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11. *Attenuation of Love Waves in Soil Layers.*

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1. Introduction

Attenuation of seismic waves due to the internal friction plays an important role in studying the behavior of media during earthquakes. The attenuation is generally discussed in terms of attenuation coefficient or Q-value. The attenuation coefficients or Q-values should be measured in a wide frequency range, because we intend to discuss the relation between the attenuation coefficients or Q-values and frequency. Although many papers have been published on the attenuation coefficients or Q-values, almost all of them are associated with the earth's crust and mantle. These studies have been reviewed, by Yamakawa and Satô (1964), by Sato (1967) and by others (e.g., Knopoff, 1964).

In a previous paper (Kudo and Shima, 1970), attenuation coefficients in various kinds of soils were determined by analysing the numerous seismograms obtained in the prospecting by means of the horizontally polarized shear waves (so-called SH waves) in the Tokyo Metropolis (Shima et al, 1968 and 1969). The frequency range analysed was 30 to 90 cps and it was concluded that Q-value of SH waves was little dependent on frequency, although the frequency range at which wave amplitudes would predominate during earthquakes is lower than the above mentioned frequency range. In an SH wave prospecting carried out at Yukigaya, Ota-ku, clear Love waves were observed (Allam, 1970). At low frequencies the spectral amplitudes of the Love waves were rather large compared with those of direct SH waves. Accordingly, it is most reasonable to employ the Love waves in order to investigate the attenuation coefficient of SH waves in a wide range of frequency.

Hirasawa and Sato (1963) derived theoretically that Q-value of Love waves was frequency dependent, even though Q-value of SH waves in each layer was independent of frequency. The frequency dependency of Q-value of Love waves is caused by the dispersion of phase velocity. Therefore, Q-value of Love waves is not necessarily associated directly with the loss mechanism of the substances.

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In the present paper, Q-value of Love waves is discussed together with the result obtained from SH wave measurement. The results are compared with the value derived from the theory of Hirasawa and Sato (1963).

2. Experimental site and apparatus

An experimental site is located at Yukigaya, Ota-ku, Tokyo Metropolis. The superficial layer and the substratum of the site are alluvial clay and Tertiary mudstone, respectively.

Waves of SH type were generated by hitting, with a hammer, the end of a slender weighted wooden plate laid down on the ground surface.

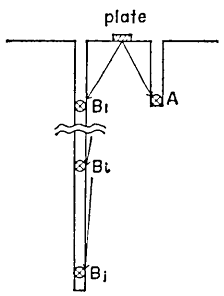

Fig. 1. Configuration of source plate and bore-hole seismometer.

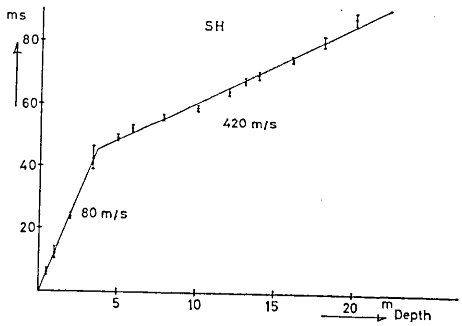

Fig. 2. Vertical travel time curves for SH waves, with 95% confidence intervals.

The underground structure at the site was determined by employing the well-shooting method. Figure 1 shows the method schematically. Notations A and B_i in this figure show the three component borehole seismometers installed at the bottom of the boreholes. The position A was fixed all through the experiments while B was installed at various depths. The waves detected by the transverse component of the seismometer were confirmed as SH waves, since the inversion of phases were observed by hitting the left and right ends of the source plate. Six vertical travel times for SH waves were obtained from independent trials at one depth. Travel times thus obtained were averaged (shown by circles) and 95% confidence intervals (shown by vertical bars) were computed. They are shown in Figure 2. The SH wave velocity and density of each layer are 80 m/s and 1.4 g/cm^3 for the superficial layer, and 420 m/s and 1.8 g/cm^3 for the substratum, respectively. The thickness of the surface layer was determined to be 3.6 m. The velocity structure of SH-waves agrees well with the geological section (Fig. 3).

