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40. *Development of the Frequency Distribution of Earthquakes with Respect to the Magnitude in the Aftershocks of the 1968 Tokachi-Oki Earthquake.*

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1. Introduction

In a previous paper[1], it was shown that the frequency distribution of earthquakes with respect to magnitude is derived from the deformation∼fracture relation of the earthquake genesis. This implies that the frequency distribution of earthquakes is nothing but a representation of the crustal deformation in the focal area. This suggests further that there is a possibility to infer the characteristics of the development of deformation from the time variation of the frequency distributions of earthquakes.

From these considerations the following questions may be raised.

Namely, how does the frequency distribution of earthquakes develop with time in the aftershock sequence of a certain great earthquake? In other words, do the earthquakes of different magnitude occur at random with time or with some regularities? If there is any regularity, is it related to the development of the aftershock area?

Hitherto, there have been made many studies about the statistical nature of aftershock sequences[2]. In those studies, however, the time variation of the frequency distribution of earthquake occurrence with respect to earthquake size has not been treated. The whole aftershock sequence has been treated as a single statistical population.

Utsu[3], however, has pointed out that, in many cases, the main shock deviates remarkably from Gutenberg-Richter's statistical formula for the frequency distribution of aftershocks. Utsu[4] and Mogi[5] have noted the statistical charateristics of M_1-M_0, where M_0 and M_1 are the

1) S. NAGUMO, *Bull. Earthq. Res. Inst.*, **47** (1969), 1024-1033.
2) T. UTSU, *Geophys. Mag.* **30** (1961), 521-605.
T. UTSU, *Jour. Fac. Sci. Hokkaido Univ., Ser. VII*, **3**, (1969), 129-196.
3) T. UTSU, *Quart. Jour. Seis.*, **28** (1963), 129-136.
4) T. UTSU, *loc. cit.*, 2)
5) K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 615-658.

magnitudes of the main shock and the largest aftershock respectively. These studies imply that the main shock is not included in the single statistical population of aftershocks.

Yamakawa[6] has pointed out that the distribution of major aftershocks indicates the existence of a linear structure in the aftershock area. This also implies the existence of some physical characteristics among the populations of aftershocks.

In this paper, the existence of reguralities in the development of the frequency distribution of earthquakes with respect to magnitude is analized in the earthquake sequence of 1968 Tokachi-Oki earthquake (M= 7.9 JMA). The relation between the development of the frequency distribution and the aftershock area will be treated in an accompanying paper[7].

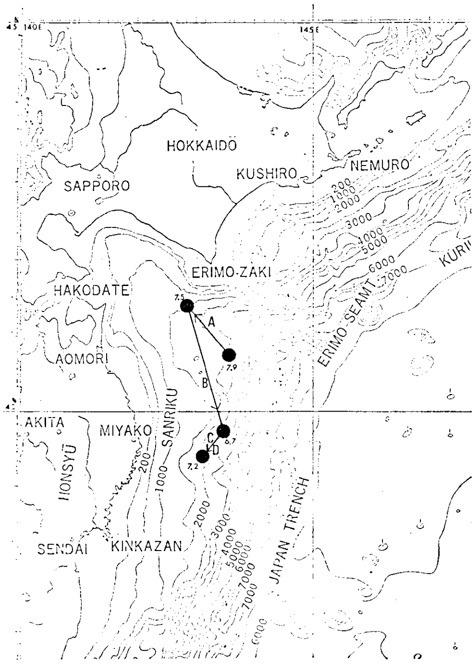

Fig. 1. The main shock and the major aftershocks of the 1968 Tokachi-Oki earthquake. The arrows indicate the direction of migration. The figures denote their magnitudes.

2. Data

Data used are the Seismological Bulletin of the Japan Meteorological Agency. For the convenience of understanding, the outline of the aftershock activity will be briefly described. In Figure 1 the major shocks are plotted which took place after the main shock of the 1968 Tokachi-Oki earthquake till the end of July. The magnitude 7.9 represents the principal shock, and other solid circles represent major aftershocks with magnitudes 6.7 or greater.

