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23. *Initial Phase and Phase Velocity of Surface Waves.*

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Abstract

The phase velocity of oceanic Love waves from the Shikotan earthquake of Jan. 29, 1968, is determined by the single-station method. The initial phase, which is expressed as the algebraic sum of the space phase, the time phase and the propagation phase, is calculated as a function of the period, based on the theory of surface wave excitation. Using the initial phase obtained above and the observed phase spectrum, the phase velocities of Love waves propagated mainly through the Pacific Ocean are obtained for the periods 40～170 seconds.

1. Introduction

The phase velocity of surface waves is usually determined by the two-station method or the single-station method. The former determines the velocity without a knowledge of the initial phase using at least two observations of surface waves at different places (SATÔ, 1955), while the latter determines the velocity using a seismogram recorded at a single station with a help of the knowledge of the initial phase (BRUNE, NAFE and OLIVER, 1960). The initial phase of waves propagating on a spherical surface was discussed by SATÔ and USAMI (1963). Generally the initial phase of surface waves is known if the solution of earthquake mechanism is determined (BEN-MENAHEM and TOKSÖZ, 1963).

In early works of phase velocity determination, the initial phase was not chosen reasonably because the theory of determining the initial phase had not been established at that time. However, we can now make use of the theory of the excitation of surface waves developed by SAITO (1967), HARKRIDER (1970) and TAKEUCHI and SAITO (1972) for the study of Rayleigh and Love waves. ODAKA and USAMI (1970) studied the initial phase of surface waves by comparing the values obtained by Fourier analysis of synthetic seismograms with the ones determined from excitation functions of the free oscillation.

The present paper aims to determine the phase velocity of Love waves from a seismogram recorded at a single station, where the initial phase is calculated as a function of period, based on the theory of the excitation of surface waves.

2. Method

Following the expression by AKI (1962), the movement of surface waves at the distance r may be written as

$$f(t, r)=\frac{1}{\pi}\int_0^\infty |f^*(p)|\cos\left(pt-\frac{pr}{c(p)}+\phi+\phi_I+\frac{m}{2}\pi\right)dp \qquad (1)$$

where

p: Angular frequency
$f^*(p)$: Fourier transform of $f(t, r)$
$c(p)$: Phase velocity
r: Epicentral distance
$\frac{m}{2}\pi$: Polar phase shift of surface waves for m polar passages (BRUNE, NAFE and ALSOP, 1961)
ϕ: Total initial phase
ϕ_I: Instrumental phase delay.

Here the total initial phase is defined as the algebraic sum of three phase factors (BEN-MENAHEM and TOKSÖZ, 1963), namely

$$\phi=\phi_S+\phi_T-\phi_P \qquad (2)$$

where

ϕ_S: Space phase, which depends on the force system
ϕ_T: Time phase, which is the Fourier transform of the source time function
ϕ_P: Propagation phase, which arises from the assumption that the rupture along the fault is moving horizontally.

The space phase together with the radiation pattern of surface waves can be obtained in terms of normal mode solutions and source functions (SAITO, 1967; TAKEUCHI and SAITO, 1972), and in terms of the far-field solutions of surface waves (HARKRIDER, 1970). According to Harkrider, the space phase in the far-field expression for the double couple force is written as

$$\phi_S=-\frac{3}{4}\pi+\arg\chi(\theta, h, p) \qquad (3)$$

where

θ: Angle between the fault line and the line from the source to the station, measured counterclockwise

h: Source depth
$\chi(\theta, h, p)$: Complex radiation pattern function.

The radiation pattern function is expressed as

$$\chi(\theta, h, p)=d_0+i(d_1 \sin\theta+d_2\cos\theta)+d_3\sin 2\theta+d_4\cos 2\theta. \quad (4)$$

The coefficients d_i in (4) for Love waves due to the double couple force are defined as

$$\begin{aligned} d_0&=0\\ d_1&=\cos\lambda\cos dG(h, p)\\ d_2&=-\sin\lambda\cos 2\delta G(h, p)\\ d_3&=\frac{1}{2}\sin\lambda\sin 2\delta V(h, p)\\ d_4&=\cos\lambda\sin\delta V(h, p) \end{aligned} \quad (5)$$

where

λ: Slip angle
δ: Dip angle.

