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# 13. A Theory of Frequency-Dependent Effect of an Island on Geomagnetic Variations.

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Summary

Self-induction effect of electric current flow around a circular hole in a thin conducting sheet is studied in an approximate way. It turns out, however, that the self-induction is not so large that marked frequency dependence of the vertical field component as observed, for example, on Hachijo-shima Island can be attributed to this origin. A phase lag amounting to several degrees seems likely to have arisen by the self-induction effect provided a periodic variation having a period of 30 min. and an island of which the diameter is 10 km are assumed.

## 1. Introduction

That a geomagnetic variation observed on an island is often affected fairly seriously by electric currents in the surrounding sea has now become established by a number of observations[1,2,3,4,5,6,7,8,9]. Such an

1) R.G. Mason, "Spatial Dependence of Time Variations of the Geomagnetic Field on Oahu, Hawaii", *Trans. Am. Geophys. Un.*, **44** (1962), 40.

2) R.G. Mason, "Spatial Dependence of Time Variation of the Geomagnetic Field in the Range 24 hrs-3 mins on Christmas Island", *Geophysics Dept., Imperial College of Sci. and Tech.*, REF. 63-3 (1963).

3) R.G. Mason, "Magnetic Effects at Canton Island of the 1962 High Altitude Nuclear Tests at Johnston Island", *Geophysics Dept., Imperial College of Sci. and Tech.*, REF 64-1 (1964).

4) D. Elvers and D. Perkins, "Geomagnetic Research on Spatial Dependence of Time Variations across Puerto Rico", *Trans. Am. Geophys. Un.*, **45** (1964), 46.

5) Y. Sasai, "Spatial Dependence of Short-period Geomagnetic Fluctuations on Oshima Island (1)", *Bull. Earthq. Res. Inst.*, **45** (1967), 137-157.

6) Y. Sasai, "Spatial Dependence of Short-period Geomagnetic Fluctuations on Oshima Island (2)", *Bull. Earthq. Res. Inst.*, **46** (1968), 907-926.

7) S. Utashiro, K. Sugiura, S. Oshima, and T. Kondo, "Conductivity Anomaly Observation on Hachijoshima Island", *Proc. Conductivity Anomaly Symp. 2.* Earthq. Res. Inst., (1968), 141-156 (*in Japanese*).

8) T. Rikitake, T. Yukutake, T. Yoshino, Y. Yamazaki and D.P. Klein, "Observation of Geomagnetic Variations of Short-period on Hawaii Island", *Proc. Conductivity Anomaly Symp. 2.* Earthq. Res. Inst., (1969), 157-162 (*in Japanese*).

9) D.P. Klein, "Spatial Characteristics of Geomagnetic Storm Time-variations in the Range of 11 to 160 Minutes on Oahu, Hawaii", *M. Sc. Thesis*, University of Hawaii, (1969), in preparation.

island effect is best demonstrated by the reversal between the vertical magnetic fields observed at stations at opposite sides of the island. The usual interpretation of the island effect is based upon the fact that electric currents induced in the sea are deflected by the poorly conducting island in such a way that the vertical magnetic field produced at one of the two stations takes a sign different from that at the other station.

Observations on Oshima[5,6] and Hawaii[8] Islands actually indicated beautiful reversals between the vertical magnetic fields observed at northern and southern stations for changes having a period range 10-60 minutes when the geomagnetic field changes in the north-south direction. Numerical calculations of the induced electric currents and magnetic fields by Sasai[6] and Kondo and Sasai[10] seem to account for the observed effects very well although the effect of self-induction has been ignored completely in their numerical work.

It has also been noticed, however, that an island effect is sometimes controlled by rapidity of geomagnetic variation. One of the best examples of such a frequency-dependent effect has been observed on Hachijo-shima Island[7]. The ratio of the change in the vertical component ($\Delta Z$) to that in the horizontal component ($\Delta H$) amounts to $-0.3$ or thereabouts for a period range 10-30 minutes at a few stations on the island. The $\Delta Z/\Delta H$ seems to approach zero as the period of geomagnetic variation gets longer and it takes on a positive value for variations having a period range 60-100 minutes.

The interpretation of island effect mentioned in the preceding paragraphs being based on a steady-state approximation, it is clear that no explanation of a frequency-dependent effect can be provided. It is the aim of this paper, therefore, to estimate the extent of frequency-dependent island effect for a simple case and to see whether the theory affords a proper explanation of the observed effects. In Section 2 will be developed an approximate theory that can possibly be applied to estimating the time-dependent behaviour of electric currents deflected by a circular obstacle in a thin conducting sheet. The later sections will be reserved for comparisons between observed and estimated results along with some discussion.

