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19. *Application of the Elasticity Theory of Dislocations to Tectonomagnetic Modelling.*

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Summary

A Green function method is developed for calculating the piezomagnetic field associated with various tectonic models. The model earth considered is the simplest in its elastic and magnetic properties; i.e. a homogeneous and isotropic semi-infinite elastic body having a uniformly magnetized top layer with a constant stress sensitivity. The experimentally as well as theoretically established law of the reversible piezomagnetic effect of rocks under uniaxial stresses is extended to include any arbitrary three-dimensional stress state. This generalized piezomagnetic law is combined with Volterra's formula in the elasticity theory of dislocations to yield a surface integral representation of the tectonomagnetic field for any dislocation models. The elementary piezomagnetic potentials, or Green's kernels in the integral formula, can be constructed by means of the Fourier transform method. As an applied example of the present theory, analytical solutions of the seismomagnetic effect due to purely vertical transcurrent faults are presented for the uniform and linear slip of an infinitely long fault as well as the uniform slip of a rectangular fault. Finally, a formal expression of the piezomagnetic potential is derived in terms of the given displacements within a strained body. In the uniform piezomagnetic medium of infinite extent, the potential is identical with the displacement itself multiplied by material constants of the medium. Hence, the tectonomagnetic total field change at the earth's surface represents, for the most part, the extension at that point in the geomagnetic field direction.

1. Introduction

Geomagnetic changes in association with tectonic events have sometimes been reported in such cases as strong earthquakes (RIKITAKE 1968), volcanic eruptions (JOHNSTON and STACEY 1969) and so on. Especially in recent years, this kind of field work has been intensively carried out in the seismically active regions with special reference to the earthquake prediction study (SMITH and JOHNSTON 1976, SUMITOMO 1977, SHAPIRO and ABDULLABEKOV 1978). This particular branch of geophysics was named "Tectonomagnetism" by NAGATA (1969). Tectonomagnetism is based upon the piezomagnetic effect of magnetized rocks. Among a variety of piezomagnetic phenomena (NAGATA 1970a), the most dominant is the reversible change against applied stresses, which is best established experimentally (KALASHNIKOV and KAPITZA 1952, OHNAKA and KINOSHITA 1968) as well as theoretically (NAGATA 1970b, STACEY and JOHNSTON 1972).

Several tectonomagnetic models have been constructed on the basis of reversible piezomagnetic effect. STACEY (1964), a forerunner of such a study, calculated possible coseismic magnetic changes due to a hypothetical distribution of the released stress by a vertical transcurrent fault. YUKUTAKE and TACHINAKA (1968) estimated geomagnetic changes associated with a dilating cylinder horizontally stretched within the earth, while DAVIS (1976) calculated the piezomagnetic field by a hydrostatically pumped sphere.

Since STEKETEE's (1958a, b) pioneer work, the elasticity theory of dislocations have provided us with a powerful means to clarify the displacement and stress field around an earthquake fault. The application of the theory to tectonomagnetic modelling was first carried out by SHAMSI and STACEY (1969), who calculated the magnetic field produced by vertical strike-slip and dip-slip faults with *infinite length*.

A vertical strike-slip fault with *semi-infinite length* was investigated by TALWANI and KOVACH (1972) in order to estimate possible magnetic changes caused by stress concentration near the fault edge. More recently, JOHNSTON (1978) interpreted some tectonomagnetic events observed along the San Andreas fault with the aid of piezomagnetic model calculations of a *rectangular* strike-slip fault.

All these model calculations require elaborate computer work, because two- or three-dimensional convolution integrals should be computed numerically. Most of these models have singularities along the edge of dislocation surface. The complicated, sometimes divergent, stress distribution near singular points prevents us from exact understanding of the fault edge effect with the limited resolution of coarce numerical grid.

SASAI (1979) presented an analytical solution of the piezomagnetic field produced by a center of dilatation within a semi-infinite solid. The algorithm is based upon the Fourier transform of convolution integrals. The method is applicable to various types of strain nuclei, and hence to dislocation models, because such models can be viewed as a distribution of some particular nuclei of strain (what are called *elementary dislocations*) on the dislocation surface. It will be shown in this paper that the piezomagnetic field associated with any dislocation models can be expressed by surface integrals of some potential functions. These functions might be called the elementary piezomagnetic potentials, which represent the piezomagnetic field produced by elementary dislocations. Once the elementary potentials are known, the surface integration can be achieved analytically for some simple distribution of dislocations. If it were not the case, numerical procedure would be greatly simplified, because the 3-dimensional convolution integral over a semi-infinite medium reduces to a 2-dimensional surface integral over a finite dislocation surface.

The main purpose of this paper is to construct the above-mentioned elementary piezomagnetic potential functions. The elastic properties of the semi-infinite medium are assumed to be homogeneous and isotropic, the uppermost layer of the medium being uniformly magnetized from the plane boundary to a depth of Currie point isotherm.

The stress-induced magnetization under any 3-dimensional stress state will be formulated as a linear combination of stress components. The stress-magnetization relationship and Volterra's formula for the dislocation stress field will be combined to give a general expression for the piezomagnetic anomaly field associated with any dislocation models. We can obtain the kernels in the integral formula, or the piezomagnetic field potentials due to elementary dislocations, by means of the Fourier transform method.

In order to show the applicability of the present theory, we will derive analytical solutions for the piezomagnetic field produced by some vertical strike-slip faults. A simple case such as an infinitely long fault will be investigated for the uniform and linear slip model, which will be compared with results obtained by SHAMSI and STACEY (1969). The piezomagnetic field due to a vertical rectangular strike-slip fault will be presented, exhibiting a remarkable fault edge effect. All these model studies cast a new light upon the seismomagnetic effect, which will be discussed briefly.

In the last section the tectonomagnetic field will be treated in a most general way. We connect the Gaussian law for the magnetic field and the equation of elastic equilibrium through the generalized piezomagnetic formula. With the help of the potential theory, we will obtain a formal representation of the piezomagnetic field as a function of the given displacement. Especially, the total field change will be shown approximately proportional to the crustal extension at the observation site measured in the geomagnetic field direction.

2. Stress-induced Magnetization

We begin with the well-established empirical law of the reversible piezomagnetic effect under uniaxial compression and tension (NAGATA 1970a, STACEY and BANERJEE 1974):

$$\left.\begin{aligned} \Delta J^{/\!/} &= \frac{J_0^{/\!/}}{1+\beta\sigma} \cong J_0^{/\!/}(1-\beta\sigma) \\ \Delta J^{\perp} &= \frac{J_0^{\perp}}{1-\frac{1}{2}\beta\sigma} \cong J_0^{\perp}\left(1+\frac{1}{2}\beta\sigma\right) \end{aligned}\right\} \tag{2·1}$$

where superscripts $/\!/$ and $\perp$ denote the magnetization component (induced plus hard remanence) parallel and perpendicular to the applied stress respectively, while the subscript 0 indicates the unstressed state. The compressive force is taken to be positive. Since the stress sensitivity β has an order of magnitude of 10^{-4} in units of bar^{-1}, the linear relationship on the right-hand side of eq. (2·1) is approximately valid within the stress range up to several hundred bars.

Eqs. (2·1) were extended to the general three dimensional stress state by STACEY, BARR and ROBSON (1965). We may resolve the magnetization J_0 into orthogonal three components $(J_1, J_2, J_3)^t$ in directions of principal stresses and apply the relations (2·1) to each component. The stress-induced magnetization in the i-th principal axis direction can be represented by (STACEY *et al.* 1965)

$$\left.\Delta J_i \boldsymbol{e}_i = \beta J_i\left(\frac{\sigma_j+\sigma_k}{2}-\sigma_i\right)\boldsymbol{e}_i \quad (i, j, k=1, 2, 3. \quad i\neq j\neq k.)\right\} \tag{2·2}$$

where J_i and ΔJ_i are the magnetization and its increment in the i-th principal axis direction, while $\boldsymbol{e}_i$ indicates a unit vector diriction of the principal stress σ_i. In the tectonomagnetic modelling hitherto made, eq. (2·2) has been directly applied; we have to work out values of principal stresses as well as their direction cosines at every point in the magnetized region. For the general three dimensional stress state, this is not an easy matter. The present writer obtained a linear relation among the magnetization change and stress components for the axially symmetric problem with respect to z axis (SASAI 1969). A similar relation for any arbitrary stress distribution will be derived here.

Let us put the stress tensor $\boldsymbol{T}$ in the Cartesian coordinates as follows:

$$\boldsymbol{T}=\begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \tau_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{pmatrix}. \tag{2·3}$$

This symmetric matrix can be transformed into a diagonal one by a modal matrix $\boldsymbol{P}$:

$$\boldsymbol{P}^{-1}\boldsymbol{T}\boldsymbol{P}=\begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{pmatrix}. \tag{2·4}$$

Column vectors of $\boldsymbol{P}$ consist of eigen vectors of $\boldsymbol{T}$ corresponding to principal stresses σ_1, σ_2 and σ_3. We may write $\boldsymbol{P}$ as

$$\boldsymbol{P}=\begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \mu_1 & \mu_2 & \mu_3 \\ \nu_1 & \nu_2 & \nu_3 \end{pmatrix}. \tag{2·5}$$

Since the modal matrix $\boldsymbol{P}$ is normalized and orthogonal, these components must satisfy the following conditions:

$$\left.\begin{array}{l} \lambda_1^2+\lambda_2^2+\lambda_3^2=1, \quad \mu_1^2+\mu_2^2+\mu_3^2=1, \quad \nu_1^2+\nu_2^2+\nu_3^2=1 \\ \lambda_1\mu_1+\lambda_2\mu_2+\lambda_3\mu_3=0, \quad \lambda_1\nu_1+\lambda_2\nu_2+\lambda_3\nu_3=0, \quad \mu_1\nu_1+\mu_2\nu_2+\mu_3\nu_3=0 \end{array}\right\}. \tag{2·6}$$

The components of any arbitrary vector expressed in the principal axis coordinates $(c_1, c_2, c_3)^t$ are related to those in the original Cartesian coordinates $(c_x, c_y, c_z)^t$ in the following way:

$$\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}=\boldsymbol{P}^{-1}\cdot\begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}=\begin{pmatrix} \lambda_1 & \mu_1 & \nu_1 \\ \lambda_2 & \mu_2 & \nu_2 \\ \lambda_3 & \mu_3 & \nu_3 \end{pmatrix}\cdot\begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}. \tag{2·7}$$

This involves the transformation relationship among orthonormal bases of both coordinates:

$$\left.\begin{array}{l} \boldsymbol{e}_1=\lambda_1\boldsymbol{e}_x+\mu_1\boldsymbol{e}_y+\nu_1\boldsymbol{e}_z \\ \boldsymbol{e}_2=\lambda_2\boldsymbol{e}_x+\mu_2\boldsymbol{e}_y+\nu_2\boldsymbol{e}_z \\ \boldsymbol{e}_3=\lambda_3\boldsymbol{e}_x+\mu_2\boldsymbol{e}_y+\nu_3\boldsymbol{e}_z \end{array}\right\}. \tag{2·8}$$

Let us first consider the magnetization changes which the horizontal magnetization in x direction suffers. Substituting $(J_1, J_2, J_3)^t=\boldsymbol{P}^{-1}\cdot(J_x, 0, 0)^t$

into the fundamental equation (2·2), we obtain each component of the magnetization change:

$$\left.\begin{aligned} \varDelta J_1\boldsymbol{e}_1&=\beta\lambda_1T_1J_x\boldsymbol{e}_1\\ \varDelta J_2\boldsymbol{e}_2&=\beta\lambda_2T_2J_x\boldsymbol{e}_2\\ \varDelta J_3\boldsymbol{e}_3&=\beta\lambda_3T_3J_x\boldsymbol{e}_3 \end{aligned}\right\} \qquad (2\cdot9)$$

where

$$\left.\begin{aligned} &T_i=\frac{1}{2}\Theta-\frac{3}{2}\sigma_i\\ &(i=1,2,3)\\ &\Theta=\sigma_1+\sigma_2+\sigma_3 \end{aligned}\right\}. \qquad (2\cdot10)$$

With the aid of relations (2·8), the total increment of the magnetization can be given as

$$\left.\begin{aligned} \varDelta\boldsymbol{M}_x&=\varDelta J_1\boldsymbol{e}_1+\varDelta J_2\boldsymbol{e}_2+\varDelta J_3\boldsymbol{e}_3\\ &=\beta J_x(S_{xx}\boldsymbol{e}_x+S_{xy}\boldsymbol{e}_y+S_{xz}\boldsymbol{e}_z) \end{aligned}\right\} \qquad (2\cdot11)$$

where

$$\left.\begin{aligned} S_{xx}&=\lambda_1^2T_1+\lambda_2^2T_2+\lambda_3^2T_3\\ S_{xy}&=\lambda_1\mu_1T_1+\lambda_2\mu_2T_2+\lambda_3\mu_3T_3\\ S_{xz}&=\lambda_1\nu_1T_1+\lambda_2\nu_2T_2+\lambda_3\nu_3T_3 \end{aligned}\right\}. \qquad (2\cdot12)$$

In quite the same manner, changes in the magnetization $(0, J_y, 0)^t$ and $(0, 0, J_z)^t$, denoted by $\varDelta\boldsymbol{M}_y$ and $\varDelta\boldsymbol{M}_z$ respectively, can be obtained:

$$\left.\begin{aligned} \varDelta\boldsymbol{M}_y&=\beta J_y(S_{yx}\boldsymbol{e}_x+S_{yy}\boldsymbol{e}_y+S_{yz}\boldsymbol{e}_z)\\ \varDelta\boldsymbol{M}_z&=\beta J_z(S_{zx}\boldsymbol{e}_x+S_{zy}\boldsymbol{e}_y+S_{zz}\boldsymbol{e}_z) \end{aligned}\right\} \qquad (2\cdot13)$$

where

$$\left.\begin{aligned} S_{yx}&=S_{xy}\\ S_{yy}&=\mu_1^2T_1+\mu_2^2T_2+\mu_3^2T_3\\ S_{yz}&=\mu_1\nu_1T_1+\mu_2\nu_2T_2+\mu_3\nu_3T_3\\ S_{zx}&=S_{xz}\\ S_{zy}&=S_{yz}\\ S_{zz}&=\nu_1^2T_1+\nu_2^2T_2+\nu_3^2T_3 \end{aligned}\right\}. \qquad (2\cdot14)$$

Next, we will investigate expressions for S_{xx}, S_{xy} and so on in terms of stress components. Since the trace of a matrix is invariant with respect to the orthogonal transformation, the following relation holds good:

$$\Theta=\sigma_1+\sigma_2+\sigma_3=\tau_{xx}+\tau_{yy}+\tau_{zz} \tag{2·15}$$

Eq. (2·4) can be rewritten as

$$\boldsymbol{T}\cdot\boldsymbol{P}=\boldsymbol{P}\cdot\begin{pmatrix}\sigma_1 & 0 & 0\\ 0 & \sigma_2 & 0\\ 0 & 0 & \sigma_3\end{pmatrix}. \tag{2·16}$$

The first row of this matrix equation is equivalent to the following equations:

$$\left.\begin{aligned}\lambda_1\tau_{xx}+\mu_1\tau_{xy}+\nu_1\tau_{xz}=\lambda_1\sigma_1\\ \lambda_2\tau_{xx}+\mu_2\tau_{xy}+\nu_2\tau_{xz}=\lambda_2\sigma_2\\ \lambda_3\tau_{xx}+\mu_3\tau_{xy}+\nu_3\tau_{xz}=\lambda_3\sigma_3\end{aligned}\right\} \tag{2·17}$$

Substitutions of (2·15) and (2·17) into (2·12) lead us to the following formulae:

$$\begin{aligned}S_{xx}&=\frac{1}{2}(\lambda_1^2+\lambda_2^2+\lambda_3^2)\Theta-\frac{3}{2}(\lambda_1^2\sigma_1+\lambda_2^2\sigma_2+\lambda_3^2\sigma_3)\\ &=\frac{1}{2}(\tau_{yy}+\tau_{zz})-\tau_{xx}\\ S_{xy}&=\frac{1}{2}(\lambda_1\mu_1+\lambda_2\mu_2+\lambda_3\mu_3)\Theta-\frac{3}{2}(\lambda_1\mu_1\sigma_1+\lambda_2\mu_2\sigma_2+\lambda_3\mu_3\sigma_3)\\ &=-\frac{3}{2}\tau_{xy}\\ S_{xz}&=\frac{1}{2}(\lambda_1\nu_1+\lambda_2\nu_2+\lambda_3\nu_3)\Theta-\frac{3}{2}(\lambda_1\nu_1\sigma_1+\lambda_2\nu_2\sigma_2+\lambda_3\nu_3\sigma_3)\\ &=-\frac{3}{2}\tau_{xz}\end{aligned}$$

in which the orthonormal conditions (2·6) are used. By making use of the second and third row of eq. (2·16), we obtain

$$\begin{aligned}S_{yy}&=\frac{1}{2}(\tau_{xx}+\tau_{zz})-\tau_{yy}\\ S_{yz}&=-\frac{3}{2}\tau_{yz}\\ S_{zz}&=\frac{1}{2}(\tau_{xx}+\tau_{yy})-\tau_{zz}\end{aligned}$$

These results can be summarized in the following compact form:

$$\left.\begin{aligned}\begin{pmatrix}\Delta J_x\\ \Delta J_y\\ \Delta J_z\end{pmatrix}&=\beta\begin{pmatrix}S_{xx} & S_{yx} & S_{zx}\\ S_{xy} & S_{yy} & S_{zy}\\ S_{xz} & S_{yz} & S_{zz}\end{pmatrix}\cdot\begin{pmatrix}J_x\\ J_y\\ J_z\end{pmatrix}\\ &=\beta\begin{pmatrix}\dfrac{\tau_{yy}+\tau_{zz}}{2}-\tau_{xx}, & -\dfrac{3}{2}\tau_{xy}, & -\dfrac{3}{2}\tau_{xz}\\ -\dfrac{3}{2}\tau_{xy}, & \dfrac{\tau_{xx}+\tau_{zz}}{2}-\tau_{yy}, & -\dfrac{3}{2}\tau_{yz}\\ -\dfrac{3}{2}\tau_{xz}, & -\dfrac{3}{2}\tau_{yz}, & \dfrac{\tau_{xx}+\tau_{yy}}{2}-\tau_{zz}\end{pmatrix}\cdot\begin{pmatrix}J_x\\ J_y\\ J_z\end{pmatrix}\end{aligned}\right\}\quad(2\cdot18)$$

The linear relationship (2·18) between the magnetization change and stress components holds even when β and $\boldsymbol{J}$ differ from place to place. However, it should be kept in mind that the formula is applicable only in a relatively low stress range of up to several hundred bars.

The generalized linear piezomagnetic formula (2·18) is equivalent to STACEY, BARR and ROBSON's (1965) formulation of eq. (2·2), which is based upon the assumption that the principle of superposition applies to the reversible piezomagnetic effect. Eq. (2·18) is convenient for tectonomagnetic model studies, because we no longer need to estimate principal stresses and principal axis directions.

This type of formulation was first proposed by RIKITAKE (1966), who suggested the following relation among magnetization changes and strain components to be obtained by experimental studies:

$$\begin{aligned}\Delta J_x&=a_{xx}e_{xx}+a_{yy}e_{yy}+a_{zz}e_{zz}+a_{yz}e_{yz}+a_{zx}e_{zx}+a_{xy}e_{xy}\\ \Delta J_y&=b_{xx}e_{xx}+b_{yy}e_{yy}+b_{zz}e_{zz}+b_{yz}e_{yz}+b_{zx}e_{zx}+b_{xy}e_{xy}\\ \Delta J_z&=c_{xx}e_{xx}+c_{yy}e_{yy}+c_{zz}e_{zz}+c_{yz}e_{yz}+c_{zx}e_{zx}+c_{xy}e_{xy}\,.\end{aligned}$$

If we replace stress components τ_{mn}'s in eq. (2·18) with strain components e_{mn}'s through Hooke's law, we will find that the generalized piezomagnetic law (2·18) is nothing but a concrete form of Rikitake's expression.

The formula (2·18) should be examined from an experiment as well as from a more microscopic viewpoint. This might be derived from the basic concept of the reversible piezomagnetic effect, namely the rotation of the spontaneous magnetization of rock-forming minerals, as employed by NAGATA (1970b) and STACEY and JOHNSTON (1972). It should also be noticed that Rikitake's expression allows of the anisotropic piezomagnetic change against applied stresses. Recent experiments brought to light some anisotropic piezomagnetic behavior of compressed rocks (HENYEY, PIKE and PALMOR 1978). We deal with, however, very macroscopic models in tectonomagnetism, and we might expect that such anisotropy vanishes as a whole in averaging randomly-oriented small scale inhomogeneities within the crust.

The piezomagnetic field potential at a point (x, y, z) outside a stressed body can be given by the dipole law of force in the following way:

$$\left.\begin{aligned} W_x(x,y,z)&=\iiint_V \beta J_x(S_{xx}U'_x+S_{xy}U'_y+S_{xz}U'_z)dx'dy'dz' \\ W_y(x,y,z)&=\iiint_V \beta J_y(S_{yx}U'_x+S_{yy}U'_y+S_{yz}U'_z)dx'dy'dz' \\ W_z(x,y,z)&=\iiint_V \beta J_z(S_{zx}U'_x+S_{zy}U'_y+S_{zz}U'_z)dx'dy'dz' \end{aligned}\right\} \tag{2·19}$$

where

$$\left.\begin{aligned} U'_x&=(x-x')/r'^3 \\ U'_y&=(y-y')/r'^3 \\ U'_z&=(z-z')/r'^3 \\ r'&=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2} \end{aligned}\right\} \tag{2·20}$$

3. Volterra's formula for the piezomagnetic field potential

STEKETEE (1958a, b) introduced the elasticity theory of dislocations into geophysics. A dislocation surface, which is a surface of discontinuity in displacement within an elastic medium, can be regarded as a distribution of 'nuclei of strain' (LOVE 1944). There exist six sets of strain nuclei to describe any type of dislocations. STEKETEE developed a Green's function method to include the effect of a stress-free plane boundary, and formulated displacement and stress field caused by a dislocation surface placed in a semi-infinite elastic medium. The analytical expressions for all these Green's functions were obtained by MARUYAMA (1964). PRESS (1965) showed that the same results are derivable from combinations of MINDLIN and CHENG's (1950) solutions for various strain nuclei in the semi-infinite solid. In this paper, only relevant results will be quoted, mainly from MARUYAMA's (1964) work. The derivation of fundamental formulae and their physical meaning should be referred to these authors (i.e. STEKETEE 1958a, b, MARUYAMA 1964, PRESS 1965).

We take the Cartesian coordinates (x_1, x_2, x_3) as shown in Fig. 1. A semi-infinite elastic body occupies $x_3>0$. The elastic properties are isotropic and homogeneous throughout the body. It is also assumed that

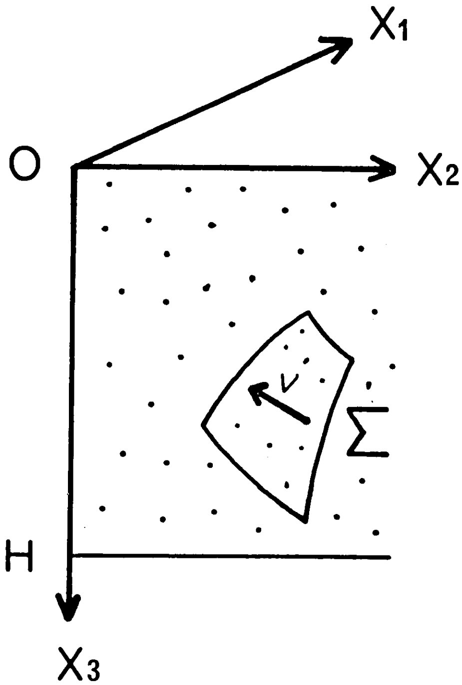

Fig. 1. The coordinate system and a dislocation surface Σ with its outward normal ν.

the top layer from the plane surface boundary $x_3=0$ to a depth of Currie point isotherm $x_3=H$ is uniformly magnetized, the stress sensitivity β being constant within the layer.

