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62. *Velocity and Response of Higher Mode Rayleigh Waves for the Pacific Ocean.*

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Abstract

Phase and group velocities and medium-response of higher mode Rayleigh waves are calculated for PC-MIN and PC-MAX, which are the upper mantle models proposed for the lowest- and the highest-velocity-areas in the Pacific (YOSHIDA, 1978). The former corresponds to the region of the ocean-floor age of 0 m.y. and the latter to that of 150 m.y. or more.

Both phase and group velocity characteristics show that 1) the velocity difference of the higher modes, calculated for the two models, does not decrease with the period, 2) and this difference is largest for the second higher mode near 200 sec in group velocity and near 150 sec in phase velocity. For a shallow earthquake the medium-response of the higher modes is remarkably small compared with the fundamental mode. For a shock with a focal depth 200 km, however, the response of the first higher mode near 40 sec is the greatest among the first four modes. This first higher mode at that period will be useful for the study of the low velocity layer under the ocean.

1. Introduction

Higher mode Rayleigh waves across the Pacific have been observed and discussed by KOVACH and ANDERSON (1964) in relation to the upper mantle structures under the ocean. The velocity characteristics of higher mode surface waves, calculated for an underground structure or an earth model possessing the low velocity layer, have been investigated by several authors (e.g., YAMAGUCHI, 1961; USAMI *et al.*, 1965; USAMI *et al.*, 1966; SATÔ, *ct al.*, 1967), and it is known that the influence of the velocity layer on the higher modes is strong.

HARKRIDER (1970) has presented amplitude spectra of higher mode surface waves for both oceanic and continental models for point sources at selected depths, and we can use these amplitude spectra of the higher modes for source mechanism studies of earthquakes.

Recently it is demonstrated in terms of group velocities and surface amplitude displacements that the so-called channel waves, such as the L_g and L_i phases, can be identified with higher mode Rayleigh waves (PANZA and CALCAGNILE, 1975).

It will be an important subject in the study of surface waves to develop knowledge on the velocity and excitation of the higher modes. The present paper is intended to calculate the upper and lower limits of velocities of higher mode Rayleigh waves in the Pacific and to estimate approximately the medium-response for these waves following the theory of surface wave excitation (SAITO, 1967; TAKEUCHI and SAITO, 1972).

2. Upper mantle models

In the previous work (YOSHIDA, 1978) two upper mantle models for the lowest-velocity-area and the highest-velocity-area, located in regions of the ocean-floor age of 0 m.y. and 150 m.y. or more, were proposed based on the group velocity data of Rayleigh waves. These two models explained observations of both group and phase velocities of the fundamental mode very well. Velocities calculated for the

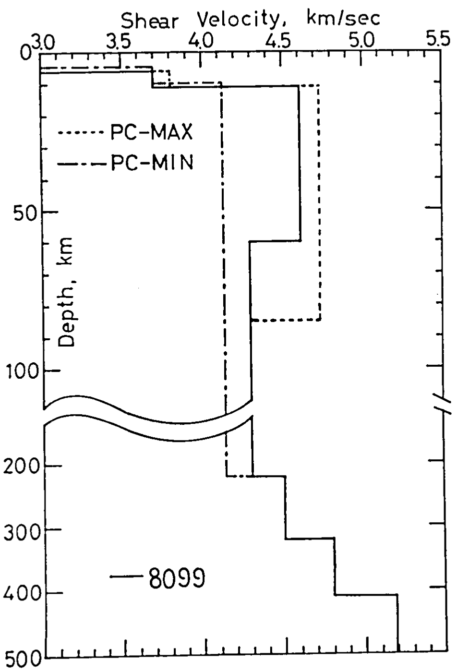

Fig. 1. Shear wave structures of the models PC-MIN and PC-MAX which correspond to the lowest- and the highest-velocity-areas in the Pacific respectively.

Table 1. Dispersion characteristics of Rayleigh waves for the models PC-MIN and PC-MAX.