## 2. Theory

### 2.1. *Steady-state solution*

Distortion of a uniform steady flow of electric currents in a sheet

10) M. KONDO and Y. SASAI, "Effect of an Island on a Geomagnetic Variation of Short-period", *Proc. Conductivity Anomaly Symp. 2.* Earthq. Res. Inst., (1969), 163-174 (*in Japanese*).

by a circular obstacle has been studied by Chapman[11], Nagata et al[12] and Ashour and Chapman[13] mostly in relation to effects of solar eclipse and island on the geomagnetic field.

According to these theories, the current function of the distorted flow, when a circular area of radius $a$ has zero conductivity, is given by

$$\Psi^{(0)} = \begin{cases} I_0(1-a^2/r^2)y & \text{for } r \geqq a \,, \\ 0 & \text{for } r \leqq a \,, \end{cases} \tag{1}$$

provided a current of intensity $I_0$ flows in the $x$-direction at points very far from the circular area, the origin of a two-dimensional polar coordinate $(r, \phi)$ being taken at the centre of the circle.

If we define the anomalous portion of current flow by the difference between the actual flow and that for a uniform sheet for which the conductivity in the circular area is the same as that for the remaining part of the sheet, the anomalous current function is given by

$$\Delta\Psi^{(0)} = \begin{cases} -I_0(a^2/r)\sin\phi & \text{for } r \geqq a \,, \\ -I_0 r \sin\phi & \text{for } r \leqq a \,. \end{cases} \tag{2}$$

In Figs. 1 and 2 are shown $\Psi^{(0)}$ and $\Delta\Psi^{(0)}$ respectively. The magnetic field components on the sheet ($z=0$) due to the anomalous current

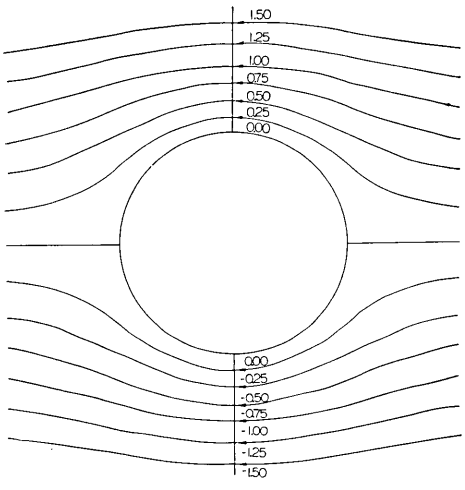


Fig. 1. Steady current function $\Psi^{(0)}$ in units of $I_0a$.

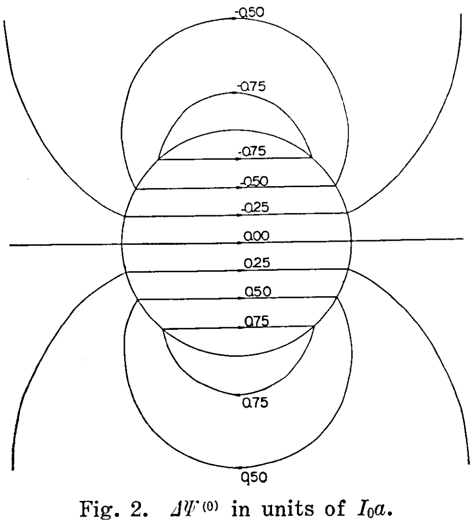


Fig. 2. $\Delta\Psi^{(0)}$ in units of $I_0a$.

11) S. Chapman, "The Effect of a Solar Eclipse on the Earth's Magnetic Field", *Terr. Magn.*, **38** (1933), 175-183.

12) T. Nagata, Y. Nakata, T. Rikitake and I. Yokoyama, "Effect of the Solar Eclipse on the Lower Parts of the Ionosphere and the Geomagnetic Field", *Rep. Ionos. Res. Japan*, **9** (1955), 121-135.

flow as shown in Fig. 2 are calculated as[13)]

$$\left.\begin{aligned}\varDelta H_{z0}^{(0)} &= -4\pi I_0 a \sin\phi \int_0^\infty J_1(\lambda a) J_1(\lambda r) d\lambda\ ,\\ \varDelta H_{r0}^{(0)} &= 4\pi I_0 a \sin\phi \int_0^\infty J_1(\lambda a) J_1'(\lambda r) d\lambda\ ,\\ \varDelta H_{\phi 0}^{(0)} &= (4\pi I_0 a/r) \cos\phi \int_0^\infty \lambda^{-1} J_1(\lambda a) J_1(\lambda r) d\lambda\ .\end{aligned}\right\} \qquad (3)$$

Figs. 3, 4, and 5 show the $\varDelta H_{x0}^{(0)}$, $\varDelta H_{y0}^{(0)}$ and $\varDelta H_{z0}^{(0)}$ distributions on the sheet in units of $I_0$. These distributions are calculated by making use of (3). $\varDelta H_{z0}^{(0)}$ becomes infinite at the edge of the circle.