Let us consider a dislocation surface Σ in the semi-infinite elastic medium. A Somigliana dislocation is defined as a discontinuity in displacements across the surface Σ, $\Delta u_k=u_k^+-u_k^-$, which may have any form as long as the forces to maintain the dislocation satisfy continuity conditions across Σ: $\tau_{kl}^+\nu_l-\tau_{kl}^-\nu_l=0$. A point on the dislocation surface is designated by $P(\xi_1, \xi_2, \xi_3)$. The stress field produced by the dislocation $\Delta u_k(P)$ at an arbitrary point $Q(x_1, x_2, x_3)$ in the elastic medium can be presented by the following Volterra's formula:

$$\left.\begin{array}{c}\tau_{mn}(Q)=\iint\limits_{\Sigma}\Delta u_k(P)H_{kl}^{mn}\nu_l(P)d\Sigma \\ (k, l=1, 2, 3. \quad m, n=x, y, z.)\end{array}\right\} \qquad (3\cdot1)$$

where $\nu_l(P)$ denotes a component of the outward normal to the surface element $d\Sigma$. The summation convention applies with respect to k and l in the above. A fourth rank tensor H_{kl}^{mn} indicates stress component of a certain strain nucleus, which is called an elementary dislocation. k and l specify the type and orientation of the elementary dislocation. Strain nuclei with $k=l$ represent displacements normal to the dislocation surface Σ, while those with $k \neq l$ describe parallel ones along Σ. STEKETEE (1958b) called the former A nuclei and the latter B nuclei. These can be expressed by combinations of double forces as shown schematically in Fig. 2.

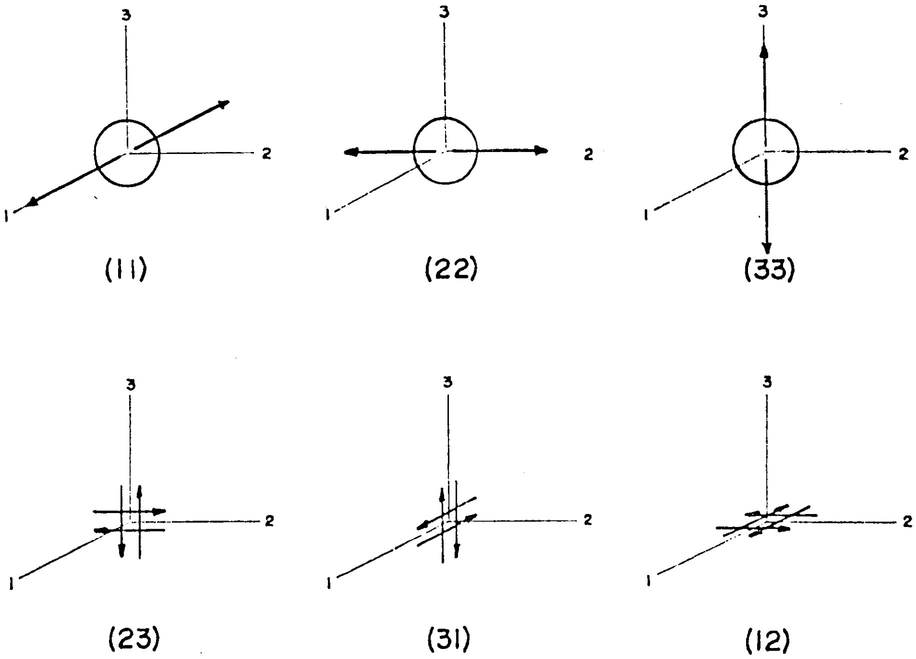

Fig. 2. A schematic representation of elementary dislocations (after MARUYAMA 1964). The A nuclei, (11), (22) and (33), correspond to the crack-forming movement, represented by the center of dilatation (circle) and the double force without moment (arrows). The B nuclei, (23), (31) and (12), can describe the shearing offset of the dislocation surface, represented by two co-planar, mutually perpendicular double forces with moment.

We substitute the stress components (3·1) into the general formula of piezomagnetic potential (2·19). Interchanging the order of integration with respect to P and Q, we obtain

$$\left.\begin{aligned} W_x &= \beta J_x \iint_{\Sigma} \Delta u_k(P) w^x_{kl} \nu_l(P) d\Sigma \\ W_y &= \beta J_y \iint_{\Sigma} \Delta u_k(P) w^y_{kl} \nu_l(P) d\Sigma \\ W_z &= \beta J_z \iint_{\Sigma} \Delta u_k(P) w^z_{kl} \nu_l(P) d\Sigma \end{aligned}\right\} \tag{3·2}$$

where

$$\left.\begin{aligned}
w_{kl}^{x}&=\int_0^H dx_3\iint_{-\infty}^{\infty}[S_{kl}^{xx}U_x'+S_{kl}^{xy}U_y'+S_{kl}^{xz}U_z']dx_1dx_2\\
w_{kl}^{y}&=\int_0^H dx_3\iint_{-\infty}^{\infty}[S_{kl}^{yx}U_x'+S_{kl}^{yy}U_y'+S_{kl}^{yz}U_z']dx_1dx_2\\
w_{kl}^{z}&=\int_0^H dx_3\iint_{-\infty}^{\infty}[S_{kl}^{zx}U_x'+S_{kl}^{zy}U_y'+S_{kl}^{zz}U_z']dx_1dx_2
\end{aligned}\right\} \quad (3\cdot3)$$

and

$$\left.\begin{aligned}
&S_{kl}^{mn}=\frac{1}{2}\delta_{mn}H_{kl}^{jj}-\frac{3}{2}H_{kl}^{mn}\\
\text{or}\quad &S_{kl}^{xx}=\frac{1}{2}(H_{kl}^{yy}+H_{kl}^{zz})-H_{kl}^{xx},\quad S_{kl}^{xy}=-\frac{3}{2}H_{kl}^{xy},\quad S_{kl}^{xz}=-\frac{3}{2}H_{kl}^{xz}\\
&\text{etc.}
\end{aligned}\right\} \quad (3\cdot4)$$

$$\left.\begin{aligned}
&U_x'=(x-x_1)/\rho'^3,\quad U_y'=(y-x_2)/\rho'^3,\quad U_z'=(z-x_3)/\rho'^3\\
&\rho'=\sqrt{(x-x_1)^2+(y-x_2)^2+(z-x_3)^2}
\end{aligned}\right\} \quad (3\cdot5)$$

Since eqs. (3·2) have the same form as eq. (3·1), we may call each of eqs. (3·2) Volterra's formula for the piezomagnetic field potential. w_{kl}^{x}, w_{kl}^{y} and w_{kl}^{z} are piezomagnetic potential due to an elementary dislocation of the type (kl). We will call them the elementary piezomagnetic potentials. Analytical expressions for these Green's functions will be derived in the next section.

4. Elementary piezomagnetic potential

We assume that the elementary dislocation is located at $(0, 0, \xi_3)$. Replacing x and y by $(x-\xi_1)$ and $(y-\xi_2)$ in the final expressions, we can arrive at corresponding results for any arbitrary point $P(\xi_1, \xi_2, \xi_3)$. We will evaluate following convolution integrals with respect to x_1 and x_2 in eqs. (3·3):

$$\left.\begin{aligned}
\omega_{kl}^{x}&=\iint_{-\infty}^{\infty}[S_{kl}^{xx}\cdot U_x'+S_{kl}^{xy}\cdot U_y'+S_{kl}^{xz}\cdot U_z']dx_1dx_2\\
\omega_{kl}^{y}&=\iint_{-\infty}^{\infty}[S_{kl}^{yx}\cdot U_x'+S_{kl}^{yy}\cdot U_y'+S_{kl}^{yz}\cdot U_z']dx_1dx_2\\
\omega_{kl}^{z}&=\iint_{-\infty}^{\infty}[S_{kl}^{zx}\cdot U_x'+S_{kl}^{zy}\cdot U_y'+S_{kl}^{zz}\cdot U_z']dx_1dx_2
\end{aligned}\right\} \quad (4\cdot1)$$

These can be solved analytically with the aid of the Fourier transform theorem of convolution integrals, which results in

$$\left.\begin{aligned}\frac{1}{2\pi}\bar{\omega}^x_{kl}&=\bar{S}^{xx}_{kl}\cdot\bar{U}_x+\bar{S}^{xy}_{kl}\cdot\bar{U}_y+\bar{S}^{xz}_{kl}\cdot\bar{U}_z\\ \frac{1}{2\pi}\bar{\omega}^y_{kl}&=\bar{S}^{yx}_{kl}\cdot\bar{U}_x+\bar{S}^{yy}_{kl}\cdot\bar{U}_y+\bar{S}^{yz}_{kl}\cdot\bar{U}_z\\ \frac{1}{2\pi}\bar{\omega}^z_{kl}&=\bar{S}^{zx}_{kl}\cdot\bar{U}_x+\bar{S}^{zy}_{kl}\cdot\bar{U}_y+\bar{S}^{zz}_{kl}\cdot\bar{U}_z\end{aligned}\right\}\tag{4·2}$$

where the bar above each function implies that it is a Fourier integral transform. $\bar{S}^{mn}_{kl}$'s can be obtained by combining $\bar{H}^{mn}_{kl}$ through eqs. (3·4). What we really need in calculating (4·1) is, therefore, not the stress components themselves, but their Fourier transforms. Instead of directly transforming H^{mn}_{kl}, we will apply the operational rules to these stress components as expressed in terms of appropriate Galerkin vector stress functions.

The displacement and stress field due to an elementary dislocation, or a particular nucleus of strain, can be derived from a Galerkin vector $\boldsymbol{\Gamma}(\Gamma_1, \Gamma_2, \Gamma_3)$. The Galerkin vector satisfies the biharmonic equation $\nabla^2\nabla^2\boldsymbol{\Gamma}=0$, which is equivalent to the equation of elastic equilibrium when there is no body force. The displacement is defined as

$$\left.\begin{aligned}&\boldsymbol{u}=(\nabla^2-\alpha\,\mathrm{grad}\cdot\mathrm{div})\boldsymbol{\Gamma}\\ \text{where}\quad&\alpha=\frac{\lambda+\mu}{\lambda+2\mu}\end{aligned}\right\}\tag{4·3}$$

while components of the stress tensor are related to the elastic displacement through Hooke's law:

$$\tau_{mn}=\lambda\delta_{mn}\,\mathrm{div}\,\boldsymbol{u}+\mu\left(\frac{\partial u_m}{\partial x_n}+\frac{\partial u_n}{\partial x_m}\right)\quad(m, n=x, y, z)\,.\tag{4·4}$$

Substituting τ_{mn} into eqs. (3·4) in place of H^{mn}_{kl}, we get the following expressions for S^{mn}_{kl} in terms of the Galerkin vector:

$$\left.\begin{aligned}&\frac{1}{\mu}S^{mn}_{kl}=\left[(1-\alpha)\delta_{mn}\nabla^2+3\alpha\frac{\partial^2}{\partial x_m\partial x_n}\right]\cdot\varDelta_{kl}-\frac{3}{2}\nabla^2\left(\frac{\partial\Gamma^m_{kl}}{\partial x_n}+\frac{\partial\Gamma^n_{kl}}{\partial x_m}\right)\\ \text{where}\quad&\varDelta_{kl}=\mathrm{div}\,\boldsymbol{\Gamma}_{kl}=\frac{\partial\Gamma^1_{kl}}{\partial x_1}+\frac{\partial\Gamma^2_{kl}}{\partial x_2}+\frac{\partial\Gamma^3_{kl}}{\partial x_3}\,.\end{aligned}\right\}\tag{4·5}$$

We will employ the following definition for the Fourier transform of a function $f(x_1, x_2)$ and its inverse as

$$\left.\begin{aligned}\bar{f}(k_1, k_2)&=\frac{1}{2\pi}\iint_{-\infty}^{\infty}f(x_1, x_2)e^{-i(k_1x_1+k_2x_2)}dx_1dx_2\\ f(x_1, x_2)&=\frac{1}{2\pi}\iint_{-\infty}^{\infty}\bar{f}(k_1, k_2)e^{i(k_1x_1+k_2x_2)}dk_1dk_2\end{aligned}\right\}\tag{4·6}$$

Under proper conditions of derivatives of $f(x_1, x_2)$ when x_1, x_2 approaches infinity, we get

$$\left.\begin{aligned} &\overline{\left(\frac{\partial^r f}{\partial x_1^r}\right)}=(ik_1)^r\bar{f}, \qquad \overline{\left(\frac{\partial^r f}{\partial x_2^r}\right)}=(ik_2)^r\bar{f} \\ &\overline{\left(\frac{\partial^r f}{\partial x_3^r}\right)}=p^r\bar{f} \qquad \left(p\equiv\frac{\partial}{\partial x_3}\right) \end{aligned}\right\} \tag{4·7}$$

The operational rules (4·7) are then applied to eqs. (4·5), yielding the Fourier transforms of S_{kl}^{mn}:

$$\left.\begin{aligned} \frac{1}{\mu}\bar{S}_{kl}^{mn}=&\left[\left\{(1-\alpha)\delta_{mn}ik_1-\frac{3}{2}E(m,n,1)\right\}(p^2-k_1^2-k_2^2)+3\alpha D(m,n)ik_1\right]\bar{\Gamma}_{kl}^1 \\ &+\left[\left\{(1-\alpha)\delta_{mn}ik_2-\frac{3}{2}E(m,n,2)\right\}(p^2-k_1^2-k_2^2)+3\alpha D(m,n)ik_2\right]\bar{\Gamma}_{kl}^2 \\ &+\left[\left\{(1-\alpha)\delta_{mn}p-\frac{3}{2}E(m,n,3)\right\}(p^2-k_1^2-k_2^2)+3\alpha D(m,n)p\right]\bar{\Gamma}_{kl}^3 \end{aligned}\right\} \tag{4·8}$$

where

$$\left.\begin{aligned} D(m,n)=&-\delta_{m1}\delta_{n1}k_1^2-\delta_{m1}\delta_{n2}k_1k_2-\delta_{m2}\delta_{n2}k_2^2 \\ &+\delta_{m1}\delta_{n3}ik_1p+\delta_{m2}\delta_{n3}ik_2p+\delta_{m3}\delta_{n3}p^2 \\ E(m,n,j)=&\,\delta_{mj}E'(n)+\delta_{nj}E'(m) \\ E'(l)=&\,\delta_{l1}ik_1+\delta_{l2}ik_2+\delta_{l3}p \end{aligned}\right\} \tag{4·9}$$

From symmetry considerations, some of the results in the following may be easily obtained by interchanging coordinates x_1 and x_2 and correspondingly k_1 and k_2. We will calculate, however, all the combinations of (kl) and (mn) independently in order to check for errors.

STEKETEE (1958a, b) showed that the displacement and stress field in the *infinite* medium due to an elementary dislocation of the type (kl) at (ξ_1, ξ_2, ξ_3) can be derived from a Galerkin vector:

$$8\pi\mu\Gamma_{kl}^m=-\lambda\delta_{kl}\frac{\partial r}{\partial x_m}-\mu\left[\delta_{mk}\frac{\partial r}{\partial x_l}+\delta_{ml}\frac{\partial r}{\partial x_k}\right] \tag{4·10}$$

where

$$r=\sqrt{(x_1-\xi_1)^2+(x_2-\xi_2)^2+(x_3-\xi_3)^2}$$

The Galerkin vector in the *semi-infinite* medium due to a dislocation (kl) at $P(0,0,\xi_3)$ consists of three parts corresponding to:

(i) double force (kl) at $P(0,0,\xi_3)$,

(ii) double force (kl) at a mirror point $P(0,0,-\xi_3)$ with equal intensity, of which a sign should be selected so as to cancel the tangential shear stress due to (i) at the surface boundary $x_3=0$,

(iii) normal load on the plane $x_3=0$, which nullifies the resultant normal stress due to (i) plus (ii).

We can readily construct (i) plus (ii) from the fundamental Galerkin vector in the infinite medium (4·10). These are as follows:

$$
\left.
\begin{array}{ll}
\left\{
\begin{aligned}
\Gamma_{11}^{1} &= -\frac{1}{8\pi}\left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{x_1}{R}+\frac{x_1}{S}\right)\\
\Gamma_{11}^{2} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_2}{R}+\frac{x_2}{S}\right)\\
\Gamma_{11}^{3} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_3-\xi_3}{R}+\frac{x_3+\xi_3}{S}\right)
\end{aligned}
\right.
&
\left\{
\begin{aligned}
\Gamma_{22}^{1} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_1}{R}+\frac{x_1}{S}\right)\\
\Gamma_{22}^{2} &= -\frac{1}{8\pi}\left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{x_2}{R}+\frac{x_2}{S}\right)\\
\Gamma_{22}^{3} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_3-\xi_3}{R}+\frac{x_3+\xi_3}{S}\right)
\end{aligned}
\right.
\\[2ex]
\left\{
\begin{aligned}
\Gamma_{33}^{1} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_1}{R}+\frac{x_1}{S}\right)\\
\Gamma_{33}^{2} &= -\frac{1}{8\pi}\left(\frac{\lambda}{\mu}\right)\left(\frac{x_2}{R}+\frac{x_2}{S}\right)\\
\Gamma_{33}^{3} &= -\frac{1}{8\pi}\left(\frac{\lambda+2\mu}{\mu}\right)\left(\frac{x_3-\xi_3}{R}+\frac{x_3+\xi_3}{S}\right)
\end{aligned}
\right.
&
\left\{
\begin{aligned}
\Gamma_{23}^{1} &= 0\\
\Gamma_{23}^{2} &= -\frac{1}{8\pi}\left(\frac{x_3-\xi_3}{R}-\frac{x_3+\xi_3}{S}\right)\\
\Gamma_{23}^{3} &= -\frac{1}{8\pi}\left(\frac{x_2}{R}-\frac{x_2}{S}\right)
\end{aligned}
\right.
\\[2ex]
\left\{
\begin{aligned}
\Gamma_{31}^{1} &= -\frac{1}{8\pi}\left(\frac{x_3-\xi_3}{R}-\frac{x_3+\xi_3}{S}\right)\\
\Gamma_{31}^{2} &= 0\\
\Gamma_{31}^{3} &= -\frac{1}{8\pi}\left(\frac{x_1}{R}-\frac{x_1}{S}\right)
\end{aligned}
\right.
&
\left\{
\begin{aligned}
\Gamma_{12}^{1} &= -\frac{1}{8\pi}\left(\frac{x_2}{R}+\frac{x_2}{S}\right)\\
\Gamma_{12}^{2} &= -\frac{1}{8\pi}\left(\frac{x_1}{R}+\frac{x_1}{S}\right)\\
\Gamma_{12}^{3} &= 0
\end{aligned}
\right.
\end{array}
\right\}
\quad (4\cdot 11)
$$

where

$$R=\sqrt{x_1^2+x_2^2+(x_3-\xi_3)^2}, \qquad S=\sqrt{x_1^2+x_2^2+(x_3+\xi_3)^2}, \qquad (4\cdot 12)$$

The third part (iii) can be obtained by solving the Boussinesq problem. The solution is expressed by a Galerkin vector of the form $(0, 0, \bar{\Gamma}_{kl})$. The problem was actually solved by the Fourier transform method (STEKETEE 1958a), so that $\bar{\Gamma}_{kl}$'s have already been worked out by MARUYAMA (1964). His results for $\bar{\Gamma}_{kl}$'s are entirely available for the present purpose (MARUYAMA, 1964, p. 315):

$$
\begin{aligned}
\bar{\Gamma}_{11} &= \frac{1}{2\pi}\Bigg[\left(2-\frac{1}{\alpha}\right)\frac{k_1^2}{k^4}+\left(2-\frac{1}{\alpha}\right)^2\frac{k_2^2}{k^4}+\left\{-\left(2-\frac{1}{\alpha}\right)\xi_3+x_3\right\}\frac{k_1^2}{k^3}\\
&\qquad +\left(2-\frac{1}{\alpha}\right)x_3\frac{k_2^2}{k^3}-\xi_3 x_3\frac{k_1^2}{k^2}\Bigg]e^{-k\xi_3}\\
\bar{\Gamma}_{22} &= \frac{1}{2\pi}\Bigg[\left(2-\frac{1}{\alpha}\right)^2\frac{k_1^2}{k^4}+\left(2-\frac{1}{\alpha}\right)\frac{k_2^2}{k^4}+\left(2-\frac{1}{\alpha}\right)x_3\frac{k_1^2}{k^3}
\end{aligned}
$$

$$\left.\begin{aligned}
&+\left\{-\left(2-\frac{1}{\alpha}\right)\xi_3+x_3\right\}\frac{k_2^2}{k^3}-\xi_3 x_3\frac{k_2^2}{k^2}\Bigg]e^{-k\zeta_2}\\
&\Gamma_{33}=\frac{1}{2\pi}\left[\left(2-\frac{1}{\alpha}\right)\frac{1}{k^2}+\left\{\left(2-\frac{1}{\alpha}\right)\xi_3+x_3\right\}\frac{1}{k}+\xi_3 x_3\right]e^{-k\zeta_2}\\
&\Gamma_{23}=\frac{i}{2\pi}\left[\left(2-\frac{1}{\alpha}\right)\frac{\xi_3 k_2}{k^2}+\frac{k_2}{k}\xi_3 x_3\right]e^{-k\zeta_2}\\
&\Gamma_{31}=\frac{i}{2\pi}\left[\left(2-\frac{1}{\alpha}\right)\frac{\xi_3 k_1}{k^2}+\frac{k_1}{k}\xi_3 x_3\right]e^{-k\zeta_2}\\
&\Gamma_{12}=\frac{1}{2\pi}\Bigg[-\left(2-\frac{1}{\alpha}\right)\left(1-\frac{1}{\alpha}\right)\frac{k_1 k_2}{k^4}-\left\{\left(2-\frac{1}{\alpha}\right)\xi_3+\left(1-\frac{1}{\alpha}\right)x_3\right\}\\
&\qquad\times\frac{k_1 k_2}{k^3}-\xi_3 x_3\frac{k_1 k_2}{k^2}\Bigg]e^{-k\zeta_2}\\
&\text{where}\qquad \zeta_2=\xi_3+x_3
\end{aligned}\right\}\quad(4\cdot 13)$$

We are now in a position to obtain the Fourier transforms of Γ^m_{kl}. However, we run into a difficulty here; the Fourier transform of a function $f(x_1, x_2)=x_1/(x_1^2+x_2^2+c^2)^{1/2}$ cannot be defined in the ordinary sense, because this function is not absolutely integrable in the domain $(-\infty<x_1, x_2<+\infty)$. Fourier transforms of such functions (what are called *slowly increasing functions*) should be considered in the meaning of *distribution*, or *generalized functions* (VLADIMIROV, 1971). We find, however, that the "distribution" term disappears in the final expression owing to the odd-

Table 1. Fourier transforms.