The quantities $G(h, p)$ and $V(h, p)$ in (5) are given in terms of the Thompson-Haskell displacement-stress vector elements (HASKELL, 1953) for the oceanic structure, and they are tabulated in Table 6 of HARKRIDER (1970).

The time phase is calculated from

$$\begin{aligned} \phi_T&=\arg[m^*(p)]\\ m^*(p)&=\int_{-\infty}^{\infty} m(t)\exp(-ipt)dt \end{aligned} \quad (6)$$

where

$m(t)$: Source time function.

The propagation phase derived by BEN-MENAHEM (1961) is defined as

$$\phi_P=\frac{bp}{2c(p)}\left(\frac{c(p)}{v}-\cos\theta\right) \quad (7)$$

where

$c(p)$: Phase velocity of Love waves near the source
b: Fault length
v: Rupture velocity.

By the method of the Fourier transform, the observed wave may be expressed as

$$f(t,r)=\frac{1}{\pi}\int_0^\infty |f^*(p)|\cos(pt+\phi')dp \tag{8}$$

where

ϕ': Phase angle of the Fourier component of the wave.

From the expressions (1) and (8) we have

$$\phi'\pm 2n\pi=-\frac{pr}{c(p)}+\phi+\phi_I+\frac{m}{2}\pi \tag{9}$$

where n is an integer. Hence the phase velocity is obtained from the expression

$$c(p)=\frac{pr}{\phi+\phi_I+(m/2)\pi-\phi'\mp 2n\pi}. \tag{10}$$

3. Data

The Shikotan earthquake of Jan. 29, 1968 (Origin time: $10^h 19^m 05.6^s$, Epicenter: 43.6°N, 146.7°E, Depth: 40 km, after USNOAA) was studied in the present analysis. This event occurred off the northwest coast of Hokkaido (Fig. 1). The fault parameters for this earthquake have been determined by SHIMAZAKI (1975) on the basis of *P*-wave first motions, radiation patterns, amplitudes of long-period surface waves and relocated aftershock distributions. The result is as follows;

Dip angle:	$\delta_1=23°$, $\delta_2=68°$
Dip direction:	$\varphi_1=$N34°W, $\varphi_2=$N123°E
Fault length:	$b=80$ km
Rupture velocity:	$v=3$ km/sec
Rise time:	$T_0=5$ sec
Rupture direction:	N146°E

where the subscripts 1 and 2 refer to the fault plane and the auxiliary plane respectively, and the source time function is assumed to have a ramp function.

Fundamental-mode Love waves observed at the stations WEL and ARE belonging to WWSSN are shown in Fig. 2, and the transverse component of movement synthesized from NS and EW components were analyzed. We could not use seismograms recorded at the other stations around the Pacific because of the saturation of amplitude. The observed data were digitized at the sampling interval 1.93 seconds.

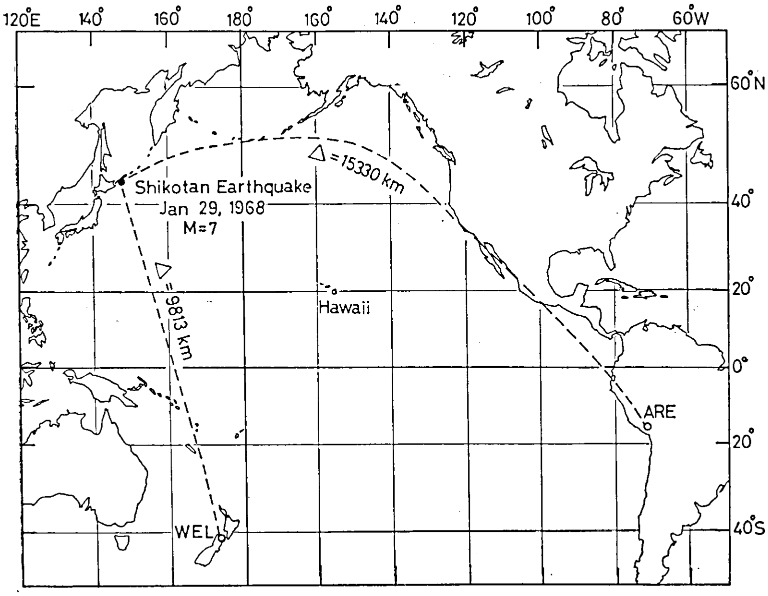
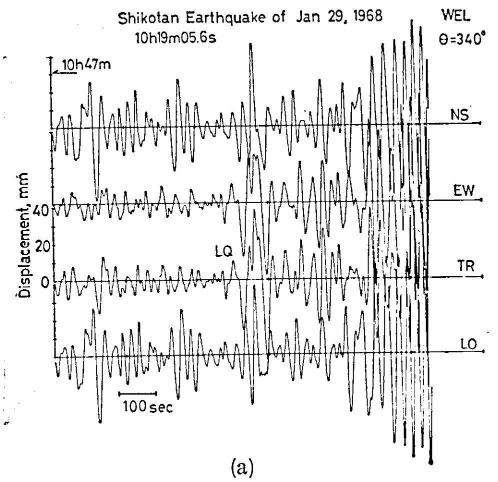