	MODEL PC-MIN		(0th)	MODEL PC-MAX		
N	T	C	U	T	C	U
30	258.7	5.073	3.613	257.4	5.098	3.680
40	210.6	4.692	3.482	208.7	4.735	3.610
50	178.1	4.449	3.460	175.5	4.515	3.640
60	154.3	4.286	3.469	151.1	4.376	3.703
80	121.6	4.088	3.511	117.7	4.223	3.816
100	100.1	3.978	3.564	95.9	4.152	3.914
120	84.8	3.914	3.613	80.6	4.120	3.996
150	68.9	3.859	3.663	64.7	4.105	4.088
200	52.3	3.815	3.694	48.5	4.112	4.167
250	42.1	3.790	3.687	38.7	4.125	4.173
300	35.3	3.771	3.666	32.2	4.130	4.130
350	30.4	3.754	3.640	27.6	4.124	4.051
	MODEL PC-MIN		(1st)	MODEL PC-MAX		
N	T	C	U	T	C	U
30	187.7	6.989	5.495	184.9	7.097	5.610
40	150.2	6.576	5.136	147.7	6.687	5.267
50	126.6	6.257	4.804	124.3	6.374	4.954
60	110.3	5.995	4.554	108.1	6.118	4.708
80	88.8	5.599	4.291	86.8	5.727	4.421
100	74.7	5.329	4.210	73.0	5.455	4.316
120	64.6	5.141	4.186	63.1	5.263	4.293
150	53.7	4.947	4.147	52.4	5.070	4.294
200	42.1	4.734	4.039	40.9	4.875	4.283
250	34.8	4.587	3.972	33.6	4.755	4.257
300	29.7	4.484	3.977	28.5	4.671	4.247
350	25.8	4.414	4.012	24.7	4.611	4.258
	MODEL PC-MIN		(2nd)	MODEL PC-MAX		
N	T	C	U	T	C	U
30	162.6	8.067	6.194	159.2	8.242	6.442
40	130.4	7.575	5.947	127.4	7.758	6.103
50	109.7	7.224	5.650	107.2	7.392	5.719
60	95.3	6.939	5.366	93.3	7.086	5.384
80	76.5	6.494	4.965	75.2	6.605	4.966
100	64.6	6.163	4.710	63.6	6.257	4.760
120	56.2	5.907	4.485	55.3	5.996	4.617
150	47.5	5.590	4.195	46.6	5.702	4.436
200	38.2	5.224	4.043	37.2	5.365	4.298
250	31.9	5.002	4.172	31.0	5.152	4.313
300	27.3	4.865	4.174	26.5	5.016	4.353
350	23.9	4.762	4.093	23.1	4.923	4.371
	MODEL PC-MIN		(3rd)	MODEL PC-MAX		
N	T	C	U	T	C	U
30	142.7	9.190	6.445	141.3	9.285	6.647
40	116.5	8.478	6.181	114.8	8.608	6.449
50	99.0	8.005	6.011	97.1	8.162	6.257
60	86.3	7.663	5.857	84.5	7.828	6.020
80	69.3	7.166	5.446	67.9	7.318	5.539
100	58.7	6.772	4.984	57.5	6.922	5.126
120	51.5	6.449	4.647	50.3	6.596	4.812
150	43.8	6.067	4.466	42.8	6.213	4.579
200	35.2	5.600	4.403	34.5	5.786	4.444
250	29.5	5.399	4.296	28.9	5.512	4.398
300	25.5	5.204	4.154	25.0	5.326	4.390
350	22.6	5.049	4.113	21.9	5.191	4.370

model PC-MIN gave the lowest velocity in the Pacific and those for the model PC-MAX gave the highest velocity.

In PC-MIN the shear wave velocity in the low velocity zone (LVZ) is 4.136 km/sec, which extends to the depth 220 km and the lid is not found (Fig. 1). PC-MIN is a model with an extremely well-developed LVZ compared to many oceanic models. The thickness of the lithosphere in PC-MAX is 85 km and the shear velocity there is about 4.73 km/sec, somewhat high compared with the standard one. According to the previous work by Yoshida it is understood that the standard oceanic model 8099 (DORMAN, EWING and OLIVER, 1960) corresponds to the region of the age of approximately 85 m.y. As is shown in Fig. 1, the shear wave structures of the two models differ considerably with that of 8099. Thus the models PC-MIN (0 m.y.) and PC-MAX (150 m.y. or more) constitute the most extreme velocity-distributions of the lithosphere and the asthenosphere under the Pacific.

3. Group and phase velocities

For PC-MIN and PC-MAX spheroidal oscillations without the gravity effect were calculated, and group and phase velocities are determined for periods less than 300 sec for the first three higher modes (Table 1) and are shown in Fig. 2. Mode designations 0th, 1st, 2nd and 3rd for the fundamental, first, second and third higher modes respectively are temporarily used. The area marked with dots

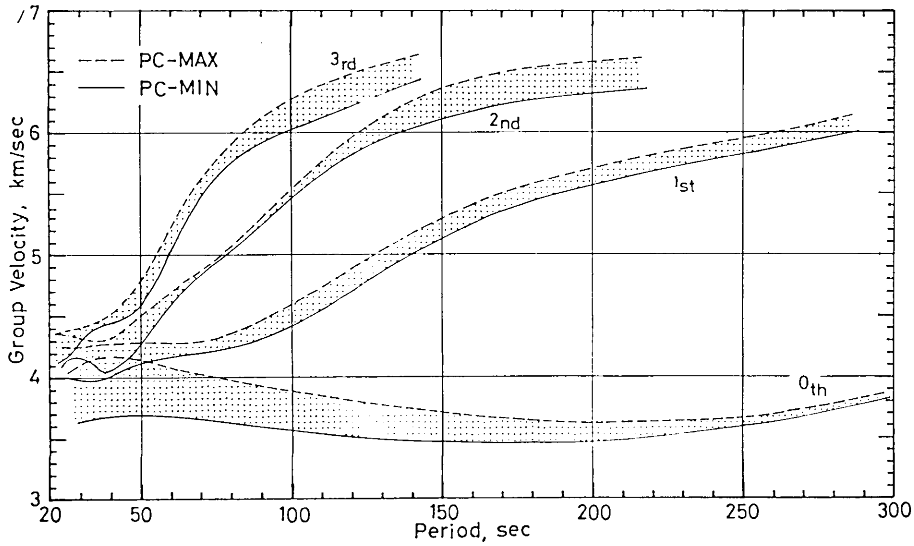

Fig. 2a.

