

EARTHQUAKES AND EARTHQUAKE SOUNDS: AS ILLUSTRATIONS OF THE GENERAL THEORY OF ELASTIC VIBRATIONS.

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(Read February 23rd, 1888.)

The first systematic application of the theory of vibrations to the problems of earthquake motion was made, I believe, by Hopkins in his Report on the Theories of Elevation and Earthquakes, presented to the British Association in 1847. During the forty years which have elapsed since then, our knowledge of earthquake phenomena has steadily grown. The labours of Mallet have been largely supplemented by the observations and experiments of a small army of enthusiasts, who have pitched their tents on the trembling soil of Italy and Japan. Their energies have been mainly directed to the perfecting of seismographs and seismometers, to the registering of all kinds of earth movements, to the study of the effects of these on buildings, and, in a limited degree, to the measurement of the velocities of propagation of disturbances due to artificial earthquakes. With all this activity on the experimental side, we have to confess that theoretic views have hardly advanced beyond the stage in which Hopkins left them in 1847. G. H. Darwin's discussion of the strains due to continental areas, and Lord Rayleigh's investigation into a special case of surface waves on an elastic solid, are perhaps the only mathematical pieces of work that have any distinct bearing on seismic phenomena. The former gives an obvious *raison d'être* for the existence of seismically sensitive regions within the earth's crust, but, being an equilibrium problem, can throw no light on that progress of the state of strain which constitutes earthquake

motion. Lord Rayleigh's results will be referred to hereafter in due course. Meanwhile, as it is my object to discuss in a general way how far earthquakes and their accompanying effects may be explained as disturbances in an elastic or sub-elastic medium, it will be convenient to reproduce here much that may be found in authoritative earthquake literature, such as Hopkins' and Mallet's *Reports*, Mallet's *Neapolitan Earthquake*, Milne's *Earthquakes*, and so on.

From the general theory of the vibrations of homogeneous elastic solids we know that there are three types of wave propagated with different velocities. If we confine our attention to an isotropic elastic solid these types reduce to two, which are kinematically easily distinguished by the relation which the direction of vibration of any particle bears to the direction of propagation of the wave. Thus in the one type the vibrations are *normal* to the wave-front—in the other they are *transverse* or *tangential*. Dynamically the types may be distinguished as the *condensational* and *distortional* waves. The former is of essentially the same character as ordinary sound waves in air; and the latter may be compared, so far as direction of motion is concerned, to waves of light in the luminiferous ether. In the condensational wave the vibrating particles move to and fro in lines parallel to the direction of motion of the wave. In the distortional wave, the particles move to and fro in lines perpendicular to the wave's direction of motion.

In all cases these two types of wave are propagated with different velocities, which depend upon the density and the elastic constants of the material. For an isotropic elastic solid there are two independent elastic moduli, known respectively as the *bulk-modulus* or resistance to compression, and the *rigidity* or resistance to distortion. The velocity of the distortional wave depends on the ratio of the rigidity to the density. The velocity of the condensational wave, however, is not so simply related to the other modulus, but depends for its value upon the rigidity as well.

Take for example a uniform cylindrical rod of iron. By giving the one end of this rod a slight twist we may set up a series of torsional vibrations, whose velocity of propagation along the rod is to be measured by the square root of the ratio of the rigidity to the density. The velocity of propagation of longitudinal vibrations, which may be supposed to be given by an impact on the end, is to be measured by the square root of the ratio of the so-called Young's Modulus to the density. Young's Modulus is a definite function of the principal moduli already mentioned, being given by the formula

$$9 n k / (3 k + n)$$

where k is the resistance to compression and n is the rigidity.

Again, if we consider the case of plane waves in an infinite solid, we find that here also the velocity of propagation of the distortional wave is given by the ratio $\sqrt{n}/\sqrt{\rho}$, while that of the condensational waves is measured in terms of a mixed modulus which is not necessarily the same as Young's Modulus. Its value is $k + \frac{3}{4} n$ which is equal to Young's Modulus only if $3 k = 2 n$.

According to Navier's and Poisson's theory of elasticity, we should have $3 k = 5 n$. This is usually expressed by saying that, if a bar is stretched under a longitudinal pull, its linear contraction at right angles to the pull is one quarter of the elongation in the direction of the pull. So far as experiments with hard metals go, this ratio may vary from .2 to .4. Nevertheless .25 may be taken to be a pretty fair mean value.

If we write m instead of $k + \frac{3}{4} n$,* we obtain for the value of the Poisson ratio the expression

$$s = \frac{m-2 n}{2 (m-n)}$$

The possible values of s range from $+\frac{1}{2}$ to -1 ;—the former being its value in an incompressible elastic body, the latter its value in a body of infinite rigidity but finite compressibility. The

* This m is not the same as the m used by Thomson and Tait, but for our present purpose it is convenient to use one symbol for the mixed modulus which determines the speed of the condensational wave.

luminiferous ether appears to be a substance of infinite resistance to compression; but of the other limiting kind of elastic material we have no example.

The velocities of the condensational and distortional waves are given respectively by the expressions $\sqrt{m/\rho}$ and $\sqrt{n/\rho}$, ρ being the density of the material.

There are experimental methods for measuring the quantities m and n ; and from them the two velocities can easily be calculated. Or if the two velocities are known, it is possible to calculate from them the two moduli. Now, it is quite obvious that m must always be greater than n ; the ratio indeed varies from ∞ for the case of the incompressible body to $4/3$ for the case of the infinitely rigid body. Of course in the latter case both waves travel with an infinite speed; but the speed of the distortional wave can never become equal to the speed of the condensational wave, however large it is made to be.

