taken into account, their values of energy would be systematically biased by a factor of 10 to 20. TAKAHASI (1951) and IIDA (1963b) appear to have taken d as the mean depth between the epicenter of earthquake and the observing location which is about 10 times larger than the depth where A_n was estimated in the case of the 1933 Sanriku tsunami. The values of E_t in the 1933 Sanriku tsunami estimated by Takahasi and Iida are 1.6×10^{23} and 1.8×10^{23} ergs, respectively, so that the probable value after removing the systematic bias of about 50 would be 3×10^{21} ergs.

A modified version of the Takahasi method was used by SOLOVIEV (1970) to compute the tsunami energy from tide gauge data. He replaced $\sum_{n=1}^{N}A_n^2\Delta\tau_n$ in (23) by $A^2\tau$ where A is the mean amplitude within the duration τ of the tsunami activity. When the maximum amplitude $A_{\max}$ is used instead of A, τ is replaced by $\tau/10$. Values of individual tsunami energy were not presented in his paper, but his Fig. 2 suggests the mean relation between his intensity i and E_t:

$$\log E_t = 0.78\, i + 20.5 \tag{29}$$

Although the intensity i is found to scatter around a mean $\bar{i}$ for a given earthquake magnitude M, SOLOVIEV gave the relation between the mean value $\bar{i}$ and the earthquake magnitude M by

$$\bar{i} = 2.52\, M - 18.3 \tag{30}$$

If (30) is substituted for (29), it follows that

$$\log E_t = 1.97\, M + 6.23 \tag{31}$$

As already mentioned, his values of A and τ are overestimated and, because of the very long duration τ he took, the reduction factor may be about 1/20~1/30.

HIRONO (1961) used the tide gauge data obtained at small islands in the Pacific to estimate the energy of the Chilean tsunami in 1960 based on (26). The values of $S(\theta)$ in the deep sea are estimated by a refraction diagram drawn from a point source lying on the Chilean coast (assuming the total reflection of waves at the coast, the factor 2π in (26) is replaced by π). The energy fluxes seem to be estimated by taking tide gauge records on the islands without modification. The values of energy thus estimated are $(1.3 \text{ and } 3.8)\times 10^{23}$ ergs from data at Christmas Island and Johnston Island, respectively. Because of a very long extension of the source region of this tsunami, the effect of directivity must be significant and the probable value of energy may be close to $1\sim 2\times 10^{23}$ ergs. Hirono also mentioned that a similar method yielded $E_t \sim 4\times 10^{23}$ ergs for the Kamchatska tsunami in 1952, but details were not available.

WATANABE (1964) also estimated the values of tsunami energy for several earthquakes taking the refractive effect into consideration. In his case, the tsunami wave height at the source region was computed from tide gauge data at several coastal stations and the energy was estimated by applying (24) assuming $R = 50$ km. Although he gave numerical values corresponding to A_n, $\Delta\tau_n$, and N, the values of E_t given by him are not reproducible. GRIGORASH and KORNEVA (1972) also stressed the importance of refractive effects in the estimation of tsunami energy.

c) Reverberation method

Another way to estimate the total energy of a very large tsunami is to make use of the reverberation of a tsunami throughout the whole

Pacific Ocean (MILLER *et al.*, 1962). If the energy density in the later stage of a very large tsunami is uniformly distributed over the whole Pacific Ocean and if the decay rate is uniform in space and time, it is possible to estimate the initial total energy by extrapolation from the knowledge of the energy density in the later stage and the decay rate together with the area of the Pacific Ocean. However, there are indications that the decay time t_0 (the time in which the energy decreases by a factor of $1/e$) of the tsunami energy observed at a coastal station increases with decreasing energy density. At the beginning, $t_0 \sim 0.5$ day and becomes larger as time elapses. On the other hand, the state of complete "diffusion" of the tsunami energy over the entire Pacific Ocean would take many days. Here we have the problem that by the time the ideal state of the diffusion of tsunami energy is attained the energy level would be already so small that it would be difficult to distinguish the tsunami energy from background wave activity (LOOMIS, 1966).