### 2.2. *Approximate solution of current function when the self-induction is taken into account*

Price[14)] has shown that a successive way of approaching the exact solution of an electromagnetic induction problem within a thin sheet can be utilized

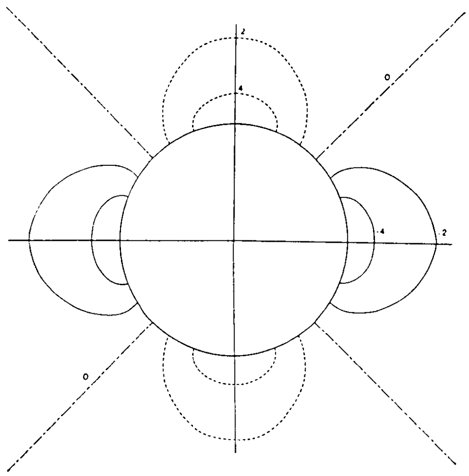

Fig. 3. $\varDelta H_{x0}^{(0)}$ in units of $I_0$.

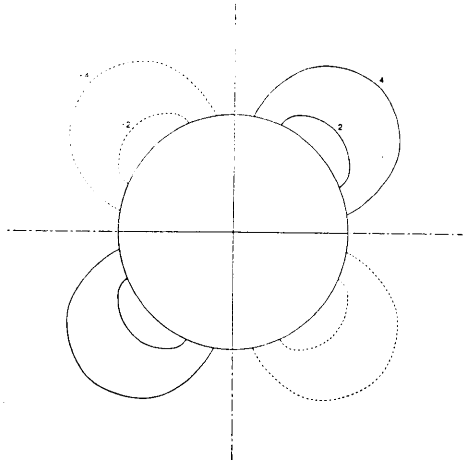

Fig. 4. $\varDelta H_{y0}^{(0)}$ in units of $I_0$.

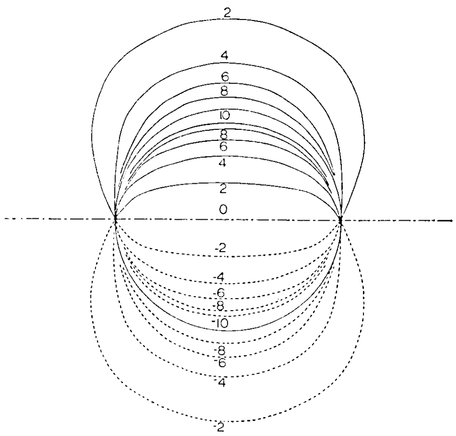


Fig. 5. $\varDelta H_{z0}^{(0)}$ in units of $I_0$.

13) A. A. ASHOUR and S. CHAPMAN, "The Magnetic Field of Electric Currents in an Unbounded Plane Sheet, Uniform Except for a Circular Area of Different Uniform Conductivity", *Geophys. J.*, **10** (1965), 31-44.

14) A. T. PRICE, "The Induction of Electric Currents in Non-uniform Thin Sheets and Shells", *Quart. J. Mech. Applied Math.*, **2** (1949), 283-310.

provided a parameter inversely proportional to the conductivity and the period of time variation and also proportional to the spatial wavelength of the inducing field does not exceed a certain limit.

In a similar fashion, an approximate solution of the present problem when the self-induction is taken into account may be obtained starting from the zeroth order approximation or steady state. It is assumed that the electromotive force that drives the electric currents in the sheet is induced somewhere very far from the circular area which represents an island. Only the influence of self-induction when these currents flow around the island will be discussed, the electromotive force arising in the vicinity of the island being entirely ignored on the assumption that the island is much smaller than the wavelength of the inducing field.

The first approximation of the current function under the influence of self-induction may be given by

$$\Psi = \Psi^{(0)} + \Psi^{(1)} , \tag{4}$$

where $\Psi^{(0)}$ has already been given in (1) and

$$\nabla^2 \Psi^{(1)} = K \partial H_{z0}^{(0)} / \partial t , \tag{5}$$

in which $K$ denotes the conductivity integrated over the thickness of the sheet. As $H_{z0}^{(0)}$ can be obtained from (3), the righthand-side of (5) is known provided the time variation is specified.

Supposing that $\Psi^{(1)}$ is obtained by solving (5) and calculating $H_{z0}^{(1)}$, the z-component of the magnetic field at $z=0$ produced by the electric currents represented by $\Psi^{(1)}$, the second approximation may become

$$\Psi = \Psi^{(0)} + \Psi^{(1)} + \Psi^{(2)} , \tag{6}$$

where

$$\nabla^2 \Psi^{(2)} = K \partial H_{z0}^{(1)} / \partial t . \tag{7}$$

Proceeding further in a similar way, we may get at an exact $\Psi$ on the condition that the series like (6) converges. It should be pointed out that the above procedure is to be applied only to the portion of the sheet of which the conductivity is finite. In contrast to that portion, $\Psi$ is always nil in the circular area where $K=0$.