In deducing the true values of m and n from the two wave velocities, we must know the density of the material. The only values for wave-velocities of both types in rocks, which I have been able to find, are those given by Messrs. Milne and Gray. These velocities were originally obtained from direct measurements of the elastic moduli of the rocks in question. The moduli themselves Professor Milne has recently furnished me with. In the following table they are given,* expressed in C.G.S. units, along with the Poisson ratio s .

Rock.	m .	n .	s .
Granite ...	4.68×10^{11}	1.44×10^{11}	+ .28
Marble.....	4.35×10^{11}	1.3×10^{11}	+ .29
Tuff	2.44×10^{11}	1.31×10^{11}	— .08
Clay Rock..	3.66×10^{11}	1.94×10^{11}	— .07
Slate.....	6.07×10^{11}	2.45×10^{11}	+ .16

* In the notes given me by Professor Milne the numbers here tabulated under m are headed "Young's Modulus." This, I am inclined to think, is a mistake. Professor Milne himself, not having the complete records in possession, is doubtful. At any rate, these numbers give the velocities of the normal vibrations as tabulated by Messrs. Milne and Gray (see *Phil. Mag.* November, 1881). Further, if they really were Young's Moduli, we should have in granite and marble examples of substances which expand when compressed!

Two of the ratios come out negative, which means physically that, if the substance is perfectly elastic, an extension of the substance by a pull in a given direction is accompanied by an extension at right angles to this direction. It also means that the ratio of the velocities of the two waves is distinctly smaller than in the other cases. This diminished ratio, it will be noticed, exists along with a diminished resistance to compression, while the rigidity continues to have much the same value as those which hold for the other rocks. In the cases of the tuff and clay-rock we may have to do with either a considerable compressibility, or a sluggishness in recovery due to the viscosity of the material. Such a viscosity might well show itself more distinctly in compression than in distortion.

If we calculate from Professor Milne's values of wave velocities obtained from his experiments on artificial earthquakes, we find for the ratio s in two different cases the values $+ .154$ and $-.152$ and for the corresponding ratios of m to n the values 2.43 and 1.76 .

In the calculations to be described presently, I have taken the following values of the several constants involved as a fair approximation to what might reasonably be regarded as somewhat near the truth, when the elastic properties of fairly solid rock are to be considered.

density	ρ	$= 3$
rigidity	n	$= 1.5 \times 10^{11}$
ratio of the wave-moduli	m/n	$= 3$
Poisson's ratio	s	$= .25$

We now pass to the consideration of the transmission of waves in an elastic solid; and first I desire to call attention to Lord Rayleigh's short paper "On Waves Propagated along the Plane Surface of an Elastic Solid."*

* Proceedings of the London Mathematical Society (Vol. XVII., 1885-6.)

To show that the paper deserves the special attention of members of the Society I need but quote the two concluding sentences. "It is not improbable that the surface waves here investigated play an important part in earthquakes, and in the collision of elastic solids. Diverging in two dimensions only, they must acquire at a great distance a continually increasing preponderance"—that is, I presume, as compared to waves diverging in three dimensions.

The purpose of the paper is "to investigate the behaviour of waves upon the plane free surface of an infinite homogeneous isotropic elastic solid, their character being such that a disturbance is confined to a superficial region of thickness comparable with the wave length. The case is thus analogous to that of deep-water waves, only that the potential energy here depends upon elastic resilience instead of upon gravity."

Starting with the usual equations of motion of a vibrating elastic solid, Lord Rayleigh obtains a general solution on the assumptions that the displacements are harmonic functions of the time and the two co-ordinates parallel to the plane free surface, but are exponential functions of negative multiples of the distance from this plane. The boundary equations are then introduced; and from the conditions for the equilibrium of a surface element the various constants of integration are determined in terms of the circumstances of the assumed motion. Two cases are discussed in detail—those, namely, of an incompressible elastic solid and of a solid for which the Poisson ratio has the value one-fourth. For both cases the results are very similar. Thus, if the displacements are supposed to be confined to one plane, a particle at the surface moves in an elliptic orbit whose major axis is perpendicular to the plane surface of the solid. For the incompressible solid the major axis is nearly twice as great as the minor axis; and for the other case it is about one and a half times as great. The displacement parallel to the plane surface penetrates but a short distance into the solid—to about one-eighth of a wave-

length for the incompressible substance, and to about one-fifth for the other case. On the other hand, there is no finite depth at which the motion perpendicular to the plane vanishes. The surface waves are propagated at a slightly slower rate than a purely distortional plane wave would be.

It would appear then that vertical motion on a level surface over which a disturbance is passing cannot exist alone. Associated with it there must always be a distinctly smaller horizontal motion, which vanishes completely at a short depth below the surface. Lord Rayleigh's formulæ also show that the amplitude of the displacement is directly as the wave-length; so that for vibrations of short period the surface motions are proportionally small.