$\bar{f}_j=ik_j\left(\zeta_1\frac{1}{k^2}+\frac{1}{k^3}\right)e^{-k\zeta_1}$	$\bar{g}_j=ik_j\left(\zeta_2\frac{1}{k^2}+\frac{1}{k^3}\right)e^{-k\zeta_2}$
$\bar{f}_3=\mp\zeta_1\frac{1}{k}e^{-k\zeta_1}\,(x_3\gtrless\xi_3)$	$\bar{g}_3=-\zeta_2\frac{1}{k}e^{-k\zeta_2}$
$p\bar{f}_j=\mp\zeta_1\frac{ik_j}{k}e^{-k\zeta_1}\,(x_3\gtrless\xi_3)$	$p\bar{g}_j=-\zeta_2\frac{ik_j}{k}e^{-k\zeta_2}$
$p\bar{f}_3=\left(-\frac{1}{k}+\zeta_1\right)e^{-k\zeta_1}$	$p\bar{g}_3=\left(-\frac{1}{k}+\zeta_2\right)e^{-k\zeta_1}$
$p^2\bar{f}_j=ik_j\left(-\frac{1}{k}+\zeta_1\right)e^{-k\zeta_1}$	$p^2\bar{g}_j=ik_j\left(-\frac{1}{k}+\zeta_2\right)e^{-k\zeta_1}$
$p^2\bar{f}_3=\mp(-2+\zeta_1 k)e^{-k\zeta_1}\ \ (x_3\gtrless\xi_3)$	$p^2\bar{g}_3=(2-\zeta_2 k)e^{-k\zeta_2}$
$p^3\bar{f}_j=\mp ik_j(-2+\zeta_1 k)e^{-k\zeta_1}\ \ (x_3\gtrless\xi_3)$	$p^3\bar{g}_j=ik_j(2-\zeta_2 k)e^{-k\zeta_2}$
$p^3\bar{f}_3=(-3k+\zeta_1 k^2)e^{-k\zeta_1}$	$p^3\bar{g}_3=(-3k+\zeta_2 k^2)e^{-k\zeta_2}$

$f_1=-x_1/R,\quad f_2=-x_2/R,\quad f_3=-(x_3-\xi_3)/R,$
$g_1=-x_1/S,\quad g_2=-x_2/S,\quad g_3=-(x_3+\xi_3)/S,$
$\zeta_1=|x_3-\xi_3|,\quad \zeta_2=x_3+\xi_3,\quad i=\sqrt{-1},$
$j=1,\ 2$

ness of $f(x_1, x_2)$. We will clarify the circumstances in Appendix A by actually deriving $\bar{f}(x_1, x_2)$ (after MARUYAMA 1980; personal communication).

Table 2. Inverse Fourier transforms.

$\bar{f}(k_1, k_2)$	$f(x, y)$
$\frac{ik_1}{k}e^{-kc}$	$-\frac{x}{\rho^3}$
e^{-kc}	$\frac{c}{\rho^3}$
$\frac{k_1^2}{k}e^{-kc}$	$\frac{1}{\rho^3}-\frac{3x^2}{\rho^5}$
$\frac{k_1k_2}{k}e^{-kc}$	$-\frac{3xy}{\rho^5}$
ik_1e^{-kc}	$-\frac{3cx}{\rho^5}$
ke^{-kc}	$-\frac{1}{\rho^3}+\frac{3c^2}{\rho^5}$
$\frac{ik_1^3}{k}e^{-kc}$	$-\frac{9x}{\rho^5}+\frac{15x^3}{\rho^7}$
$\frac{ik_1^2k_2}{k}e^{-kc}$	$-\frac{3y}{\rho^5}+\frac{15x^2y}{\rho^7}$
$k_1k_2e^{-kc}$	$-\frac{15cxy}{\rho^7}$
$k_1^2e^{-kc}$	$\frac{3c}{\rho^5}-\frac{15cx^2}{\rho^7}$
k^2e^{-kc}	$-\frac{9c}{\rho^5}+\frac{15c^3}{\rho^7}$
ik_1ke^{-kc}	$\frac{3x}{\rho^5}-\frac{15c^2x}{\rho^7}$
$\frac{k_1^2}{k^2}e^{-kc}$	$\frac{1}{\rho(\rho+c)}-\frac{x^2(2\rho+c)}{\rho^3(\rho+c)^2}$
$\frac{k_1k_2}{k^2}e^{-kc}$	$-\frac{xy(2\rho+c)}{\rho^3(\rho+c)^2}$
$\frac{ik_1^3}{k^2}e^{-kc}$	$-\frac{3x(2\rho+c)}{\rho^3(\rho+c)^2}+\frac{2x^3}{\rho^3(\rho+c)^3}+\frac{3x^3(2\rho+c)}{\rho^5(\rho+c)^2}$
$\frac{ik_1^2k_2}{k^2}e^{-kc}$	$-\frac{y(2\rho+c)}{\rho^3(\rho+c)^2}+\frac{2x^2y}{\rho^3(\rho+c)^3}+\frac{3x^2y(2\rho+c)}{\rho^5(\rho+c)^2}$
$\frac{ik_1^3}{k^3}e^{-kc}$	$-\frac{3}{4}\frac{x}{\rho^3}+\frac{1}{4}\frac{x(x^2-3y^2)(3\rho+c)}{\rho^3(\rho+c)^3}$
$\frac{ik_1^2k_2}{k^3}e^{-kc}$	$-\frac{1}{4}\frac{y}{\rho^3}+\frac{1}{4}\frac{y(3x^2-y^2)(3\rho+c)}{\rho^3(\rho+c)^3}$
$\bar{f}(k_2, k_1)$	$f(y, x)$

Thus Fourier transforms (in the sense of distribution) of Γ^1_{kl}, Γ^2_{kl} and Γ^3_{kl} are obtained by the formulae in Table 1. We may conduct operational calculus by substituting Γ^1_{kl}, Γ^2_{kl} and $(\Gamma^3_{kl}+\Gamma_{kl})$ into eqs. (4·8) to yield $\bar{S}^{mn}_{kl}$. They are summarized in Appendix B1.

On the other hand, Fourier transforms of U_x, U_y and U_z are defined as

$$\left.\begin{aligned}&\bar{U}_x=-\frac{ik_1}{k}e^{-k\zeta}, \qquad \bar{U}_y=-\frac{ik_2}{k}e^{-k\zeta}, \qquad \bar{U}_z=-e^{-k\zeta}\\ &\text{where}\\ &\zeta=x_3-z\end{aligned}\right\} \tag{4·14}$$

Combining results in Appendix B1 and (4·14) through eqs. (4·2), we have $\bar{\omega}^x_{kl}$, $\bar{\omega}^y_{kl}$ and $\bar{\omega}^z_{kl}$, as shown in Appendix B2. Integrating those with respect to x_3 from 0 to H, we obtain $\bar{w}^x_{kl}$, $\bar{w}^y_{kl}$ and $\bar{w}^z_{kl}$, which will be found in Appendix B3. Finally, by making use of the inverse Fourier transform formulae in Table 2, we arrive at analytical expressions for the elementary piezomagnetic potential:

$(kl)=(11)$

$$\begin{aligned}\frac{2}{\mu}w^x_{11}=&\frac{3}{2}\left[-\frac{x}{\rho^3}\right]^1_3-\frac{(4\alpha-1)(\alpha-2)}{2\alpha}\left[-\frac{1}{4}\frac{x}{\rho^3}+\frac{1}{4}\frac{x(3y^2-x^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]^1_3\\
&+(\alpha-1)\xi_3\left[-\frac{3x(2\rho+c)}{\rho^3(\rho+c)^2}+\frac{2x^3}{\rho^3(\rho+c)^3}+\frac{3x^3(2\rho+c)}{\rho^5(\rho+c)^2}\right]^1_3\\
&+9\alpha H\left\{-\frac{3x(2\rho_3+c_3)}{\rho_3^3(\rho_3+c_3)^2}+\frac{2x^3}{\rho_3^3(\rho_3+c_3)^3}+\frac{3x^3(2\rho_3+c_3)}{\rho_3^5(\rho_3+c_3)^2}\right\}\\
&-6(1-2\alpha)H\left\{-\frac{x(2\rho_3+c_3)}{\rho_3^3(\rho_3+c_3)^2}+\frac{2xy^2}{\rho_3^3(\rho_3+c_3)^3}+\frac{3xy^2(2\rho_3+c_3)}{\rho_3^5(\rho_3+c_3)^2}\right\}\\
&-6\alpha H\xi_3\left\{-\frac{9x}{\rho_3^5}+\frac{15x^3}{\rho_3^7}\right\}\\
&+\begin{cases}(1-\alpha)H\left\{-\dfrac{3x(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^3}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^3(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\} & (H<\xi_3)\\[2ex]
\dfrac{1-4\alpha}{2}\left[-\dfrac{3}{4}\dfrac{x}{\rho^3}+\dfrac{1}{4}\dfrac{x(x^2-3y^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]^1_2\\
\quad+6\alpha\left[-\dfrac{x}{\rho^3}\right]^1_2\\
\quad+(1-\alpha)\xi_3\left\{-\dfrac{3x(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^3}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^3(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\}\\
\quad+3\alpha(H-\xi_3)\left\{-\dfrac{3x(2\rho_2+c_2)}{\rho_2^3(\rho_2+c_2)^2}+\dfrac{2x^3}{\rho_2^3(\rho_2+c_2)^3}+\dfrac{3x^3(2\rho_2+c_2)}{\rho_2^5(\rho_2+c_2)^2}\right\} & (H>\xi_3),\end{cases}\end{aligned} \tag{4·15a}$$

$$
\begin{aligned}
\frac{2}{\mu} w^{y}_{11} = & -\frac{3}{2}\left[-\frac{1}{4}\frac{y}{\rho^3}+\frac{1}{4}\frac{y(3x^2-y^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]_3^1 \\
& +\frac{(2\alpha-1)(1-\alpha)}{\alpha}\left[-\frac{3}{4}\frac{y}{\rho^3}+\frac{1}{4}\frac{y(y^2-3x^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]_3^1 \\
& +(\alpha-1)\xi_3\left[-\frac{y(2\rho+c)}{\rho^3(\rho+c)^2}+\frac{2x^2y}{\rho^3(\rho+c)^3}+\frac{3x^2y(2\rho+c)}{\rho^5(\rho+c)^2}\right]_3^1 \\
& +9\alpha H\left\{-\frac{y(2\rho_3+c_3)}{\rho_3^3(\rho_3+c_3)^2}+\frac{2x^2y}{\rho_3^3(\rho_3+c_3)^3}+\frac{3x^2y(2\rho_3+c_3)}{\rho_3^5(\rho_3+c_3)^2}\right\} \\
& -6(1-2\alpha)H\left\{-\frac{3y(2\rho_3+c_3)}{\rho_3^3(\rho_3+c_3)^2}+\frac{2y^3}{\rho_3^3(\rho_3+c_3)^3}+\frac{3y^3(2\rho_3+c_3)}{\rho_3^5(\rho_3+c_3)^2}\right\} \\
& -6\alpha H\xi_3\left\{-\frac{3y}{\rho_3^5}+\frac{15x^2y}{\rho_3^7}\right\} \\
& +\begin{cases}
(1-\alpha)H\left\{-\dfrac{y(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^2y}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^2y(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\} & (H<\xi_3) \\
\dfrac{1-4\alpha}{2}\left[-\dfrac{1}{4}\dfrac{y}{\rho^3}+\dfrac{1}{4}\dfrac{y(3x^2-y^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]_2^1 & \\
\quad +3(2\alpha-1)\left[-\dfrac{y}{\rho^3}\right]_2^1 & \\
\quad +(1-\alpha)\xi_3\left\{-\dfrac{y(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^2y}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^2y(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\} & \\
\quad +3\alpha(H-\xi_3)\left\{-\dfrac{y(2\rho_2+c_2)}{\rho_2^3(\rho_2+c_2)^2}+\dfrac{2x^2y}{\rho_2^3(\rho_2+c_2)^3}+\dfrac{3x^2y(2\rho_2+c_2)}{\rho_2^5(\rho_2+c_2)^2}\right\} & (H>\xi_3),
\end{cases} \qquad (4\cdot15\text{b})
\end{aligned}
$$

$$
\begin{aligned}
\frac{2}{\mu} w^{z}_{11} = & \frac{3}{2}(1-2\alpha)\left[\frac{1}{\rho(\rho+c)}-\frac{x^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1 \\
& +\frac{(2\alpha-1)(1-\alpha)}{\alpha}\left[\frac{1}{\rho(\rho+c)}-\frac{y^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1 \\
& +(\alpha-1)\xi_3\left[\frac{1}{\rho^3}-\frac{3x^2}{\rho^5}\right]_3^1 \\
& -9\alpha H\left\{\frac{1}{\rho_3^3}-\frac{3x^2}{\rho_3^5}\right\} \\
& +6(1-2\alpha)H\left\{\frac{1}{\rho_3^3}-\frac{3y^2}{\rho_3^5}\right\} \\
& +6\alpha H\xi_3\left\{\frac{3c_3}{\rho_3^5}-\frac{15c_3x^2}{\rho_3^7}\right\} \\
& +\begin{cases}
(1-\alpha)H\left\{\dfrac{1}{\rho_1^3}-\dfrac{3x^2}{\rho_1^5}\right\} & (H<\xi_3) \\
\dfrac{2\alpha-5}{2}\left[\dfrac{1}{\rho(\rho+c)}-\dfrac{x^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_2^1 &
\end{cases}
\end{aligned}
$$

$$+3(1-2\alpha)\left[\frac{c}{\rho^3}\right]_2^1$$

$$+(1-\alpha)\xi_3\left\{\frac{1}{\rho_1^3}-\frac{3x^2}{\rho_1^5}\right\}$$

$$-3\alpha(H-\xi_3)\left\{\frac{1}{\rho_2^3}-\frac{3x^2}{\rho_2^5}\right\} \qquad (H>\xi_3)\,. \tag{4·15c}$$

$(kl)=(22)$

$$w_{22}^x(x,y)=w_{11}^y(y,x)\,, \tag{4·16a}$$

$$w_{22}^y(x,y)=w_{11}^x(y,x)\,, \tag{4·16b}$$

$$w_{22}^z(x,y)=w_{11}^z(y,x)\,. \tag{4·16c}$$

$(kl)=(33)$

$$\frac{2}{\mu}w_{33}^x=\frac{1-4\alpha}{2}\left[-\frac{x}{\rho^3}\right]_3^1+(1-\alpha)\xi_3\left[-\frac{3cx}{\rho^5}\right]_3^1$$

$$+3\alpha H\left\{-\frac{3c_3x}{\rho_3^5}\right\}+6\alpha H\xi_3\left\{\frac{3x}{\rho_3^5}-\frac{15c_3^2x}{\rho_3^7}\right\}$$

$$+\begin{cases}(\alpha-1)H\left\{-\dfrac{3c_1x}{\rho_1^5}\right\} & (H<\xi_3)\\[2ex] \dfrac{4\alpha-1}{2}\left[-\dfrac{x}{\rho^3}\right]_2^1 & \\[2ex] \quad +(\alpha-1)\xi_3\left\{-\dfrac{3c_1x}{\rho_1^5}\right\}-3\alpha(H-\xi_3)\left\{-\dfrac{3c_2x}{\rho_2^5}\right\} & (H>\xi_3)\,,\end{cases} \tag{4·17a}$$

$$w_{33}^y(x,y)=w_{33}^x(y,x)\,, \tag{4·17b}$$

$$\frac{2}{\mu}w_{33}^z=\frac{1+2\alpha}{2}\left[\frac{c}{\rho^3}\right]_3^1+(1-\alpha)\xi_3\left[-\frac{1}{\rho^3}+\frac{3c^2}{\rho^5}\right]_3^1$$

$$-3\alpha H\left\{-\frac{1}{\rho_3^3}+\frac{3c_3^2}{\rho_3^5}\right\}-6\alpha H\xi_3\left\{-\frac{9c_3}{\rho_3^5}+\frac{15c_3^3}{\rho_3^7}\right\}$$

$$+\begin{cases}(\alpha-1)H\left\{-\dfrac{1}{\rho_1^3}+\dfrac{3c_1^2}{\rho_1^5}\right\} & (H<\xi_3)\\[2ex] -\dfrac{1+2\alpha}{2}\left[\dfrac{c}{\rho^3}\right]_2^1 & \\[2ex] \quad +(\alpha-1)\xi_3\left\{-\dfrac{1}{\rho_1^3}+\dfrac{3c_1^2}{\rho_1^5}\right\} & \\[2ex] \quad +3\alpha(H-\xi_3)\left\{-\dfrac{1}{\rho_2^3}+\dfrac{3c_2^2}{\rho_2^5}\right\} & (H>\xi_3)\,.\end{cases} \tag{4·17c}$$

$(kl)=(23)$

$$\frac{2}{\mu}w_{23}^{x}=(\alpha-1)\left[-\frac{xy(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1+(\alpha-1)\xi_3\left[-\frac{3xy}{\rho^5}\right]_3^1$$
$$+3\alpha H\left\{-\frac{3xy}{\rho_3^5}\right\}-6\alpha H\xi_3\left\{-\frac{15c_3xy}{\rho_3^7}\right\}$$
$$+\begin{cases}(1-\alpha)H\left\{-\dfrac{3xy}{\rho_1^5}\right\} & (H<\xi_3)\\ (1-\alpha)\left[-\dfrac{xy(2\rho+c)}{\rho^3(\rho+c)^2}\right]_2^1 & \\ \quad+(1-\alpha)\xi_3\left\{-\dfrac{3xy}{\rho_1^5}\right\}-3\alpha(H-\xi_3)\left\{-\dfrac{3xy}{\rho_2^5}\right\} & (H>\xi_3)\,,\end{cases} \tag{4·18a}$$

$$\frac{2}{\mu}w_{23}^{y}=\frac{3}{2}\left[\frac{1}{\rho(\rho+c)}-\frac{x^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1$$
$$+\frac{2\alpha+1}{2}\left[\frac{1}{\rho(\rho+c)}-\frac{y^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1$$
$$+(\alpha-1)\xi_3\left[\frac{1}{\rho^3}-\frac{3y^2}{\rho^5}\right]_3^1$$
$$+3\alpha H\left\{\frac{1}{\rho_3^3}-\frac{3y^2}{\rho_3^5}\right\}-6\alpha H\xi_3\left\{\frac{3c_3}{\rho_3^5}-\frac{15c_3y^2}{\rho_3^7}\right\}$$
$$+\begin{cases}(1-\alpha)H\left\{\dfrac{1}{\rho_1^3}-\dfrac{3y^2}{\rho_1^5}\right\} & (H<\xi_3)\\ -\dfrac{3}{2}\left[\dfrac{1}{\rho(\rho+c)}-\dfrac{x^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_2^1 & \\ \quad-\dfrac{2\alpha+1}{2}\left[\dfrac{1}{\rho(\rho+c)}-\dfrac{y^2(2\rho+c)}{\rho^3(\rho+c)^2}\right]_2^1 & \\ \quad+(1-\alpha)\xi_3\left\{\dfrac{1}{\rho_1^3}-\dfrac{3y^2}{\rho_1^5}\right\} & \\ \quad-3\alpha(H-\xi_3)\left\{\dfrac{1}{\rho_2^3}-\dfrac{3y^2}{\rho_2^5}\right\} & (H>\xi_3)\,,\end{cases} \tag{4·18b}$$

$$\frac{2}{\mu}w_{23}^{z}=\frac{4\alpha-1}{2}\left[-\frac{y}{\rho^3}\right]_3^1+(1-\alpha)\xi_3\left[-\frac{3cy}{\rho^5}\right]_3^1$$
$$+3\alpha H\left\{-\frac{3c_3y}{\rho_3^5}\right\}-6\alpha H\xi_3\left\{\frac{3y}{\rho_3^5}-\frac{15c_3^2y}{\rho_3^7}\right\}$$
$$+\begin{cases}(\alpha-1)H\left\{-\dfrac{3c_1y}{\rho_1^5}\right\} & (H<\xi_3)\\ \dfrac{1-4\alpha}{2}\left[-\dfrac{y}{\rho^3}\right]_2^1 & \end{cases}$$

$$+(\alpha-1)\xi_3\left\{-\frac{3c_1y}{\rho_1^5}\right\}-3\alpha(H-\xi_3)\left\{-\frac{3c_2y}{\rho_2^5}\right\}\qquad(H>\xi_3)\,.\tag{4·18c}$$

$(kl)=(31)$

$$w_{31}^x(x,y)=w_{23}^y(y,x)\,,\tag{4·19a}$$

$$w_{31}^y(x,y)=w_{23}^x(y,x)\,,\tag{4·19b}$$

$$w_{31}^z(x,y)=w_{23}^z(y,x)\,.\tag{4·19c}$$

$(kl)=(12)$

$$\begin{aligned}
\frac{2}{\mu}w_{12}^x=&\frac{3}{2}\left[-\frac{y}{\rho^3}\right]_3^1+\frac{(4\alpha-1)(\alpha-2)}{2\alpha}\left[-\frac{1}{4}\frac{y}{\rho^3}+\frac{1}{4}\frac{y(3x^2-y^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]_3^1\\
&+(\alpha-1)\xi_3\left[-\frac{y(2\rho+c)}{\rho^3(\rho+c)^2}+\frac{2x^2y}{\rho^3(\rho+c)^3}+\frac{3x^2y(2\rho+c)}{\rho^5(\rho+c)^2}\right]_3^1\\
&+3(2-\alpha)H\left\{-\frac{y(2\rho_3+c_3)}{\rho_3^3(\rho_3+c_3)^2}+\frac{2x^2y}{\rho_3^3(\rho_3+c_3)^3}+\frac{3x^2y(2\rho_3+c_3)}{\rho_3^5(\rho_3+c_3)^2}\right\}\\
&-6\alpha H\xi_3\left\{-\frac{3y}{\rho_3^5}+\frac{15x^2y}{\rho_3^7}\right\}\\
&+\begin{cases}(1-\alpha)H\left\{-\dfrac{y(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^2y}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^2y(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\} & (H<\xi_3)\\[2ex]
\dfrac{3}{2}\left[-\dfrac{y}{\rho^3}\right]_2^1+\dfrac{1-4\alpha}{2}\left[-\dfrac{1}{4}\dfrac{y}{\rho^3}+\dfrac{1}{4}\dfrac{y(3x^2-y^2)(3\rho+c)}{\rho^3(\rho+c)^3}\right]_2^1 & \\
\quad+(1-\alpha)\xi_3\left\{-\dfrac{y(2\rho_1+c_1)}{\rho_1^3(\rho_1+c_1)^2}+\dfrac{2x^2y}{\rho_1^3(\rho_1+c_1)^3}+\dfrac{3x^2y(2\rho_1+c_1)}{\rho_1^5(\rho_1+c_1)^2}\right\} & \\
\quad+3\alpha(H-\xi_3)\left\{-\dfrac{y(2\rho_2+c_2)}{\rho_2^3(\rho_2+c_2)^2}+\dfrac{2x^2y}{\rho_2^3(\rho_2+c_2)^3}+\dfrac{3x^2y(2\rho_2+c_2)}{\rho_2^5(\rho_2+c_2)^2}\right\} & (H>\xi_3)\,.\end{cases}
\end{aligned}\tag{4·20a}$$

$$w_{12}^y(x,y)=w_{12}^x(y,x)\,,\tag{4·20b}$$

$$\begin{aligned}
\frac{2}{\mu}w_{12}^z=&\frac{\alpha^2-9\alpha+2}{2\alpha}\left[-\frac{xy(2\rho+c)}{\rho^3(\rho+c)^2}\right]_3^1+(\alpha-1)\xi_3\left[-\frac{3xy}{\rho^5}\right]_3^1\\
&+6\alpha H\xi_3\left\{-\frac{15c_3xy}{\rho_3^7}\right\}\\
&+\begin{cases}(1-\alpha)H\left\{-\dfrac{3xy}{\rho_1^5}\right\} & (H<\xi_3)\\[2ex]
\dfrac{2\alpha-5}{2}\left[-\dfrac{xy(2\rho+c)}{\rho^3(\rho+c)^2}\right]_2^1 & \\
\quad+(1-\alpha)\xi_3\left\{-\dfrac{3xy}{\rho_1^5}\right\}-3\alpha(H-\xi_3)\left\{-\dfrac{3xy}{\rho_2^5}\right\} & (H>\xi_3)\,.\end{cases}
\end{aligned}\tag{4·20c}$$

The square brackets with numerals should be read as

$$[f(\rho, c)]_j^i = f(\rho_i, c_i) - f(\rho_j, c_j)\,, \tag{4·21}$$

and

$$\left.\begin{aligned} \rho_1 &= \sqrt{x^2+y^2+c_1^2}\,, & c_1 &= \xi_3 - z \\ \rho_2 &= \sqrt{x^2+y^2+c_2^2}\,, & c_2 &= 2H - \xi_3 - z\,, \\ \rho_3 &= \sqrt{x^2+y^2+c_3^2}\,, & c_3 &= 2H + \xi_3 - z\,. \end{aligned}\right\} \tag{4·22}$$

According to the general theory of multipoles (STRATTON 1941), all these elementary piezomagnetic potentials can be viewed as a set of multipoles placed at $(0, 0, \xi_3)$, $(0, 0, 2H-\xi_3)$ and $(0, 0, 2H+\xi_3)$, together with lines of multipoles connecting these points. This feature is similar to the piezomagnetic field of the Mogi model (SASAI 1979). We can construct the solution for the Mogi model, which is essentially a point force source of the center of dilatation, by summing up w_{11}, w_{22} and w_{33} in the above. This simple nature of the elementary potential is of much help for physically understanding the piezomagnetic changes associated with dislocation models. The magnetic field is equivalent to that produced by the distribution of multipoles along the dislocation surface plus its mirror image magnets below the Currie depth.