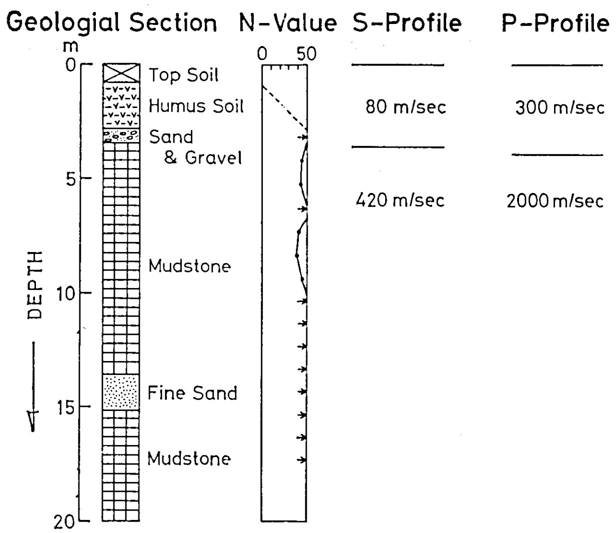

Fig. 3. Geological section and S- and P-profiles (Yukigaya Primary School).

The seismometers used in this experiment were of the moving coil type with one second in the natural period. DC amplifiers and 100 cps galvanometers were employed. Thus the frequency characteristics of the apparatus were flat in the frequency range from 2 to 60 cps.

3. Preparatory analyses of *Q*-value of Love waves

Only a few papers have been published on the attenuation of waves in soil layers, in spite of the importance from the view-point of geophysics as well as engineering seismology. Recently, Maekawa (1969) studied the Q-value of Rayleigh and apparent Love waves generated by small

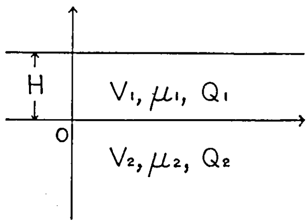

Fig. 4. Geometry.

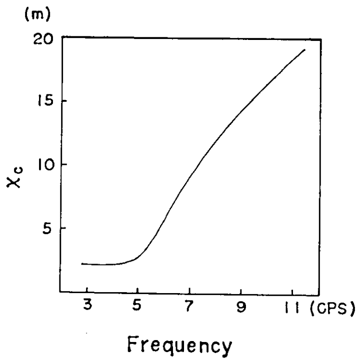

Fig. 5. Relation between the critical distance X_c and the frequency.

explosions, assuming the Q-value as being constant, in media similar to the authors'. However, the aim, method and frequency range in his analysis are a little different from those of the authors. Furthermore, no result related to the body waves was presented in his paper. It is noted that the attenuation of surface waves differs from that of body waves, as will be seen in the latter part of this section. As a result, some further discussion to investigate the loss mechanism of substances is required.