If we consider the features of earthquake motions, we find that the vertical motion when it is appreciable is always very much smaller than the horizontal motion. Hence we cannot have here merely the surface disturbance discussed by Lord Rayleigh. If his investigation touches upon any earthquake phenomenon, this phenomenon is never met with by itself alone. Horizontal displacements exist, at any rate along with it, of a magnitude greater far than Lord Rayleigh's result requires. The simple conclusion is that ordinary earthquakes cannot be regarded as due to the propagation of surface waves. Professor Milne has, at various times, speculated upon the existence of such surface-waves outstripping the vibrations transmitted through the mass. There never has seemed to me sufficient reason for calling in the aid of these surface-waves, as distinct from the mass-waves. Lord Rayleigh's investigation shows besides that the velocity of a surface disturbance is somewhat less than the velocity of the distortional plane wave travelling through the mass. There is no evidence of a quickened velocity. These two facts, namely, the comparative minuteness of the vertical motion in all earthquakes, and the somewhat slower speed of Lord Rayleigh's surface wave, seem to show that we can expect very little towards the elucidation of earth-

quake phenomena by taking into account the so-called surface wave.

I now pass to the consideration of the reflection and refraction of plane waves at the surface of separation of two elastic media. In doing so I shall direct more especial attention to the case in which the one medium is rock and the other water. The case in which both media are solid substances is a good deal more troublesome to deal with ; and so far I have not had time to work out any detailed calculations concerning it. A few general considerations will show the nature of the problem.

The reflection and refraction of plane waves at the bounding surface of two media have been very closely studied by many mathematicians. Especially have their efforts been directed towards the explanation of the ordinary phenomena of light upon a purely dynamic basis. Cauchy, Green, Maccullagh, Lorenz, Rayleigh, Thomson, may be mentioned in this connection. It is sufficient here to point out that, when the problem is worked out for the case of two incompressible elastic substances of equal rigidities but different densities, results are obtained in fair accordance with observation. The media being incompressible, no wave of condensation can be propagated through them. Distortional waves only can exist. Thus an incident distortional wave falling on the bounding surface will, in general, be broken up into two waves, one reflected into the first medium and the other refracted into the second medium. But although distortional waves alone exist in the media, the correct solution of the problem in elastic solids requires us to take account of something existing at the bounding surface of the nature of a condensational wave. We must bear in mind, indeed, what the physical meaning of incompressibility is. It is not that the condensational wave vanishes but that it is transmitted with infinite velocity. By taking this surface disturbance into account—this *pressural* wave as Thomson has called it—we are able partially to explain certain

phenomena of reflection and refraction of polarised light in terms of the theory of elastic solids.

Now in this special problem we begin with a distortional wave incident on the bounding surface; and, although the media are taken as incompressible, we must not neglect the effect of the pressural wave. Hence, if our methods of attack are to be the same in all cases, we must admit the possibility of true waves of compression being started in media of finite compressibility, when upon their boundary a single distortional wave impinges. In other words, an incident distortional wave may be broken up into four parts;—a reflected distortional, a refracted distortional, a reflected condensational, and a refracted condensational. In like manner, an incident condensational wave will in general give rise to reflected and refracted distortional waves as well as to reflected and refracted condensational waves.

The various angles of reflection and refraction are easily calculated in terms of the angles of incidence, it being noted that the surface trace is common to all the waves. In other words each wave velocity is, so to speak, the component in its direction of the velocity of propagation of the surface trace.

Thus, let a condensational wave be incident at an angle θ to the normal to the bounding surface; let m , n , ρ , and m' , n' , ρ' be the wave moduli and densities of the two media, in the first of which the incident wave is given. Then if θ' be the angle of refraction of the condensational wave, and ϕ ϕ' the angles of reflection and refraction of the distortional wave, the above condition gives these equations:—

$$\frac{m}{\rho} \operatorname{cosec}^2 \theta = \frac{n}{\rho} \operatorname{cosec}^2 \phi = \frac{m'}{\rho} \operatorname{cosec}^2 \theta' = \frac{n'}{\rho'} \operatorname{cosec}^2 \phi'$$

Now, as n is less than m , there will always be a reflected distortional wave, except of course at normal incidence when $\theta=0^\circ$, or at grazing incidence when $\theta=90^\circ$. There will be refracted waves at all except the limiting incidences if m/ρ is greater than m'/ρ' . If m/ρ should be intermediate in value between m'/ρ' and n'/ρ' there will always be a refracted dis-

tortional wave, but for angles of incidence higher than a certain critical value a refracted condensational wave is impossible. Further if m/ρ should be less than n'/ρ' , then, for each refracted wave, there is a special critical angle of incidence at and above which the wave vanishes. When the critical value corresponding to the refracted distortional wave is reached, there will be total reflection, and the whole energy of the incident wave will be divided between the two reflected waves.

If the incident wave is a distortional wave, there must always be a critical angle of incidence for and above which the reflected condensational wave vanishes. The existence of such critical angles for the refracted waves will depend upon the relative values of the quantities n/ρ , m'/ρ' , n'/ρ' ,—the condition for the possibility of total reflection being that n/ρ is less than n'/ρ' .

If one of the media is a fluid, there can, of course, be no distortional wave in it. It is this somewhat simple case I propose to discuss in detail. I shall not here enter into the purely mathematical method by what the energies of the various possible waves have been determined. It is sufficient to say that it is the usual mode of treatment of plane waves, a harmonic form being assumed and the constants determined so as to satisfy the equations of motion and the boundary conditions.

We shall take then, as the one medium, water; and, as the other, rock of density 3, rigidity 1.5×10^{11} and Poisson ratio .25. The density of the water is taken as unity and the value of the bulk-modulus, which in this case is also the wave-modulus, 2.2×10^{10} . The quantities are given in C.G.S. units. The manner in which, for different angles of incidence in the rock, the energy of the incident wave is distributed amongst the reflected and refracted waves is shown in the following tables. The first refers to the case of the incident wave being condensational; the second to the case of the incident wave being distortional. The quantities A A_1 A' re-

present the energies of the incident, reflected and refracted condensational waves; B , B_1 , B' the energies of the similar set of distortional waves. The corresponding angles of incidence, reflection and refraction are given in contiguous columns— θ referring to the condensational and ϕ to the distortional waves.