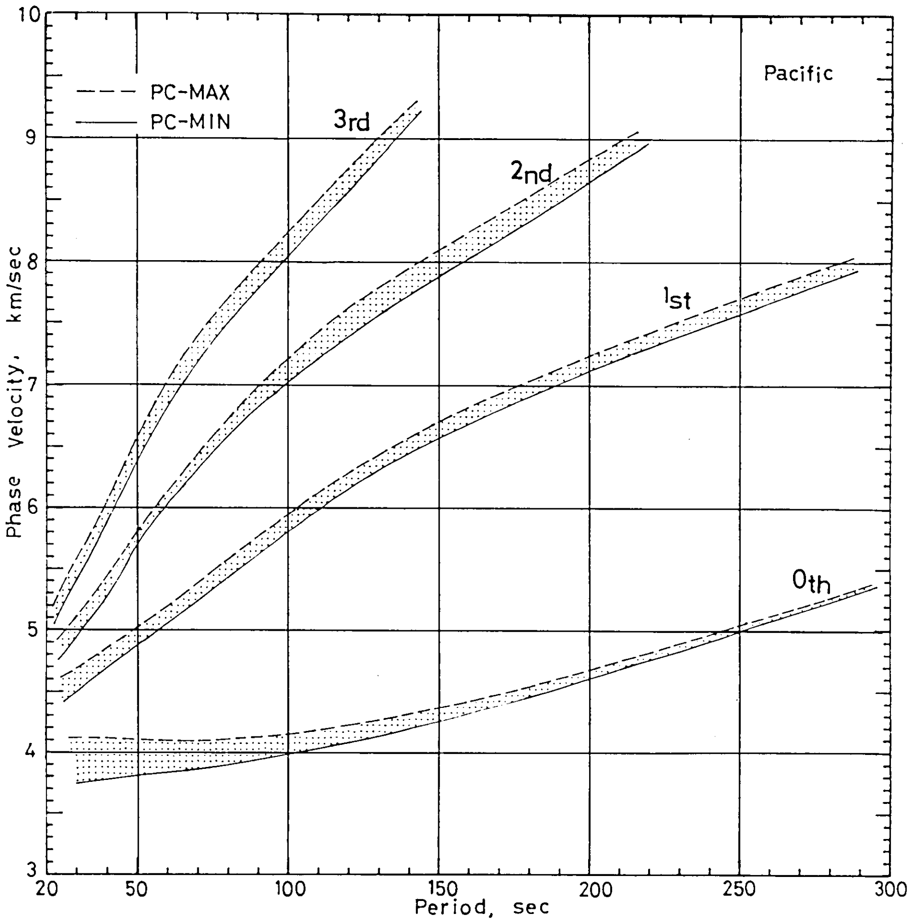

Fig. 2b.

Fig. 2. Group (a) and phase (b) velocities for the models PC-MAX (broken line) and PC-MIN (solid line). Dot-marks are given to the parts between two curves.

shows the velocity range of Rayleigh waves across the Pacific, and the width of this area at any period will be interpreted as the measure of the regionality of velocities. Hence hereafter the word 'regionality' will be used to imply the velocity difference for two extreme models.

We see from Fig. 2 that the largest difference of group velocities between PC-MIN and PC-MAX is about 0.31 km/sec (4.8%) at the period 200 sec for the second higher mode, which is smaller than that of 0.5 km/sec (13%) around 40 sec for the fundamental mode. The extremely small difference of velocities near 75 sec for the second higher mode is a conspicuous feature of the group velocity characteristics. We also see that the regionality of group velocity does not decrease with the period monotonously for the higher modes, while it does for the fundamental mode, and that the group velocities for all the four modes are very close to one another for periods of less than 50 sec.

In the phase velocity, the largest difference between the two models for the higher modes is about 0.2 km/sec (2.5%) near 150 sec for the second higher mode (the difference for the fundamental mode

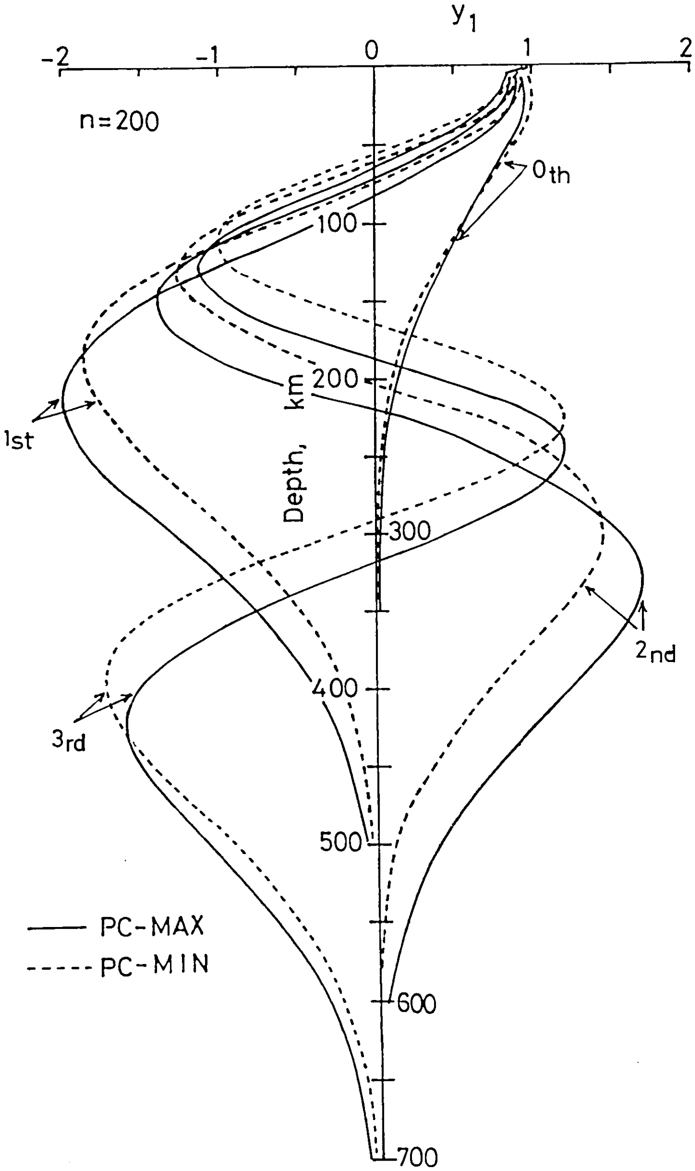

Fig. 3a.