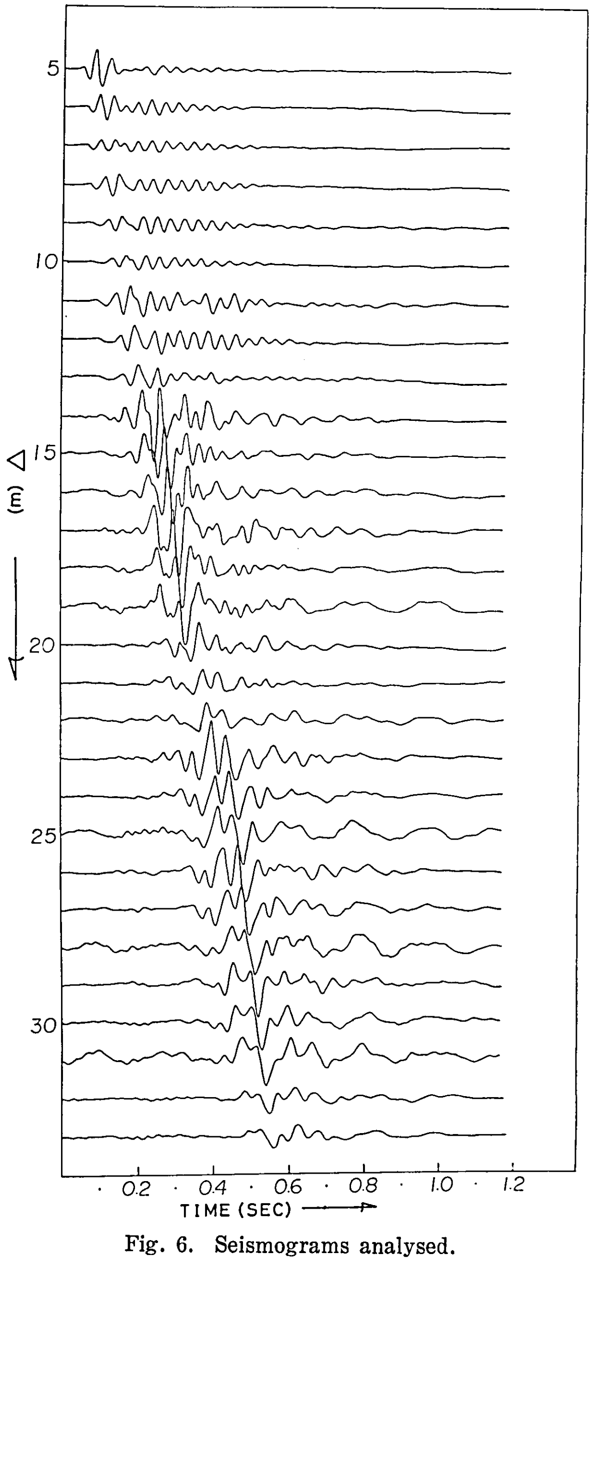

Fig. 6. Seismograms analysed.

The loss mechanism cannot be clarified till the attenuation coefficients or Q-values are obtained in a wide range of frequency at the same site.

Close attention must be paid to the generation and propagation of Love waves when studying their amplitudes or phases. Satô (1952) obtained the critical appearance-distance of Love waves X_c where the Love waves can clearly be observed. The critical distance is expressed, when the source is located on the surface, as

$$X_c = 3H/\sqrt{c^2/V_1^2 - 1}\ , \quad (1)$$

where H and V_1 are

thickness and shear velocity of the superficial layer, and c is the phase velocity of the Love wave. For example, in the S-profile shown in Fig. 3, the relation between the critical distance X_c and the frequency is given by Fig. 5. Figure 6 shows the records observed at the ground surface. Fourier spectra were computed from each complete seismogram, and examples of them are shown in Fig. 7. From the figure we can easily find that the spectrum at near distance, $\Delta=6$ m, shows the largest amplitude at about 30 cps and the amplitudes at the low frequency around several cycles per second are less than 10% of its maximum value. Waves observed at the point are mainly of the body wave, as seen in Fig. 5. In contrast with it, for example, at a remote distance, $\Delta=23$ m, the spectral amplitudes at lower frequencies are large. The observed waves may contain several modes of Love waves (Allam, 1970). The cut-off frequency of the first mode of Love waves is about 11 cps. Thus, in order to avoid the influence of the higher modes of Love waves and of body waves, frequencies less than 11 cps are taken into account in the following analyses.

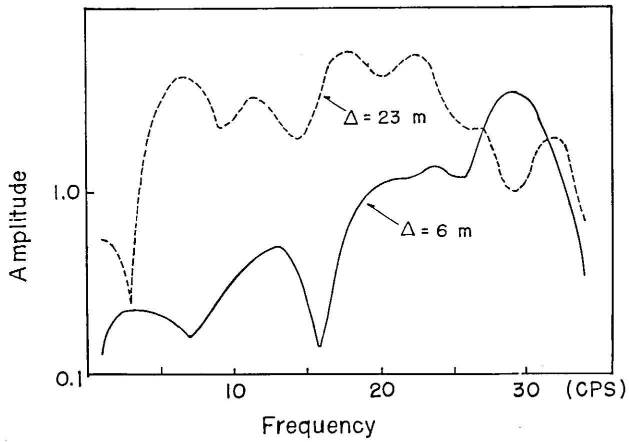

Fig. 7. Example of spectra (arbitrary scale).