INCIDENT WAVE CONDENSATIONAL.					
INCIDENT.		REFLECTED.	REFRACTED.		REFLECTED.
θ	A	A_1	θ'	A'	ϕ B_1
0°	I	.599	0°	.401	0° .000
10°	I	.536	3° 49'	.397	5° 45' .071
20°	I	.377	7° 32'	.370	11° 23' .254
30°	I	.195	11° 2'	.333	16° 35' .456
40°	I	.056	14° 15'	.293	21° 47' .660
50°	I	.006	17° 4'	.244	26° 15' .753
60°	I	.014	19° 22'	.206	30° .775
70°	I	.031	21° 5'	.188	32° 51' .783
80°	I	.000	22° 9'	.182	34° 39' .818
89°	I	.616	22° 31'	.069	35° 16' .314
90°	I	I		.000	.000

INCIDENT WAVE DISTORTIONAL.					
INCIDENT.		REFLECTED.	REFLECTED.		REFRACTED.
ϕ	B	B_1	θ_1	A_1	θ' A'
0°	I	.1	0°	.0	0° .0
10° 23'	I	.711	20°	.253	7° 32' .036
21° 47'	I	.222	40°	.656	14° 15' .126
30°	I	.014	60°	.779	19° 22' .206
34° 39'	I	.027	80°	.815	22° 9' .157
35° 16'	I	.679	89°	.311	22° 31' .007
35° 16'	I	I	90°	.000	22° 31' .000
36°	I	.584	non-existent.		22° 56' .415
40°	I	.461			25° 14' .539
50°	I	.504			30° 33' .495
60°	I	.506			35° 4' .494
70°	I	.520			38° 34' .480
80°	I	.634			40° 47' .366
89° 45'	I	.818			41° 34' .183
90°	I	I			.000

In the first table B and B' of course do not appear; and in the second table A and B' do not appear.

It should be mentioned that each wave energy is calculated independently; and a test of the accuracy of the calculations is afforded by the condition that the energy of the incident wave must be fully accounted for. That is, since, in every case the incident wave (either A or B) is taken as unity, the sum of all the others must also be unity.

The chief peculiarities embodied in these tables are shown graphically in the corresponding curves. Any one curve represents the manner in which the energy of each wave depends on the angle of incidence. The angles of incidence are measured off along the horizontal line; and the corresponding energies are represented by the ordinates perpendicular thereto. The energy of the incident wave is of course represented by a straight line at unit distance from the line along which the angles of incidence are measured off.

The first set of curves shows the state of things for an incident condensational wave. For the sake of brevity, we shall occasionally refer to the different waves by the letters A , A_1 , A' , B , B_1 , chosen to represent their energies. At perpendicular incidence condensational waves only are started at the bounding surface; and as the angle of incidence increases the energies of both of these diminish. A' , which we may also call the water wave, seems to fall off continuously until it vanishes at grazing incidence. The A -wave, however, vanishes at two distinct incidences, and after 80° is reached begins to increase till at 90° it attains unity. The behaviour of this reflected condensational wave is extremely curious, being indeed practically non-existent for incidences between 50° and 80° . The greater part of the energy of the incident wave is then accounted for by the B_1 or reflected distortional wave. For incidences higher than 45° , three-quarters of the whole incident energy is so transformed. It will be noticed that up to pretty high angles of incidence the water-wave does not suffer any very rapid falling off.

Turning now to the second set of curves, which show the state of things for an incident distortional wave, we meet with some very curious relations. For reasons already discussed, the A_1 -wave cannot exist for incidences higher than a certain critical value, which depends only on the rock itself. This wave however attains a considerable maximum value for an angle of incidence slightly below this critical value. Almost for the same incidence, the B_1 -wave falls to a very low minimum, almost vanishing indeed. Comparing this first portion of the second set of curves with the first set of curves as a whole, we see a general resemblance between the two. That is, the reflected wave of the same type as the incident wave rapidly falls off to a minimum, as the angle of incidence grows, while the reflected wave of the other type rapidly increases to a maximum. Finally, however, the reflected wave of the same type, in both cases quite abruptly, runs up to equality with the incident wave. In the second set of curves this happens at the angle of total reflection; for, not only does the A_1 -wave vanish, but so also does the A' -wave*—which indeed never attains any great significance at the lower incidences. After the critical angle of incidence is passed, however, the A' -wave soon reaches a maximum, being then of greater importance than the B_1 -wave, and gradually falls away to zero, while the B_1 -wave as gradually rises to unity.

In trying in some way to bring these results into correspondence with earthquake phenomena, we notice first of all that, if an earthquake is to be regarded as a progressive wave in an elastic solid, the angles of emergence of the waves will generally be small—that is, the angles of incidence large. Hence we need pay but little attention to the state of affairs at the lower incidences. For higher incidences we see that whether the incident wave is condensational or distortional, the energy is reflected either wholly or almost wholly in the

* This seems to be a result as novel as it is curious from a purely theoretical point of view, although it has no special bearing on earthquake phenomena.

distortional wave form. Suppose for example that a disturbance begins at some region below the bottom of the sea, say at the point *C* in the figure; and let us assume that what starts from *C* is a simple wave of compression—that is a condensational wave. Then to any point *P* suitably placed, there will come not only a purely condensational but also a distortional wave produced by reflection from some part of the surface separating the sea and the land.