MILLER *et al.* speculated the total energy of the Chilean tsunami in 1960 to be on the order of 3×10^{23} ergs by inferring the initial energy density (12 hours after the occurrence of the earthquake) in the open sea from data obtained at La Jolla on the California coast by taking the local enhancement factor into account. They apparently used the decay time of the order of 1.5 days (Fig. 7, MILLER *et al.*, 1962) in the interval of about 5 days.

VAN DORN (1963) also applied the same idea to estimate the tsunami energy of the 1957 Aleutian earthquake on the basis of data obtained at Wake Island. He used the decay time on the order of 4 days (Fig. 7, VAN DORN, 1963) estimated from data in several days after 40 hours from the beginning of the record and obtained the energy E_t as 2.5×10^{22} ergs.

It should be noted that the decay rates used in these two studies are different, suggesting that the estimated decay rates were of the regional feature. Furthermore, it is extremely difficult to estimate the mean energy density for the entire Pacific Ocean from data obtained at a single station. As already noted by MILLER *et al.* (1962), there exist many ambiguities in the chain of reasoning to arrive at the value of total energy and the value should be trusted by the order of magnitude only.

d) Spectral inversion method

VAN DORN (1963) also attempted to estimate the initial wave form of the 1957 Aleutian tsunami by a spectral inversion method of the observed dispersive wave pattern at Wake Island. To make the inversion unique, he assumed an axially symmetric initial disturbance and obtained the bell shaped depression with a slight elevation around the fringe of the depression. The potential energy was computed from this initial disturbance to be 2.7×10^{22} ergs. The deficiency of this method when applied to a tsunami problem is that the assumed axisymmetric original disturbance may not be valid because the effect of directivity is significant. In this respect, a deduction from a single station data is quite questionable because the wave patterns are expected to differ considerably depending on the azimuth of the observing site with respect to the tsunami source. The estimated values of tsunami energy by the reverberation method and the spectral inversion method for the 1957 Aleutian tsunami turned out to be very close, but this coincidence seems to be fortuitous.

e) Potential energy method

This method of estimating energy was already discussed in relation to the earthquake fault model. Instead of estimating the crustal deformation from a fault model, the tsunami energy may be computed from empirical data of crustal uplift estimated geodetically. When a rough idea about the vertical displacement $D(x, y)$ and the total area S are obtained by a field survey, it is possible to estimate E_t by the formula $E_t=(1/2)\rho g D_*^2 S$, with $D_*^2=\frac{1}{S}\int D^2 dxdy$. This kind of computation was carried out for the 1964 Alaska tsunami by several investigators. The derived values of energy are (in ergs): 2.3×10^{21} (VAN DORN, 1964), 5.88×10^{21} (PARARAS-CARAYANNIS, 1967), 1.4×10^{22} (HATORI, 1970), and 2.2×10^{22} (BERG *et al.*, 1972). It is noticed that the differences between the earliest and the latest values amount to a factor of 10. Since the last two figures are based on detailed data of crustal uplift, the true energy would be somewhere around 2×10^{22} ergs. HATORI (1970) also computed the tsunami energy for several Japanese earthquakes by this method.

The difficulty in applying this method is that the crustal uplift occurring underwater is difficult to estimate. As seen in the example of the 1964 Alaska earthquake, the uncertainty may amount to a factor of 10 if no reliable data is available.

Now let us apply the present method to the 1960 Chilean tsunami. According to PLAFKERS's (1972) investigation on the crustal deformation and also from data of tsunami height distribution along the Chilean coast (SIEVERS *et al.*, 1963) the source area of the tsunami may be roughly estimated to be confined between the coast and the trench axis and to be of the dimensions 900 km $\times$ 120 km with the longer axis parallel to the coast. In view of the maximum uplift of 6 m observed at Isla Guanbin, and also that the mean inundation height of the tsunami was about 10 m, it may be reasonable to assume that the mean maximum uplift was about 7 m along the offshore side of the source area and decreased to zero in cosine form toward the Chilean coastline. With this geometry of the uplift, the total potential energy E_t was

$$E_t = \rho g \frac{1}{4} H_{\max}^2 S \sim 1.3 \times 10^{23} (\text{ergs}) \tag{32}$$