The dispersion curves of phase velocities of Love waves for two cases were computed. In Figure 8, the solid and broken lines correspond to the dispersion curves when the shear velocity in the superficial layer is 80 and 75 m/s, respectively. The phase velocities obtained from the observed phase spectra are also plotted in this figure. The experimental results shown in the figure were obtained by means of the regression line analysis. The mean values are shown by circles. These values fall between two theoretical curves. However, considering the precision of determining the shear velocity the authors adopted 80 m/s as the shear velocity of the superficial layer.

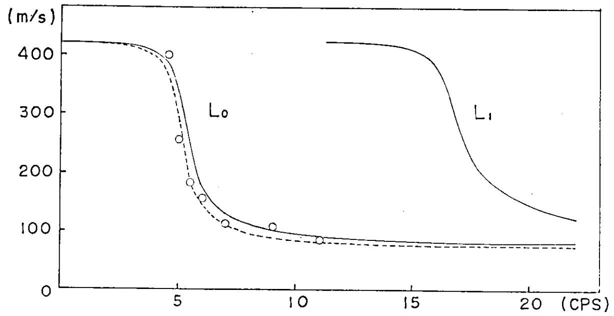

Fig. 8. Dispersion curves for Love waves. Solid and broken lines correspond to the cases when the shear wave velocities in the surface layer are 80 m/s and 75 m/s respectively. Observed values are shown by circles.

Hirasawa and Sato (1963) showed that Q-value of Love waves Q_L is of frequency dependence, even though Q-values of shear waves in each layer Q_1, Q_2 are independent of frequency. Neglecting the order of Q^{-2}, the period equation and the relation among Q_L, Q_1 and Q_2 are expressed as follows,

$$\xi_2=(\mu_1/\mu_2)\xi_1 \tan \xi_1 \ , \tag{2}$$

$$\frac{Q_L}{Q_1}=\frac{\xi_1+A\xi_2}{\xi_2\left(AV^2-\frac{2}{a^2}\,\xi_1\xi_2\right)+\frac{Q_1}{Q_2}\left(\frac{V_1^2}{V_2^2}\cdot V^2+\frac{2}{a^2}\xi_2^2\right)}\ , \tag{3}$$

where

$$\left.\begin{aligned}
&\xi_1 \fallingdotseq a\sqrt{V^2-1}\ ,\\
&\xi_2 \fallingdotseq a\sqrt{1-V'^2}\ ,\\
&a^2=\omega^2H^2/c^2\\
&V^2=c^2/V_1^2\ ,\\
&V'^2=c^2/V_2^2\ ,\\
&A\equiv(\mu_2/\mu_2)(\xi_1+\sin\xi_1\cos\xi_1)/\cos^2\xi_1\ ,
\end{aligned}\right\} \tag{4}$$

The equation (2) is the same as the one for the perfect elastic media. The phase and group velocities c, U of the fundamental mode of Love waves are shown in Fig. 9-a. Figure 9-b shows the amplitude factor in the case of a line source. The ratios Q_L/Q_1 when Q_2/Q_1 is 2, 3/2, 1, 3/4 and 1/2 were computed by means of equation (3) and these are shown in Fig. 9-c. It is clear from the figure that the Q_L-value depends remarkably on frequency. Especially, the Q_L-value at the frequency corresponding to the minimum group velocity is fairly small, in the model shown in Fig. 3, regardless of Q_2/Q_1.