Fig. 1. Location of the earthquake (solid circle) and stations (open circles).



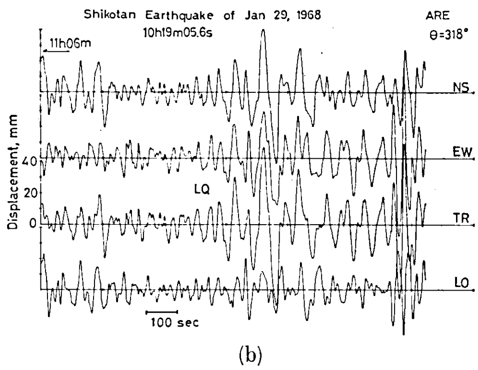


Fig. 2. Seismograms of NS and EW components of the observed waves involving Love waves. Transverse (TR) and longitudinal (LO) components are synthesized from them. Azimuths from station to epicenter are given in the upper right.

4. Analysis

The radiation patterns were calculated from the formula (4) and they are shown in Fig. 3 for the periods 10, 20, 30, 50, 100 and 200 seconds. In these calculations, the focal mechanism solution of the slip angle 107° and the dip angle 23° is used with the notations defined by

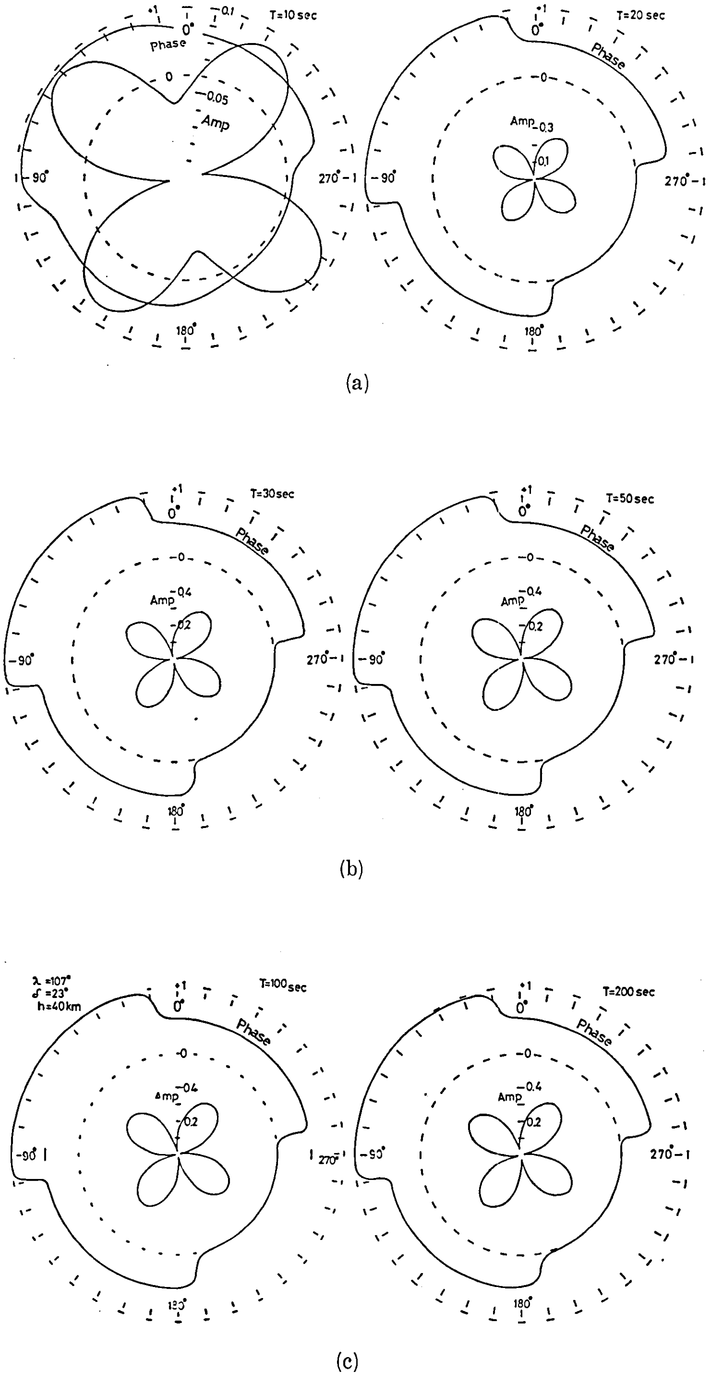


Fig. 3. Radiation patterns of the amplitude and the phases for six periods. +1 means 2π radian. (Fundamental mode of Love wave)