Let us now think of an electromagnetic induction by $H_{z0}^{(1)}$ within an entirely uniform sheet for which a non-conducting area as has been considered in the preceding paragraphs is non-existing. The current function $\Psi_0^{(1)}$ of the induced currents for such a case can be obtained by solving

$$\nabla^2 \Psi_0^{(1)} = K \partial H_{z0}^{(0)} / \partial t , \tag{8}$$

on the condition that the sheet extends to infinity.

The first approximation of the anomalous current function given rise to by the presence of the circular obstacle is then defined by

$$\Delta\Psi = \Delta\Psi^{(0)} + \Delta\Psi^{(1)} , \tag{9}$$

where

$$\Delta\Psi^{(1)} = \Psi^{(1)} - \Psi_0^{(1)} . \tag{10}$$

In a similar way is obtained $\Psi_0^{(2)}$ from

$$\nabla^2\Psi_0^{(2)} = K\partial H_{z0}^{(1)}/\partial t , \tag{11}$$

and the second approximation becomes

$$\Delta\Psi = \Delta\Psi^{(0)} + \Delta\Psi^{(1)} + \Delta\Psi^{(2)} , \tag{12}$$

where

$$\Delta\Psi^{(2)} = \Psi^{(2)} - \Psi_0^{(2)} . \tag{13}$$

It is therefore possible, at least in theory, to arrive at the anomalous current function provided the series like (12) converges.

2.2.1. $\Delta\Psi^{(1)}$ *for periodic induction*

In the case of a periodic induction with period $T$, we may put

$$\partial/\partial t = i\alpha \quad (i=\sqrt{-1},\ \alpha=2\pi/T) . \tag{14}$$

A solution of (5) appropriate to the present problem can be expressed as

$$\Psi^{(1)} = \int_0^\infty \psi^{(1)}(r\,;\lambda)d\lambda \sin\phi , \tag{15}$$

whereas $\psi^{(1)}$ satisfies

$$d^2\psi^{(1)}/dr^2 + (1/r)d\psi^{(1)}/dr - \psi^{(1)}/r^2 = AJ_1(\lambda a)J_1(\lambda r) , \tag{16}$$

where

$$\left.\begin{aligned} A &= -4\pi i I_0 C , \\ C &= \alpha K a . \end{aligned}\right\} \tag{17}$$

The solution of (16) which becomes vanishingly small for an infinitely large $r$ is given by

$$\psi^{(1)}(r\,;\lambda) = C^{(1)}r^{-1} + (A/2)J_1(\lambda a)[r\int_0^r J_1(\lambda r)dr - r^{-1}\int_0^r r^2 J_1(\lambda r)dr] , \tag{18}$$

which can be rewritten as

$$\psi^{(1)}(r;\ \lambda) = C^{(1)}r^{-1} - A\lambda^{-2}J_1(\lambda a)J_1(\lambda r) . \tag{19}$$

The boundary condition that the electric current normal to the circular boundary vanishes implies

$$\psi^{(1)}(a;\ \lambda)=0\ , \tag{20}$$

so that we obtain

$$C^{(1)}=Aa\lambda^{-2}[J_1(\lambda a)]^2\ . \tag{21}$$

Putting (21) into (19), we obtain

$$\psi^{(1)}(r;\ \lambda)=A\lambda^{-2}J_1(\lambda a)[(a/r)J_1(\lambda a)-J_1(\lambda r)]\ . \tag{22}$$

Going back to (15) and putting (22) into it, the analytical expression of $\Psi^{(1)}$ is obtained as

$$\Psi^{(1)}=\begin{cases} A\int_0^\infty \lambda^{-2}J_1(\lambda a)[(a/r)J_1(\lambda r)-J_1(\lambda r)]d\lambda \sin\phi & \text{for } r\geqq a\ ,\\ 0 & \text{for } r\leqq a\ . \end{cases} \tag{23}$$

It is obvious that the solution of (8) for the whole area of the sheet including $r=0$ and $r=\infty$ is given by

$$\Psi_0^{(1)}=\int_0^\infty \psi_0^{(1)}(r;\ \lambda)d\lambda \sin\phi\ , \tag{24}$$

where

$$\psi_0^{(1)}(r;\ \lambda)=-A\lambda^{-2}J_1(\lambda a)J_1(\lambda r)\ \text{ for } 0\leqq r\leqq\infty\ , \tag{25}$$

so that the first approximation of the anomalous current function as defined by

$$\Delta\Psi^{(1)}=\int_0^\infty [\psi^{(1)}(r;\ \lambda)-\psi_0^{(1)}(r;\ \lambda)]d\lambda \sin\phi \tag{26}$$

becomes

$$\Delta\Psi^{(1)}=\begin{cases} A(a/r)\int_0^\infty \lambda^{-2}[J_1(\lambda a)]^2 d\lambda \sin\phi & \text{for } r\geqq a\ ,\\ A\int_0^\infty \lambda^{-2}J_1(\lambda a)J_1(\lambda r)d\lambda \sin\phi & \text{for } r\leqq a\ . \end{cases} \tag{27}$$