CARRIER and GREENSPAN (1958) showed that for a special type of an initial mound of water with the maximum height H_m the runup height on a linearly sloping bottom is 1.45 H_m. Therefore, the maximum uplift of 7 m for the inundation height of 10 m may be acceptable, but the mean maximum uplift of 7 m all along the length of the source region seems to be too large. The values of tsunami energy for the Chilean earthquake estimated by various methods are (in ergs): 4.5×10^{23} (IIDA, 1963b), $1.3 \sim 3.8 \times 10^{23}$ (HIRONO, 1961), 3×10^{23} (MILLER *et al.*, 1962), and 10^{23} (present study). All estimates involve uncertainty but the most probable value would be about 10^{23} ergs.

f) Numerical inversion method

Presently the most reliable energy estimate may be obtained by means of numerical experiments of the tsunami generation and propagation on the basis of shallow water equations. By the trial and error method, starting from a known earthquake fault model, it is possible to find a probable model of a tsunami source which can best explain the observed coastal tsunami height distribution (AIDA, 1972, 1974, 1978a, 1978b, 1979a, 1979b). In most of these studies, Aida gave numerical values of tsunami energy computed by the potential energy method based on the derived tsunami source models. Since the uncertainty in the source estimation of tsunamis expressed by the relative standard deviation between the computed and observed wave amplitudes along the coast is about 0.4 (AIDA, 1979a), the accuracy of energy estimation is probably better than a factor of 2. He extended the numerical method to several tsunamis in the past and estimated the total tsunami energy as well as the earthquake moment which generated the tsunami (AIDA, 1977). The values of tsunami energy for the Izu-Oshima-Kinkai earthquake of 1978 (AIDA, 1978a) and the Tonankai earthquake of 1944 (AIDA, 1979b) are not stated explicitly in Aida's papers, but they are 10^{17} and 2.3×10^{20} ergs, respectively (AIDA, personal communication). There results are considered, at present, to be the best estimate of tsunami energy.

g) Empirical tsunami energy as a function of the moment-magnitude of earthquakes

The tsunami energy E_t estimated by various methods is plotted as a function of the moment-magnitude M_w as far as possible. For most of large earthquakes considered here, the earthquake moment M_0 is already known (see, for example, OHNAKA, 1978), and for smaller earthquakes ($M \leqq 7.5$) the difference of M_J(JMA magnitude) and M_w is usually small so that M_J is used for M_w if M_0 is not available. Figure 6 shows the total energy of tsunamis estimated by the energy flux method in the vicinity of Japan (IIDA, 1963b; HATORI, 1966, 1971, 1972). The dashed line corresponds to (31) derived from Soloviev's data (1970) with M substituted for M_w. In Fig. 7 the total energy estimated by the potential energy method (BERG *et al.*, 1972; HATORI, 1970, present paper) and the numerical inversion method (AIDA, 1972, 1977, 1978b, 1979a, unpublished data) are plotted as a function of M_w. The solid lines in these figures correspond to (14). It is seen that the general trend of $\log E_t$ with respect to M_w in Fig. 6 is well represented by (14), provided that the values estimated

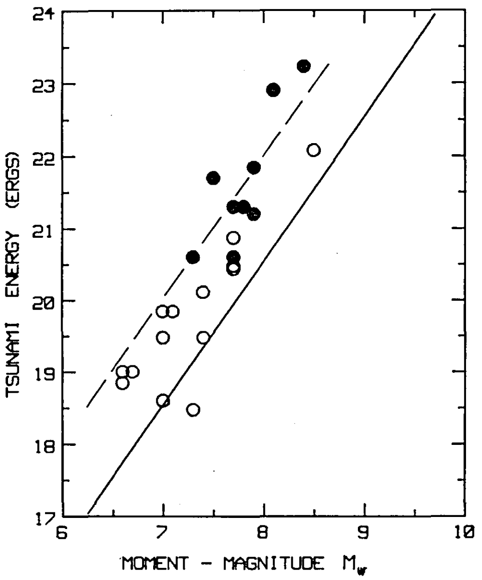

Fig. 6. Tsunami energy E_t estimated by the energy flux method as a function of the moment-magnitude of earthquake M_w: solid circles by Iida, open circles by Hatori and a dashed line by Soloviev expressed by (31). The solid line corresponds to (14) which is supposed to be the upper limit of tsunami energy in the present study.