JAROSCH and ABOODI (1970). The peripheral curves in Fig. 3 show the azimuthal variation of the phase of the radiation pattern function in a unit ring (HASKELL, 1963; BEN-MENAHEM and HARKRIDER, 1964) and they nearly hold the same pattern for the periods longer than 20 seconds. It is also seen from the figures that the phases change rapidly at the azimuth where the excitation is minimum. The space phases in (4) are given in Fig. 4 for the azimuthal angles 77° (WEL) and 174° (ARE).

The ramp function, which was discussed by HASKELL (1964) in relation to the total energy of elastic wave radiation from a propagating fault, is written in the form

$$m(t)=\begin{cases} 0 & t<0 \\ \dfrac{t}{T_0} & 0\leqq t\leqq T_0. \\ 1 & t>T_0 \end{cases} \tag{11}$$

The Fourier transform of this function is

$$m^*(p)=\frac{1}{p^2T_0}(\exp(-ipT_0)-1) \tag{12}$$

whose phase angle defined in (6) is

$$\phi_T=-\frac{\pi}{2}-\frac{pT_0}{2}. \tag{13}$$

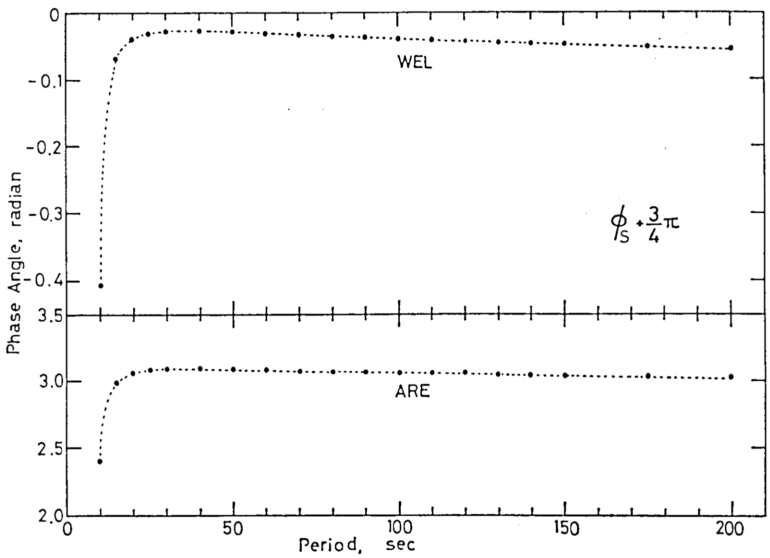


Fig. 4. The space phases for WEL and ARE.