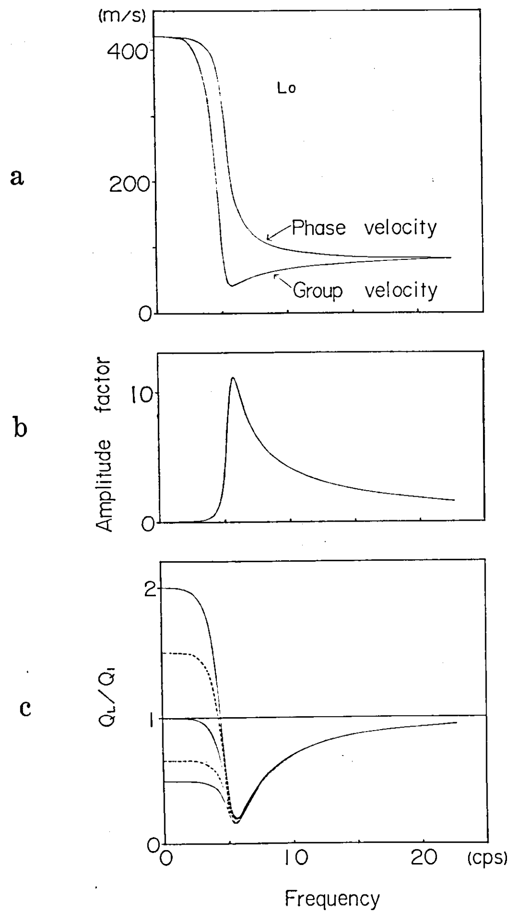

Fig. 9. Characteristics of fundamental mode of Love waves.
a) Phase and group velocities.
b) The amplitude factor (in the case of a line source).
c) Q_L/Q_1, in terms of Q_2/Q_1. Parameters Q_2/Q_1 are 2, 3/2, 1, 1/2 and 1/3 from top to bottom, respectively.

4. Results of analysis

After Onda and Komaki (1968), the waves observed in the direction bisecting the source plate are of SH type alone. And in this experiment, the effect of the plate length is neglected at a distance farther than about 10 m. In other words, the observed wave can be regarded as originating from a point source. Accordingly, the observed spectral amplitude should be corrected with respect to the factor of spherical spreading of waves. Figure 10 shows the examples of the corrected spectra observed at various distances from the source.

Attenuation coefficients were determined for each source independently applying the regression line analysis, since the source spectrum might be different from each other. An example of the relation between the distance and spectral density is shown in Fig. 11. Six such values are averaged and shown in Fig. 12 by circles.

In the previous paper (Kudo and Shima, 1970), Q_2 was determined

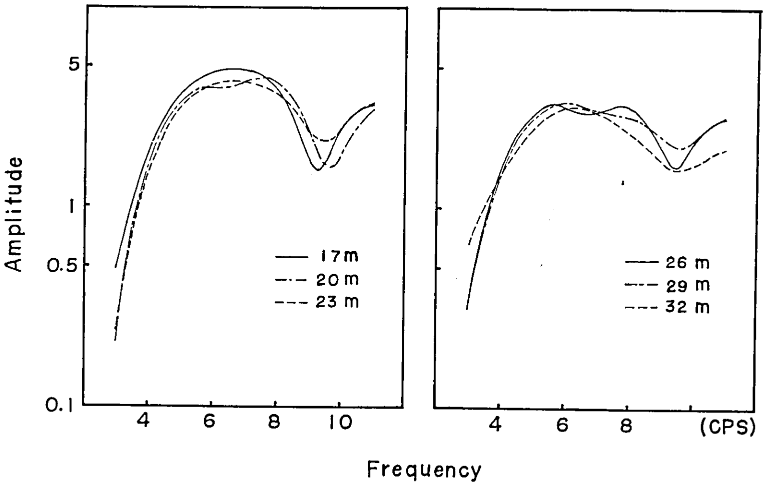

Fig. 10. Example of spectra corrected with regard to spherical spreading.