It is easy to see, however, that this transformation of condensational into distortional straining will accompany all similar cases of reflection at the boundary of two different media, whether the one medium is water or some other substance—air, say, or mud, or rock. Also we may safely assume that during *refraction* across a boundary separating two media, both being of the category of elastic solids, an incident condensational wave will give rise to a distortional as well as to a condensational refracted wave. In the light of these results, then, it is little wonder that no definite relation has ever been shown to exist between the manner of motion of a particle and the direction of propagation of an earthquake. I should also be inclined to regard as absolutely futile any attempt to infer the nature of the movement in which the shock originates from the nature of the motion of any surface particle.

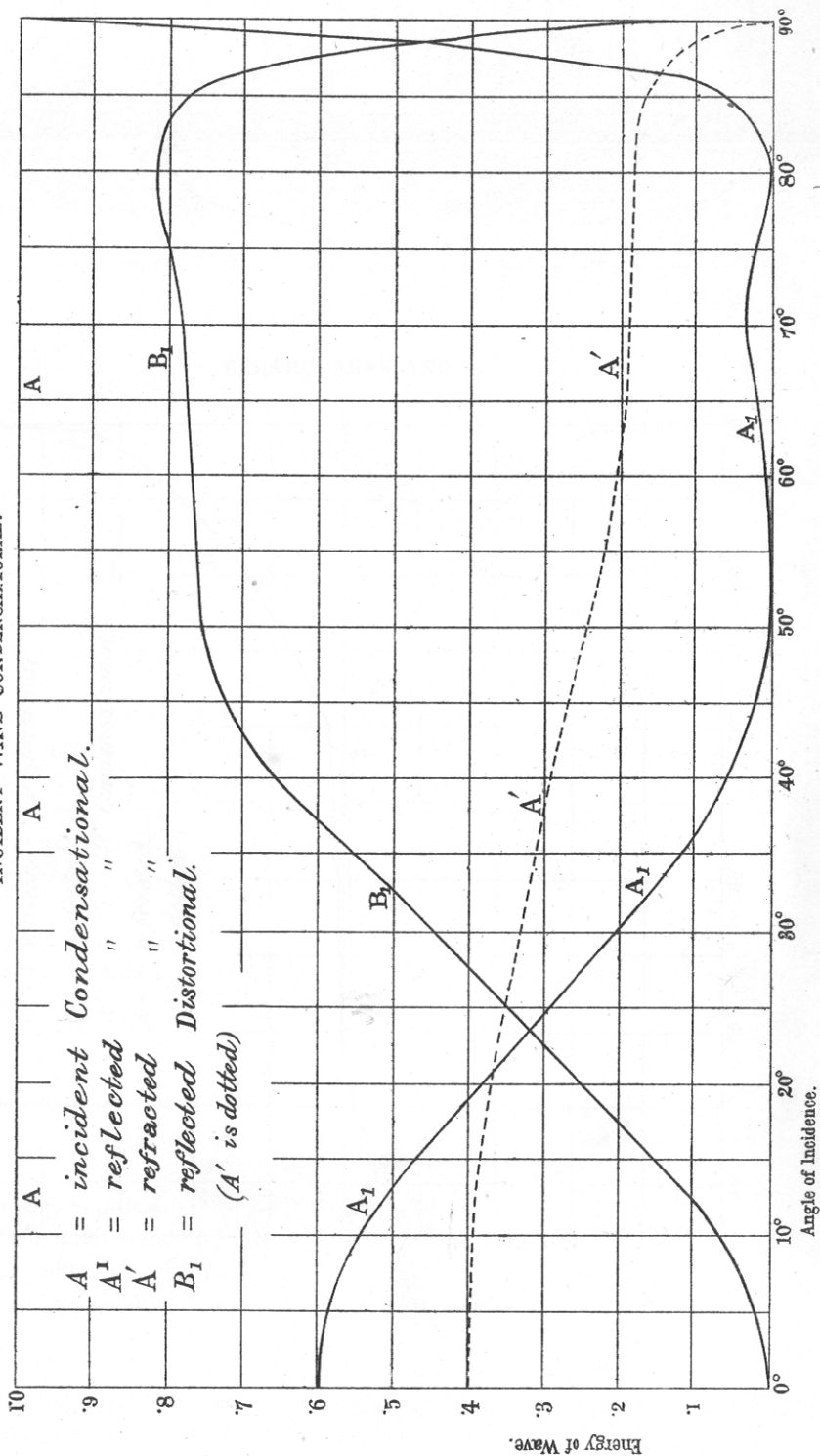
Even in the extremely simple case of an isotropic elastic solid, we see how a single reflection (and probably refraction) is sufficient to alter the type of wave motion, or rather to bring into existence the other type. How much more will this be true in such a heterogeneous mass as we know the earth's crust to be! And if the large earth shiftings, which certainly mean straining beyond the limits of elasticity, differ essentially from the purely elastic disturbances we have just been considering, it will not be in the direction of simplicity.

EARTHQUAKE SOUNDS, ETC.

FROM ROCK TO WATER.

(1)

INCIDENT WAVE CONDENSATIONAL.