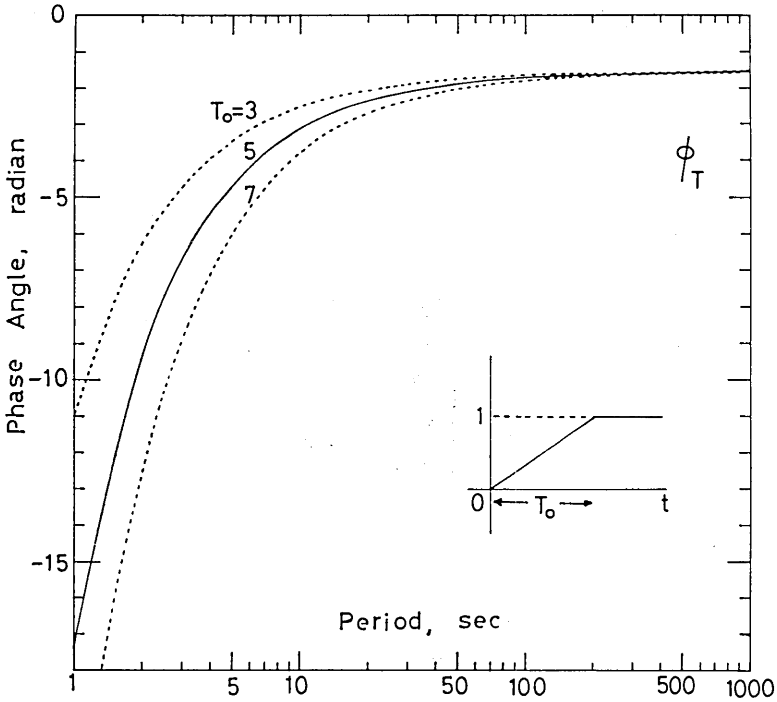


Fig. 5. The time phases for the ramp function with the rise time T_0. For the Shikotan earthquake T_0=5 sec (solid line) is adopted. Other phases (dashed lines) are given for comparison.

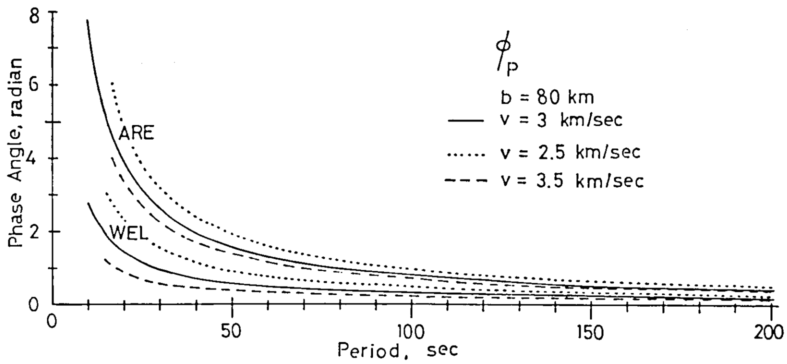


Fig. 6. The propagation phases for WEL and ARE. The phases expressed by solid lines are used in the computation. Other phases (dashed and dotted lines) are given for comparison.

Introducing the rise time T_0=3, 5 and 7 seconds into expression (13), the time phase is calculated and is shown in Fig. 5. This figure shows that the difference among the three time phases is small for periods longer than 40 seconds, but it is conspicuous at short periods.

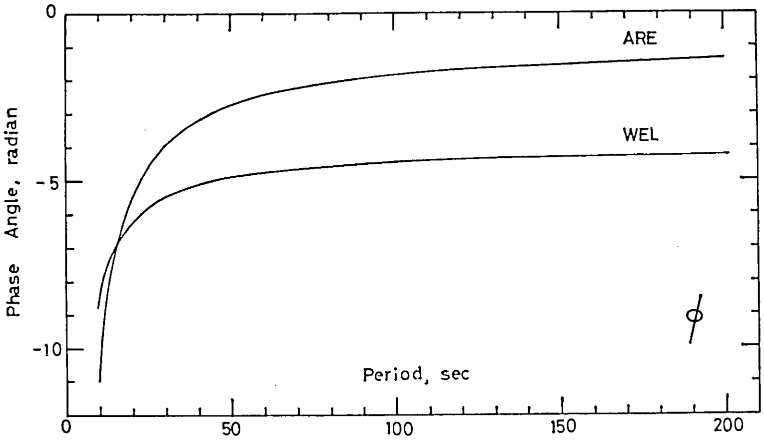


Fig. 7. The total initial phases for WEL and ARE.