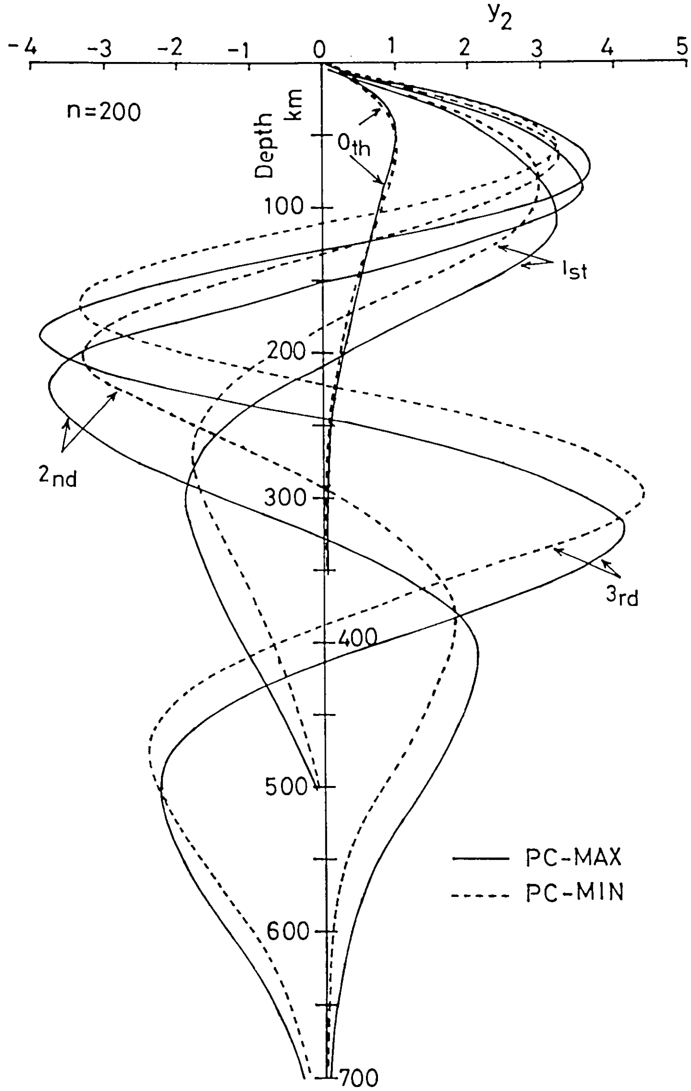

Fig. 3b.

is about 0.3 km/sec at 40 sec, 7.9% of the velocity for PC-MIN).

4. Higher modes around 40 sec

KOVACH and ANDERSON (1964) observed the first higher mode from deep earthquakes in the period range of 20 to 50 sec and discussed the layer parameters near the lower part of the LVZ under the ocean. However the group velocities of the three higher modes of that period are close, as was shown above. If, therefore, we have knowledge of the relationship between the upper mantle structures and the excita-

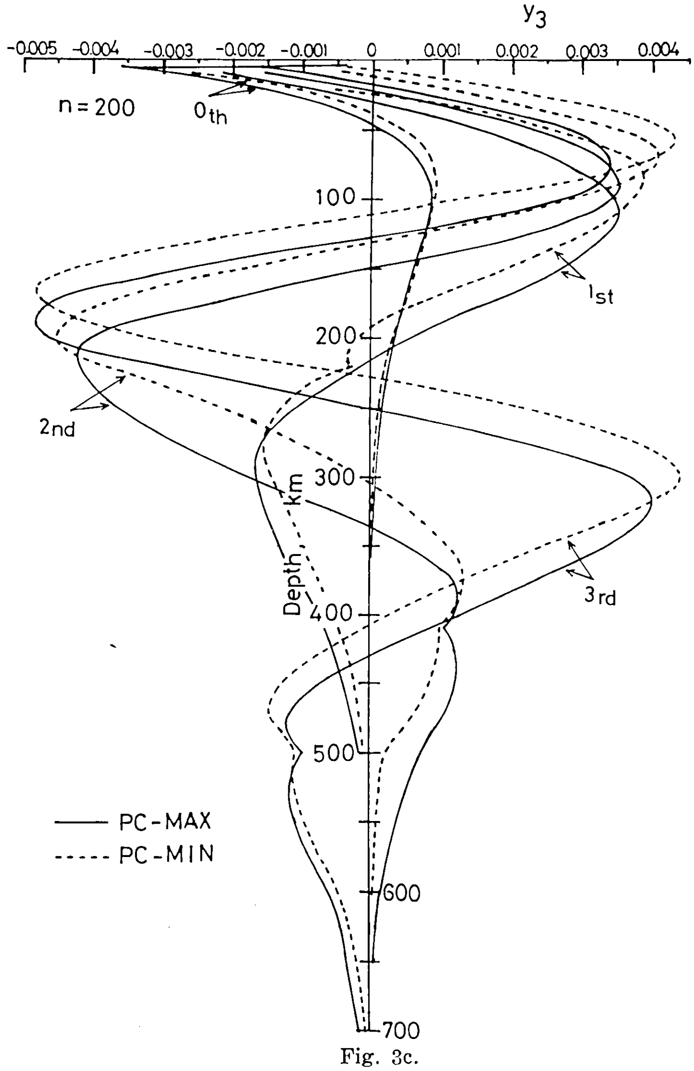

Fig. 3c.