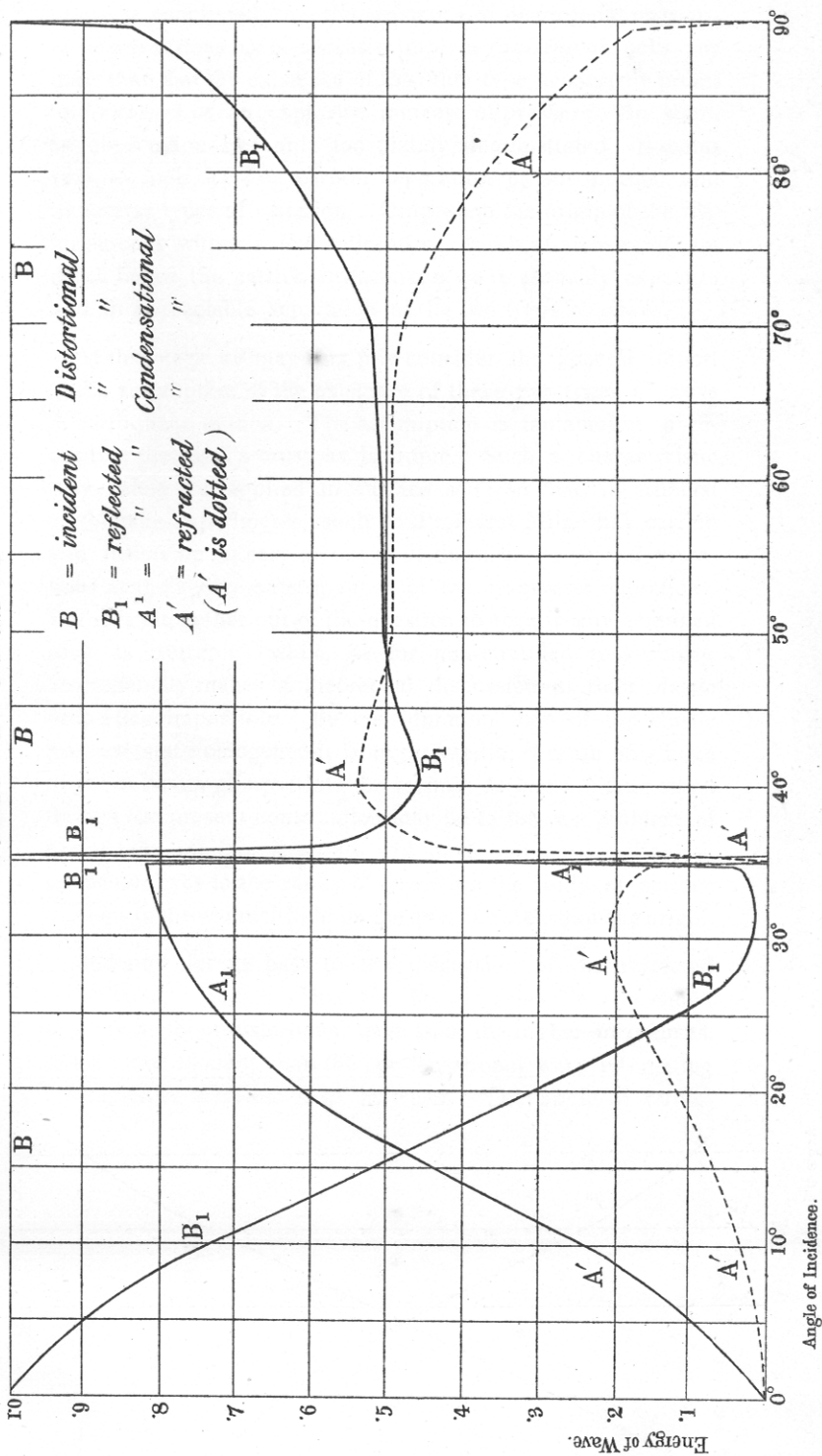


EARTHQUAKES AND

(2)

FROM ROCK TO WATER.

INCIDENT WAVE DISTORTIONAL.



Angle of Incidence.

It seems reasonable to expect in these also somewhat analogous, although much more complex, relations. Hence it may safely be concluded that the existence of distortional or transverse waves does not of necessity imply a *faulting* of rocks, any more than that the existence of the other type necessarily points to a *rupture* or an explosive increase of pressure. In short, as observation has only too plainly demonstrated, it seems vain to look for any certain separation of the normal and transverse types of vibration. Only when the origin of the disturbance is within a few miles of us, and is at an insignificant depth below the earth's surface, can we reasonably expect to find an appreciable separation of the two types of waves.

At this stage we may very fitly consider the general import of the assumption of the existence of these two types of wave in earthquake motion. The assumption is tantamount to regarding the earth's crust as isotropic. Such a characteristic may safely be applied to surface soil; so that, in artificial earthquake experiments, such as Professor Milne has carried out, it may be an easy matter to distinguish the normal vibrations as their wave outstrips that of the transverse vibrations. But it is altogether out of the question to regard any stratified rock as isotropic; while, as for non-stratified rocks, their heterogeneity makes a theoretical discussion of their elastic properties impossible. By consideration then of the elastic properties of homogeneous isotropic media, we can only hope to get at best a glimpse into the seismic darkness. And small though the present contribution may be to the vast problem of earthquake motion, it surely will have some value if only it opens our eyes to the vanity of expecting the study of surface motions to throw much light on the question of earthquake origin.