In Table 1 data of the principal shock and major earthquakes are tabulated.

As is well known, the biggest aftershock (M=7.5) took place about 10 hours after the principal shock (7.9) in the direction towards the northwest of the main shock, off Urakawa. After about 23 hours after the

6) N. YAMAKAWA, *Zisin*, (ii), **18** (1965), 25-40.
N. YAMAKAWA, *Papers Met. Geophys.*, **18** (1967), 15-26.
7) S. NAGUMO, *Bull Earthq. Res. Inst.*, **48** (1970), 759-778.

Table 1. Data of the main shock and the major aftershocks of the 1968 Tokachi-Oki earthquake.

	M	Date			Epicenter λ	Epicenter φ	Deph H
Main Shock	7.9	1969	May 16	09^{h} 49^{m}	143°35′E	40°44′N	00 km
After Shock	7.5	〃	〃 16	19 39	142 51	41 25	40
〃	6.7	〃	〃 17	08 05	143 29	39 46	30
〃	7.2	〃	June 12	22 42	143 08	39 25	00

principal shock, an aftershock with M=6.7 took place in the direction towards the south-west of the main shock.

After these big aftershocks, aftershock activity has occurred in the wide area along the continental side of the Japan trench. However, about 27 days after the principal shock, on July 12, the 3rd largest aftershock (M = 7.2) occurred in the southern part of the aftershock area. Epicenters of more than about 300 aftershocks are registered in the Bulletin of the Japan Meteorological Agency during the period of eighty days from May 16 till July 30, 1969.

3. Development of aftershocks

Division of the period

In order to study the time variation of the frequency distribution of earthquakes with respect to magnitude, the period of the aftershock activity is divided into several intervals. The division is made by several

Table 2. Division of the period of aftershocks.

Period		Interval	Number of Earthquakes
	Main Shock, 7.9, May 16, 09^{h} 49^{m}		
A		0.4 Days	18
	After Shock, 7.5, May 16, 19 39		
B		0.9	47
	〃 6.7, May 17, 08 05		
C		7	114
	〃 6.3, May 23, 04 29		
D		27	178
	〃 7.2, June 12, 22 42		
E		76	340
	〃 July 31, 23 59		

major aftershocks as shown in Table 2. This is done so as to know the role of the major aftershocks.

The development of the frequency distribution of aftershocks is shown in Fig. 2 (A)∼(E);

Fig. 2 (A) shows the frequency distribution of aftershocks with respect to the magnitude at the time before the occurrence of the biggest aftershock (M=7.5) on May 16, 19h 36m, which is shown by the arrow in the figure. The interval is about 10 hours after the main shock, (M = 7.9 of May 16, 09h 46m).

This figure shows that the biggest aftershock M=7.5 took place after the occurrence of about 20 aftershocks with magnitudes smaller