tion of the higher modes, those modes will be effectively used for the study of the underground structure.

In Fig. 3 the displacement and stress distributions with depth for PC-MAX and PC-MIN are shown for the spheroidal oscillations of the order number of the associated Legendre function 200 (period around 40 sec). y_1 and y_3 are the radial factors of the vertical and horizontal components of displacement, namely

$$u_r = y_1(r)\, Y_n^m e^{i\omega t} \,,$$

$$u_\theta = y_3(r) \frac{\partial Y_n^m}{\partial \theta} e^{i\omega t} \,, \qquad (1)$$

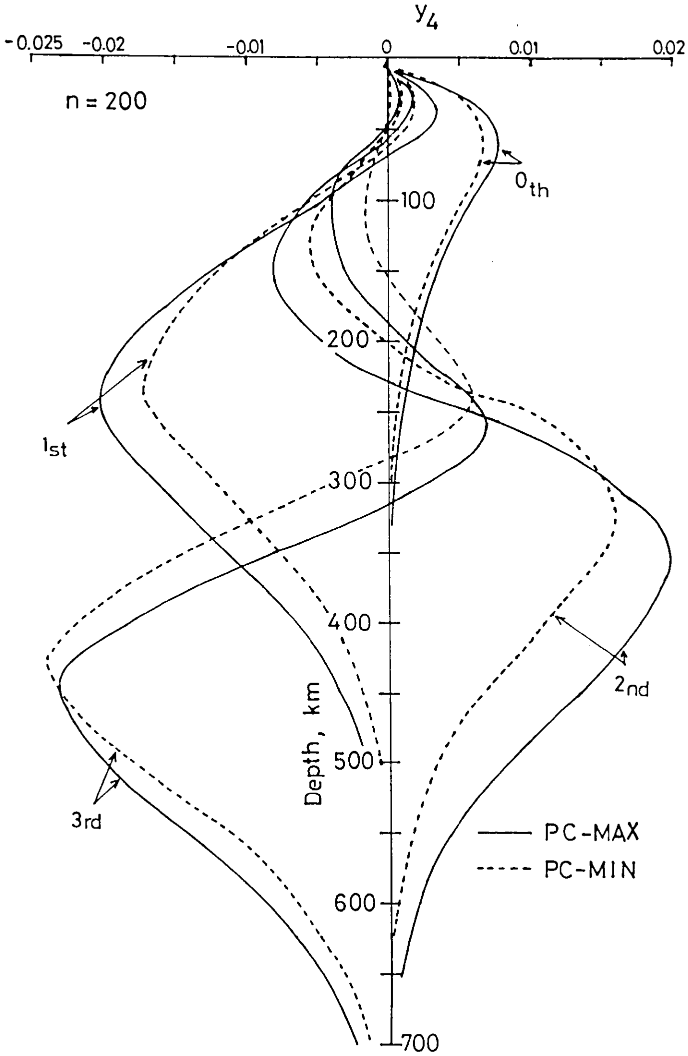

Fig. 3d.

Fig. 3. The radial factors of vertical (y_1) and horizontal (y_3) components of displacement and those of (y_2) and (y_4) of stress. The unit of y_1 and y_3 is cm and that of y_2 and y_4 is 10^5 dyne/cm^2. These are normalized so as to give y_1=1 cm at the earth's surface.

$$u_\phi = \frac{y_3(r)}{\sin\theta}\frac{\partial Y_n^m}{\partial\phi}e^{i\omega t},$$

and y_2 and y_4 are those of stress given by

$$P_{rr} = y_2(r)Y_n^m e^{i\omega t},$$

$$P_{r\theta} = y_4(r)\frac{\partial Y_n^m}{\partial\theta}e^{i\omega t}, \qquad (2)$$

$$P_{r\phi}=\frac{y_4(r)}{\sin\theta}\frac{\partial Y_n^m}{\partial\phi}e^{i\omega t} ,$$

where

$$Y_n^m(\theta, \phi)=P_n^m(\cos\theta)\binom{\cos}{\sin} m\phi . \qquad (3)$$

We see in Fig. 3 that, for the first higher mode, the maximum of y_1 and y_4 occurs near the depth 200 km, close to the bottom of the LVZ (see also Fig. 1), and the maximum of y_2 and y_3 occurs near the depth 100 km. For other higher modes the maxima of the radial factors are distributed deeper than the first higher mode. This fact suggests that the Rayleigh wave of the first higher mode with a period around 40 sec is the most useful for the study of the physical properties in the LVZ, since this mode at that period is the most sensitive to that zone.