And now let us pass to the discussion of the refracted water-wave. Here a glance at the two sets of curves shows that the incident distortional wave is, at the higher incidences, much more efficient than the condensational wave in creating a progressive disturbance in the water. The angle of refrac-

tion can never exceed 42° ; so that even for very high incidences the water wave will travel upwards to the surface tolerably directly. Here I think we may have the explanation of the curious bumpings which have sometimes been felt at sea. These must not be confounded with the tidal waves so frequently the companions of earthquakes, and due almost without a doubt to large displacements of the ocean bottom. What I refer to here are the jerks or shakings (sometimes accompanied by sounds) discussed by Professor Milne in the opening paragraph of Chapter IX. of his book on *Earthquakes*. Sounds of course will be heard if the periodic time of any of the components in the wave motion is short enough, and if at the same time the intensity is sufficient to give rise to *audible* sound waves in the air, either directly, or indirectly through the medium of such a solid as a ship. According to Colladon's experiments at the Lake of Geneva, the speed of sound in water at $8^\circ.1$ C. is 1435 metres per second. This gives 14'35 metres (or about 8 fathoms) for the wave-length of a wave whose pitch is 100 vibrations per second. A slower vibration will of course give a longer wave-length; and a quicker a shorter. But enough has been said to show that in such a wave of condensation we have something quite fitted to affect even a large ship as a whole.

Now all that has been said regarding the transference of vibrations from rock to water will, in a general way, hold true of their transference from rock to air. For all angles of incidence in the rock, the angle at which the refracted ray passes out into the air is very small. Thus, returning to the equation

$$\frac{m}{\rho} \operatorname{cosec}^2 \theta = \frac{m'}{\rho'} \operatorname{cosec}^2 \theta'$$

and giving m' , the wave-modulus in air, the value 1.41×10^6 , and ρ' the value .0013, we find, with the same values as formerly for the rock constants,

$$\operatorname{cosec}^2 \theta = .00242 \operatorname{cosec}^2 \theta'$$

Hence of $\theta = 90^\circ$, $\theta' = 2^\circ 50'$ nearly.

In the same way, calculating for the incident distortional wave, we obtain

$$\operatorname{cosec}^2 \phi = .00726 \operatorname{cosec}^2 \theta'$$

Hence if $\phi = 90^\circ$, $\theta' = 4^\circ 53'$ fully. Thus, whatever the incidence, the refracted wave goes off in a direction never more than 5° removed from the normal.

Into a detailed calculation regarding the distribution of the energy, it is not necessary to go. The amount of energy which gets into the air as a condensational wave is extremely small compared to the vibratory energy existing in the rock. With the constants as given above it is doubtful if for any incidence as much as the thousandth part of the original energy is so transmitted into the air. For most incidences it is distinctly less.

It is thus easy to see why in earthquakes which may be accompanied by considerable mechanical violence, there may be no audible sound phenomena. The essential condition for the production of earthquake sounds is a sufficiently pronounced vertical motion with a sufficiently rapid period. According to Professor Sekiya's recent analysis,* vertical motion as measured on the seismographs is absent from most of the earthquakes that shake Japan. When vertical motion is apparent, it is in the more intense shocks. We cannot assume of course that the vertical motion is absent in those cases in which the seismograph shows no trace of it. It is always much smaller than the horizontal motion, being on the average only one-sixth of it. Hence when the horizontal motion is itself very small, as in the weaker shocks, the vertical motion may be too small to affect the seismograph. Or, as is more than likely, it may have too short a period to make itself felt, even though its amplitude may be large enough to be otherwise apparent. We must be careful indeed not to confuse the seismograph indications with the rapid elastic vibrations which seem a necessity for the production of sound phenomena.

* See the present volume of Transactions Vol. XII.; also the Journal of the College of Science, Imperial University, Vol. II.

That the quick short period motions that precede the big wave as shown on our seismographs may co-exist along with vertical vibrations sufficiently rapid to cause audible sounds is highly probable; but in no other sense can they be regarded as "connected" with those sounds, as seems to be suggested by Professor Milne. These rapid sinuosities appear on all the best diagrams showing the horizontal motion; but, as, I believe, it is the vertical motion we must look to specially.

Another point brought out strongly by Professor Sekiya's analysis is that in no case has he found the vertical motion precede the horizontal motion. The vertical seems always to show itself later. It is certain, however, that earthquake sounds are often heard before the earthquake shock is felt. This simply means that the big earth shiftings which affect our seismographs are preceded by rapid vibratory motions which, however large they may be, cannot have any mechanical effect on the instruments. The case is exactly similar to what happens if we pass alternating electric currents through the coil of an ordinary galvanometer. No matter how sensitive the galvanometer, or how intense the alternating current,—so long as the alternation is rapid enough, no effect is observed on the galvanometer needle. So it cannot fail to be with ordinary seismographs as regards rapid vibrations. I doubt if a seismograph, mechanically capable of registering vibrations occurring at even so slow a rate as 10 per second, has been as yet imagined. It is very questionable also if those sinuous records which the seismograph tracings show as precursors of the large slow waves really indicate what is taking place in the soil. For, exactly as very rapid vibrations will not show at all on the seismograph trace, so somewhat less rapid vibrations will not show to their full. There must always be a lagging of the record behind the motion recorded. Thus before a given motion has its full effect on the seismograph, the rapid reverse motion may set in and prevent anything like a complete record. Not until a comparatively slow "swing-swang" of the ground takes place can we hope to have a tracing even ap-

proximately true as to amplitude. It is therefore well, I think, that seismologists should bear this point in mind. It is highly probable that an earthquake is preceded by rapid vibratory motions. That we should expect; and the early sinuosities of earthquake tracings certainly suggest the same. But that these sinuosities can be taken as an approximate representation of the amplitudes or periods of the rapid vibration to which they are due may well be matter of grave doubt.