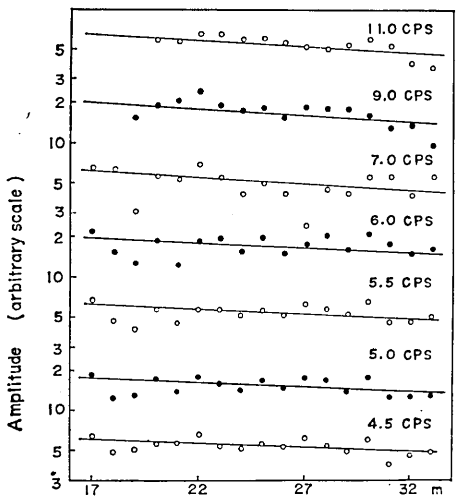

Fig. 11. An example of the relation between distance and spectral amplitude.

to be 6.5 in the frequency range from 30 to 90 cps. We assume that Q's of SH waves in each layer are independent of frequency. Under this assumption, attenuation coefficients of Love waves associated with our model were computed when Q_2/Q_1 was 1, 1/2 and 1/3, in order to compare the observed values directly with the values expected from the theory (Hirasawa and Sato, 1963). These results are also shown in Fig. 12, the averaged values close to the theoretical curve for Q_2/Q_1 being 1/3.

The values of Q_L were calculated using the observed phase velocities and attenuation coefficients. They are shown in Fig. 13. It is clear from the figure that the observed

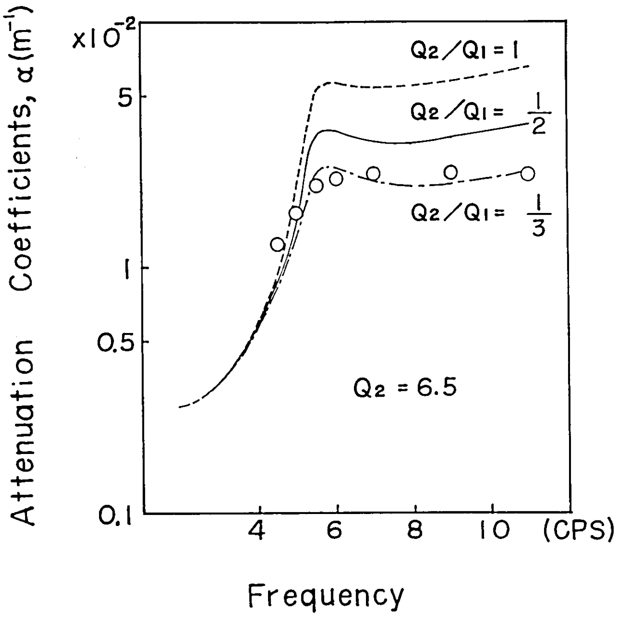

Fig. 12. Attenuation coefficients of Love waves.

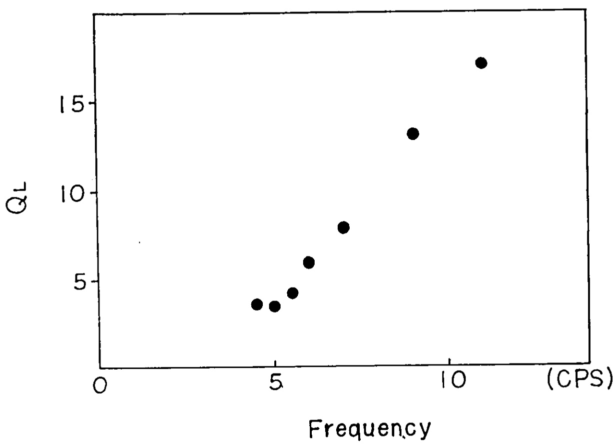

Fig. 13. Observed Q-values of Love waves.