5. The Piezomagnetic Field Associated with a Vertical Strike-slip Fault—Application of the Theory

Let us now apply the present theory to a simple tectonic model, i.e. a vertical rectangular strike-slip fault. The displacement and stress field of this type of fault was investigated in detail by CHINNERY (1961, 1963). The fault geometry is illustrated in Fig. 3. The piezomagnetic field potential at a point (x, y, z) in this case reduces to

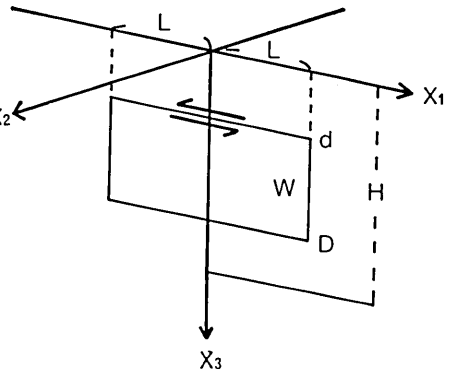

Fig. 3. The geometry of a vertical rectangular strike-slip fault.

$$
\left.
\begin{aligned}
W_x &= \frac{1}{2}\mu\beta J_x \int_d^L \int_{-L}^L \Delta u(\xi_1, \xi_3) w_{12}^x(x-\xi_1, y, z, \xi_3) d\xi_1 d\xi_3 \\
W_y &= \frac{1}{2}\mu\beta J_y \int_d^D \int_{-L}^L \Delta u(\xi_1, \xi_3) w_{12}^y(x-\xi_1, y, z, \xi_3) d\xi_1 d\xi_3 \\
W_z &= \frac{1}{2}\mu\beta J_z \int_d^D \int_{-L}^L \Delta u(\xi_1, \xi_3) w_{12}^z(x-\xi_1, y, z, \xi_3) d\xi_1 d\xi_3
\end{aligned}
\right\} \quad (5\cdot 1)
$$

in which $\Delta u(\xi_1, \xi_3)$ indicates the dislocation along the fault as a function of position.

Infinitely Long Fault

When the fault length is much greater than its width and when we observe the magnetic field near its center, we may regard it as a two-dimensional one with infinite length. Such a fault was investigated by SHAMSI and STACEY (1969) in order to estimate the possible seismo-magnetic effect accompaying the 1906 San Francisco earthquake. They proposed three kinds of fault slip distributions, namely (a) uniform, (b) linear and (c) sinusoidal slip models as shown in Fig. 4. Although SHAMSI and STACEY conducted numerical integrations for the linear slip model, we will present here analytical solutions for the first two models.

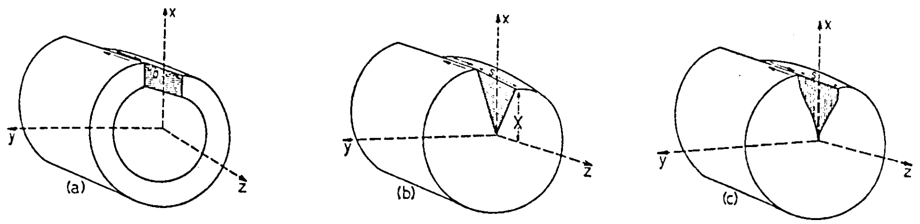

Fig. 4. Sections of three types of the fault slippage along an infinitely long vertical strike-slip fault as proposed by SHAMSI and STACEY (1969) : (a) uniform slip, (b) linear slip and (c) sinusoidal slip model respectively.

Since the dislocation slip at a given depth $(\xi_1, 0, \xi_3)$ is uniform in the ξ_1 direction, we can obtain the piezomagnetic potential due to a line of screw dislocation as follows:

$$
\left.
\begin{aligned}
w_x &= \int_{-\infty}^{\infty} w_{12}^x d\xi_1 = -\frac{2y}{y^2+(\xi_3-z)^2} + \frac{2y}{y^2+(2H+\xi_3-z)^2} \\
&\quad + \begin{cases} 0 & (\xi_3 > H) \\ -\dfrac{2y}{y^2+(\xi_3-z)^2} + \dfrac{2y}{y^2+(2H-\xi_3-z)^2} & (\xi_3 < H) \end{cases} \\
w_y &= \int_{-\infty}^{\infty} w_{12}^y d\xi_1 = 0 \\
w_z &= \int_{-\infty}^{\infty} w_{12}^z d\xi_1 = 0
\end{aligned}
\right\} \quad (5\cdot 2)
$$

In case of such a simplified fault geometry, only the horizontal magnetization parallel to the fault strike produces the observable magnetic field. The non-zero component w_x consists of lines of magnetic dipoles polarized in the y direction, at depths $z=\xi_3$, $2H+\xi_3$ and $2H-\xi_3$.

The slip distıibution function Δu might be given as follows;

(a) uniform slip model:

$$\left.\begin{aligned}&\Delta u=U_0 \qquad (d\leqq\xi_3\leqq D)\\&(U_0=\text{const.}),\end{aligned}\right\} \tag{5·3a}$$

(b) linear slip model:

$$\left.\begin{aligned}&\Delta u=U_0\cdot\frac{D-\xi_3}{D-d} \qquad (d\leqq\xi_3\leqq D)\\&(U_0\text{: the maximum displacement at the fault top}),\end{aligned}\right\} \tag{5·3b}$$

(c) sinusoidal slip model:

$$\left.\begin{aligned}&\Delta u=U_0\sin\left(\frac{\pi}{2}\cdot\frac{D-\xi_3}{D-d}\right) \qquad (d\leqq\xi_3\leqq D)\\&(U_0\text{: the maximum displacement at the fault top}).\end{aligned}\right\} \tag{5·3c}$$

The piezomagnetic field potential of these models will be shown as

$$W=\frac{1}{2}\mu\beta J_x\int_d^D \Delta u\cdot w_x d\xi_3. \tag{5·4}$$

The physical pictures of the magnetic field represented by eq. (5·4) would be easily imagined as shown schematically in Fig. 5: they are equivalent to the magnetic field produced by infinitely long plate magnets placed along the fault plane polarized perpendicularly to the fault surface plus some mirror image magnets with opposite polarity, their magnetic intensity being proportional to the slip discontinuity Δu.

The integration of eq. (5·4) can be achieved without any difficulty for the uniform and linear slip model, results of which will be found in Appendix C. The integral for the sinusoidal slip model is no longer expressible by elementary functions. However, numerical calculations of eq. (5·4) for the sinusoidal slip model would be less laborious than the 2-dimensional convolution integrals as carried out by SHAMSI and STACEY (1969).

The surface magnetic components due to the uniform and linear slip models are computed along the intersection perpendicular to the fault trace, as shown in Fig. 6(a). All the model parameters are the same as those given by SHAMSI and STACEY (1969), i. e. $d=0\,km$, $D=5\,km$, $U_0=-5\,m$ (right lateral), average magnetic dip $I_0=60°$, fault strike $\varphi=45°$

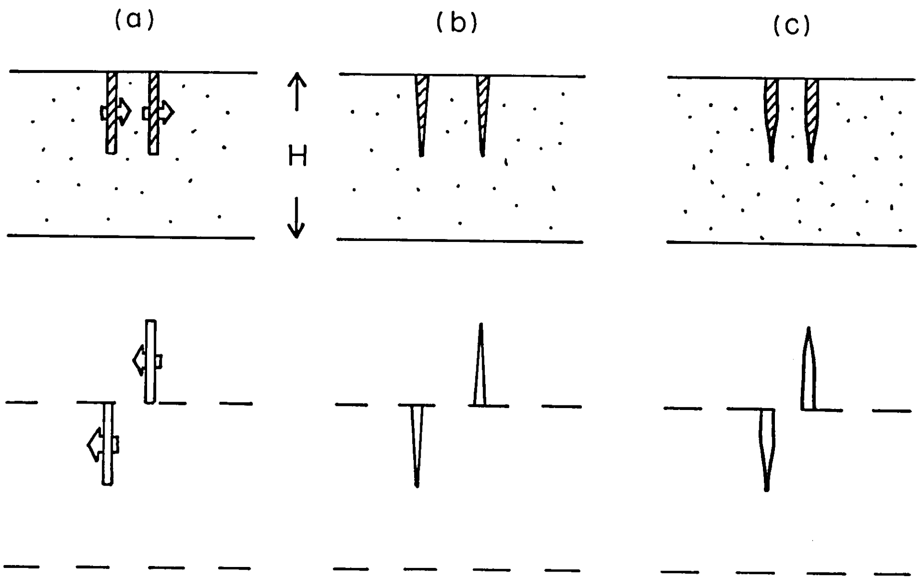

Fig. 5. The vertical cross section of equivalent plate magnets which produce the surface magnetic field by (a) uniform, (b) linear and (c) sinusoidal slip model respectively. The shadowed and hollow magnets have opposite polarity with each other. The thickness of each plate magnet is proportional to the intensity of magnetization.

west of magnetic north, Currie depth $H=20\,km$, stress sensitivity $\beta=1.0\times10^{-4}\,bar^{-1}$, average magnetization $J_0=1.0\times10^{-3}\,emu$ and the rigidity μ is assumed here to be $3.5\times10^{11}\,cgs$.

The magnetic field just on the surface trace of the fault becomes infinite in the present calculation. This result is unlike that of SHAMSI and STACEY. They fixed the upper limit of stress change near the singular fault edge up to 100 bars in numerically calculating the piezomagnetic field. It should be emphasized, however, that the magnetic field singularity is not brought about by the anomalous stress field around the fault top itself. As we have seen in the derivation process of eq. (5·4), the stress-induced magnetization change throughout the magnetized crust contributes to such a divergent magnetic field along the dislocation surface. As long as the fault top with a finite amount of slip discontinuity lies at the earth's surface, there is always a singularity in the magnetic field along the fault line.

For the burried fault, the field no longer diverges. An example for the case of $d=0.5\,km$ is shown in Fig. 6(b). This figure resembles fairly well the results of SHAMSI and STACEY, except for the total force component. (See Fig. 6 in their paper.) The resultant F component in their

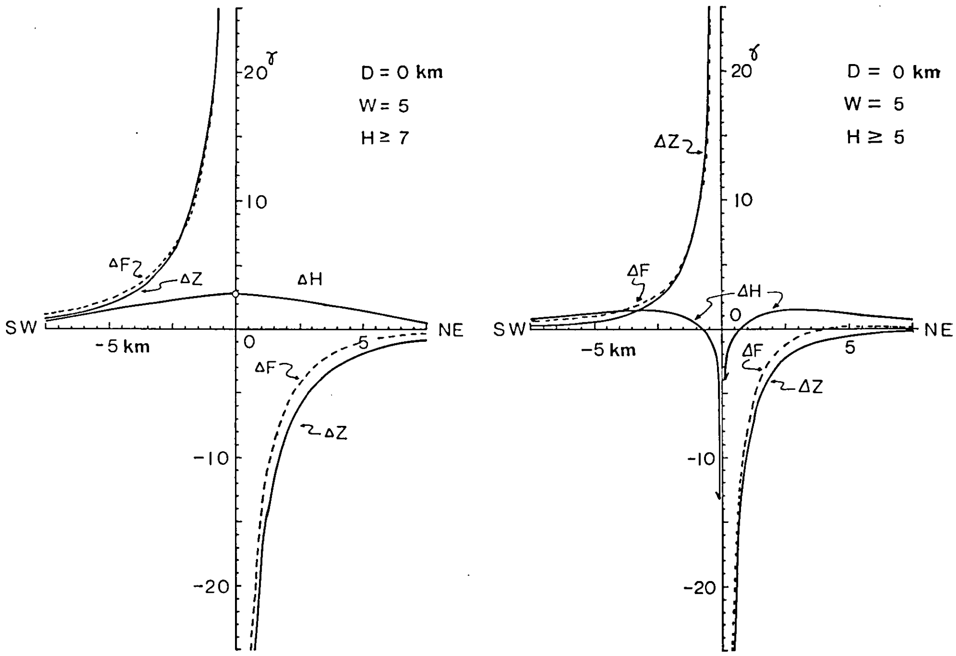

Fig. 6(a). The surface magnetic field along a line perpendicular to the fault accompanying the uniform (left) and linear slip model (right) repectively, when the fault top d=0 km.

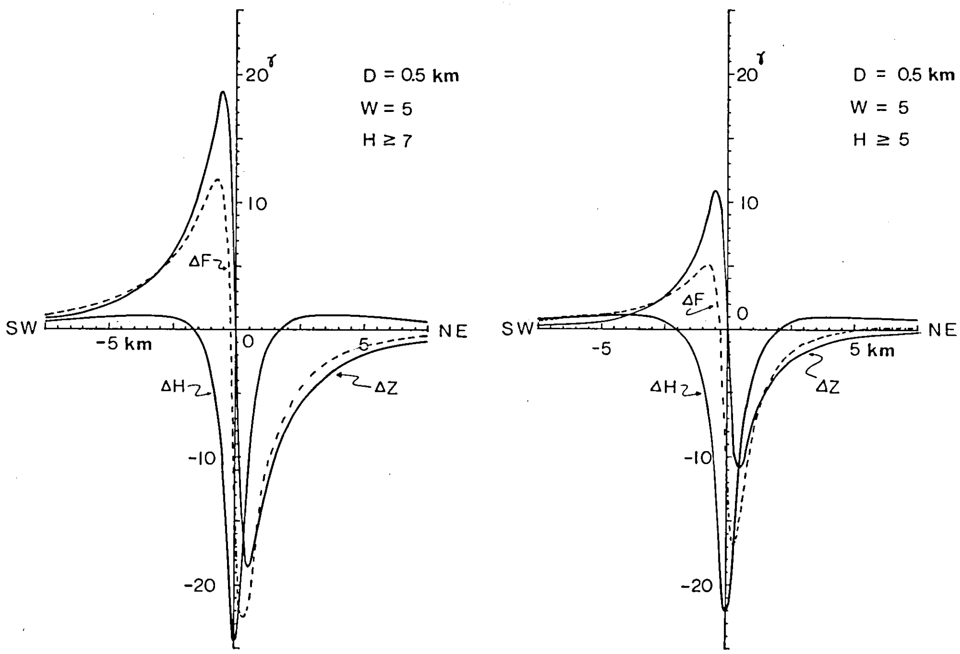

Fig. 6(b). The surface magnetic field along a line perpendicular to the fault accompanying the uniform (left) and linear slip model (right) respectively, when the fault top d=0.5 km.

figure seems too small in comparison with the H and Z component. This might come from simply mistaking the sign of the Z component in composing the total field.

The solution for the uniform slip model may be, from another viewpoint, regarded as a combination of stationary line currents flowing in the x direction at depths $z=d$, D, $2H\pm d$ and $2H\pm D$. The surface magnetic field is produced for the most part by the "equivalent line current" at $z=d$. This explains why observable magnetic changes are found merely in a very narrow area above the fault top in Fig. 6.

The position as well as the polarity of the "equivalent plate magnets" vary with the spatial configuration of the underground fault against the Currie depth H, as schematically depicted in Fig. 7. In any case the

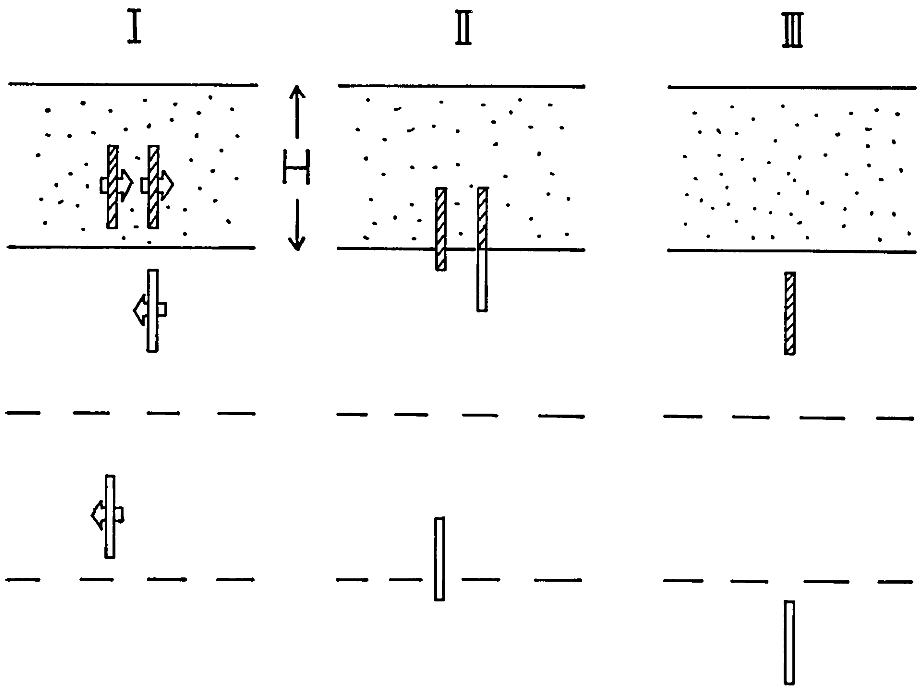

Fig. 7. The vertical cross section of equivalent plate magnets for three representative spatial configurations of the fault and the Currie depth.

coseismic magnetic change would be hardly detectable at teleseismic distances (say, several to ten times as far as the Currie depth H), because the equivalent magnets and their negative images are so closely situated that their magnetic field would cancel each other at these distances.

Rectangular Fault

When the fault slip Δu is uniform over a rectangular fault, integrals in eq. (5·1) can be evaluated analytically. This may be achieved efficiently with the aid of the formulae in Table 2, because the primitive function of each integrand would be found in the table by taking into account the operational rules (4·7). The results are summarized in Appendix D. Expressions for the magnetic field components are so lengthy that only those for the piezomagnetic potential are given.

When we normalize the fault dimension with the fault width W, the overall intensity of the seismomagnetic effect is determined by a parameter

$$C=\frac{1}{2}\beta J_0\mu\frac{\Delta U}{W}. \qquad (5\cdot 5)$$

This expression is reasonable in view of the fact that the stress drop along a fault is proportional to $\mu\Delta U/W$ (KNOPOFF 1958). Since the seismometrically determined stress drop is roughly the same for a number of earthquakes within a wide range of magnitude, the seismomagnetic coefficient C does not depend so much on the earthquake magnitude. Apart from the regional variety of average magnetization J_0, a governing factor of the seismomagnetic effect is the depth of fault top d/W, as we have already seen in the 2-dimensional fault.

An example of the seismomagnetic effect will be shown for three representative fault orientations: magnetic North-South fault in Fig. 8(a)-(d), mag. NW-SE fault in Fig. 9(a)-(d), and mag. E-W fault in Fig. 10(a)-(d) respectively. Fault parameters as well as magnetic constants are listed in Table 3. According to an empirical relation between the seismic moment and earthquake magnitude (OHNAKA 1976), such a fault movement corresponds approximately to a magnitude 6.3 earthquake. Although these figures show results for the left-lateral movement, we

Table 3. Parameters of a vertical rectangular strike-slip fault.

fault length	$2L$	10 km
fault width	W	5 km
depth of burial	d	0.5 km
dislocation (left-lateral)	ΔU	1 m
rigidity	μ	3.5×10^{11} cgs
average magnetization	J_0	1.0×10^{-3} emu/cc
stress sensitivity	β	1.0×10^{-4} bar^{-1}
average mag. dip	I_0	45 deg.
Currie depth	H	15 km

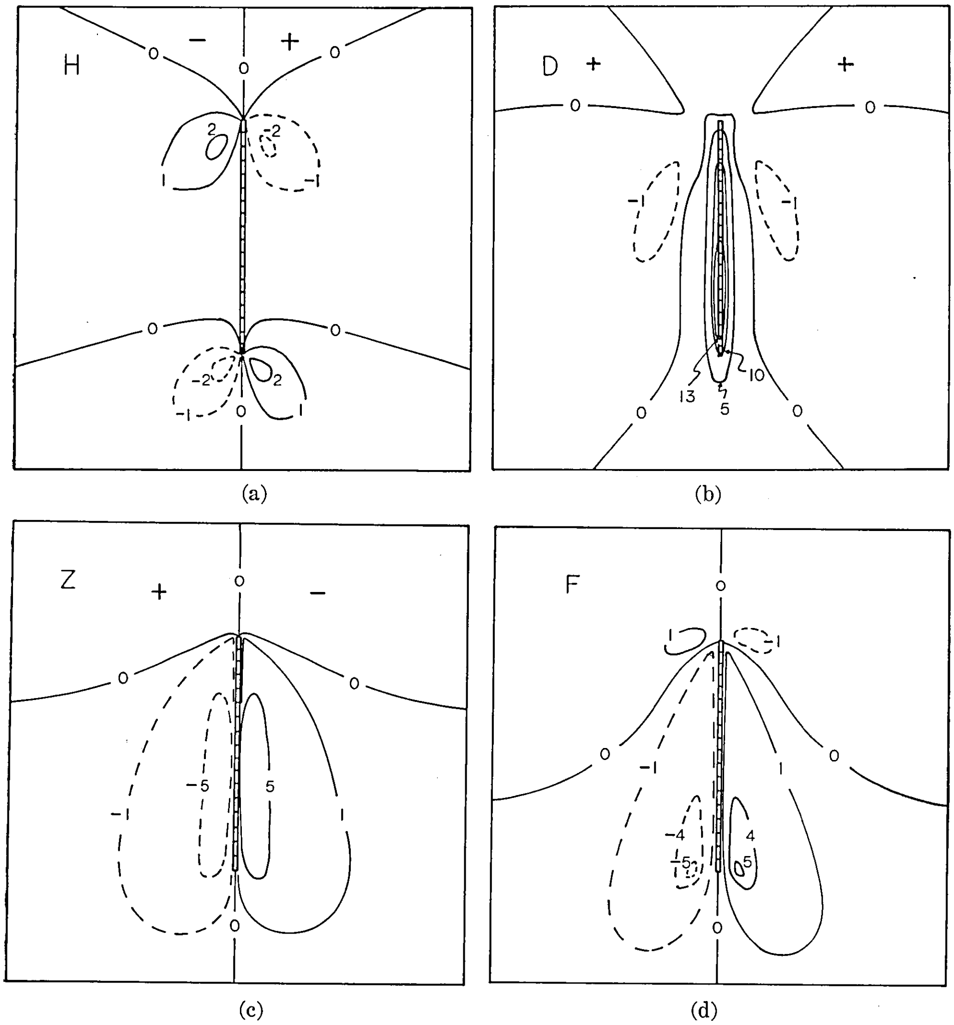

Fig. 8. The magnetic field accompanying the magnetic North-South fault: (a) the H, (b) the D, (c) the Z and (d) the F component respectively, in units of gammas.

may simply exchange signs for the right-lateral one.