2.2.2. *Numerical calculation of integrals involved in (23) and (27)*

Gegenbauer's integral as given in text books of Bessel function[15] leads to

$$\int_0^\infty \lambda^{-2}J_1(\lambda a)J_1(\lambda r)d\lambda=\frac{\pi^{1/2}}{4}\ \frac{ar}{(a^2+r^2)^{1/2}}F(1/4,\ 3/4,\ 2,\ \beta)\ , \tag{28}$$

15) e.g. G.N. WATSON, *A Treatise on the Theory of Bessel Functions* (Cambridge Univ. Press., 1922), p. 407.

where $F$ is a hypergeometric function and

$$\beta = 4a^2r^2/(a^2+r^2)^2 \, . \tag{29}$$

When $r=a$, (28) reduces to

$$\int_0^\infty \lambda^{-2}[J_1(\lambda a)]^2 d\lambda = (\pi/2)^{1/2}(a/4)F(1/4,\ 3/4,\ 2,\ 1)$$
$$= (\pi/2)^{1/2}(a/4)/[\Gamma(7/4)\Gamma(5/4)] \, . \tag{30}$$

From the definition of hypergeometric function, (28) can be written in a form of infinite series as

$$\int_0^\infty \lambda^{-2}J_1(\lambda a)J_1(\lambda r)d\lambda$$
$$= \frac{\pi^{1/2}}{4}\frac{ar}{(a^2+r^2)^{1/2}}\left[1+\frac{1.3}{4^2 2}\beta+\frac{1.3.5.7}{4^4 2!3!}\beta^2+\frac{1.3.5.7.9.11}{4^6 3!4!}\beta^3+\cdots\right].$$

Since the series on the righthand-side of (31) converges fairly rapidly, $\Psi^{(1)}$ and $\Delta\Psi^{(1)}$ can readily be evaluated numerically. Their distributions in the first quadrant are shown in Figs. 6 and 7 in units of $iCI_0a$.

### 2.3. *Magnetic fields due to* $\Delta\Psi^{(1)}$

The first approximation of the magnetic fields at $z=0$ due to the

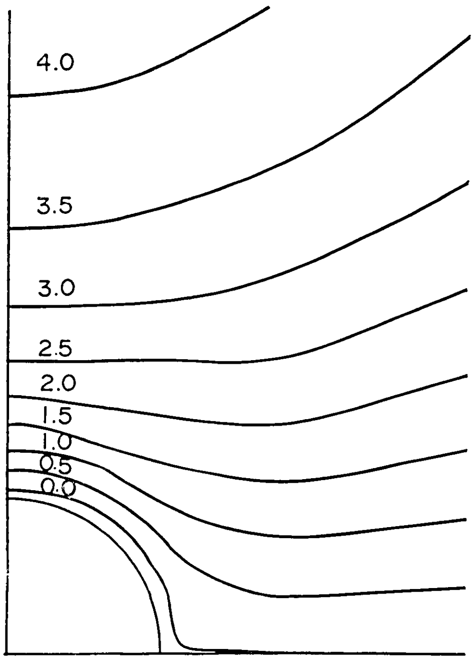


Fig. 6. $\Psi^{(1)}$ in units of $iCI_0a$.

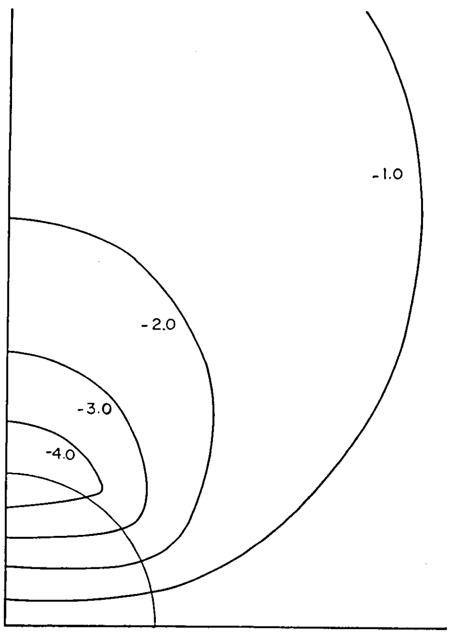


Fig. 7. $\Delta\Psi^{(1)}$ in unit of $iCI_0a$.

anomalous currents can be obtained by

$$\Delta H_{x0}^{(1)} = -2\pi\partial(\Delta\Psi^{(1)})/\partial x \ , \tag{32}$$

$$\Delta H_{y0}^{(1)} = -2\pi\partial(\Delta\Psi^{(1)})/\partial y \ , \tag{33}$$

$$\Delta H_{z0}^{(1)} = -\int_0^{2\pi}\int_0^{\infty}[\Delta\Psi^{(1)}(0, r, \phi) - \Delta\Psi_0^{(1)}](d\rho/\rho^2)d\theta \ , \tag{34}$$

in which $\Delta\Psi_0^{(1)}$ denotes the value of $\Delta\Psi^{(1)}(0, r, \phi)$ at a certain point on the sheet which is the origin of a polar coordinate $\rho$ and $\theta$.