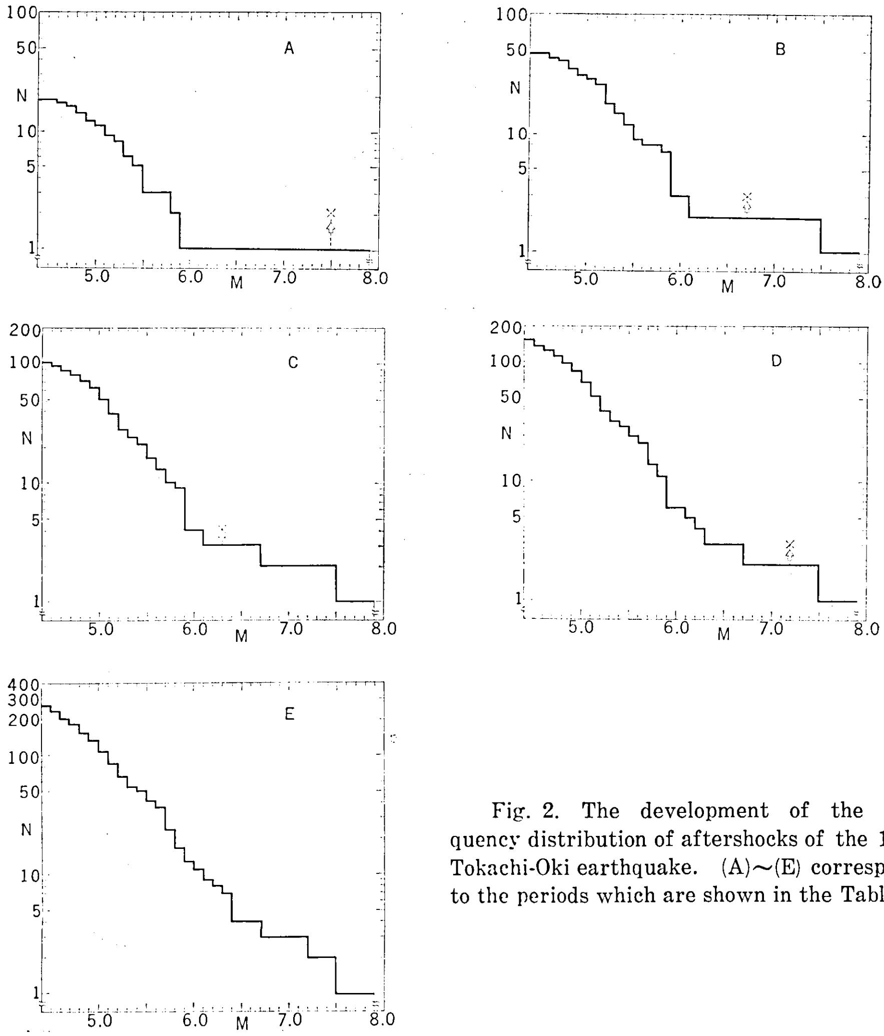

Fig. 2. The development of the frequency distribution of aftershocks of the 1968 Tokachi-Oki earthquake. (A)∼(E) correspond to the periods which are shown in the Table 2.

than 6. This figure also shows that the occurrence of the biggest aftershock is independent of the frequency distribution of smaller aftershocks with magnitude smaller than 6.

One can hardly infer the necessity of the occurrence of the earthquake with M=7.5 in this figure, from the distribution of the smaller earthquakes. The shook with M=7.5 will not be regarded as included in the population of smaller earthquakes. Instead, this biggest aftershock will be regarded as belonging to the population which will be controlled by the main shock, under Gutenberg-Richter's statistical formula.

Fig. 2 (B) shows the cumulative frequency distribution prior to the occurrence of the major aftershock (M=6.7) of 1968 May 17, 08h 05m, which is indicated by the arrow in the figure, beginning with the occurrence of main shock.

This figure shows that the major aftershock of M=6.7 occurred after about 50 aftershocks had taken place. It is seen in this figure that the occurrence of the aftershock with magnitude 6.7 is independent of the frequency distribution of the smaller earthquakes. In other words, the occurrence of the aftershock with magnitude 6.7 is not thought to satisfy Gutenberg-Richter's statistical formula, which will fit only the smaller earthquakes. This aftershock with M = 6.7 is thought to belong to the same population to which also the main shock and the biggest aftershock belong.

Fig. 2 (C) shows the cumulative frequency distribution of the aftershocks prior to the occurrence of the aftershock (M=6.3) on May 23, 04h 29m, which is indicated by an arrow in the figure. The period is about one week after the main shock. It is seen in this figure that the aftershock with M=6.3 seems to belong to the population containing about 80 smaller earthquakes.

Fig. 2 (D) shows the cumulative frequency curve of aftershocks prior to the occurrence of the second biggest aftershock with M=7.2 in June 12, 22h 42m. The period is about 27 days after the main shock. It is seen in this figure that this large aftershock, which is shown by an arrow, does not belong to the population of the smaller earthquakes, but it does belong to the same population of the main shock and large aftershocks.

Fig. 2 (E) shows the cumulative frequency curve for all aftershocks from the main shock till the end of July. The period is about 80 days and the total number of aftershocks is about 300. These aftershocks, including the main shock, seem to satisfy a single statistical formula of Gutenberg and Richter.