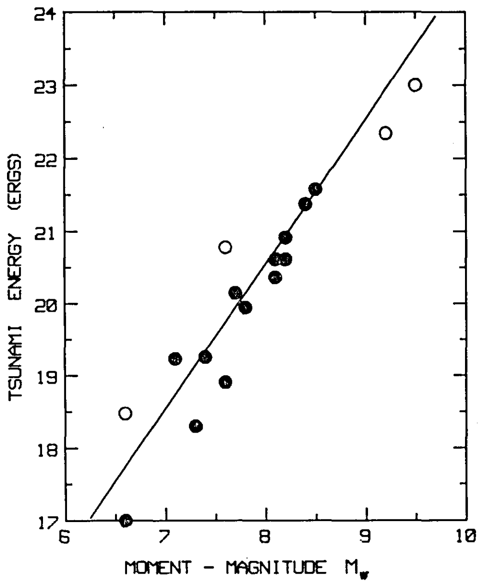

Fig. 7. Tsunami energy E_t estimated by the potential energy method (open circles) and numerical inversion method (solid circles). The solid line corresponds to (14) which is supposed to be the upper limit of tsunami energy in the present study.

by the energy flux method are roughly overestimated by a factor 10 to 20. It may be natural that the tsunami energy estimated by the numerical inversion method follows closely the relation given by (14) as seen in Fig. 7, because the tsunami generation by ordinary earthquakes and the observed tsunami behaviors are found to be explained, at least as a first approximation, by fault models seismically determined, and (14) is based on the relation of idealized fault models to the moment-magnitude M_w of earthquakes. For very large earthquakes ($M \geqq 9.0$), however, the empirical values are somewhat lower than those expected from (14).

5. Summary and discussion

On the basis of a simple kinematic similarity model of the earthquake fault, tsunami energy is estimated as the potential energy of the initial sea-surface distubance which is estimated to be equivalent to the static displacement of the sea-bottom due to the fault dislocation. The relation between the tsunami energy E_t and parameters (M_w, δ, λ, h^*) of the earthquake fault is expressed by (13). The dependence of E_t on the moment-magnitude M_w is $\log E_t \sim 2\,M_w$ if δ, λ, h^* are held constant, so that $\log (E_t/E_s) \sim 0.5\,M_w$.

The dependence of E_t on δ, λ, and h^* is given by a function $F(\delta, \lambda, h^*)$ which is independent of M_w. Behaviors of F with respect to δ, λ, and h^* are shown in Figs. 2 to 4. Roughly speaking, the variation of F is within one order of magnitude for each change of parameters in the intervals: $0° \leqq \delta \leqq 90°$, $0° \leqq \lambda \leqq 90°$, and $h^* < 1.0$. The largest value of $F(=0.11)$ is found for $\delta \sim 60°$, $\lambda \sim 90°$ and $h \sim 0.04$ (the computed shallowest depth of the upper rim of the fault). In general, for any fixed values of h^* and δ, E_t decreases with the decrease of λ and the range of variation is the largest when $\delta = 90°$ (vertical fault). The dependence of E_t on δ shows an opposite trend with respect to λ being larger or smaller than about 45°: for small λ, E_t decreases as δ increases, and for large λ, E_t increases as δ increases (if the relative depth h^* is larger than, say, 0.5, E_t increases with the increase of δ up to about 45° and then slightly decreases as δ approaches 90°). The range of variation of E_t with respect to δ is largest when $\lambda = 0°$ (strike slip).