In the two dimensional case, only the dipole term with respect to a horizontal magnetization in the fault-strike direction, J_x, contributes to the surface magnetic field. As for the three dimensional fault, multipole terms as well as two other magnetization components J_y and J_z play a significant role. The most outstanding feature is the fault edge effect. Especially, the surface points just upon the fault edge become many-valued and singular. This arises from inappropriate modelling of the slip termination at fault edges for the sake of mathematical simplicity.

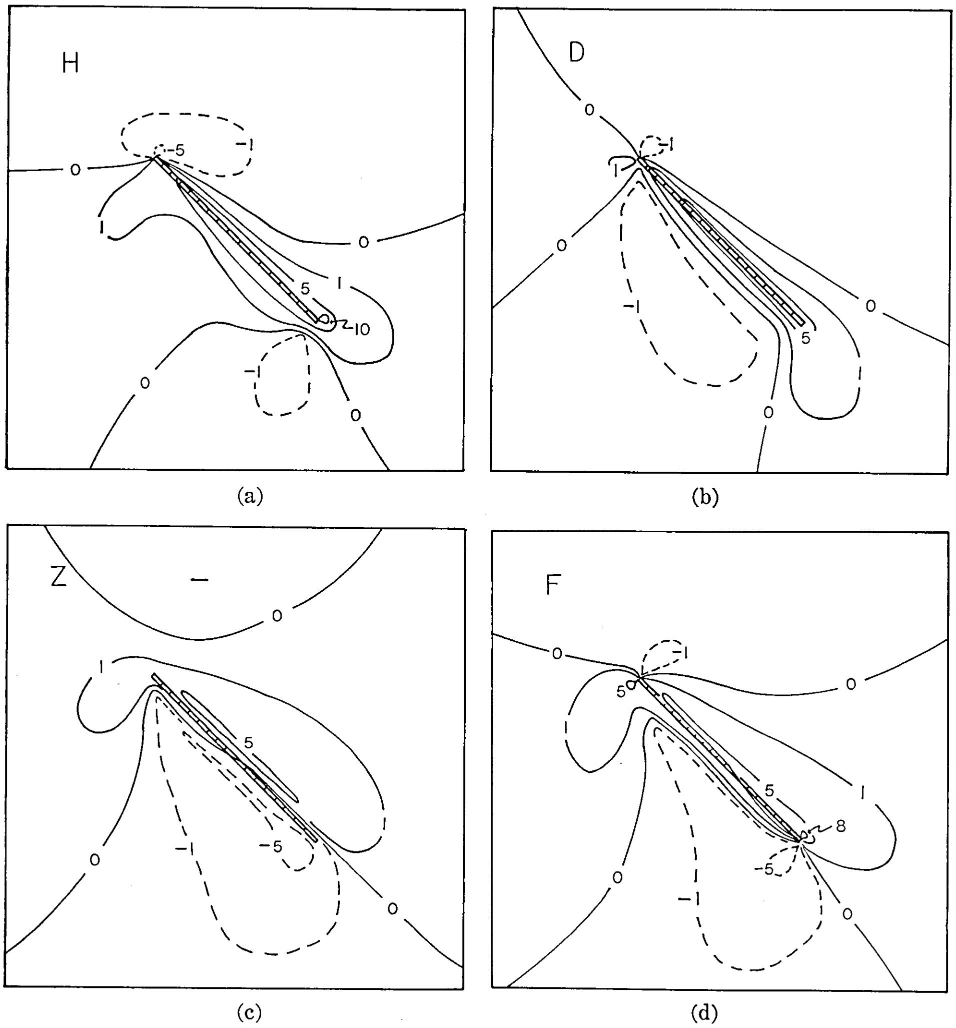

Fig. 9. The magnetic field accompanying the magnetic NW-SE fault: (a) the H, (b) the D, (c) the Z and (d) the F component respectively, in units of gammas.

A more realistic model with the gradual fade-out of slip discontinuity might enable us to avoid these singularities.

A comparison will be made here between STACEY's (1964) model and the present dislocation fault. Typical examples of the Stacey model are given in Fig. 11(a), (b) and (c), illustrating the coseismic F component changes after STACEY (1964). Patterns of piezomagnetic changes on the basis of the dislocation theory are very different from Stacey's model. Discrepancies are as follows:

(a) Magnetic changes due to the Stacey model are monotonous and wide-spread, while those of the dislocation model are much more local-

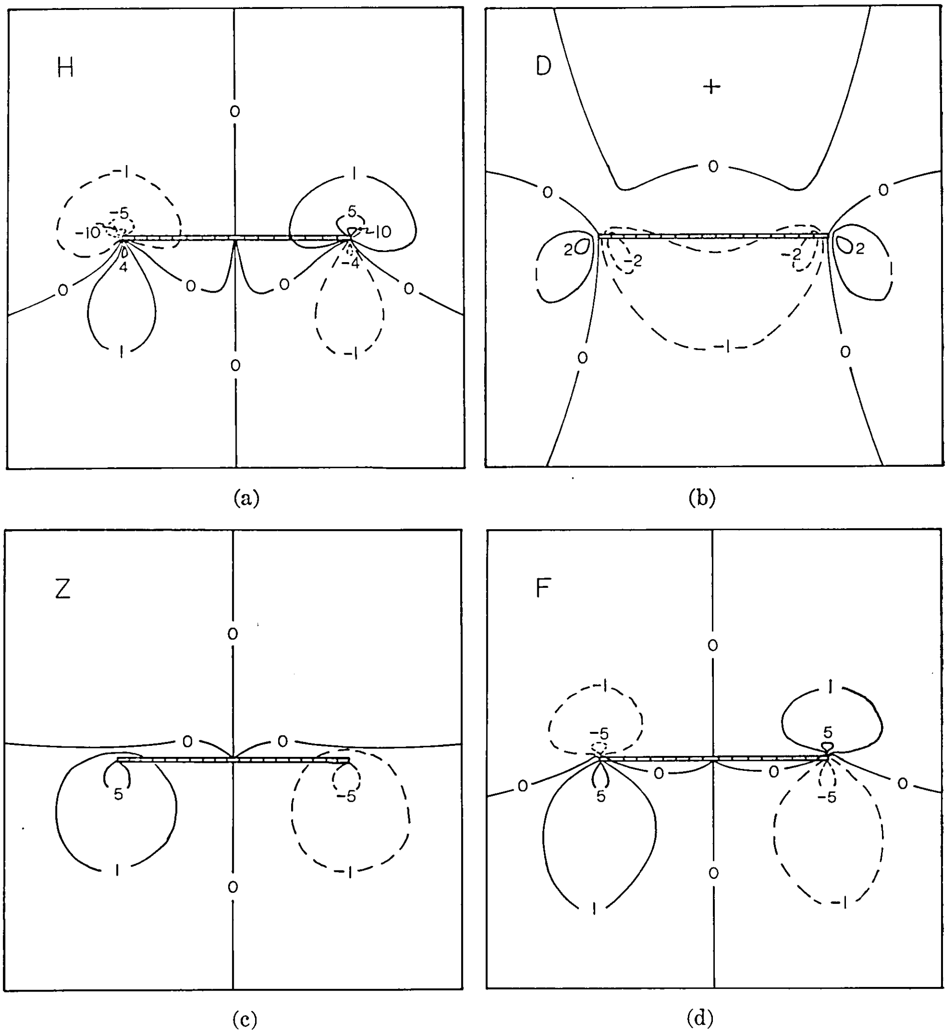

Fig. 10. The magnetic field accompanying the magnetic East-West fault: (a) the H, (b) the D, (c) the Z and (d) the F component respectively, in units of gammas.

ized near the fault. Stacey dealt with the coseismic stress release of the τ_{xy} component, i. e. the shearing stress in a plane parallel to the fault surface, and neglected other stress components. According to CHINNERY (1963), the major stress change associated with a dislocation fault model is likewise the τ_{xy} component. However, intensive changes in the τ_{xy} component are confined to around the dislocation fault and not so extended away from the fault as Stacey's hypothetical stress distribution.

(b) The Stacey model lacks the fault edge effect.

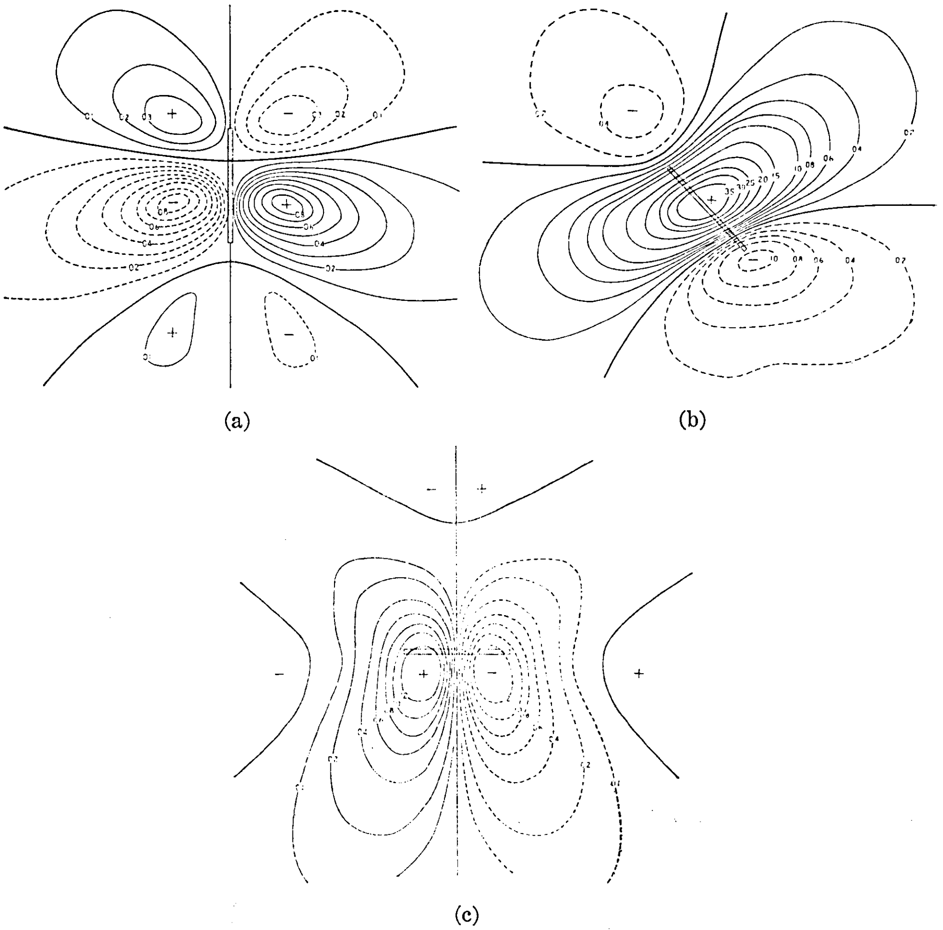

Fig. 11. The F component changes associated with the Stacey model with fault orientations of (a) magnetic N-S, (b) mag. NW-SE and (c) mag. E-W direction respectively (after STACEY 1964).

(c) No change occurs in the vertical magnetization J_z in the case of Stacey's model, in contrast to the substantial contribution from the J_z component in the dislocation fault. The discrepancy is attributed to the ignorance of other stress components except for the major shear stress τ_{xy} in the Stacey model.

In these respects, the Stacey model might nowadays be regarded as rather classical, although its proposal was the most important starting point in tectonomagnetic studies.

Much effort has been made to observe the seismomagnetic effect in the context of the earthquake prediction study. Results in this section tell us that the seismomagnetic effect is expected only in the epicentral region and that its detectability is highly dependent on whether the top of the seismic fault is shallow or not.

6. Tectonomagnetism as a tool for monitoring crustal strain changes

Looking back to the piezomagnetic field potential for the uniform slip model of a vertical strike-slip fault with infinite length, we see that its functional form is quite similar to the displacement field of the same model as obtained by CHINNERY (1961), except for additional terms involving the effect of Currie depth H in the tectonomagnetic model. We also find that the elementary piezomagnetic potential contains some identical functions as compared with MARUYAMA's (1964) solutions for the displacement due to each elementary dislocation, although they do not completely coincide with each other. This suggests a close relationship between the piezomagnetic potential and the displacement field, and hence between their spatial derivatives, namely the magnetic field and the strain component. To make this clear, we will develop here a formal representation of the piezomagnetic potential in terms of displacements, and will investigate its physical meaning with the aid of the theory of Newtonian potential (KELLOGG 1929, VLADIMILOV 1971).

We start from the generalized linear piezomagnetic formula (2·18). Each component of the matrix $\boldsymbol{S}$ in eq. (2·18) can be rewritten in terms of displacements through Hooke's law as

$$\left.\begin{aligned} S_{mn} &= \frac{1}{2}\delta_{mn}\Theta - \frac{3}{2}\tau_{mn} \\ &= \mu\left\{\delta_{mn}\Delta - \frac{3}{2}\left(\frac{\partial u_m}{\partial x_n} + \frac{\partial u_n}{\partial x_m}\right)\right\} \end{aligned}\right\} \tag{6·1}$$

where

$$\left.\begin{aligned} \Theta &= \tau_{xx} + \tau_{yy} + \tau_{zz} \\ \Delta &= \mathrm{div}\,\boldsymbol{u} \end{aligned}\right\}. \tag{6·2}$$

We denote the incremental magnetization $\Delta \boldsymbol{M}_k$ in the J_k component as

$$\Delta \boldsymbol{M}_k = \beta \boldsymbol{S} \cdot \boldsymbol{J}_k \tag{6·3}$$

where $\boldsymbol{J}_k$ implies $(J_x, 0, 0)^t$ and the like.
Then we obtain

$$\left.\begin{aligned}\frac{1}{\beta J_k\mu}\operatorname{div}\Delta\boldsymbol{M}_k&=-\frac{1}{2}\frac{\partial\Delta}{\partial x_k}-\frac{3}{2}\nabla^2u_k\\&=-\frac{2(\lambda+\mu)}{3\lambda+2\mu}\nabla^2u_k\end{aligned}\right\}. \tag{6·4}$$

The last expression in eq. (6·4) comes from the equation of equilibrium to be satisfied by the homogeneous and isotropic elastic body when there acts no body force:

$$(\lambda+\mu)\frac{\partial\Delta}{\partial x_k}+\mu\nabla^2u_k=0\,. \tag{6·5}$$

In the previous sections, we have considered the magnetic potential only in the free space as expressed by eq. (2·19). The magnetic potential can be defined, however, even within the magnetized body, because there exists no conduction current. Then W_k should satisfy the following Poisson's equation:

$$\nabla^2W_k(\boldsymbol{r})=\begin{cases}4\pi\operatorname{div}\Delta\boldsymbol{M}_k=-4\pi C_k\nabla^2u_k(\boldsymbol{r}) & (\boldsymbol{r}\in V)\\ 0 & (\boldsymbol{r}\,\overline{\in}\,V)\end{cases} \tag{6·6}$$

where

$$C_k=\frac{1}{2}\beta J_k\mu\frac{3\lambda+2\mu}{\lambda+\mu}. \tag{6·7}$$

We assume that $u_k(\boldsymbol{r})$ is already known by solving the equation (6·5). The problem reduces to represent $W_k(\boldsymbol{r})$ in terms of the given $u_k(\boldsymbol{r})$.

By making use of a Green function which satisfies

$$\nabla^2G(\boldsymbol{r},\boldsymbol{r}')=-\delta(\boldsymbol{r}-\boldsymbol{r}')\,, \tag{6·8}$$

the solution of eq. (6·6) can be given as

$$W_k(\boldsymbol{r})=4\pi C_k\int_V G(\boldsymbol{r},\boldsymbol{r}')\nabla'^2u_k(\boldsymbol{r}')dV'\,.$$

$G(\boldsymbol{r},\boldsymbol{r}')$ is actually a well-known function, i. e.

$$\left.\begin{aligned}G(\boldsymbol{r},\boldsymbol{r}')&=\frac{1}{4\pi|\boldsymbol{r}-\boldsymbol{r}'|}=\frac{1}{4\pi\rho}\\ \rho&=\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}\end{aligned}\right\}. \tag{6·10}$$

With the aid of Green's theorem together with eq. (6·8), we obtain the following expression:

$$\left.\begin{aligned}W_k(\boldsymbol{r})=&-4\pi C_ku_k(\boldsymbol{r})\theta(\boldsymbol{r}\in V)\\&+4\pi C_k\int_S\left[\left\{\frac{\partial u_k(\boldsymbol{r}')}{\partial n'}\right\}G(\boldsymbol{r},\boldsymbol{r}')-\{u_k(\boldsymbol{r}')\}\frac{\partial G(\boldsymbol{r},\boldsymbol{r}')}{\partial n'}\right]dS'\end{aligned}\right\} \tag{6·11}$$

where

$$\theta(\boldsymbol{r}\in V)=\begin{Bmatrix} 1 & (\boldsymbol{r}\in V) \\ 0 & (\boldsymbol{r}\overline{\in} V) \end{Bmatrix}. \tag{6·12}$$

The first term in (6·11) appears only when the observation point is located within the magnetic body, and represents the contribution from the magnetization immediately at that point. The latter convolution integral implies the influence of the boundary of magnetic body. If we can neglect the boundary effect and/or when we consider the piezomagnetic potential in an infinite medium, the potential can be simply given as

$$W_k(\boldsymbol{r})=-4\pi C_k u_k(\boldsymbol{r}). \tag{6·13}$$

The basic concept of the elastic dislocation has sometimes been interpreted in analogy with the double layer in the potential theory. The double layer, which is a surface distribution of dipoles, can be viewed as a gap in the potential value across the surface. Similarly, the discontinuity in displacements across a surface is equivalent to the distribution of some particular force sources in the elasticity theory of dislocations, where the displacement corresponds to the potential. Eq. (6·13) tells us that the correspondence is most straightforward within the ideal (which means *linear* and *reversible*) piezomagnetic body: the magnetic potential is identical to the displacement itself multiplied by some material constants.

Let us examine the boundary effect on the righthand side of (6·11). The former convolution integral represents a single layer potential with the surface density $\{\partial u_k(\boldsymbol{r}')/\partial n'\}_S$, while the latter a double layer having a density distribution of $\{u_k(\boldsymbol{r}')\}_S$. Across the boundary surface of magnetic body, it appears as if a gap in the potential value $4\pi C_k\{u_k(\boldsymbol{r})\}$ might occur owing to the vanishment of the first term in eq. (6·11) outside the body. This is completely compensated for by a jump in the potential across the double layer, of which the amount is equal to its intensity $-4\pi C_k u_k(\boldsymbol{r})$ (KELLOGG 1929). Hence the magnetic potential W_k is continuous at the boundary surface.

Applying the formula (6·11) to the simple geometry hitherto considered, namely a magnetic layer bounded by $z=0$ and $z=H$, we obtain the piezomagnetic potential in the free space ($z<0$):

$$\left.\begin{aligned} W_k(\boldsymbol{r})=&C_k\iint_{-\infty}^{\infty}\left[\left\{-\frac{\partial u_k(\boldsymbol{r}')}{\partial z'}\right\}\frac{1}{\rho}+\{u_k(\boldsymbol{r}')\}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=+0}dx'dy' \\ &+C_k\iint_{-\infty}^{\infty}\left[\left\{\frac{\partial u_k(\boldsymbol{r}')}{\partial z'}\right\}\frac{1}{\rho}-\{u_k(\boldsymbol{r}')\}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=H}dx'dy' \end{aligned}\right\} \tag{6·14}$$

Especially when $\boldsymbol{r}$ approaches the plane surface $z=0$, the double layer term in the former integral of (6·14) becomes

$$\lim_{z\to-0} C_k\iint_{-\infty}^{\infty}\left[\{u_k(\boldsymbol{r}')\}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=+0}dx'dy'=-2\pi C_k\{u_k(\boldsymbol{r}')\}_{z'=+0} \qquad (6\cdot15)$$

This explains why we come across identical functions in the piezomagnetic potential at $z=-0$ and the displacement field at $z=+0$ of any static dislocation models. Moreover, in the case of the two-dimensional fault, the single layer term at the earth's surface vanishes owing to traction-free boundary conditions, so that the surface magnetic potential is completely coincidental with the displacement at that surface point, as stated in the beginning of this section.

The formula (6·14) implies that the piezomagnetic potential in the free space can be determined solely by the displacement field at the earth's surface and that at the Currie point isotherm, any other knowledge is not required on displacements within the magnetic crust. We are now led to an easier way of evaluating the elementary piezomagnetic potential. Displacements due to the elementary dislocation at any depth have been obtained by MARUYAMA (1964). Thus it will suffice to apply the Fourier transform theorem of convolution integrals to eq. (6·14) and then to take its inverse. The formula (6·14) was, however, figured out by the present writer at the final stage of this study. Although the procedure developed in section 4 is rather unrefined and laborious, results given in Appendices B1, B2 and B3 might be useful for further studies of tectonomagnetic models in cases where the magnetic property of the crust varies with depth.

Let us now investigate the magnetic field at the earth's surface. We will recall here a useful theorem for derivatives of the surface layer potential when a point approaches the source layer. Under proper conditions for the smoothness of the surface density distribution $\sigma(p)$, the normal derivative of a single layer potential $U^{(s)}$ on the positive and negative side of the layer can be given by (KELLOGG 1929)

$$\left.\begin{aligned}\frac{\partial U^{(s)}}{\partial n_+}&=-2\pi\sigma(p)+\iint_S\sigma(p')\frac{\partial}{\partial n'}\left(\frac{1}{\rho}\right)dS'\\ \frac{\partial U^{(s)}}{\partial n_-}&=2\pi\sigma(p)+\iint_S\sigma(p')\frac{\partial}{\partial n'}\left(\frac{1}{\rho}\right)dS'\end{aligned}\right\} \qquad (6\cdot16)$$

while the tangential derivatives are continuous on both sides of the layer. Similar relations hold good for the double layer. In this case, tangential derivatives of a double layer potential $U^{(d)}$ with a surface moment density $\mu(p)$ become discontinuous across the layer as shown by

$$\left.\begin{aligned}\frac{\partial U^{(d)}}{\partial t_+}&=-2\pi\frac{\partial\mu(p)}{\partial t}+\iint_S\mu(p')\frac{\partial^2}{\partial t\partial n'}\left(\frac{1}{\rho}\right)dS'\\ \frac{\partial U^{(d)}}{\partial t_-}&=\ \ 2\pi\frac{\partial\mu(p)}{\partial t}+\iint_S\mu(p')\frac{\partial^2}{\partial t\partial n'}\left(\frac{1}{\rho}\right)dS'\end{aligned}\right\}\tag{6·17}$$

Normal derivatives are continuous at the double layer. (The formula (6·17) is not explicitly described in KELLOGG (1929). However, this can be verified in the same manner as the proof of the continuity of normal derivatives under the assumptions that the double layer is sufficiently smooth and that $\mu(p)$ has continuous partial derivatives of second order on S.) In case that the source layer geometry is simply a plane, integral terms in eqs. (6·16) and (6·17) vanish, which represent the curvature effects.