It is of course possible to get at analytical expressions of the magnetic field components on the basis of (27). But the writer does not see points in doing so because (32), (33) and (34) enable us to compute the field components fairly easily with the help of an electronic computer. In fact, the analytical expressions involve double integrations from 0 to $\infty$, so that they are too complicated to be handled properly.

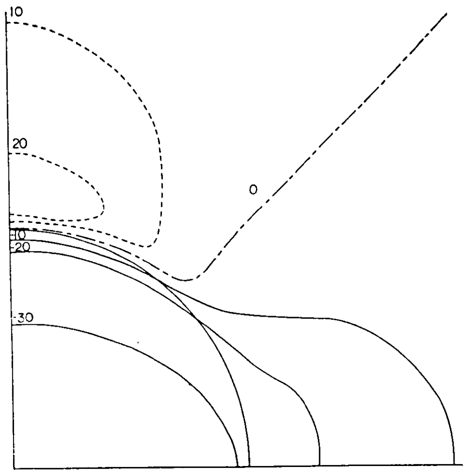


Fig. 8. $\Delta H_{x0}^{(1)}$ in units of $iCI_0$

In Figs. 8, 9 and 10 are illustrated the three components of magnetic field due to $\Delta\Psi^{(1)}$ in units of $iCI_0a$. We see that the distribution patterns are not greatly different from those for

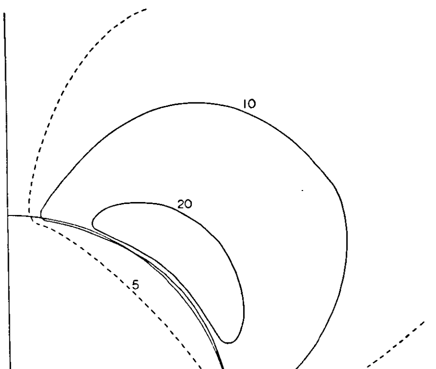


Fig. 9. $\Delta H_{y0}^{(1)}$ in units of $iCI_0$.

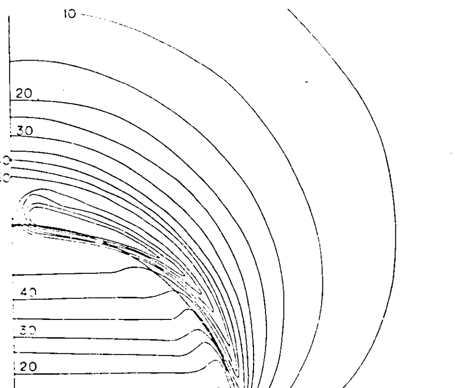


Fig. 10. $\Delta H_{z0}^{(1)}$ in units of $iCI_0$

$\Delta\Psi^{(0)}$ as shown in Figs. 3, 4 and 5.

### 2.4. *Further approximation*

It is possible to get at $\Psi^{(2)}$ by solving (7) numerically because $H_{z0}^{(1)}$ on its righthand-side can be obtained in a fashion similar to what we studied in the last section. A preliminary calculation indicates that $H_{z0}^{(1)}$ in units of $iCI_0$ is of the same order as $H_{z0}^{(0)}$ in units of $I_0$ and also that the distribution patterns of the two fields are not largely different from one another. It is therefore seen that $\Psi^{(2)}$ and consequently $\Delta\Psi^{(2)}$ in units of $(iC)^2I_0a$ would be of the same order as those for the first approximation. For a relatively slow variation for which $C\ll 1$ may be assumed, $\Delta\Psi^{(2)}$ becomes very small. In this paper, only variations for which the above condition holds good will be studied, so that no influence of the second as well as higher approximations is taken into account.

## 3. Over-all field

Let us assume that the electromotive force, which gives rise to the island effect, is induced in an infinitely extending sheet by an external magnetic field of which the potential is given as

$$W_e = A_0\, e^{i\alpha t}\, e^{\lambda z} \sin \lambda y\ . \tag{35}$$

According to the theory of electromagnetic induction within a sheet[4], the induced potential is obtained as

$$W_i = \begin{cases} B_0\, e^{i\alpha t}\, e^{-\lambda z} \sin \lambda y & \text{for } z>0\ , \\ -B_0\, e^{i\alpha t}\, e^{\lambda z} \sin \lambda y & \text{for } z<0\ , \end{cases} \tag{36}$$

where

$$B_0 = 2\pi i\alpha(\rho_0\lambda + 2\pi i\alpha)^{-1} A_0\ . \tag{37}$$

Let us further put

$$\left.\begin{aligned} B_0 &= (U+iV)A_0\ , \\ U &= 1/(1+\gamma^2)\ , \\ V &= \gamma/(1+\gamma^2)\ , \\ \gamma &= \rho_0\lambda/2\pi\alpha\ . \end{aligned}\right\} \tag{38}$$

The conductivity of sea-water being taken as $\sigma(=4\times10^{-11}$ e.m.u.), the sheet resistance is given by

$$\rho_0 = 1/(\sigma d)\ , \tag{39}$$

in which $d$ denotes the thickness of the sheet.