For small values of h^*, the depth dependence of E_t varies according to the values of δ and λ: for small δ and λ, there exists an optimum depth h^* at $0.2 \sim 0.3$ when E_t is the maximum, but for large δ and λ, there is no such optimum depth and the maximum value of E_t occurs for shallowest faults. For large h^* (say $h^* \geqq 0.5$), the decrease of E_t with the increase of h^* is roughly exponential with decay constant of about 2.4. Since the depth dependence of E_t is expressed in terms of the relative depth ($h^* = H^*/L$), the decrease of E_t with respect to the increase of the actual depth H^* is more rapid for earthquakes of smaller magnitude M_w.

The empirically estimated values of tsunami energy, except those deduced by the numerical inversion method, can be trusted by the order of magnitude only because of many ambiguities in the derivation of final values. Even so, the general feature of the $E_t \sim M_w$ relation is consistent with the present study, but there seems to be a systematic bias by a factor of 10 or more in the tsunami energy deduced by the conventional energy flux method.

There are many problems concerning the source parameters of earthquakes, even if we admit a purely elastic dislocation model as the earthquake mechanism. The aspect ratio W/L may differ considerably from 1/2 and there seems to be a tendency that this ratio decreases for a very large M_w. The rigidity μ may vary by a factor of 2 to 3, and the constant α in (4) may be about 3 times larger than that adopted ($\alpha = 1.23 \times 10^{23}$ dyne cm^{-2}). Taking these uncertainties into account, the constant term in (13) may vary considerably and the estimate of energy E_t may vary be a factor of 2. To be more realistic in a model of the earthquake fault, we may take the layering of the crust and/or the non-uniform distribution of the stress drop into consideration. Furthermore, the rectangular plane fault is also an idealization. For very large earthquakes, there may exist an imbricate fault which has a large dip angle δ. In view of these features of very large shallow earthquakes along interplate boundaries, the actual static displacement of the ocean bottom may deviate considerably from that deduced from a simple rectangular plane fault with uniform rigidity and dislocation. For example, according to (14) the possible maximum value of E_t is 3.5×10^{23} ergs for $M_w=9.5$. In reality, however, the most probable value of the tsunami energy of the Chilean earthquake in 1960 ($M_w=9.5$) is estimated to be on the order of 10^{23} ergs from empirical data. In the present state of the art concerning the understanding of earthquake mechanism, however, it may not be fruitful in the tsunami problem to take a complicated fault model into consideration.

Another problem is the assumption that the initial water surface disturbance is equivalent to the static vertical displacement of the ocean bottom. This assumption is generally valid only for large scale bottom deformations. For moderate to small earthquakes ($M\leqq7.0$) with the epicenter located in deep ocean ($d\sim5$ km), this assumption may become a fair approximation. Because, in this case, the width of the fault is about 20 km for $M_w\sim7.0$ and a considerable amount of power in the static bottom deformation may be present in small scale components which would be filtered out in the actual sea-surface disturbance by a factor of $1/\cosh(kd)$. From this point of view, tsunami energy for small values of M_w ($\leqq7.0$) given by (13) may be overestimated. Values of tsunami energy for small earthquakes are expected to be more sensitive than for large earthquakes to the depth of water d and the mean depth H^* of the fault so that the empirically determined values of energy would be scattered over a wide range.

Taking all these uncertainties into consideration, it seems to be very difficult to determine the tsunami energy beyond the accuracy of a factor of 2 to 3 when we use the parameters of a simple earthquake fault. It is also mentioned that the accuracy of the seismic moment determination, which is said to be better than a factor of 2 (AKI, 1972), limits the reliability of the tsunami energy estimation by a factor of about 2.

Acknowledgements

I thank Miss I. Kato for her assistance in numerical computations and for typing the manuscript.

References

ABE, K., 1975, Reliable estimation of the seismic moment of large earthquakes. *J. Phys. Earth*, **23**, 381-390.

ABE, K., 1979, Size of great earthquakes of 1837-1974 inferred from tsunami data. *J. Geophys. Res.*, 84, 1561-1568.

ADAMS, W. M., 1974, Tsunami anomalies and precursory phenomena having potential value as predictors. *Proc. 5th World Conf. Earthq. Eng.*, **11**, 2754-2762.