We may take the x axis in the magnetic north direction without loss of generality. Applying these formulae to derivatives of the piezo-magnetic potential (6·14), we obtain formal expressions of the tectono-magnetic field in terms of displacements:

$$\left.\begin{aligned}\frac{\Delta H(x,y,0)}{C_0}&=2\pi\left\{\frac{\partial u_f}{\partial x}\right\}_{z=+0}+\left.\frac{\partial U}{\partial x}\right|_{z=0}-\left.\frac{\partial W}{\partial x}\right|_{z=0}\\ \frac{\Delta D(x,y,0)}{C_0}&=2\pi\left\{\frac{\partial u_f}{\partial y}\right\}_{z=+0}+\left.\frac{\partial U}{\partial y}\right|_{z=0}-\left.\frac{\partial W}{\partial y}\right|_{z=0}\\ \frac{\Delta Z(x,y,0)}{C_0}&=2\pi\left\{\frac{\partial u_f}{\partial z}\right\}_{z=+0}-\left.\frac{\partial V}{\partial z}\right|_{z=0}-\left.\frac{\partial W}{\partial z}\right|_{z=0}\end{aligned}\right\}\tag{6·18}$$

where

$$u_f=u_x\cos I_0+u_z\sin I_0\tag{6·19}$$

$$\left.\begin{aligned}U&=\iint_{-\infty}^{\infty}\left[\left\{\frac{\partial u_f}{\partial z'}\right\}\frac{1}{\rho}\right]_{z'=+0}dx'dy'\\ V&=\iint_{-\infty}^{\infty}\left[\{u_f\}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=+0}dx'dy'\\ W&=\iint_{-\infty}^{\infty}\left[\left\{\frac{\partial u_f}{\partial z'}\right\}\frac{1}{\rho}-\{u_f\}\frac{\partial}{\partial z'}\left(\frac{1}{\rho}\right)\right]_{z'=H}dx'dy'\end{aligned}\right\}\tag{6·20}$$

and

$$C_0=\frac{1}{2}\beta J_0\mu\frac{3\lambda+2\mu}{\lambda+\mu}\tag{6·21}$$

u_f, as defined by (6·19), is nothing but a projection of the displacement vector $\boldsymbol{u}$ on the direction of the earth's magnetic field.

In the current tectonomagnetic field work, reliable results have been brought out mostly by the total intensity measurements. The resultant total field change ΔF can be expressed by

$$\frac{\Delta F(x,y,0)}{C_0}=2\pi\left\{\frac{\partial u_f}{\partial f}\right\}_{z=+0}+\left(\frac{\partial U}{\partial x}\cos I_0-\frac{\partial V}{\partial z}\sin I_0\right)\bigg|_{z=0}-\frac{\partial W}{\partial f}\bigg|_{z=0} \tag{6·22}$$

in which the differential sign with respect to the geomagnetic field direction is written as

$$\left.\begin{aligned}\frac{\partial}{\partial f}&=\boldsymbol{e}_f\cdot\nabla=\cos I_0\frac{\partial}{\partial x}+\sin I_0\frac{\partial}{\partial z}\\ \boldsymbol{e}_f&=(\cos I_0,\ 0,\ \sin I_0)\end{aligned}\right\} \tag{6·23}$$

Eq. (6·22) tells us that the tectonomagnetic total field change is a close indication of the simple extension or contraction just at the observation site in the direction of the geomagnetic field. This is particularly so in the source region of tectonic events, as we have already seen in the case of the seismomagnetic effect. We can then roughly estimate the detectability of crustal strain changes through the magnetic method. In consideration of only the first term in eq. (6·22), a unit change in the total field corresponds to a strain change of $(2\pi C_0)^{-1}$. For a model earth with parameters $\beta=1.0\times10^{-4}\,bar^{-1}$, $J_0=1.0\times10^{-3}\,emu/cc$ and $\lambda=\mu=3.5\times10^{11}$ *cgs*, this amounts to 3.6×10^{-5} per gammas.

Thus a proton precession magnetometer, a familiar tool in tectonomagnetic studies, might be regarded as a sort of extensometer stretched along the direction of the main geomagnetic field. The proton magnetometer as a "strain gauge" has a rather poor resolution, and can measure only one strain component in a particular direction. However, the use of this instrument would have merits as compared with the ordinary crustal strain measurement systems, because of its excellent drift-free characteristics. All these discussed in this section might be useful for qualitatively understanding tectonomagnetic changes, if any, so long as they were ascribable to the reversible piezomagnetic effect.

In conclusion, Volterra's formula for the piezomagnetic field (3·2) makes it possible to estimate magnetic changes associated with any static dislocation models, such as an inclined fault, the dike formation by intrusive magmas and so on. At the present stage of the tectonomagnetic study, it might be useful to increase a stock of knowledge on various types of tectonomagnetic models even if magnetic and elastic properties of the model earth under consideration are simplified too much.

Acknowledgements

I would like to express my hearty thanks to associate Professor T. Maruyama, who kindly read the manuscript to examine the basic idea in this study, on the basis of a communication with whom an insufficient understanding on some aspects of Fourier transforms in the first draft was supplemented as given in Appendix A. The present study was initiated under useful suggestions and advices of Professor Y. Hagiwara, whom I acknowledge sincerely.

Appendix A. The Fourier transform of $x_1/(x_1^2+x_2^2+c^2)^{1/2}$ $(c>0)$.

We write the Fourier transform of a function $f(x_1, x_2)$ as

$$F[f(x_1, x_2)]=\bar{f}(k_1, k_2)=\frac{1}{2\pi}\iint_{-\infty}^{\infty} f(x_1, x_2)e^{-i(k_1x_1+k_2x_2)}dx_1dx_2 \tag{A·1}$$

Differentiating both sides of (A·1) with respect to k_1 under the integral sign, we obtain the following formula:

$$F[x_1f(x_1, x_2)]=i\frac{\partial}{\partial k_1}\bar{f}(k_1, k_2). \tag{A·2}$$

In the ordinary Fourier transform theory, this formula is applicable only to a severely restricted class of functions: the function $f(x_1, x_2)$ as well as $x_1f(x_1, x_2)$ should be absolutely integrable in $(-\infty<x_1,\ x_2<+\infty)$. By introducing the concept of the *distribution*, we can consider the Fourier transform of a more general class of functions, i. e. the *slowly increasing fuction* (the ***s. i.*** function) which is everywhere differentiable by any number of times and such that it and all its derivatives are $O(|x|^N)$ as $|x|\to\infty$ for some N. Since the function $x_1/(x_1^2+x_2^2+c^2)^{1/2}$ is an ***s. i.*** function, we will investigate here its Fourier transform in the sense of distribution, following VLADIMILOV (1971).

The Fourier transform of a distribution $f(x_1, x_2)$ can be defined on the basis of the *good function* $\varphi(x_1, x_2)$ (or the *testing function* which is finite and any times differentiable) as

$$(\bar{f}, \varphi)=(f, \bar{\varphi}) \tag{A·3}$$

which implies

$$\iint_{-\infty}^{\infty} F[f](k_1, k_2)\varphi(k_1, k_2)dk_1dk_2=\iint_{-\infty}^{\infty} f(x_1, x_2)F[\varphi](x_1, x_2)dx_1dx_2. \tag{A·4}$$

Substituting $x_1f(x_1, x_2)$ for $f(x_1, x_2)$ in eq. (A·4) and integrating by part, we get

$$(F[x_1 f], \varphi) = -i\left(F[f], \frac{\partial \varphi}{\partial k_1}\right) = \left(i\frac{\partial \bar{f}}{\partial k_1}, \varphi\right) \tag{A·5}$$

We may write this relation symbolically as follows:

$$F[x_1 f] = i\frac{\partial \bar{f}}{\partial k_1} \tag{A·5'}$$

which is apparently quite the same as eq. (A·2). However, the distributional differential sign on the right of (A·5') is no longer coincident exactly with that in the classical meaning.

When $\bar{f}(k_1, k_2)$ in eq. (A·5) has a discontinuity along a line s, the distributional differentiation can be represented in the following way:

$$\left(\frac{\partial \bar{f}}{\partial k_1}, \varphi\right) = \iint \left\{\frac{\partial \bar{f}}{\partial k_1}\right\} \varphi dk_1 dk_2 + \int [\bar{f}]_s \cos(\boldsymbol{n}, k_1) \varphi ds \tag{A·6}$$

where $\{\partial \bar{f}/\partial k_1\}$ denotes the ordinary derivative, $[\bar{f}]_s$ the gap in the value of $\bar{f}$ across s outward from inside, while $\boldsymbol{n}$ the outward normal to s. (A·6) might be written symbolically as

$$\frac{\partial \bar{f}}{\partial k_1} = \left\{\frac{\partial \bar{f}}{\partial k_1}\right\} + [\bar{f}]_s \cos(\boldsymbol{n}, k_1)\delta_s \tag{A·7}$$

δ_s indicates a *distribution* called "*single layer*", corresponding to an extension of δ function along the line s. δ_s is defined to satisfy the relation:

$$(\mu\delta_s, \varphi) = \iint \mu(s)\delta_s \varphi(k_1, k_2) dk_1 dk_2 = \int_s \mu(s)\varphi(s) ds \tag{A·8}$$

Now we put $f(x_1, x_2) = 1/(x_1^2 + x_2^2 + c^2)^{1/2}$. The ordinary Fourier transform of $f(x_1, x_2)$ becomes

$$\bar{f}(k_1, k_2) = \frac{1}{k} e^{-ck}, \qquad k \neq 0 \qquad (k = \sqrt{k_1^2 + k_2^2}) \tag{A·9}$$

Let us take a region $G = \{(k_1, k_2)\,;\, \varepsilon < k < K\}$ as shown in Fig. A-1. In the region $K \leqq k$ for sufficiently large K, we may put $\varphi(k_1, k_2) = 0$. We assume that $\bar{f}(k_1, k_2)$ is zero within a circle $L_\varepsilon = \{(k_1, k_2)\,;\, k = \varepsilon\}$. Putting (A·9) into (A·6), and making integrations on the right hand side of (A·6) over the region G, we seek to obtain

$$\left(\frac{\partial \bar{f}}{\partial k_1}, \varphi\right) = \lim_{\varepsilon \to +0} \left[\iint_G \left\{\frac{\partial \bar{f}}{\partial k_1}\right\} \varphi dk_1 dk_2 + \int_{L_\varepsilon} [\bar{f}]_s \cos(\boldsymbol{n}, k_1) \varphi ds\right] \tag{A·10}$$

Taking into account that

$$[\bar{f}]_s = -\frac{1}{\varepsilon} e^{-c\varepsilon}, \qquad \cos(\boldsymbol{n}, k_1) = -\cos\theta \quad \text{and} \quad ds = \varepsilon d\theta\,,$$

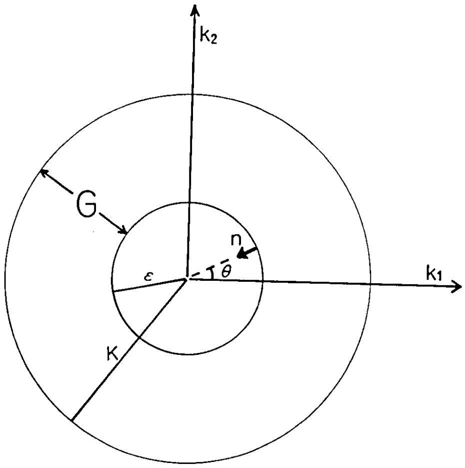

Fig. A-1.

we see the contour integral in (A·6), i.e.

$$\int_{L_\varepsilon}[\bar{f}]_s \cos(\boldsymbol{n}, k_1)\varphi ds=\int_0^{2\pi} e^{-c\varepsilon}\varphi(k_1, k_2)\cos\theta d\theta$$

approaches $\varphi(0,0)\left[\int_0^{2\pi}\cos\theta d\theta\right]=0$ as $\varepsilon\to+0$. Thus eq. (A·10) is led to

$$\left(\frac{\partial\bar{f}}{\partial k_1}, \varphi\right)=\iint\left\{\frac{\partial\bar{f}}{\partial k_1}\right\}\varphi dk_1dk_2=\left(\left\{\frac{\partial\bar{f}}{\partial k_1}\right\}, \varphi\right) \tag{A·11}$$

Going back to (A·5), we arrive at the final result:

$$F\left[\frac{x_1}{\sqrt{x_1^2+x_2^2+c^2}}\right]=-i\left(\frac{1}{k^3}+\frac{c}{k^2}\right)k_1e^{-ck} \tag{A·12}$$

which agrees with the one to be obtainable through the formal application of the formula (A·2).

The Fourier transform in the sense of distribution is introduced in this paper merely for the sake of mathematical convenience so as to apply the operational rules. In fact, we could do without this concept, if we dare to evaluate all the sets of Fourier transforms of H_{kl}^{mn}. As we have seen in (4·11), Γ_{kl}^{mn}'s are composed of only two kinds of functions: i.e. $x_j/(x_1^2+x_2^2+c^2)^{1/2}$ $(j=1, 2)$ and $c/(x_1^2+x_2^2+c^2)^{1/2}$. Hence eq. (A·9) and (A·12) are sufficient to derive $\bar{S}_{kl}^{mn}$ through the formulae in Table 1.

Appendix B1: Fourier transforms of S_{kl}^{mn}'s.

$(kl)=(11)$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{xx}=\left[-(1+5\alpha)\frac{k_1^4}{k^3}-(1+8\alpha)\frac{k_1^2k_2^2}{k^3}\right]e^{-k\zeta_1}$$

$$+\left[3\alpha\frac{k_1^4}{k^2}\right]\zeta_1 e^{-k\zeta_1}$$

$$+\left[-3(1+\alpha)\frac{k_1^4}{k^3}+\frac{2-3\alpha-8\alpha^2}{\alpha}\frac{k_1^2k_2^2}{k^3}+\frac{4(1-\alpha)(2\alpha-1)}{\alpha}\frac{k_2^4}{k^3}\right]e^{-k\zeta_2}$$

$$+\left[(2+\alpha)\frac{k_1^4}{k^2}+4(\alpha-1)\frac{k_1^2k_2^2}{k^2}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[9\alpha\frac{k_1^4}{k^2}+6(2\alpha-1)\frac{k_1^2k_2^2}{k^2}-6\alpha\xi_3\frac{k_1^4}{k}\right]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{xy}=\left[-3\alpha\frac{k_1^3k_2}{k^3}-6\alpha\frac{k_1k_2^3}{k^3}\right]e^{-k\zeta_1}$$

$$+\left[3\alpha\frac{k_1^3k_2}{k^2}\right]\zeta_1 e^{-k\zeta_1}$$

$$+\left[3(\alpha-2)\frac{k_1^3k_2}{k^3}+\frac{6(\alpha^2-3\alpha+1)}{\alpha}\frac{k_1k_2^3}{k^3}\right]e^{-k\zeta_2}+\left[3(\alpha-2)\frac{k_1^3k_2}{k^2}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[9\alpha\frac{k_1^3k_2}{k^2}+6(2\alpha-1)\frac{k_1k_2^3}{k^2}-6\alpha\xi_3\frac{k_1^3k_2}{k}\right]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{xz}=\mp[6\alpha ik_1]e^{-k\zeta_1}\mp\left[-3\alpha\frac{ik_1^3}{k}\right]\zeta_1 e^{-k\zeta_1}\qquad (x_3\gtrless\xi_3)$$

$$+[-6\alpha ik_1]e^{-k\zeta_2}$$

$$+\left[3\alpha\frac{ik_1^3}{k}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[9\alpha\frac{ik_1^3}{k}+6(2\alpha-1)\frac{ik_1k_2^2}{k}-6\alpha\xi_3 ik_1^3\right]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{yx}=\frac{4\pi}{\mu}\bar{S}_{11}^{xy},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{yy}=\left[2(1-\alpha)\frac{k_1^4}{k^3}+5(1-\alpha)\frac{k_1^2k_2^2}{k^3}+3(1-2\alpha)\frac{k_2^4}{k^3}\right]e^{-k\zeta_1}$$

$$+\left[3\alpha\frac{k_1^2k_2^2}{k^2}\right]\zeta_1 e^{-k\zeta_1}$$

$$+\left[6(1-\alpha)\frac{k_1^4}{k^3}+\frac{(1-\alpha)(11\alpha-4)}{\alpha}\frac{k_1^2k_2^2}{k^3}+\frac{(1-2\alpha)(\alpha+2)}{\alpha}\frac{k_2^4}{k^3}\right]e^{-k\zeta_2}$$

$$+\left[4(\alpha-1)\frac{k_1^4}{k^2}+(2+\alpha)\frac{k_1^2k_2^2}{k^2}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[9\alpha\frac{k_1^2k_2^2}{k^2}+6(2\alpha-1)\frac{k_2^4}{k^2}-6\alpha\xi_3\frac{k_1^2k_2^2}{k}\right]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{yz}=\mp\left[3(2\alpha-1)ik_2-3\alpha\frac{ik_1^2k_2}{k}\zeta_1\right]e^{-k\zeta_1} \qquad (x_3\gtrless\xi_3)$$

$$+[3(1-2\alpha)ik_2]e^{-k\zeta_2}+\left[3\alpha\frac{ik_1^2k_2}{k}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[9\alpha\frac{ik_1^2k_2}{k}+6(2\alpha-1)\frac{ik_2^3}{k}-6\alpha\xi_3 ik_1^2k_2\right]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{zx}=\frac{4\pi}{\mu}\bar{S}_{11}^{xz}, \qquad \frac{4\pi}{\mu}\bar{S}_{11}^{zy}=\frac{4\pi}{\mu}\bar{S}_{11}^{yz},$$

$$\frac{4\pi}{\mu}\bar{S}_{11}^{zz}=\left[(7\alpha-1)\frac{k_1^2}{k}+3(2\alpha-1)\frac{k_2^2}{k}\right]e^{-k\zeta_1}$$

$$+[-3\alpha k_1^2]\zeta_1 e^{-k\zeta_1}$$

$$+\left[3(3\alpha-1)\frac{k_1^2}{k}+\frac{(5\alpha-2)(2\alpha-1)}{\alpha}\frac{k_2^2}{k}\right]e^{-k\zeta_2}$$

$$+[(2-5\alpha)k_1^2]\xi_3 e^{-k\zeta_2}$$

$$+[-9\alpha k_1^2+6(1-2\alpha)k_2^2+6\alpha\xi_3 k_1^2 k]x_3 e^{-k\zeta_2}.$$

$(kl)=(22)$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{xx}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{yy}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{xy}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{xy}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{xz}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{yz}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{yx}=\frac{4\pi}{\mu}\bar{S}_{22}^{xy},$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{yy}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{xx}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{yz}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{xz}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{zx}=\frac{4\pi}{\mu}\bar{S}_{22}^{xz}, \qquad \frac{4\pi}{\mu}\bar{S}_{22}^{zy}=\frac{4\pi}{\mu}\bar{S}_{22}^{yz},$$

$$\frac{4\pi}{\mu}\bar{S}_{22}^{zz}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{11}^{zz}(k_2,k_1).$$

$(kl)=(33)$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{xx}=\left[(1-\alpha)\frac{k_1^2}{k}+2(\alpha-1)\frac{k_2^2}{k}\right]e^{-k\zeta_1}$$

$$+[-3\alpha k_1^2]\zeta_1 e^{-k\zeta_1}$$

$$+\left[(\alpha-1)\frac{k_1^2}{k}+2(1-\alpha)\frac{k_2^2}{k}\right]e^{-k\zeta_2}$$

$$+[-(\alpha+2)k_1^2+4(1-\alpha)k_2^2]\xi_3 e^{-k\zeta_2}$$

$$+[3\alpha k_1^2+6\alpha\xi_3 k_1^2 k]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{xy}=\left[3(1-\alpha)\frac{k_1k_2}{k}\right]e^{-k\zeta_1}+[-3\alpha k_1k_2]\zeta_1 e^{-k\zeta_1}$$

$$+\left[3(\alpha-1)\frac{k_1k_2}{k}\right]e^{-k\zeta_2}+[3(\alpha-2)k_1k_2]\xi_3 e^{-k\zeta_2}$$

$$+[3\alpha k_1k_2+6\alpha\xi_3 k_1k_2k]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{xz}=\mp[3\alpha ik_1k\zeta_1]e^{-k\zeta_1}\qquad (x_3\gtrless\xi_3)$$

$$+[-3\alpha ik_1k]\xi_3 e^{-k\zeta_2}$$

$$+[3\alpha ik_1k+6\alpha\xi_3 ik_1k^2]x_3 e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{yx}=\frac{4\pi}{\mu}\bar{S}_{33}^{xy},$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{yy}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{33}^{xx}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{yz}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{33}^{xz}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{zx}=\frac{4\pi}{\mu}\bar{S}_{33}^{xz},\qquad \frac{4\pi}{\mu}\bar{S}_{33}^{zy}=\frac{4\pi}{\mu}\bar{S}_{33}^{yz},$$

$$\frac{4\pi}{\mu}\bar{S}_{33}^{zz}=[(1-\alpha)k]e^{-k\zeta_1}+[3\alpha k^2]\zeta_1 e^{-k\zeta_1}$$

$$+[(\alpha-1)k]e^{-k\zeta_2}+[(5\alpha-2)k^2]\xi_3 e^{-k\zeta_2}$$

$$+[-3\alpha k^2-6\alpha\xi_3 k^3]x_3 e^{-k\zeta_2}.$$

$(kl)=(23)$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{xx}=\mp\left[2(\alpha-1)ik_2-3\alpha\frac{ik_1^2k_2}{k}\zeta_1\right]e^{-k\zeta_1}\qquad (x_3\gtrless\xi_3)$$

$$+[2(\alpha-1)ik_2]e^{-k\zeta_2}$$

$$+\left[-(\alpha+2)\frac{ik_1^2k_2}{k}+4(1-\alpha)\frac{ik_2^3}{k}\right]\xi_3 e^{-k\zeta_2}$$

$$+\left[-3\alpha\frac{ik_1^2k_2}{k}+6\alpha\xi_3 ik_1^2k_2\right]x_3e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{xy}=\mp\left[\frac{3}{2}ik_1-3\alpha\frac{ik_1k_2^2}{k}\zeta_1\right]e^{-k\zeta_1} \qquad (x_3\gtrless\xi_3)$$

$$+\left[\frac{3}{2}ik_1+3(\alpha-2)\xi_3\frac{ik_1k_2^2}{k}\right]e^{-k\zeta_2}$$

$$+\left[-3\alpha\frac{ik_1k_2^2}{k}+6\alpha\xi_3 ik_1k_2^2\right]x_3e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{xz}=\left[\frac{3}{2}(2\alpha-1)\frac{k_1k_2}{k}-3\alpha k_1k_2\zeta_1\right]e^{-k\zeta_1}$$

$$+\left[-\frac{3}{2}(2\alpha-1)\frac{k_1k_2}{k}+3\alpha\xi_3k_1k_2\right]e^{-k\zeta_2}$$

$$+[3\alpha k_1k_2-6\alpha\xi_3k_1k_2k]x_3e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{yx}=\frac{4\pi}{\mu}\bar{S}_{23}^{xy},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{yy}=\mp\left[(2\alpha+1)ik_2-3\alpha\frac{ik_2^3}{k}\zeta_1\right]e^{-k\zeta_1} \qquad (x_3\gtrless\xi_3)$$

$$+[(2\alpha+1)ik_2]e^{-k\zeta_2}$$

$$+\left[4(1-\alpha)\frac{ik_1^2k_2}{k}-(\alpha+2)\frac{ik_2^3}{k}\right]\xi_3e^{-k\zeta_2}$$

$$+\left[-3\alpha\frac{ik_2^3}{k}+6\alpha\xi_3 ik_2^3\right]x_3e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{yz}=\left[\frac{3}{2}\frac{k_1^2}{k}+3\alpha\frac{k_2^2}{k}\right]e^{-k\zeta_1}+[-3\alpha k_2^2]\zeta_1e^{-k\zeta_1}$$

$$+\left[-\frac{3}{2}\frac{k_1^2}{k}-3\alpha\frac{k_2^2}{k}\right]e^{-k\zeta_2}+[3\alpha k_2^2]\xi_3e^{-k\zeta_2}$$

$$+[3\alpha k_2^2-6\alpha\xi_3k_2^2k]x_3e^{-k\zeta_2},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{zx}=\frac{4\pi}{\mu}\bar{S}_{23}^{xz}, \qquad \frac{4\pi}{\mu}\bar{S}_{23}^{zy}=\frac{4\pi}{\mu}\bar{S}_{23}^{yz},$$

$$\frac{4\pi}{\mu}\bar{S}_{23}^{zz}=\mp[(1-4\alpha)ik_2+3\alpha ik_2k\zeta_1]e^{-k\zeta_1} \qquad (x_3\gtrless\xi_3)$$

$$+[(1-4\alpha)ik_2+(5\alpha-2)\xi_3ik_2k]e^{-k\zeta_2}$$

$$+[3\alpha ik_2k-6\alpha\xi_3ik_2k^2]x_3e^{-k\zeta_2}.$$

$(kl)=(31)$

$$\frac{4\pi}{\mu}\bar{S}_{31}^{xx}(k_1,k_2)=\frac{4\pi}{\mu}\bar{S}_{23}^{yy}(k_2,k_1),$$

$$\frac{4\pi}{\mu}\bar{S}^{xy}_{31}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{xy}_{23}(k_2, k_1)\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{xz}_{31}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{yz}_{23}(k_2, k_1)\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{yx}_{31}=\frac{4\pi}{\mu}\bar{S}^{xy}_{31}\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{yy}_{31}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{xx}_{23}(k_2, k_1)\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{yz}_{31}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{xz}_{23}(k_2, k_1)\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{zx}_{31}=\frac{4\pi}{\mu}\bar{S}^{xz}_{31}\,,\qquad \frac{4\pi}{\mu}\bar{S}^{zy}_{31}=\frac{4\pi}{\mu}\bar{S}^{yz}_{31}\,,$$

$$\frac{4\pi}{\mu}\bar{S}^{zz}_{31}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{zz}_{23}(k_2, k_1)\,.$$