It should be noted that for the three higher modes all the radial factors $y_1, \cdots, y_4$ for PC-MAX are shifted approximately about 30 km deeper than those for PC-MIN (Fig. 3), and that the eigen-periods of the spheroidal oscillations of the order number 200 for PC-MAX are about 1 sec shorter than those for PC-MIN (Table 1). For the fundamental mode, the radial distributions of displacement and stress for the two models are not so different as those for the higher modes, however the discrepancy in the eigen-periods reaches about 4 sec.

5. Medium response

In the observation of mixed-mode Rayleigh waves the information with respect to the excitation is useful for mode-separation. In this section the medium response of Rayleigh waves for PC-MAX and PC-MIN are considered numerically for a displacement dislocation earthquake source which is equivalent to the usual point-source double couple without moment in an unfaulted medium (MARUYAMA, 1963; BURRIDGE and KNOPOFF, 1964). Using a formula given in SAITO (1967) and TAKEUCHI and SAITO (1972), the displacement vector for the spheroidal oscillation excited by an arbitrary source for a unit moment, can be written as,

$$U(r, t)=\frac{1}{2}\sum_n\sum_m [r^2(f_2y_1-f_1y_2)+n(n+1)r^2(f_4y_3-f_3y_4)]_{r=r_s}$$
$$\times F(\omega_n)\{y_1(r)S_{1,n}^m(\theta, \phi)+y_3(r)S_{2,n}^m(\theta, \phi)\}e^{i\omega_n t} , \qquad (4)$$

where the source is located on a surface $r=r_s$, and the time dependence is assumed to be a step function. ω_n is the eigen-frequency of the normal mode, and f_i is the source function which depends on the force system exerted at the source. $S^m_{1,n}$ and $S^m_{2,n}$ are the vector spherical harmonics defined by

$$S_{1,n}^{m}(\theta, \phi)=(Y_n^m, 0, 0) ,$$
$$S_{2,n}^{m}(\theta, \phi)=\left(0, \frac{\partial Y_n^m}{\partial \theta}, \frac{1}{\sin\theta}\frac{\partial Y_n^m}{\partial \phi}\right) . \tag{5}$$

The transfer function $F(\omega_n)$ multiplied by the source time spectrum (Fig. 4), which is inversely proportional to the average kinetic energy and depends neither on the source function nor on the receiver location, is given as

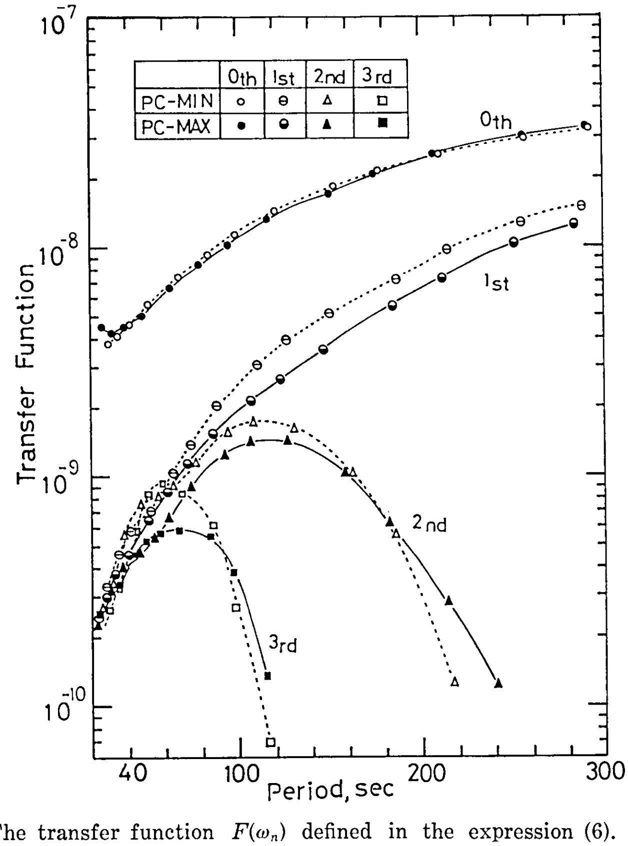

Fig. 4. The transfer function $F(\omega_n)$ defined in the expression (6). The unit is 10^{-15} dyne^{-1} cm^{-1}.

$$F(\omega_n)=1/\omega_n^2 \int_0^a \rho r^2[y_1^2+n(n+1)y_3^2]dr . \tag{6}$$

Here ρ is the density and a is the radius of the earth.