AIDA, I., 1972, Numerical estimation of a tsunami source. *Zisin, 2nd Ser.*, 25, 343-352 (in Japanese).

AIDA, I., 1974, Numerical computation of a tsunami based on a fault origin model of an earthquake. *Zisin, 2nd Ser.*, **27**, 141-154 (in Japanese).

AIDA, I., 1977, Simulations of large tsunamis occurring in the past off the coast of the Sanriku district. *Bull. Earthq. Res. Inst.*, **52**, 71-101 (in Japanese).

AIDA, I., 1978a, A numerical experiment for the tsunami accompanying the Izu-Oshima-Kinkai earthquake of 1978. *Bull. Earthq. Res. Inst.*, **53**, 863-873 (in Japanese).

AIDA, I., 1978b, Numerical experiments for the tsunami accompanying the Miyagiken-oki earthquake of 1978. *Bull Earthq. Res. Inst.*, **53**, 1167-1175 (in Japanese).

AIDA, I., 1979a, Reliability of a tsunami source model derived from fault parameters. *J. Phys. Earth*, **26**, 57-73.

AIDA, I., 1979b, A source model of the tsunami accompanying the Tonankai earthquake of 1944. *Bull Earthq. Res. Inst.*, **54**, 329-341 (in Japanese).

AKI, K., 1972, Earthquake mechanism. *Tectonophysics*, **13**, 423-446.

ALEXEEV, A. S. and V. K. GUSIAKOV, 1974, Numerical modelling of tsunami and seismic surface wave generation by a submarine earthquake. *Tsunami Research Symposium 1974*, Bull. 15, Roy. Soc. N. Z., 243-251, Royal Soc. New Zealand and the UNESCO Press, Paris.

BERG, E., D. C. COX, A. S., FURUMOTO, K. KAJIURA, H. KAWASUMI and E. SHIMA, 1972, Source of the major tsunami. *The Great Alaska Earthquake of 1964, Oceanography and Coastal Engineering*, 122-139, NAS Pub. 1605, Nat. Acad. Sci., Washington D. C.

CARRIER, G. F. and H. P. GREENSPAN, 1953, Water waves of finite amplitude on a sloping beach. *J. Fluid Mech.*, **4**, 97-109.

GELLER, R. J., 1976, Scaling relation for earthquake source-parameters and magnitudes. *Bull. Seism. Soc. Am.*, **66**, 1501-1523.

GRIGORASH, Z. K. and L. A. KORNEVA, 1972, Wave-field charts and energy of tsunamis in the Black Sea. *Atmos. Oceanic Phys.*, **78**, 320-323 (transl. from Russian by A. G. U.).

HASKELL, N., 1964, Total energy and energy spectral density of elastic wave radiation from propagating faults. *Bull. Seism. Soc. Am.*, 54, 1811-1842.

HATORI, T., 1966, Vertical displacement in a tsunami source area and the topography of the sea bottom. *Bull. Earthq. Res. Inst.*, 44, 1449-1464.

HATORI, T., 1970, Vertical crustal deformation and tsunami energy. *Bull. Earthq. Res. Inst.*, 48, 171-188.

HATORI, T., 1971, The magnitude of tsunamis generated in Hiuganada during the years, 1926-1970. *Zisin, 2nd Ser.*, 24, 95-106 (in Japanese).

HATORI, T., 1972, An investigation of the tsunami of Sept. 6, 1971 generated off southwestern Sakhalin. *Zisin, 2nd Ser.*, **25**, 218-224 (in Japanese).

HATORI, T., 1973, A method for determining tsunami magnitude. *Tsunami, Acad. Sci. USSR, Yuzuno-Sakhalinsk* 32, 86-96, Proc. I. U. G. G. Tsunami Symp., Moscow, 1971 (in Russian).

Hirono, T., 1961, The report on the tsunami of the Chilean earthquake 1960. *Tech Rep. J.M.A.*, No. 8, 217-225 (in Japanese).

Iida, K., 1958, Magnitude and energy of earthquakes accompanied by tsunami, and tsunami energy. *J. Earth Sci., Nagoya Univ.* **6**, 101-112.