$(kl)=(12)$

$$\begin{aligned}\frac{4\pi}{\mu}\bar{S}^{xx}_{12}=&\left[(\alpha-1)\frac{k_1^3k_2}{k^3}-(2\alpha+1)\frac{k_1k_2^3}{k^3}\right]e^{-k\zeta_1}+\left[3\alpha\frac{k_1^3k_2}{k^2}\right]\zeta_1e^{-k\zeta_1}\\&+\left[\frac{(\alpha-1)(2-\alpha)}{\alpha}\frac{k_1^3k_2}{k^3}+\frac{(2\alpha-1)(\alpha-4)}{\alpha}\frac{k_1k_2^3}{k^3}\right]e^{-k\zeta_2}\\&+\left[(\alpha+2)\frac{k_1^3k_2}{k^2}+4(\alpha-1)\frac{k_1k_2^3}{k^2}\right]\xi_3e^{-k\zeta_2}\\&+\left[3(2-\alpha)\frac{k_1^3k_2}{k^2}-6\alpha\xi_3\frac{k_1^3k_2}{k}\right]x_3e^{-k\zeta_2}\,,\end{aligned}$$

$$\begin{aligned}\frac{4\pi}{\mu}\bar{S}^{xy}_{12}=&\left[3\alpha\frac{k_1^2k_2^2}{k^3}-\frac{3}{2}k\right]e^{-k\zeta_1}+\left[3\alpha\frac{k_1^2k_2^2}{k^2}\right]\zeta_1e^{-k\zeta_1}\\&+\left[3\frac{-\alpha^2+4\alpha-2}{\alpha}\frac{k_1^2k_2^2}{k^3}-\frac{3}{2}k\right]e^{-k\zeta_2}\\&+\left[3(2-\alpha)\frac{k_1^2k_2^2}{k^2}\right]\xi_3e^{-k\zeta_2}\\&+\left[3(2-\alpha)\frac{k_1^2k_2^2}{k^2}-6\alpha\xi_3\frac{k_1^2k_2^2}{k}\right]x_3e^{-k\zeta_2}\,,\end{aligned}$$

$$\begin{aligned}\frac{4\pi}{\mu}\bar{S}^{xz}_{12}=&\mp\left[\frac{3}{2}ik_2-3\alpha\frac{ik_1^2k_2}{k}\zeta_1\right]e^{-k\zeta_1}\qquad(x_3\gtrless\xi_3)\\&+\left[-\frac{3}{2}ik_2+3\alpha\xi_3\frac{ik_1^2k_2}{k}\right]e^{-k\zeta_2}\\&+\left[3(2-\alpha)\frac{ik_1^2k_2}{k}-6\alpha\xi_3ik_1^2k_2\right]x_3e^{-k\zeta_2}\,,\end{aligned}$$

$$\frac{4\pi}{\mu}\bar{S}^{yx}_{12}=\frac{4\pi}{\mu}\bar{S}^{xy}_{12},$$

$$\frac{4\pi}{\mu}\bar{S}^{yy}_{12}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{xx}_{12}(k_2, k_1),$$

$$\frac{4\pi}{\mu}\bar{S}^{yz}_{12}(k_1, k_2)=\frac{4\pi}{\mu}\bar{S}^{xz}_{12}(k_2, k_1),$$

$$\frac{4\pi}{\mu}\bar{S}^{zx}_{12}=\frac{4\pi}{\mu}\bar{S}^{xz}_{12}, \qquad \frac{4\pi}{\mu}\bar{S}^{zy}_{12}=\frac{4\pi}{\mu}\bar{S}^{yz}_{12},$$

$$\begin{aligned}\frac{4\pi}{\mu}\bar{S}^{zz}_{12}=&\left[(\alpha+2)\frac{k_1k_2}{k}-3\alpha k_1k_2\zeta_1\right]e^{-k\zeta_1}\\&+\left[\frac{-\alpha^2+6\alpha-2}{\alpha}\frac{k_1k_2}{k}+(2-5\alpha)\xi_3k_1k_2\right]e^{-k\zeta_2}\\&+[3(2-\alpha)k_1k_2+6\alpha\xi_3k_1k_2k]x_3e^{-k\zeta_2},\end{aligned}$$

where $\zeta_1=|x_3-\xi_3|$

$\zeta_2=x_3+\xi_3$.

Appendix B2: Fourier transforms of ω^m_{kl}'s.

$(kl)=(11)$

$$\begin{aligned}\frac{2}{\mu}\bar{\omega}^x_{11}=&\left[3(1+3\alpha)ik_1-\frac{(\alpha-1)(\alpha-2)}{\alpha}\frac{ik_1k_2^2}{k^2}\right]e^{-kz_3}\\&+\left[-2(1+2\alpha)\frac{ik_1^3}{k}\right]\xi_3e^{-kz_3}\\&+\left[-18\alpha\frac{ik_1^3}{k}+12(1-2\alpha)\frac{ik_1k_2^2}{k}+12\alpha\xi_3ik_1^3\right]x_3e^{-kz_3}\\&+\begin{cases}\left[(1-\alpha)\dfrac{ik_1^3}{k^2}\right]e^{-kz_1} & (0<x_3<\xi_3)\\ \left[(1-\alpha)\dfrac{ik_1^3}{k^2}+12\alpha ik_1\right]e^{-kz_2} & \\ \quad+\left[-6\alpha\dfrac{ik_1^3}{k}\right](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3),\end{cases}\end{aligned}$$

$$\begin{aligned}\frac{2}{\mu}\bar{\omega}^y_{11}=&\left[3(3\alpha-1)\frac{ik_1^2k_2}{k^2}+\frac{2}{\alpha}(4\alpha^2-1)\frac{ik_2^3}{k^2}\right]e^{-kz_3}\\&+\left[-2(1+2\alpha)\frac{ik_1^2k_2}{k}\right]\xi_3e^{-kz_3}\\&+\left[-18\alpha\frac{ik_1^2k_2}{k}+12(1-2\alpha)\frac{ik_2^3}{k}+12\alpha\xi_3ik_1^2k_2\right]x_3e^{-kz_3}\end{aligned}$$

$$+\begin{cases} \left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]e^{-kz_1} & (0<x_3<\xi_3) \\ \left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}+6(2\alpha-1)ik_2\right]e^{-kz_2} & \\ \quad +\left[-6\alpha\dfrac{ik_1^2k_2}{k}\right](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3)\,, \end{cases}$$

$$\frac{2}{\mu}\bar{\omega}_{11}^z=\left[3(1-5\alpha)\frac{k_1^2}{k}+\frac{2(1-2\alpha)(4\alpha-1)}{\alpha}\frac{k_2^2}{k}\right]e^{-kz_3}$$

$$+[2(4\alpha-1)k_1^2]\xi_3e^{-kz_3}$$

$$+[18\alpha k_1^2+12(2\alpha-1)k_2^2-12\alpha\xi_3k_1^2k]x_3e^{-kz_3}$$

$$+\begin{cases} \left[(1-\alpha)\dfrac{k_1^2}{k}\right]e^{-kz_1} & (0<x_3<\xi_3) \\ \left[(1-\alpha)\dfrac{k_1^2}{k}+6\dfrac{k_2^2}{k}-12\alpha k\right]e^{-kz_2} & \\ \quad +[6\alpha k_1^2](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3)\,. \end{cases}$$

$(kl)=(22)$

$$\frac{2}{\mu}\bar{\omega}_{22}^x(k_1,k_2)=\frac{2}{\mu}\bar{\omega}_{11}^y(k_2,k_1)$$

$$\frac{2}{\mu}\bar{\omega}_{22}^y(k_1,k_2)=\frac{2}{\mu}\bar{\omega}_{11}^x(k_2,k_1)$$

$$\frac{2}{\mu}\bar{\omega}_{22}^z(k_1,k_2)=\frac{2}{\mu}\bar{\omega}_{11}^z(k_2,k_1)$$

$(kl)=(33)$

$$\frac{3}{\mu}\bar{\omega}_{33}^x=[(1-\alpha)ik_1]e^{-kz_3}+[2(1+2\alpha)ik_1k]\xi_3e^{-kz_3}$$

$$+[-6\alpha ik_1k-12\alpha\xi_3ik_1k^2]x_3e^{-kz_3}$$

$$+\begin{cases} [(\alpha-1)ik_1]e^{-kz_1} & (0<x_3<\xi_3) \\ [(\alpha-1)ik_1]e^{-kz_2} & \\ \quad +[6\alpha ik_1k](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3)\,, \end{cases}$$

$$\frac{2}{\mu}\bar{\omega}_{33}^y(k_1,k_2)=\frac{2}{\mu}\bar{\omega}_{33}^x(k_2,k_1)\,,$$

$$\frac{2}{\mu}\bar{\omega}_{33}^z=[(1-\alpha)k]e^{-kz_3}+[2(1-4\alpha)k^2]\xi_3e^{-kz_3}$$

$$+[6\alpha k^2+12\alpha\xi_3 k^3]x_3 e^{-kz_3}$$

$$+\begin{cases}[(\alpha-1)k]e^{-kz_1} & (0<x_3<\xi_3)\\ [(\alpha-1)k]e^{-kz_2} & \\ \quad +[-6\alpha k^2](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3).\end{cases}$$

$(kl)=(23)$

$$\frac{2}{\mu}\bar{\omega}^x_{23}=\left[(5\alpha-2)\frac{k_1k_2}{k}\right]e^{-kz_3}+[-2(2\alpha+1)k_1k_2]\xi_3 e^{-kz_3}$$

$$+[-6\alpha k_1k_2+12\alpha\xi_3 k_1k_2k]x_3e^{-kz_3}$$

$$+\begin{cases}\left[(1-\alpha)\frac{k_1k_2}{k}\right]e^{-kz_1} & (0<x_3<\xi_3)\\ \left[(2-5\alpha)\frac{k_1k_2}{k}\right]e^{-kz_2}+[6\alpha k_1k_2](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3),\end{cases}$$

$$\bar{\omega}^y_{23}=\left[3\frac{k_1^2}{k}+(5\alpha+1)\frac{k_2^2}{k}\right]e^{-kz_3}$$

$$+[-2(2\alpha+1)k_2^2]\xi_3e^{-kz_3}$$

$$+[-6\alpha k_2^2+12\alpha\xi_3k_2^2k]x_3e^{-kz_3}$$

$$+\begin{cases}\left[(1-\alpha)\frac{k_2^2}{k}\right]e^{-kz_1} & (0<x_3<\xi_3)\\ \left[-3\frac{k_1^2}{k}-(5\alpha+1)\frac{k_2^2}{k}\right]e^{-kz_2} & \\ \quad +[6\alpha k_2^2](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3),\end{cases}$$

$$\frac{2}{\mu}\bar{\omega}^z_{23}=[(7\alpha-1)ik_2]e^{-kz_3}+[2(1-4\alpha)ik_2k]\xi_3e^{-kz_3}$$

$$+[-6\alpha ik_2k+12\alpha\xi_3ik_2k^2]x_3e^{-kz_3}$$

$$+\begin{cases}[(\alpha-1)ik_2]e^{-kz_1} & (0<x_3<\xi_3)\\ [(1-7\alpha)ik_2]e^{-kz_2} & \\ \quad +[6\alpha ik_2k](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3).\end{cases}$$

$(kl)=(31)$

$$\frac{2}{\mu}\bar{\omega}^x_{31}(k_1,k_2)=\frac{2}{\mu}\bar{\omega}^y_{23}(k_2,k_1),$$

$$\frac{2}{\mu}\bar{\omega}^y_{31}(k_1,k_2)=\frac{2}{\mu}\bar{\omega}^x_{23}(k_2,k_1),$$

$$\frac{2}{\mu}\bar{\omega}^z_{31}(k_1, k_2) = \frac{2}{\mu}\bar{\omega}^z_{23}(k_2, k_1)\,.$$

$(kl)=(12)$

$$\frac{2}{\mu}\bar{\omega}^x_{12} = \left[\frac{(\alpha-1)(\alpha-2)}{\alpha}\frac{ik_1^2k_2}{k^2} + 3ik_2\right]e^{-kz_3}$$
$$+\left[-2(1+2\alpha)\frac{ik_1^2k_2}{k}\right]\xi_3 e^{-kz_3}$$
$$+\left[6(\alpha-2)\frac{ik_1^2k_2}{k} + 12\alpha\xi_3 ik_1^2k_2\right]x_3 e^{-kz_3}$$
$$+\begin{cases}\left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]e^{-kz_1} & (0<x_3<\xi_3)\\[2mm] \left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}+3ik_2\right]e^{-kz_2} + \left[-6\alpha\dfrac{ik_1^2k_2}{k}\right](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3)\,,\end{cases}$$

$$\frac{2}{\mu}\bar{\omega}^y_{12}(k_1, k_2) = \frac{2}{\mu}\bar{\omega}^x_{12}(k_2, k_1)\,,$$

$$\frac{2}{\mu}\bar{\omega}^z_{12} = \left[\frac{\alpha^2-9\alpha+2}{\alpha}\frac{k_1k_2}{k}\right]e^{-kz_3}$$
$$+[2(4\alpha-1)k_1k_2]\xi_3 e^{-kz_3}$$
$$+[-12\alpha\xi_3 k_1k_2k]x_3 e^{-kz_3}$$
$$+\begin{cases}\left[(1-\alpha)\dfrac{k_1k_2}{k}\right]e^{-kz_1} & (0<x_3<\xi_3)\\[2mm] \left[-(\alpha+5)\dfrac{k_1k_2}{k}\right]e^{-kz_2} + [6\alpha k_1k_2](x_3-\xi_3)e^{-kz_2} & (x_3>\xi_3)\,,\end{cases}$$

where $z_1=\xi_3-z\,,$

$z_2=2x_3-\xi_3-z\,,$

$z_3=2x_3+\xi_3-z\,.$

Appendix B3: Fourier transforms of w^m_{kl}'s.

$(kl)=(11)$

$$\frac{2}{\mu}\bar{w}^x_{11} = \left[\frac{3}{2}\frac{ik_1}{k} + \frac{(2-\alpha)(4\alpha-1)}{2\alpha}\frac{ik_1k_2^2}{k^3}\right](e^{-kC_1}-e^{-kC_3})$$
$$+\left[(\alpha-1)\frac{ik_1^3}{k^2}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$

$$+\left[9\alpha\frac{ik_1^3}{k^2}-6(1-2\alpha)\frac{ik_1k_2^2}{k^2}-6\alpha\frac{ik_1^3}{k}\xi_3\right]He^{-kC_3}$$

$$+\begin{cases}\left[(1-\alpha)\dfrac{ik_1^3}{k^2}\right]He^{-kC_1} & (H<\xi_3)\\ \left[\dfrac{1-4\alpha}{2}\dfrac{ik_1^3}{k^3}+6\alpha\dfrac{ik_1}{k}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\dfrac{ik_1^3}{k^2}\right]\xi_3e^{-kC_1}+\left[3\alpha\dfrac{ik_1^3}{k^2}\right](H-\xi_3)e^{-kC_2} & (H>\xi_3),\end{cases}$$

$$\frac{2}{\mu}\overline{w}_{11}^y=\left[-\frac{3}{2}\frac{ik_1^2k_2}{k^3}+\frac{(2\alpha-1)(1-\alpha)}{\alpha}\frac{ik_2^3}{k^3}\right](e^{-kC_1}-e^{-kC_3})$$

$$+\left[(\alpha-1)\frac{ik_1^2k_2}{k^2}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$

$$+\left[9\alpha\frac{ik_1^2k_2}{k^2}-6(1-2\alpha)\frac{ik_2^3}{k^2}-6\alpha\frac{ik_1^2k_2}{k}\xi_3\right]He^{-kC_3}$$

$$+\begin{cases}\left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]He^{-kC_1} & (H<\xi_3)\\ \left[\dfrac{1-4\alpha}{2}\dfrac{ik_1^2k_2}{k^3}+3(2\alpha-1)\dfrac{ik_2}{k}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]\xi_3e^{-kC_1}+\left[3\alpha\dfrac{ik_1^2k_2}{k^2}\right](H-\xi_3)e^{-kC_2} & (H>\xi_3),\end{cases}$$

$$\frac{2}{\mu}\overline{w}_{11}^z=\left[\frac{3}{2}(1-2\alpha)\frac{k_1^2}{k^2}+\frac{(2\alpha-1)(1-\alpha)}{\alpha}\frac{k_2^2}{k^2}\right](e^{-kC_1}-e^{-kC_3})$$

$$+\left[(\alpha-1)\frac{k_1^2}{k}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$

$$+\left[-9\alpha\frac{k_1^2}{k}+6(1-2\alpha)\frac{k_2^2}{k}+6\alpha k_1^2\xi_3\right]He^{-kC_3}$$

$$+\begin{cases}\left[(1-\alpha)\dfrac{k_1^2}{k}\right]He^{-kC_1} & (H<\xi_3)\\ \left[\dfrac{2\alpha-5}{2}\dfrac{k_1^2}{k^2}+3(1-2\alpha)\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\dfrac{k_1^2}{k}\right]\xi_3e^{-kC_1}+\left[-3\alpha\dfrac{k_1^2}{k}\right](H-\xi_3)e^{-kC_2} & (H>\xi_3).\end{cases}$$

$(kl)=(22)$

$$\frac{2}{\mu}\overline{w}_{22}^x(k_1,k_2)=\frac{2}{\mu}\overline{w}_{11}^y(k_2,k_1),$$

$$\frac{2}{\mu}\overline{w}_{22}^y(k_1,k_2)=\frac{2}{\mu}\overline{w}_{11}^x(k_2,k_1),$$

$$\frac{2}{\mu}\overline{w}^z_{22}(k_1, k_2)=\frac{2}{\mu}\overline{w}^z_{11}(k_2, k_1)\,.$$

$(kl)=(33)$

$$\begin{aligned}\frac{2}{\mu}\overline{w}^x_{33}=&\left[\frac{1-4\alpha}{2}\frac{ik_1}{k}\right](e^{-kC_1}-e^{-kC_3})\\&+[(1-\alpha)ik_1]\xi_3(e^{-kC_1}-e^{-kC_3})\\&+[3\alpha ik_1+6\alpha\xi_3 ik_1k]He^{-kC_3}\\&+\begin{cases}[(\alpha-1)ik_1]He^{-kC_1} & (H<\xi_3)\\ \left[\frac{4\alpha-1}{2}\frac{ik_1}{k}\right](e^{-kC_1}-e^{-kC_2})\\ \quad+[(\alpha-1)ik_1]\xi_3e^{-kC_1}+[-3\alpha ik_1](H-\xi_3)e^{-kC_3} & (H>\xi_3)\,,\end{cases}\end{aligned}$$

$$\frac{2}{\mu}\overline{w}^y_{33}(k_1, k_2)=\frac{2}{\mu}\overline{w}^x_{33}(k_2, k_1)\,,$$

$$\begin{aligned}\frac{2}{\mu}\overline{w}^z_{33}=&\left[\frac{1+2\alpha}{2}\right](e^{-kC_1}-e^{-kC_3})\\&+[(1-\alpha)k]\xi_3(e^{-kC_1}-e^{-kC_3})\\&+[-3\alpha k-6\alpha\xi_3k^2]He^{-kC_3}\\&+\begin{cases}[(\alpha-1)k]He^{-kC_1} & (H<\xi_3)\\ \left[-\frac{1+2\alpha}{2}\right](e^{-kC_1}-e^{-kC_2})\\ \quad+[(\alpha-1)k]\xi_3e^{-kC_1}+[3\alpha k](H-\xi_3)e^{-kC_2} & (H>\xi_3)\,.\end{cases}\end{aligned}$$