From Fig. 4 we see that for the fundamental mode the transfer

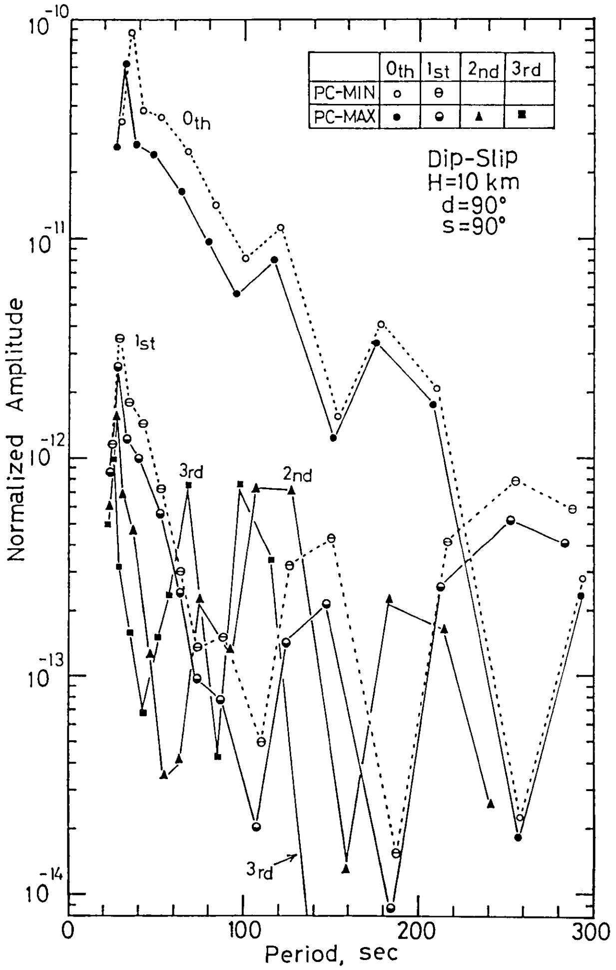

Fig. 5a.

functions for PC-MIN and PC-MAX are nearly equal to each other in the period range 30~300 sec. For the first higher mode, however, the function for PC-MIN is larger than that for PC-MAX in the same period range. For the second higher mode the former is larger than the latter in the range 30~140 sec as also for the third higher mode in the range 40~80 sec. Thus the difference of the transfer functions defined in (6), for the two models, is larger for the higher modes than for the fundamental mode.

Making use of the equation (4), the vertical component of displacement for a double couple point source of the pure dis-slip motion on a vertical fault plane is expressed as

$$u(r, \theta, \phi, t)=\sum_{n} \bar{u}(r, \theta, \phi)\cdot\cos \omega_n t \ , \tag{7}$$

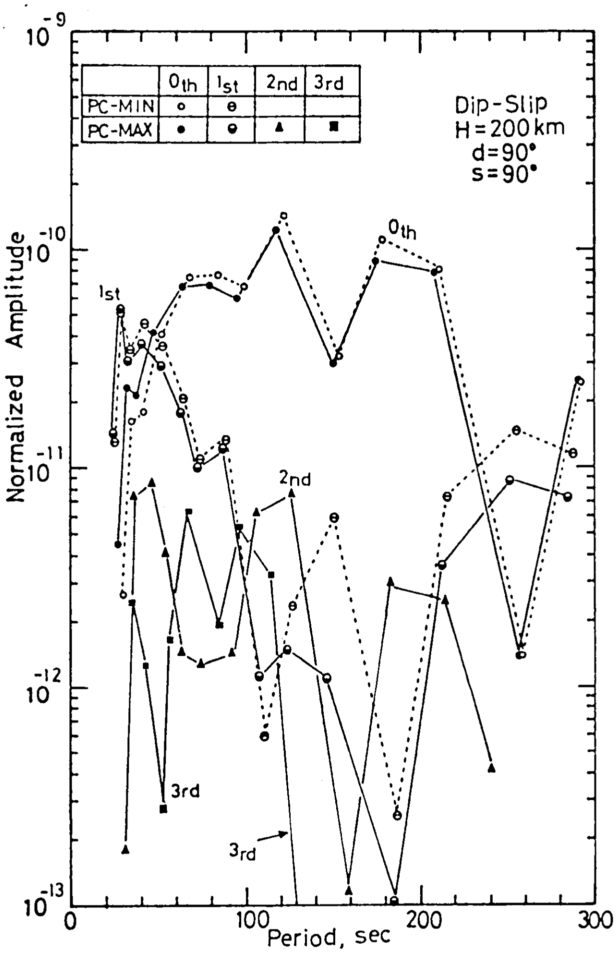

Fig. 5b.

Fig. 5. The normalized amplitude $\bar{u}(r, \theta, \phi)$ of the four modes excited by the dip-slip source. d and s represent the dip angle and the slip angle respectively. The source is assumed to be located at the depth (a) H=10 km and (b) H=200 km, and the receiver at $\theta=66°$ and $\phi=90°$. The unit is 10^{-29} cm.

where

$$\bar{u}(r, \theta, \phi)=\sin\phi\cdot\frac{2n+1}{4\pi}\cdot\frac{1}{\mu_s}\cdot y_1(r)\cdot F(\omega_n)\cdot P_n^1(\cos\theta)\cdot y_4(r_s)\ . \tag{8}$$

The medium response calculated numerically through this expression for the two models is shown in Fig. 5.

For a shallow seismic source (Fig. 5a), the response of the three higher modes is very low, approximately a figure less than the fundamental mode for periods less than 200 sec. However the excitation pattern varies strongly for a deep seismic source (Fig. 5b). For the fundamental mode, the response increases for periods longer than 50 sec, but decreases rapidly for periods shorter than 50 sec, compared with that of a shallow source. It should be noted that all the higher modes are excited approximately a figure larger than those for a shallow one and that the response of the first higher mode is the largest of the four modes at around 40 sec. This property of the high radiation of the first higher mode from deep shocks suggests the possibility of the observation of this mode in the Pacific, and this first higher mode will be useful for the study not only of the upper mantle structure but also of the earthquake mechanism, as applied by FUKAO and ABE (1971).

We also notice from Fig. 5 that, for a shallow source, the response for the hard model PC-MAX is smaller than that for the soft model PC-MIN for both the fundamental and first higher modes and this relation also holds for a deep source at long periods. However for a deep source at periods shorter than 60 sec for the fundamental mode and shorter than 30 sec for the first higher mode, PC-MAX dominates PC-MIN.