Iida, K., 1963a, Magnitude, energy and generation mechanisms of tsunamis and a catalogue of earthquakes associated with tsunamis. *Proc. tsunami meetings associated 10th Pac. Sci. Congr.*, Honolulu, edited by D.C. Cox, *I.U.G.G. Monogr.* 24, 7-18.

Iida, K., 1963b, A relation of earthquake energy to tsunami energy and the estimation of the vertical displacement in a tsunami source. *J. Earth Sci., Nagoya Univ.* **7**, 49-67.

Iida, K., 1970, The generation of tsunamis and the focal mechanism of earthquakes. *Tsunamis in the Pacific Ocean*, edited by W.M. Adams, 3-18, East Wast Center Press, Honolulu.

Iida, K., D.C. Cox and G. Pararas-Carayannis, 1967, Preliminary catalog of tsunamis occurring in the Pacific Ocean. *Univ. Hawaii, Inst. Geophys. Data Rep.* 5, HIG-67-10.

Imamura, A., 1942, History of Japanese tsunamis. *Kaiyo-no Kagaku* (Oceanography), **2**, (in Japanese).

Imamura, A., 1949, List of tsunami in Japan. *Zisin, 2nd Ser.*, **2**, 23-28.

Kajiura, K., 1970, Tsunami source, energy and the directivity of wave radiation. *Bull. Earthq. Res. Inst.*, 48, 835-869.

Kanamori, H., 1977, The energy release in great earthquakes. *J. Geophys. Res.*, **82**, 2981-2987.

Kanamori, H. and D.L. Anderson, 1975, Theoretical basis of some empirical relations in seismology. *Bull. Seism. Soc. Am.*, **65**, 1073-1095.

Loomis, H.G., 1966, Spectral analysis of tsunami records from stations in the Hawaiian Islands. *Bull. Seism. Soc. Am.*, **56**, 697-713.

Mansinha, L. and D.E. Smylie, 1971, The displacement fields of inclined faults. *Bull. Seism. Soc. Am.*, **61**, 1433-1440.

Miller, G.R., W.H. Munk and F.E. Snodgrass, 1962, Long-period waves over California's continental borderland. Part II. Tsunamis. *J. Mar. Res.*, **20**, 31-41.

Murty, T.S. and H.G. Loomis, 1980, A new objective tsunami magnitude scale. Submitted to *Marine Geodesy*.

Ohnaka, M., 1978, Earthquake-source parameters related to magnitude. *Geophys. J. R. astr. Soc.*, **55**, 45-66.

Pararas-Carayannis, G., 1967, A study of the source mechanism of the Alaska earthquake and tsunami of March 27, 1964. Part I. Water waves. *Pac. Sci.*, **21**, 301-310.

Plafker, G., 1972, Alaskan earthquake of 1964 and Chilean earthquake of 1960: Implications for arc tectonics. *J. Geophys. Res.*, **77**, 901-925.

Podyapolsky, G.S., 1970, Generation of the tsunami wave by the earthquake. *Tsunamis in the Pacific Ocean*, edited by W.M. Adams, 19-32, East West Center Press, Honolulu.

Purcaru, G. and H. Berckhemer, 1978, A magnitude scale for very large earthquakes. *Tectonophysics*, **49**, 189-198.

Sievers, H.A., G. Villegas and G. Barros, 1963, The seismic sea waves of May 22, 1960 along the Chilean coast. *Bull. Seism. Soc. Am.*, **53**, 1125-1190 (Transl. from Spanish by P.St. Amand).

Singh, S.K. and J. Havskov, 1980, On moment-magnitude scale. *Bull. Seism. Soc. Am.*, **70**, 379-383.

Soloviev, S.L., 1970, Recurrence of tsunamis in the Pacific. *Tsunamis in the Pacific*

Ocean, edited by W. M. ADAMS, 149-163, East West Center Press, Honolulu.

TAKAHASI, R., 1951, An estimate of future tsunami damage along the Pacific coast of Japan. *Bull. Earthq. Res. Inst.*, **29**, 71-95.