The frequency distribution which is shown in the Fig. 2 (D) will not be regarded as indicating a single population. The aftershoks which took place from June 12 till the end of July seem to occure so as to complete a shape of the population.

In Fig. 3 the successive development of the frequency distribution of aftershocks are shown in a single frame. From the time variations of the frequency distribution, several important characteristics are derived.

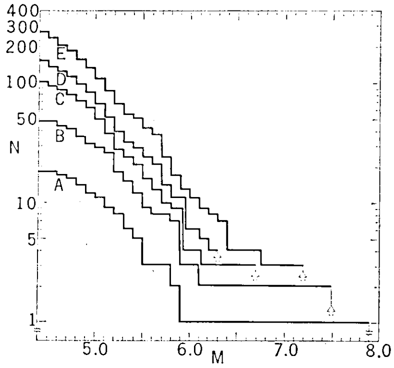

Fig. 3. The development of the frequency distribution of aftershocks. From the principal shock of May 16, 09h49m to (A) May 16, 19h39m, (B) May 17, 08h05m, (C) May 23, 04h29m, (D) June 12, 22h42m, (E) July 31, 23h59m.

1) The three major aftershocks seem to be induced by the principal shock so as to satisfy Gutenberg-Richter's statistical formula. In other word, these three major aftershocks are not induced by the occurrence of the numerous smaller aftershocks.
2) The main aftershocks occur first, and then smaller earthquakes follow.
3) The smaller aftershocks with magnitudes smaller than 6 seem to take place so as to form a population which satisfies Gutenberg-Richter's statistical formula.
4) Aftershocks of intermediate size (M $\fallingdotseq$ 6) seem to occur so as to connect the different populations of earthquakes, namely the population of the main shock and major aftershocks with the population of smaller aftershocks.
5) There seems to be three different groups in the sequence of aftershocks. One is the group of larger shocks which belong to the same sequence as the main shock. The other is the small aftershock swarm with magnitudes smaller than 6. The third group is the one with earthquakes having intermediate magnitudes and occurring so as to interconnect the two ensembles.

4. Discussions

The time variation of the cumulative frequency distribution of earthquakes with respect to the magnitude in the case of the 1968 Tokachi-Oki earthquake is a particular example of the development of the sequence of aftershocks. However, to aid a generalization of the pattern of the development of aftershocks, the development process is schematically summarized and represented in Fig. 4.

There seems to be several distinct stages. The first stage is the occurrence of the main shock. This is the typical pattern of the main shock type earthquake. The 2nd stage is the occurence of small after shocks. The magnitudes of these small aftershocks are distinctly smaller than the ones which are expected from Gutenberg-Richter's formula governed by the magnitude of the main shock. The magnitudes of these small aftershocks satisfy their own statistical formula. The 3rd stage is the occurrence of major aftershocks. The major aftershocks take place so as to form the population which is governed by the main shock. In the case of the 1968 Tokachi-Oki earthquake, the main shock and the three major aftershocks are regarded as belonging to this population. These three stages seems to occur at the early stage of the aftershock activity. In this case, however, the earthquake with M =7.5 on June 12 took place nearly a month after the occurrence of the main shock. These major shocks seem to play a role in exciting the area of aftershock activity. The relation of these shocks to the development of the aftershock area will be studied in the accompaning paper[8].