Q_L depends remarkably on frequency. Furthermore, in frequencies higher than the frequency of the minimum group velocity, the values of Q_L/Q_1 do not differ much regardless of the ratio Q_2/Q_1, as can be seen in Fig. 14. In this figure, the observed values of Q_L were reduced to agree with the theoretical curves, given in terms of Q_L/Q_1, at 11 cps. One can realize from Fig. 14 that the reduced values close to the theoretical curves of Q_2/Q_1 being 1/3, and the assumption that Q_1 as well as Q_2 is

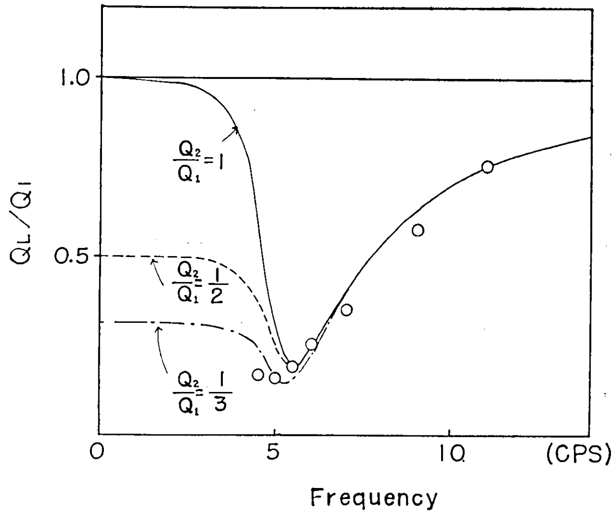

Fig. 14. Reduced Q-values compared with the theoretical curves.

independent of frequency, is valid at least in the frequency range from 4.5 to 11 cps.

5. Concluding remarks

The Q-value of Love waves was investigated in the frequency range from 4.5 to 11 cps. Through the analysis, the following conclusions can be drawn:

1) The observed Q_L depends remarkably on frequency, especially near the frequency corresponding to the minimum group velocity, as expected from the theory of Hirasawa and Sato (1963). Thus it was also proved experimentally that the Q_L itself does not necessarily signify the intrinsic attenuation in media.

2) Analyses were done under the assumptions that Q_1 and Q_2 are independent of frequency and Q_2 is 6.5. No discrepancies were found between the observational results and theory. It follows that the Q-value in the substratum was approximately constant in the frequency range from 4.5 to 90 cps, and the Q-value in superficial layer was estimated to be 20. It is noted that the Q-value in the superficial soil layer is higher than that of the substratum, although it sounds somewhat unfamiliar.

3) The frequency dependence of Q-value of shear waves in soils may be fairly small, if any.

As it is hard to determine the Q-value in the thin surface layer by means of body waves, this method of analysis employing the Love waves is effective. The predominant frequencies of body waves are frequently higher than those of Love waves not only in the events from small scale experiments but also in the case of earthquakes. Consequently the use of both Love and body waves are apposite to the investigation of the loss mechanism in a wide frequency range.

Finally, the authors express their thanks to Mr. M. Yanagisawa for his kind advice and assistance in the field and laboratory, and acknowledge the assistance of Mrs. S. Miyagi.

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11. 軟弱な地層における Love 波の減衰

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二層構造での Love 波の Q を，4.5 から 11 cps の周波数範囲で求めた．その結果；

1) 平沢・佐藤の理論から予想されるように，Love 波の Q は周波数に依存している．特に，minimum group velocity を与える周波数付近でそれが著しい．
2) 下層の S 波の Q は，30 から 90 cps の周波数範囲で，ほぼ周波数に依らず 6.5 であつた (Kudo and Shima, 1970)．今回の解析では，平沢・佐藤の理論と比較するため，上層及び下層の S 波の Q は周波数に依存しないという仮定を設けたが，理論とのくい違いは見られなかつた．したがつて下層の S 波の Q は 4.5 から 90 cps の周波数範囲では，ほとんど周波数に依存していないと言えよう．また上層の S 波の Q は 20 となる．上層の Q が下層の Q より大きいことは多少奇異な感を受ける．

Love 波の Q は，Q の本質とは異なる地下構造に大きく依存しているため，取り扱いには注意が必要である．しかしながら，減衰機構を調べる目的には，Love 波は実体波では測定困難な周波数にその卓越した振幅を持つ場合が多いので，大いに利用されてしかるべきである．