TAKAHASI, R., 1963, On the spectra and the mechanism of generation of tsunamis. *Proc. tsunami meetings associated 10th Pac. Sci. Congr.*, Honolulu, edited by D. C. COX, *I.U.G.G. Monogr.* **24**, 19-25.

VAN DORN, W. G., 1963, The source motion of the tsunami of March 9, 1957 as deduced from wave measurements at Wake Island. *Proc. tsunami meetings associated 10th Pac. Sci. Congr., Honolulu*, edited by D. C. COX, *I.U.G.G. Monogr.* **24**, 39-48.

VAN DORN, W. G., 1964, Source mechanism of the tsunami of March 28, 1964, in Alaska. *Proc. 9th Conf. Coastal Eng.*, Lisbon, 166-190.

WARD, S. N., 1980, Relationships of tsunami generation and an earthquake source. *J. Phys. Earth*, **28**, 441-474.

WATANABE, H., 1964, Studies on the tsunamis on the Sanriku coast of the northeastern Honshu in Japan. *Geophys. Mag., J. M. A.*, **32**, 1-65.

WATANABE, H., 1970, Statistical studies of tsunami sources and tsunamigenic earthquakes occurring in and near Japan. *Tsunamis in the Pacific Ocean*, edited by W. M. ADAMS, 99-117, East West Center Press, Honolulu.

WILSON, B., 1964, Generation and dispersion characteristics of tsunamis. *Studies on Oceanography*, edited by K. YOSHIDA, 413-444. Univ. of Tokyo Press, Tokyo.

YAMASHITA, T. and R. SATO, 1974, Generation of tsunami by a fault model. *J. Phys. Earth*, **22**, 415-440.

---

20. 津波エネルギーと地震断層モデルのパラメータとの関係

地震研究所　梶浦欣二郎

地震によっておこる津波のエネルギーを，幾何学的相似を仮定した簡単な地震断層モデルにもとずいて推定した．その結果によると津波エネルギー E_t （エルグ）は，

$$\mathrm{Log}\, E_t = 2M_w + \mathrm{Log}\, F + 5.50$$

で与えられる．ここで，M_w は地震のモーメント・マグニチュードであり，F は地震モーメントと直接関係しない断層パラメータの関数である．長さ L，幅 W の地震断層を考えると，F に含まれるパラメータは断層面の傾斜角 δ，断層面上のすべり方向 λ，断層面の相対深さ $h^*(h^*=H^*/L$；H^* は断層面の平均深さ，L は断層の長さ)，および断層面のたてよこ比 W/L であるが，このうち $W/L=1/2$ を仮定した．

これらのパラメータの変化により，F は最大１桁くらい変るが，最大の F の値は約0.1である．鉛直の断層で，たてずれのときとよこずれのときの津波エネルギーの違いが最も大きい．

断層面の深さによる津波エネルギーの違いは，相対深さ h^* の関数であり，やや深い地震では $E_t \sim \exp(-h^*/2.4)$ の程度にエネルギーは深さとともに減少する．h^* はもともと断層の長さ L で無次元化されているので，M_w が小さくなると L が減少するため，小さな地震ほど津波エネルギーの深さ H^* による減少率は大きくなる．

これらの結果を，今までの津波エネルギーの推測値と比較する．まず，いろいろの方法によって推定された津波エネルギーの信頼度を検討し，エネルギー推定の不確かさを明らかにした．最も普通に行なわれる，エネルギー・フラックスを利用する方法では，今までの値は系統的に10倍以上の過大評価であるが M_w に対する依存性は，$\mathrm{Log}\, E_t \sim 2M_w$ と矛盾しない．津波数値計算と実測波高との比較から求めた津波エネルギーは，もともと断層モデルから出発していることもあって上式でよくあらわされる．最大級の地震津波であるチリ津波について，地殻変動の推定から求めた津波エネルギーは 10^{23} エルグの程度と思われ，上式の方が過大評価ぎみである．これは，断層モデルを簡単化しすぎていることにも原因があるであろう．