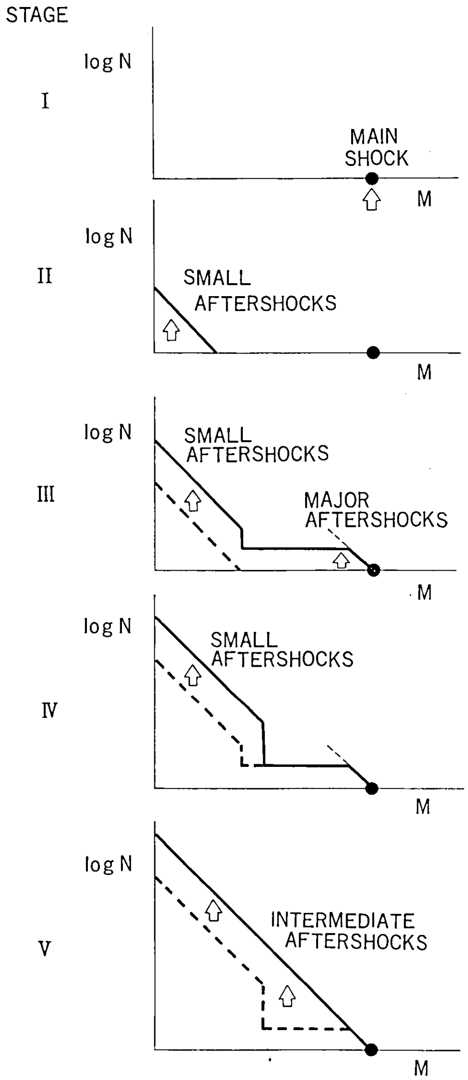

Fig. 4. Schematic representation of the development of the frequency distribution of aftershocks with respect to magnitude in the 1968 Tokachi-Oki earthquake.

The 4th stage of the development of the aftershock is characterized by the increase of the number of smaller aftershocks. The population of smaller aftershocks developes independently without the occurrence of larger aftershocks. There seems to be two different populations of aftershocks at this stage. The 5th stage will be characterized by the

8) S. Nagumo, *loc. cit.*, 7).

occurrence of aftershocks of intermediate size. These shocks seem to occur so as to combine these two different populations of larger and smaller earthquakes to a single population. This may be the final stage of the aftershock activity.

Utsu[9] has already pointed out that the magnitude of the principal shock often deviates from Gutenberg-Richter's statistical formula for the aftershocks. Utsu[10] also pointed out that the difference between the magnitude of the main shock and the biggest aftershock M_1, is a function of M_0. Mogi[11] has remarked that there exist regional characteristics in the distribution of M_0-M_1 and has given same physical meaning to this hetrogeneity.

Utsu[12] has developed his study and remarked that there seem to be two different groups in the frequency distribution of earthquakes and that there is a great difference in the value of Gutenberg-Richters statistical formula for the populations of larger aftershocks and for the smaller aftershocks. According to him, the example of the 1968 Tokachi-Oki earthquakes is rather a special one. In many great earthquakes, the development of aftershocks ends at the stage 4 and the appearance of stage 5 is not common. (The great Kwanto Earthquake of 1923 also has a small M_0-M_1)

Concerning the physical meaning of the magnitude frequency distribution of earthquakes, there are two different view points. One regards it as a probability distribution, and the other regards it as a physical, causal law. If this relation is a probability distribution, the occurrence of earthquakes of different magnitude should be at random with respect to a random variable.

However, as is seen in the above analyses, there evidently exist regularities for the time sequence of the occurrence. This fact is another evidence which supports the latter view.

5. Development of the cumulative number of aftershocks

The development of the cumulative number of aftershocks is shown in Fig. 5. The cumulative number of aftershocks increases in the form of a power function during a month after the main shock. There is a remarkable jump in the $\sum N \sim T$ curve at the major aftershock with $M=7.2$ in June 12. This jump corresponds to the formation of a new aftershock area.

9) T. UTSU, *loc. cit.*, 3).
10) T. UTSU, *loc. cit.*, 2).
11) K. MOGI, *loc. cit.*, 5).
12) T. UTSU, *Geophys. Bull. Hokkaido Univ.*, **23** (1970), 49–71.

In Fig. 6 $\sum N$ is plotted as a function of $\log T$. This representation is very similar to that of Benioff[13]. According to Utsu[14] $\sum \sqrt{E}$ approaches the value of $\sum N$ when the magnitude of earthquake of which energy is computed becomes smaller and smaller.

In the case of this earthquake, the change of the gradient in the figure will correspond to the development of the aftershock area. It is not necessary to assume that it is due to change of creep characteristics.