6. Conclusions

The dispersion curves of the three higher modes of Rayleigh waves, for the PC-MAX and PC-MIN models, have been presented for periods less than 300 sec (Fig. 2). It is shown that the regionality of phase and group velocities of the higher modes does not decrease with the period for the range considered here.

Among the three higher modes, the greatest regionality of group velocity occurs near the period of 200 sec for the second higher mode and the velocity difference is about 0.31 km/sec (4.8%). The regionality of phase velocity is also large for the same mode, at periods longer than 100 sec; the difference is about 0.2 km/sec (2.5%) near 150 sec.

The transfer function, which is independent of the source function and the receiver location, is shown for PC-MAX and PC-MIN (Fig. 4). The difference of the function values between the two models for the higher modes is larger than that for the fundamental mode.

For a shallow earthquake the response of the higher modes is very low compared with that of the fundamental mode for periods less than 200 sec. However, if the seismic source is near the depth of 200 km, the first higher mode with a period around 40 sec, whose group velocity is in the range 4.0 to 4.3 km/sec, is expected to be strongly excited (Fig. 5). This first higher mode at that period is very sensitive to the LVZ and will provide us with useful information for the study of physical properties in that zone. The study of the first higher mode of oceanic Reyligh waves is now in progress with reference to the wave form and spectrum.

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References

BURRIDGE, R. and L. KNOPOFF, 1964, Body force equivalents for seismic dislocations, *Bull. Seism. Soc. Amer.*, 54, 1875–1888.

DORMAN, J., M. EWING and J. OLIVER, 1960, Study of shear-velocity distribution in the upper mantle by mantle Rayleigh waves, *Bull. Seism. Soc. Amer.*, **50**, 87–115.

FUKAO, Y. and K. ABE, 1971, Multi-mode Love waves excited by shallow and deep earthquakes, *Bull. Earthq. Res. Inst.*, **49**, 1–12.

HARKIDER, G. H., 1970, Surface waves in multilayered elastic media. Part II. Higher mode spectra and spectral ratios from point sources in plane layered earth models, *Bull. Seism. Soc. Amer.*, **60**, 1937–1987.

KOVACH, R. L. and D. L. ANDERSON, 1964, Higher mode surface waves and their bearing on the structure of the earth's mantle, *Bull. Seism. Soc. Amer.*, **54**, 161–182.

MARUYAMA, T., 1963, On the force equivalents of dynamical elastic dislocations with reference to the earthquake mechanism, *Bull. Earthq. Res. Inst.*, **41**, 467–486.

PANZA, G. F. and G. CALCAGNILE, 1975, L_g, L_i and R_g from Rayleigh Modes, *Geophys. J. R. Astr. Soc.*, **40**, 475–487.

SAITO, M., 1967, Excitation of free oscillations and surface waves by a point source in a vertically heterogeneous earth, *J. Geophys. Res.*, **72**, 3689–3699.

SATÔ, Y., T. USAMI and M. LANDISMAN, 1967, Theoretical seismograms of spheroidal type on the surface of a gravitating elastic sphere. II. Case of Gutenberg-Bullen A′ earth model, *Bull. Earthq. Res. Inst.*, **45**, 601–624.

TAKEUCHI, H. and M. SAITO, 1972, Seismic surface waves, in *Methods in Computational Physics*, edited by B. A. Bolt, 11, 217–295, Academic Press, New York.

USAMI, T., Y. SATÔ and M. LANDISMAN, 1965, Theoretical seismograms of spheroidal type on the surface of a heterogeneous spherical earth, *Bull. Earthq. Res. Inst.*, 43, 641–660.

USAMI, T., Y. SATÔ and M. LANDISMAN, 1966, Preliminary study of the propagation of spheroidal disturbances on the surface of a heterogeneous spherical earth, *Geophys. J. R. Astr. Soc.*, **11**, 243–251.

YAMAGUCHI, R., 1961, Surface waves and layered structures, Part I. Influence of a low velocity layer and some study on L_g and R_g waves, *Bull. Earthq. Res. Inst.*, **39**, 653–668.

YOSHIDA, M., 1978, Group velocity distributions of Rayleigh waves and two upper mantle models in the Pacific Ocean, *Bull. Earthq. Res. Inst.*, **53**, 319–338.

62. 太平洋における高次モードレイリー波の速度と応答

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太平洋における速度の最も低い地域と最も高い地域に対して提唱された上部マントルモデル (Yoshida, 1978), PC-MIN と PC-MAX, に対する高次モードレイリー波の位相及び群速度そして媒質の応答が計算されている．前者は海底年代 0 m.y. の地域に相当し，後者は 150 m.y. 又はそれ以上の地域に相当する．

位相及び群速度のどちらも次の様な特性を示す； 1) 二つのモデルに対して計算された高次モードの速度差は周期と共に減少しない，2) この速度差は二次高次モードで最大であり，その周期は群速度が 200 秒付近で，位相速度が 150 秒付近に於てである．浅い地震では高次モードの媒質応答は基本モードのそれよりはるかに小さい．震源が深さ 200 km の場合は，しかしながら，周期 40 秒付近の一次高次モードの応答は最初の四つのモードの中では最大である．上記の周期におけるこの一次高次モードは海洋下の低速度層の研究に有用であると思われる．