As for $\lambda$, we may assume

$$\lambda = 10^{-8}\,\mathrm{cm}^{-1}\ , \tag{40}$$

on the assumption that we are concerned with geomagnetic bays and similar changes in middle latitudes.

Assuming a sea having a depth of 1 km, for instance, $U$ and $V$ are calculated for various values of period as given in Table 1.

We see, therefore, that the imaginary parts of the inducing field are so small that the phase difference between the real and imaginary parts amounts to only 10° or so even for 1-hour period variation. In other words, the sea behaves as though its conductivity is nearly perfect. If we deal with a sea of a greater depth, the perfect conductor approximation becomes even better.

Table 1. $U$ and $V$ for 1 km depth.

| Period | $U$ | $V$ |
|---|---|---|
| 10 sec. | 1.00000 | 0.00063 |
| 30 | 1.00000 | 0.00190 |
| 60 | 0.99999 | 0.00380 |
| 300 | 0.99964 | 0.01899 |
| 600 | 0.99856 | 0.03794 |
| 1800 | 0.98717 | 0.11252 |
| 3600 | 0.95060 | 0.21671 |

In the following treatment, we shall assume $y \doteqdot 0$. The assumption leads to a condition that the magnetic field normal to the sheet is always very small. The condition does not conflict with the magnetic variation field in low or middle latitudes.

The current function of the induced currents is given by

$$\Psi_{00} = (A_0/2\pi)(U+iV)\sin\lambda y\ , \tag{41}$$

which may be rewritten as

$$\Psi_{00} = (A_0\lambda/2\pi)(U+iV)y\ , \tag{42}$$

for a small value of $\lambda y$.

Comparing (42) to $\Psi^{(0)}$ in (1) for a large value of $r$, we may put

$$A_0\lambda/2\pi = I_0 a\ . \tag{43}$$

It is therefore seen that a magnetic field, of which the components are given by

$$\left.\begin{aligned} H_{x0} &= 0\ , \\ H_{y0} &= 2\pi a I_0(1+U+iV)\ , \\ H_{z0} &= 0\ , \end{aligned}\right\} \tag{44}$$

is accompanied by the electromagnetic induction. As the writer pointed out before, the electromotive force arising in the neighbourhood of the island is not important for the induction by a field having such a long wavelength.

What have been studied in Section 2 now lead to

$$\left.\begin{aligned} H_x &= H_{x0} + \Delta H_{x0}^{(0)} + \Delta H_{x0}^{(1)} , \\ H_y &= H_{y0} + \Delta H_{y0}^{(0)} + \Delta H_{y0}^{(1)} , \\ H_z &= H_{z0} + \Delta H_{z0}^{(0)} + \Delta H_{z0}^{(1)} , \end{aligned}\right\} \tag{45}$$

the additional field components being given in (3) and (32), (33) and (34).

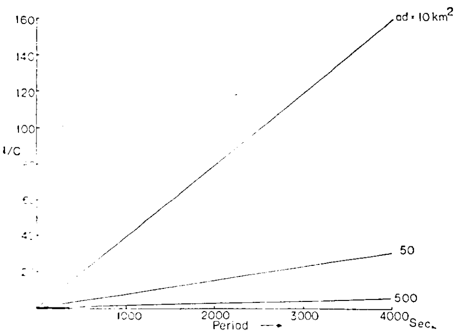


Fig. 11. Changes in $1/C$ for various $ad$'s as the period increases.

The first approximation of the field is proportional to $C$ which is defined by (17). Taking $\sigma = 4 \times 10^{-11}$ e.m.u. for the conductivity of sea-water, changes in $1/C$ as the period increases are shown in Fig. 11 taking the product of the radius of the island ($a$) and the depth of the sea ($d$) as the parameter. Typical values of $C$ for $ad = 10\,\text{km}^2$ are listed in Table 2 for a number of periods. From Fig. 11 and Table 2, it is seen that the present method may not work for combinations of large $ad$ and small period because $C$ exceeds 1.

An example of over-all field components is given in Table 3 on the basis of a calculation indicated in (45). It is assumed that $ad = 10\,\text{km}^2$ and $T = 1800$ sec. and the calculation is made only for points on the $y$ axis. In this case, it is obvious that $H_x = 0$ on the $y$ axis.