$(kl)=(23)$

$$\begin{aligned}\frac{2}{\mu}\overline{w}^x_{23}=&\left[(\alpha-1)\frac{k_1k_2}{k^2}\right](e^{-kC_1}-e^{-kC_3})\\&+\left[(\alpha-1)\frac{k_1k_2}{k}\right]\xi_3(e^{-kC_1}-e^{-kC_3})\\&+\left[3\alpha\frac{k_1k_2}{k}-6\alpha\xi_3k_1k_2\right]He^{-kC_3}\\&+\begin{cases}\left[(1-\alpha)\frac{k_1k_2}{k}\right]He^{-kC_1} & (H<\xi_3)\\ \left[(1-\alpha)\frac{k_1k_2}{k^2}\right](e^{-kC_1}-e^{-kC_2})\\ \quad+\left[(1-\alpha)\frac{k_1k_2}{k}\right]\xi_3e^{-kC_1}+\left[-3\alpha\frac{k_1k_2}{k}\right](H-\xi_3)e^{-kC_2} & (H>\xi_3)\,,\end{cases}\end{aligned}$$

$$\frac{2}{\mu}\overline{w}^{y}_{23}=\left[\frac{3}{2}\frac{k_1^2}{k^2}+\frac{2\alpha+1}{2}\frac{k_2^2}{k^2}\right](e^{-kC_1}-e^{kC_3})$$
$$+\left[(\alpha-1)\frac{k_2^2}{k}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$
$$+\left[3\alpha\frac{k_2^2}{k}-6\alpha\xi_3k_2^2\right]He^{-kC_3}$$
$$+\begin{cases}\left[(1-\alpha)\frac{k_2^2}{k}\right]He^{-kC_1} & (H<\xi_3)\\ \left[-\frac{3}{2}\frac{k_1^2}{k^2}-\frac{2\alpha+1}{2}\frac{k_2^2}{k^2}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\frac{k_2^2}{k}\right]\xi_3e^{-kC_1}+\left[-3\alpha\frac{k_2^2}{k}\right](H-\xi_3)e^{-kC_2} & (H>\xi_3)\,,\end{cases}$$

$$\frac{2}{\mu}\overline{w}^{z}_{23}=\left[\frac{4\alpha-1}{2}\frac{ik_2}{k}\right](e^{-kC_1}-e^{-kC_3})$$
$$+[(1-\alpha)ik_2]\xi_3(e^{-kC_1}-e^{-kC_3})$$
$$+[3\alpha ik_2-6\alpha\xi_3ik_2k]He^{-kC_3}$$
$$+\begin{cases}[(\alpha-1)ik_2]He^{-kC_1} & (H<\xi_3)\\ \left[\frac{1-4\alpha}{2}\frac{ik_2}{k}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+[(\alpha-1)ik_2]\xi_3e^{-kC_1}+[-3\alpha ik_2](H-\xi_3)e^{-kC_2} & (H>\xi_3)\,.\end{cases}$$

$(kl)=(31)$

$$\frac{2}{\mu}\overline{w}^{x}_{31}(k_1,k_2)=\frac{2}{\mu}\overline{w}^{y}_{23}(k_2,k_1)\,,$$

$$\frac{2}{\mu}\overline{w}^{y}_{31}(k_1,k_2)=\frac{2}{\mu}\overline{w}^{x}_{23}(k_2,k_1)=\frac{2}{\mu}\overline{w}^{x}_{23}(k_1,k_2)\,,$$

$$\frac{2}{\mu}\overline{w}^{z}_{31}(k_1,k_2)=\frac{2}{\mu}\overline{w}^{z}_{23}(k_2,k_1)\,.$$

$(kl)=(12)$

$$\frac{2}{\mu}\overline{w}^{x}_{12}=\left[\frac{(4\alpha-1)(\alpha-2)}{2\alpha}\frac{ik_1^2k_2}{k^3}+\frac{3}{2}\frac{ik_2}{k}\right](e^{-kC_1}-e^{-kC_3})$$
$$+\left[(\alpha-1)\frac{ik_1^2k_2}{k^2}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$
$$+\left[3(2-\alpha)\frac{ik_1^2k_2}{k^2}-6\alpha\xi_3\frac{ik_1^2k_2}{k}\right]He^{-kC_3}$$

$$+\begin{cases}\left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]He^{-kC_1} & (H<\xi_3)\\ \left[\dfrac{1-4\alpha}{2}\dfrac{ik_1^2k_2}{k^3}+\dfrac{3}{2}\dfrac{ik_2}{k}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\dfrac{ik_1^2k_2}{k^2}\right]\xi_3e^{-kC_1}+\left[-3\alpha\dfrac{ik_1^2k_2}{k^2}\right](\xi_3-H)e^{-kC_2} & (H>\xi_3)\,,\end{cases}$$

$$\frac{2}{\mu}\bar{w}_{12}^y(k_1,k_2)=\frac{2}{\mu}\bar{w}_{12}^x(k_2,k_1)\,,$$

$$\frac{2}{\mu}\bar{w}_{12}^z=\left[\frac{\alpha^2-9\alpha+2}{2\alpha}\frac{k_1k_2}{k^2}\right](e^{-kC_1}-e^{-kC_3})$$

$$+\left[(\alpha-1)\frac{k_1k_2}{k}\right]\xi_3(e^{-kC_1}-e^{-kC_3})$$

$$+[6\alpha\xi_3k_1k_2]He^{-kC_3}$$

$$+\begin{cases}\left[(1-\alpha)\dfrac{k_1k_2}{k}\right]He^{-kC_1} & (H<\xi_3)\\ \left[\dfrac{2\alpha-5}{2}\dfrac{k_1k_2}{k^2}\right](e^{-kC_1}-e^{-kC_2}) & \\ \quad+\left[(1-\alpha)\dfrac{k_1k_2}{k}\right]\xi_3e^{-kC_1}+\left[3\alpha\dfrac{k_1k_2}{k}\right](\xi_3-H)e^{-kC_2} & (H>\xi_3)\,.\end{cases}$$

Appendix C: Piezomagnetic field potential accompanying a vertical strike-slip fault with infinite length.

(a) uniform slip model

$$\frac{2}{\beta J_x\mu U_0}W_x=W_0^x+\begin{cases}W_{\mathrm{I}}^x & (H\geqq D)\\ W_{\mathrm{II}}^x & (D>H\geqq d)\\ W_{\mathrm{III}}^x & (d>H)\end{cases}$$

$$W_0^x=3\left[\tan^{-1}\frac{d-z}{y}-\tan^{-1}\frac{D-z}{y}+\tan^{-1}\frac{2H+D-z}{y}-\tan^{-1}\frac{2H+d-z}{y}\right]$$

$$W_{\mathrm{I}}^x=3\left[\tan^{-1}\frac{d-z}{y}-\tan^{-1}\frac{D-z}{y}-\tan^{-1}\frac{2H-D-z}{y}+\tan^{-1}\frac{2H-d-z}{y}\right]$$

$$W_{\mathrm{II}}^x=3\left[\tan^{-1}\frac{d-z}{y}-2\tan^{-1}\frac{H-z}{y}+\tan^{-1}\frac{2H-d-z}{y}\right]$$

$$W_{\mathrm{III}}^x=0$$

(b) linear slip model

$$\frac{2(D-d)}{\beta J_x \mu U_0} W_x = W_0^x + \begin{cases} W_{\mathrm{I}}^x & (H \geqq D) \\ W_{\mathrm{II}}^x & (D > H \geqq d) \\ W_{\mathrm{III}}^x & (d > H) \end{cases}$$

$$W_0^x = -2(D-z)\left\{\tan^{-1}\frac{D-z}{y} - \tan^{-1}\frac{d-z}{y}\right\} + y \log\frac{y^2+(D-z)^2}{y^2+(d-z)^2}$$

$$+2(2H+D-z)\left\{\tan^{-1}\frac{2H+D-z}{y} - \tan^{-1}\frac{2H+d-z}{y}\right\} - y \log\frac{y^2+(2H+D-z)^2}{y^2+(2H+d-z)^2}$$

$$W_{\mathrm{I}}^x = -2(D-z)\left\{\tan^{-1}\frac{D-z}{y} - \tan^{-1}\frac{d-z}{y}\right\} + y \log\frac{y^2+(D-z)^2}{y^2+(d-z)^2}$$

$$+2(2H-D-z)\left\{\tan^{-1}\frac{2H-D-z}{y} - \tan^{-1}\frac{2H-d-z}{y}\right\} - y \log\frac{y^2+(2H-D-z)^2}{y^2+(2H-d-z)^2}$$

$$W_{\mathrm{II}}^x = -2(D-z)\left\{\tan^{-1}\frac{H-z}{y} - \tan^{-1}\frac{d-z}{y}\right\} + y \log\frac{y^2+(H-z)^2}{y^2+(d-z)^2}$$

$$+2(2H-D-z)\left\{\tan^{-1}\frac{H-z}{y} - \tan^{-1}\frac{2H-d-z}{y}\right\} - y \log\frac{y^2+(H-z)^2}{y^2+(2H-d-z)^2}$$

$$W_{\mathrm{III}}^x = 0 .$$

Appendix D: Piezomagnetic field potential accompanying a vertical rectangular strike-slip fault.

(a) horizontal magnetization in the x direction

$$\frac{2}{\beta J_x \mu \Delta U} W_x = W_0^x + \begin{cases} W_{\mathrm{I}}^x & (H \geqq D) \\ W_{\mathrm{II}}^x & (D > H \geqq d) \\ W_{\mathrm{III}}^x & (d > H) \end{cases}$$

$$W_0^x = \frac{3}{2}\left[\left\{\tan^{-1}\frac{tp_1}{yS_1}\right\}\Big\|_d^D - \left\{\tan^{-1}\frac{tp_3}{yS_3}\right\}\Big\|_d^D\right]$$

$$+\frac{(4\alpha-1)(\alpha-2)}{2\alpha}\left[\left\{\frac{ty}{r^2}\cdot\frac{p_1}{S_1+p_1}\right\}\Big\|_d^D - \left\{\frac{ty}{r^2}\cdot\frac{p_3}{S_3+p_3}\right\}\Big\|_d^D\right]$$

$$+(\alpha-1)\left[\left\{\frac{ty}{r^2}\cdot\frac{p_1^2}{S_1(S_1+p_1)}\right\}\Big\|_d^D - z\left\{\frac{ty}{S_1(S_1+p_1)^2}\right\}\Big\|_d^D\right]$$

$$-(\alpha-1)\left[\left\{\frac{ty}{r^2}\cdot\frac{p_3^2}{S_3(S_3+p_3)}\right\}\Big\|_d^D - (z-2H)\left\{\frac{ty}{S_3(S_3+p_3)^2}\right\}\Big\|_d^D\right]$$

$$+3(2-\alpha)H\left[\left\{-\frac{ty}{S_3(S_3+p_3)^2}\right\}\Big\|_d^D\right]$$

$$-6\alpha H\left[\left\{-\frac{ty}{S_3^3}\right\}\Big\|_d^D+(z-2H)\left\{\frac{ty}{r^4}\left(\frac{3p_3}{S_3}-\frac{p_3^3}{S_3^3}\right)\right\}\Big\|_d^D\right]$$

$$W_{\mathrm{I}}^x=\frac{3}{2}\left[\left\{\tan^{-1}\frac{tp_1}{yS_1}\right\}\Big\|_d^D+\left\{\tan^{-1}\frac{tp_2}{yS_2}\right\}\Big\|_d^D\right]$$

$$+\frac{1-4\alpha}{2}\left[\left\{\frac{ty}{r^2}\cdot\frac{p_1}{S_1+p_1}\right\}\Big\|_d^D+\left\{\frac{ty}{r^2}\frac{p_2}{S_2+p_2}\right\}\Big\|_d^D\right]$$

$$+(1-\alpha)\left[\left\{\frac{ty}{r^2}\cdot\frac{p_1^2}{S_1(S_1+p_1)}\right\}\Big\|_d^D-z\left\{\frac{ty}{S_1(S_1+p_1)^2}\right\}\Big\|_d^D\right]$$

$$-3\alpha\left[\left\{\frac{ty}{r^2}\cdot\frac{p_2^2}{S_2(S_2+p_2)}\right\}\Big\|_d^D-(z-2H)\left\{\frac{ty}{S_2(S_2+p_2)^2}\right\}\Big\|_d^D\right]$$

$$+3\alpha H\left[\left\{\frac{ty}{S_2(S_2+p_2)^2}\right\}\Big\|_d^D\right]$$

$$W_{\mathrm{II}}^x=W_{\mathrm{I}}^x(D\to H)+(1-\alpha)H\left[\left\{-\frac{ty}{S_1(S_1+p_1)^2}\right\}\Big\|_H^D\right]$$

$$W_{\mathrm{III}}^x=(1-\alpha)H\left[\left\{\frac{-ty}{S_1(S_1+p_1)^2}\right\}\Big\|_d^D\right]$$

where $t=x-x_1$,

$p_1=x_3-z$, $\quad p_2=2H-x_3-z$, $\quad p_3=2H+x_3-z$,

$r^2=t^2+y^2$, $\quad S_1=\sqrt{r^2+p_1^2}$, $\quad S_2=\sqrt{r^2+p_2^2}$, $\quad S_3=\sqrt{r^2+p_3^2}$

and $\quad f(x_1, x_3)\|_a^b=f(L, b)-f(L, a)-f(-L, b)+f(-L, a)$.

(b) horizontal magnetization in the y direction

$$\frac{2}{\beta J_y\mu\Delta U}W_y=W_0^y+\begin{cases}W_{\mathrm{I}}^y & (H\geqq D)\\ W_{\mathrm{II}}^y & (D>H\geqq d)\\ W_{\mathrm{III}}^y & (d>H)\end{cases}$$

$$W_0^y=\frac{3}{2}[-\{\log(S_1+p_1)\}\|_d^D+\{\log(S_3+p_3)\}\|_d^D]$$

$$+\frac{(4\alpha-1)(\alpha-1)}{2\alpha}\left[-\frac{1}{2}\left\{\log(S_1+p_1)+\frac{t^2-y^2}{r^2}\cdot\frac{p_1}{S_1+p_1}\right\}\Big\|_d^D\right.$$

$$\left.+\frac{1}{2}\left\{\log(S_3+p_3)+\frac{t^2-y^2}{r^2}\cdot\frac{p_3}{S_3+p_3}\right\}\Big\|_d^D\right]$$

$$+(\alpha-1)\left[\left\{-\frac{1}{2}\log(S_1+p_1)+\frac{1}{2}\cdot\frac{p_1}{S_1+p_1}+\frac{y^2}{r^2}\cdot\frac{p_1^2}{S_1(S_1+p_1)}\right\}\Big\|_d^D\right.$$

$$\left.+z\left\{\frac{1}{S_1+p_1}-\frac{y^2}{S_1(S_1+p_1)^2}\right\}\Big\|_d^D\right]$$

$$-(\alpha-1)\left[\left\{-\frac{1}{2}\log(S_3+p_3)+\frac{1}{2}\cdot\frac{p_3}{S_3+p_3}+\frac{y^2}{r^2}\cdot\frac{p_3^2}{S_3(S_3+p_3)}\right\}\bigg\|_d^D\right.$$

$$\left.+(z-2H)\left\{\frac{1}{S_3+p_3}-\frac{y^2}{S_3(S_3+p_3)^2}\right\}\bigg\|_d^D\right]$$

$$+3(2-\alpha)H\left[\left\{\frac{1}{S_3+p_3}-\frac{y^2}{S_3(S_3+p_3)^2}\right\}\bigg\|_d^D\right]$$

$$-6\alpha H\left[\left\{\frac{1}{S_3}-\frac{y^2}{S_3^3}\right\}\bigg\|_d^D+(z-2H)\left\{\frac{1}{r^2}\left(\frac{3y^2-r^2}{r^2}\cdot\frac{p_3}{S_3}-\frac{y^2}{r^2}\cdot\frac{p_3^3}{S_3^3}\right)\right\}\bigg\|_d^D\right]$$

$$W_{\mathrm{I}}^y=\frac{3}{2}\left[-\{\log(S_1+p_1)\}\|_d^D-\{\log(S_2+p_2)\}\|_d^D\right]$$

$$+\frac{1-4\alpha}{2}\left[-\frac{1}{2}\left\{\log(S_1+p_1)+\frac{t^2-y^2}{r^2}\cdot\frac{p_1}{S_1+p_1}\right\}\bigg\|_d^D\right.$$

$$\left.-\frac{1}{2}\left\{\log(S_2+p_2)+\frac{t^2-y^2}{r^2}\cdot\frac{p_2}{S_2+p_2}\right\}\bigg\|_d^D\right]$$

$$+(1-\alpha)\left[\left\{-\frac{1}{2}\log(S_1+p_1)+\frac{1}{2}\frac{p_1}{S_1+p_1}+\frac{y^2}{r^2}\frac{p_1^2}{S_1(S_1+p_1)}\right\}\bigg\|_d^D\right.$$

$$\left.+z\left\{\frac{1}{S_1+p_1}-\frac{y^2}{S_1(S_1+p_1)^2}\right\}\bigg\|_d^D\right]$$

$$-3\alpha\left[\left\{-\frac{1}{2}\log(S_2+p_2)+\frac{1}{2}\frac{p_2}{S_2+p_2}+\frac{y^2}{r^2}\cdot\frac{p_2^2}{S_2(S_2+p_2))}\right\}\bigg\|_d^D\right.$$

$$\left.+(z-2H)\left\{\frac{1}{S_2+p_2}-\frac{y^2}{S_2(S_2+p_2)^2}\right\}\bigg\|_d^D\right]$$

$$+3\alpha H\left[\left\{-\frac{1}{S_2+p_2}+\frac{y^2}{S_2(S_2+p_2)^2}\right\}\bigg\|_d^D\right]$$

$$W_{\mathrm{II}}^y=W_{\mathrm{I}}^y(D\to H)+(1-\alpha)H\left[\left\{\frac{1}{S_1+p_1}-\frac{y^2}{S_1(S_1+p_1)^2}\right\}\bigg\|_H^D\right]$$

$$W_{\mathrm{III}}^y=(1-\alpha)H\left[\left\{\frac{1}{S_1+p_1}-\frac{y^2}{S_1(S_1+p_1)^2}\right\}\bigg\|_d^D\right]$$

(c) vertical magnetization

$$\frac{2}{\beta J_z\mu\varDelta U}W_z=W_0^z+\begin{cases}W_{\mathrm{I}}^z & (H\geqq D)\\ W_{\mathrm{II}}^z & (D>H\geqq d)\\ W_{\mathrm{III}}^z & (d>H)\end{cases}$$

$$W_0^z=\frac{\alpha^2-9\alpha+2}{2\alpha}\left[\left\{\frac{y}{S_1+p_1}\right\}\bigg\|_d^D-\left\{\frac{y}{S_3+p_3}\right\}\bigg\|_d^D\right]$$

$$+(\alpha-1)\left[\left\{\frac{y}{S_1}\right\}\bigg\|_d^D-z\cdot\left\{\frac{y}{r^2}\cdot\frac{p_1}{S_1}\right\}\bigg\|_d^D\right]$$

$$-(\alpha-1)\left[\left\{\frac{y}{S_3}\right\}\Big\|_d^D-(z-2H)\cdot\left\{\frac{y}{r^2}\cdot\frac{p_3}{S_3}\right\}\Big\|_d^D\right]$$

$$+6\alpha H\left[\left\{-\frac{y}{r^2}\cdot\frac{p_3^3}{S_3^3}\right\}\Big\|_d^D+(z-2H)\cdot\left\{\frac{y}{S_3^3}\right\}\Big\|_d^D\right]$$

$$W_{\mathrm{I}}^z=\frac{2\alpha-5}{2}\left[\left\{\frac{y}{S_1+p_1}\right\}\Big\|_d^D+\left\{\frac{y}{S_2+p_2}\right\}\Big\|_d^D\right]$$

$$+(1-\alpha)\left[\left\{\frac{y}{S_1}\right\}\Big\|_d^D-z\cdot\left\{\frac{y}{r^2}\cdot\frac{p_1}{S_1}\right\}\Big\|_d^D\right]$$

$$+3\alpha\left[-\left\{\frac{y}{S_2}\right\}\Big\|_d^D+(z-2H)\cdot\left\{\frac{y}{r^2}\cdot\frac{p_2}{S_2}\right\}\Big\|_d^D\right]$$

$$-3\alpha H\left[\left\{\frac{y}{r^2}\cdot\frac{p_2}{S_2}\right\}\Big\|_d^D\right]$$

$$W_{\mathrm{II}}^z=W_{\mathrm{I}}^z(D\to H)+(1-\alpha)H\left[\left\{-\frac{y}{r^2}\cdot\frac{p_1}{S_1}\right\}\Big\|_H^D\right]$$

$$W_{\mathrm{III}}^z=(1-\alpha)H\left[\left\{-\frac{y}{r^2}\cdot\frac{p_1}{S_1}\right\}\Big\|_d^D\right]$$

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19. くい違いの弾性論を用いた地殻変動モデルに伴うピエゾ磁気変化

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地震や火山活動などに伴う地殻変動は，くい違いの弾性論を用いてモデル化されている。本論文では，この種のモデルによるピエゾ磁気変化を，グリーン関数を使って求める方法を定式化した。先ず単軸圧縮実験に基づいた岩石の可逆的ピエゾ磁気変化の公式を，任意の三次元応力状態における帯磁と応力の関係式に拡張する。この一般化したピエゾ磁気公式と，半無限弾性体内のくい違い面による応力を与える Volterra の公式を組み合わせて，任意のくい違いモデルに伴うピエゾ磁気変化のポテンシャルを与える式を得る。これはピエゾ磁気ポテンシャルに対する Volterra の公式とも呼ぶべきもので，地磁気変化はモデルのくい違い変位量を重みとしたポテンシャル関数の面積分の形で表現される。ただし，地球は等方で均質な半無限弾性体であり，地表からある深さ（キュリー点等温面）までは一様に帯磁し，かつこの部分の磁気応力係数も一定という，最も簡単な場合を考えた。前述したポテンシャル関数がグリーン関数に他ならないが，物理的にはくい違い面上の各点に置かれた複双力源が作るピエゾ磁気ポテンシャルと見なせる。任意のくい違いモデルを記述するに必要な6種類の歪核によるピエゾ磁気ポテンシャルは，たたみ込み積分のフーリエ変換定理を利用して，解析的に得られる。

従来のピエゾ磁気モデルの計算では，帯磁した弾性体の各点で主応力と主軸方向から帯磁変化を求め，これを全体積にわたって数値積分する，という手続きがとられていた。本稿の方法によれば，半無限領域にわたる体積分は有限な広がりの面積分に帰着され，各点における応力成分を求める必要も無いので，数値計算が極めて簡単化される。くい違い面の形と変位不連続の分布が単純な場合には，解析解が容易に得られる。簡単な応用例として，垂直な横ずれ断層に伴うピエゾ磁気変化の解析解を求めた。先ず無限に長い二次元断層について，Shamsi and Stacey (1969) の提案した uniform slip および linear slip モデルの解を求め，これ等が断層の位置とそのキュリー点深度に関する鏡像の位置に置かれた板磁石の作る磁場と等価であることを示した。更に断層面が長方形の場合の解を，Stacey (1964) のモデルと比較した。くい違いの弾性論に基づく断層モデルでは，断層の末端効果が卓越することと，鉛直磁化成分の変化からの寄与が加わるため，地磁気変化の分布が Stacey モデルと著しく異なる。いわゆる地震地磁気効果は，①震央域でのみ検出可能である，②その変化量は地震のマグニチュードにはあまり依存しないが，断層面上端の深さに大きく依存する，③浅い地震に限っては，従来の Stacey モデルに比べてかなり大きな変化が期待される，等が結論された。

最後に，等方均質で一様磁化した弾性体が表面力又は内部応力源によって変形した場合，そのピエゾ磁気ポテンシャルを，磁性体内の変位分布の関数として表現する一般式を導いた。特に無限媒質中のポテンシャルは，その点における変位に媒質の物質定数をかけたものと一致することが示される。この事実はくい違いの弾性論の基本概念が，ポテンシャル論における二重層との類推から説明されることを考えあわせると，興味深い。磁性体外部の磁気ポテンシャルは，磁性体表面の変位とその法線微分を知るだけで，一義的に定まる。ニュートン・ポテンシャルの知識を借りて，地表面での磁場変化，特に全磁力変化の一般的表現を求めてみた。これは観測点における地球磁場方向への変位成分の同じ方向への微分に，地表面とキュリー点等温面からの付加的寄与をあわせたものになる。従って全磁力変化の検出は，その観測点における地磁気方向への地殻の伸縮率を測ることと，大体において等価である。