In conclusion, I would draw special attention to the following point which seems to be of some importance.

In the discussion of the propagation of seismic disturbances through the earth's crust, a clear distinction should be drawn between purely elastic and QUASI-elastic phenomena. So long as the materials constituting the earth's crust are not strained distinctly beyond the limits of elasticity, we have to do with purely elastic vibrations. These generally speaking will be transmitted with considerable speed, comparable to that of sound in steel wires. Such high speeds have indeed been observed, their existence depending upon a small compressibility (or high rigidity) combined with a comparatively small density. The destructive effects of earthquakes are, however, due to the propagation of *quasi-elastic* disturbances. In them the material is distinctly strained beyond the limits of elasticity; or, at all events, so strained as to bring about conditions in which other strain-coefficients than the usual ones of rigidity and compressibility play the important part. It is quite to be expected that these *quasi-elastic* disturbances should travel much more slowly than the purely elastic ones. The investigation given above into the effect upon the type of elastic waves as they suffer reflection at the boundary of two isotropic elastic media suggests the existence of analogous effects in the propagation of all seismic disturbances. The aeolotropy and discontinuity of the earth's crust will transform a disturbance of an originally simple type into one or more of excessive complexity. Furthermore, wherever a *quasi-elastic* disturbance suffers transformation at some region of discontinuity, it will give rise to a

new set of elastic disturbances. And again, as the *quasi*-elastic disturbances lose energy per unit volume, partly because of radiation, partly because of dissipation, they will gradually lose their *quasi*-character, and become of a purely elastic nature. It is quite conceivable, then, that under certain circumstances the speed of a disturbance might undergo strange variations, appearing even to be accelerated as its intensity diminished. Such a phenomenon was observed by Lieut. Col. Abbot at Flood Rock explosion in 1885. Of course a peculiar change of this kind might easily be due to the different elastic properties of successive portions of rock travelled through. It is quite possible, however, that the other explanation is the true one. It is known that a very intense sound travels faster in air than one of less intensity; and the same will be true of vibrations in elastic solids. But there must be a superior limit to the intensity, for intensities above which this relation will cease to hold. Viscosity, friction, and the little understood effects of permanent strain will make themselves more and more strongly felt as the strains increase beyond the limits of elasticity. I understand, indeed, that, in the case of cannon reports, the sound has been observed to travel somewhat less rapidly during its early than during its after stages. Here the very large initial aërial disturbances bring in conditions, either thermodynamic or elastic, under which the ordinary theory fails even in approximate application. If such a phenomenon is met with in the comparatively simple case of sound waves in air, similar phenomena are certain to exist in the more complex cases that correspond to earth shakings.

Another point, which this explicit recognition of purely elastic and *quasi*-elastic disturbances suggests, is in relation to the measurement of earthquake velocities by comparison of the effects at distant stations. Thus the purely elastic tremors felt at stations far distant from the centre of seismic disturbance have probably not come as such directly therefrom. They are, so to speak, the feebler descendants of the *quasi*-elastic disturbances, which may have caused havoc at localities

nearer to the seismic centre. The initial elastic tremors felt at these nearer stations will reach the further distant ones with intensities so diminished as no longer to be appreciable. Thus in the very usual method of timing the arrival of a tremor by the blurring of an image reflected from the surface of mercury, it is evident that the speed, as estimated between two stations in the line of propagation of the disturbance, must be somewhat smaller than the true value. For, before the particular tremors which sufficiently blurred the image at the first station have reached the second one, their intensity has become diminished. Hence the sufficient blurring of the image at the second station is due to the diminished violence of tremors, which passed through the first station subsequently to the blurring of the image there. Now the same reasoning will apply with even greater force to other than mere tremors; and especially will it apply to the case of the propagation of the quasi-elastic disturbances which constitute dangerous earthquakes.

If the views so far expressed are correct there is no difficulty in understanding the nature of earthquake sounds. As already pointed out, they are to be traced to rapid vertical vibrations of the ground, so rapid as to be inappreciable on our seismographs. Sometimes they may be due to transverse vibrations of walls caused by horizontal displacements of the ground; or, as suggested by Mallet, they may be transmitted through the framework of the body. That these sounds should frequently precede the coming of the true earthquake shock is simply due to the running ahead of the purely elastic waves. The nature of the rock or soil through which these waves proceed will have a powerful influence upon their final intensity. Thus in soft rock, viscosity will soon destroy the vibrations of short period. In such circumstances there will be less chance of hearing earthquake sounds than when the rock is hard and solid. Very frequently earthquake sounds die away before the earth swayings have ceased—a fact which is probably connected with the short-lived character of the vertical

motion as compared with the horizontal motions traced out by our seismographs.

With the air pulsations, which if rapid enough constitute audible sounds, the following curious effect of earthquakes may have some connection. I am indebted to Professor Sekiya for the information. It seems that, at the time an earthquake shock passes, or it may be a little sooner, birds flying in the air have been seen to drop suddenly, as if for an instant paralysed, and then to recover themselves. This effect might be sufficiently explained as due to a momentary mental paralysis produced by fear. Perhaps, however, we have a sufficient physical cause in the air pulsations, a slight change of density being enough to disturb the delicate poise of the hovering bird.