The angle θ in the expression (7) is measured counterclockwise from N146°E, which is the direction of rupture, and the propagation phases due to the fault length of 80 km and the rupture velocity of 3 km/sec are given in Fig. 6 for the angles $\theta=347°$ (WEL) and $\theta=84°$ (ARE). The phase velocity $c(p)$ in (7) is taken from the table given by HARKRIDER (1970). We see from the figure that the phase variation is strong at short periods and the phase difference due to the different rupture velocities decreases with the period. The total initial phases defined in (2) are shown in Fig. 7 for two stations WEL and ARE.

5. Result

The instrumental phase delay in the expressions (1) and (10) was corrected making use of the theoretical phase characteristics for the WWSSN long-period instrument computed by MITCHEL and LANDISMAN (1969).

Finally substituting the total initial phase, the instrumental phase delay and the observed phase into expression (10), the phase velocities of Love waves were calculated for the periods 40~170 seconds for ARE and 60~140 seconds for WEL (Fig. 8). The group velocity U in the figure is calculated using the formula

$$U=\frac{dp}{dk} \tag{14}$$

where k is the wave number. Fig. 8 shows that the phase velocity for WEL is slightly higher than that for ARE for the period shorter than

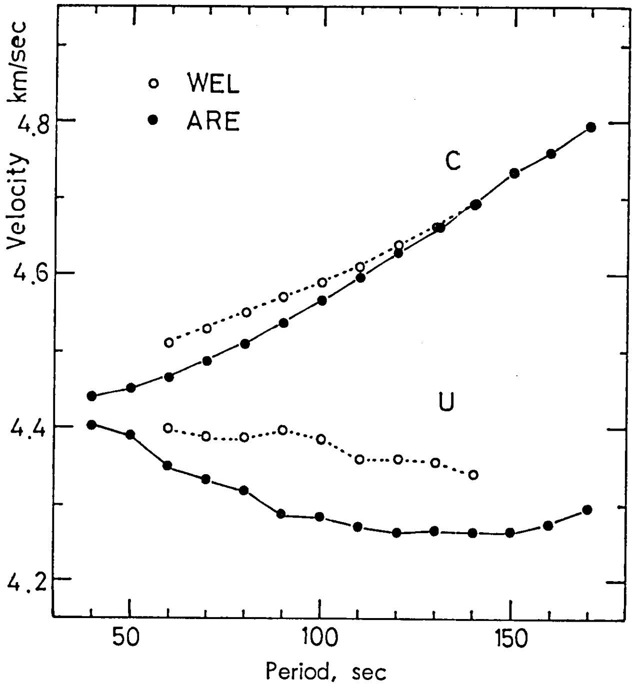


Fig. 8. Calculated phases (C) and Group (U) velocity curves.

140 seconds. The difference is more remarkable in the group velocity curves. As a whole, the phase velocities above obtained for WEL and ARE are somewhat low compared with the standard oceanic ones (for example, the phase velocities for the model CIT 11A constructed by ANDERSON and TOKSÖZ, (1963) and KOVACH (1965) are 4.60 and 4.83 km/sec at periods of 72 and 164 seconds respectively). This fact might have resulted from the wave paths, which are partly pseudo-oceanic for both stations.