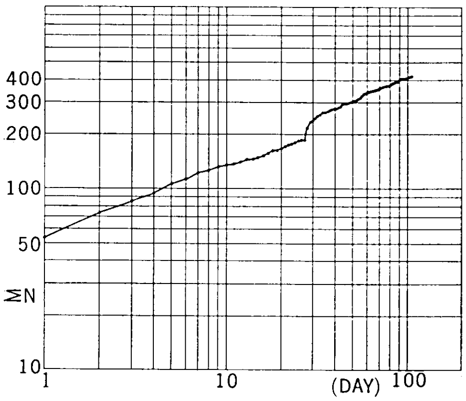

Fig. 5. Cumulative number of aftershocks in the 1968 Tokachi-Oki earthquakes.

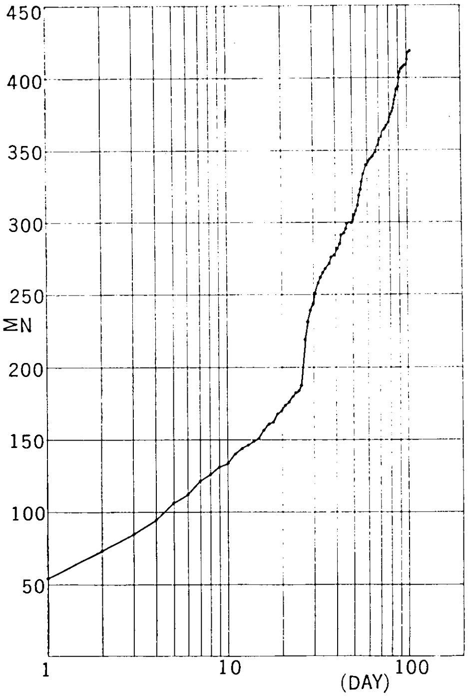

Fig. 6. Cumulative number of aftershocks in the 1968 Tokachi-Oki earthquakes.

6. Summary and Conclusions

Several regularities are revealed for the development of the freqnency distribution of aftershocks with respect to the magnitude in the 1968 Tokachi-Oki earthquake.

The main results are as follows.

1) The major aftershocks occur first and then smaller aftershocks are generated. This is the characteristics of the main shock type earthquake.

2) The major aftershocks occur in the early stage of the aftershock activity so as to satisfy Gutenberg-Richter's statistical formula which will be regulated by the principal shock.

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13) H. Benioff, *Bull. Seis. Soc. Amer.*, **41** (1951), 31-62.

14) T. Utsu, *Geophys. Bull. of Hokkaido Univ.*, **18** (1967), 53-69.

3) Smaller aftershocks which are excited by a certain major aftershock occur so as to satisfy the statistical formula by themselves.
4) In the later stage of the aftershock development, aftershocks of intermediate size, with magnitudes around M=6 in this case, occur so as to unify the two populations of smaller earthquakes and major aftershocks to a single earthquake population governed by the principal shock.

The correspondence of these characteristics to the development of the aftershock area will be studied in the accompanying paper. The existence of such regularity in the time sequence is another evidence of the fact that Gutenberg and Richter's statistical formula is a physical law and is not a probability distribution.

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40. 1968 年十勝沖地震余震におけるマグニチュード別度数分布の発達

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1968 年十勝沖地震の余震においてマグニチュード別度数分布の発達にいくつかの規則性が認められる．主な結果は次の通りである．

(1) 主な余震が先づ発生し，次に小さい余震が発生する．これは主震型地震の特徴であろう．
(2) 主な余震は余震活動の早い時期に発生し，主震で規定されるところの Gutenberg-Richter の統計式を満たすように起る．
(3) 或る主な余震によって誘起される小さい余震は，それ自身で Gutenberg-Richter の統計式を満たすように起る．
(4) 余震活動発達の後期において，中間的規模の地震が起り，小さい余震の母集団と主震および主な余震の母集団の 2 つを 1 つの母集団に統一するように起る．

これらのマグニチュード別度数分布発達についての特徴が余震域の発達といかに対応するかという問題は次の論文にて取扱われている．マグチュード別度数分布が時間系列において，このように規則性があるということは，統計式が物理法則を表すものであって，確率分布ではないことのもう 1 つの証拠であろう．