6. Discussion

In the solution of the Shikotan earthquake mechanism the ramp function was assumed as the source time function by Shimazaki. However if a force is exerted stepwise in time at the source, the time phase is simply $-(\pi/2)$, independent of period. An exponential function (Fig. 9), which is another form of the source time function, was deduced by BEN-MENAHEM and TOKSÖZ (1963) for the Kamchatka earthquake of November 4, 1952. In Figs. 5 and 9 we see that the phases between the ramp function and the exponential function differ strongly at short periods. Namely the phase of the ramp function changes rapidly for periods shorter than 10 seconds while that of the exponential function converges to

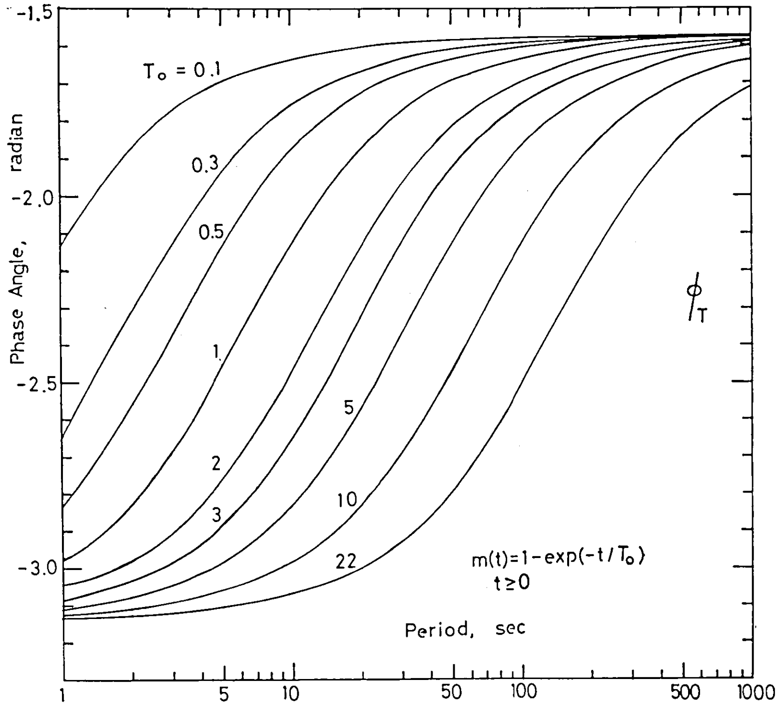


Fig. 9. The time phases for the exponential function with the rise time T_0.

$-2.0\sim-3.0$ radians at short periods. However, for a long period 50(150) seconds, the phase angles for the ramp and exponential functions with a rise time of 5 seconds show no large discrepancy and are $-1.885(-1.676)$ and $-2.132(-1.777)$ radians respectively. These values are not so different from the time phase of the step function $-(\pi/2)$, either.

The accuracy of the phase velocities obtained is estimated as follows. In the present analysis the error of 0.1 or 0.2 radians may be involved in the individual phase factor; the space phase, the time phase, the propagation phase and the observed phase of Love waves, therefore, the total error will be at most around half a radian or less. The phase error of 0.8 radians leads to the phase velocity error of 0.009 km/sec (0.2%), 0.017 km/sec (0.5%) and 0.04 km/sec (0.8%) for periods of 50, 100 and 200 seconds, respectively.

7. Conclusion

The initial phase of Love waves excited by the Shikotan earthquake has been calculated (Fig. 7) making use of the theory of the excitation of surface waves, and the phase velocity of the oceanic Love waves was determined (Fig. 8). So far as Love waves from the Shikotan earthquake are concerned, the single-station method seems to be applicable for the periods longer than 40 or 50 seconds, taking into account the characteristics of the initial phase. The determined phase velocities for WEL are slightly higher than those for ARE, but both of them are lower than the standard oceanic velocities.

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23. 表面波のイニシヤルフェイズと位相速度

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1968 年 1 月 29 日の色丹地震によって励起された海洋性ラブ波の位相速度が 1 観測点法（1 点で観測された波から速度を求める）によって求められた．震源における力源の空間的分布によって決まる空間位相，震源時間関数によって決まる時間位相，及び破壊伝播によって生ずる伝播位相の代数的な和であるイニシヤルフェイズが，表面波の励起理論に基づき周期の関数として計算された．このようにして得られたイニシヤルフェイズと観測された位相スペクトルから，主として太平洋を伝わったラブ波の位相速度が周期 40～170 秒の範囲で求められた．