The real and imaginary parts of $H_z/H_y$ ratio at a few points on the $y$ axis are shown in Fig. 12, $1/C$ being taken as the abscissa. It may be said that the real part of the ratio takes on a nearly constant value for $C$'s smaller than 0.05, while its imaginary part is so small that the phase lag amounts to only several degrees. As far as the present theory holds good, therefore, it does not seem possible to account for the results that $\Delta Z/\Delta H$ depends largely on the period of geomagnetic variation

Table 2. $C$ for $ad = 10\,\text{km}^2$.

| Period | $C$ | Period | $C$ |
|---|---|---|---|
| 10 sec. | 2.5133 | 600 sec. | 0.0419 |
| 30 | 0.8378 | 1800 | 0.0140 |
| 60 | 0.4189 | 3600 | 0.0070 |
| 300 | 0.0838 | | |

Table 3. $H_y$ and $H_z$ in units of $I_0 a$ on the $y$ axis when $\sigma d = 10\,\text{km}^2$ and $T = 1800\,\text{sec}$.

| $r/a$ | $H_y$ | | $H_z$ | | $H_z/H_y$ | |
|---|---|---|---|---|---|---|
| | Real pt. | Imag. pt. | Real pt. | Imag. pt. | Real pt. | Imag. pt. |
| 0.9 | 12.454 | 0.983 | 9.849 | 1.765 | 0.797 | 0.079 |
| 0.8 | 12.428 | 1.217 | 7.026 | 1.443 | 0.571 | 0.060 |
| 0.7 | 12.418 | 1.300 | 5.513 | 1.201 | 0.499 | 0.050 |
| 0.6 | 12.410 | 1.369 | 4.318 | 0.981 | 0.352 | 0.040 |
| 0.5 | 12.407 | 1.396 | 3.431 | 0.782 | 0.280 | 0.032 |
| 0.4 | 12.404 | 1.424 | 2.611 | 0.605 | 0.213 | 0.024 |
| 0.3 | 12.401 | 1.451 | 1.957 | 0.460 | 0.160 | 0.018 |
| 0.2 | 12.399 | 1.465 | 1.236 | 0.295 | 0.101 | 0.012 |
| 0.1 | 12.399 | 1.465 | 0.733 | 0.153 | 0.060 | 0.005 |
| 0.0 | 12.398 | 1.479 | 0 | 0 | 0 | 0 |

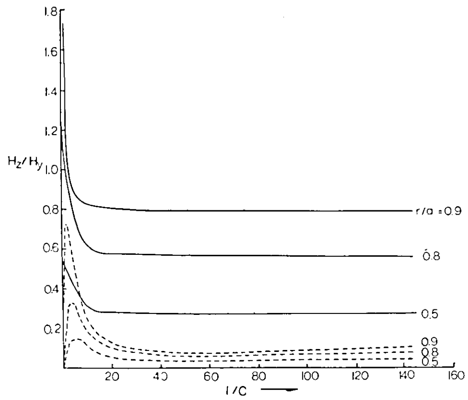


Fig. 12. Changes in $H_z/H_y$ ratio at a few points on the $y$ axis as estimated for various values of $1/C$.

and that it sometimes changes its sign at a certain period as observed on Hachijo-shima Island.

## 4. Discussion and conclusions

The change in amplitude and sign of $\Delta Z/\Delta H$ of geomagnetic variations observed at stations on Hachijo-shima Island does not seem to be accounted for by a simple theory of island effect even if the influence of self-induction is taken into account in an approximate way. When the geomagnetic field changes very rapidly, it is obvious that the present theory of which the zeroth order approximation relies on the steady state does not work. In such a case a theory, in which the zeroth order approximation of current function should be that for an infinitely extending sheet of perfect conductor with a circular hole, is needed for getting at a solution. Although a few theories[16,17] for such an electromagnetic induction exist, they are not formulated in a way applicable to the present problem.

It should be pointed out, however, that the theory developed in this paper may be applicable to the period range for which we observe a remarkable frequency dependence of island effect on Hachijo-shima Island. The writer is of the opinion, therefore, that the cause of such a frequency-dependent island effect should be sought elsewhere rather than in the effect of self-induction.

One of the possible causes of the frequency-dependent effect would be the possible difference in site for effective flow of induced currents between short and long period variations. As has been shown by Ashour[18], the intensity of electric currents induced in oceans is almost uniform throughout the depth for geomagnetic variations having a period of 1 hour or longer, while, for variations of short period, 10 sec. say, the intensity decreases exponentially with depth just as for ordinary skin effect. The depth of the sea around an island usually getting deeper gradually as the distance from the shore gets larger, it is readily surmised, therefore, that the induced currents for a rapid variation flow closer to the island than those for a slow variation. We therefore think that the effective radius of an island is large for a slow variation, so that $\Delta Z$ anomaly actually observed on the island is to be small because the observation stations are concentrated over the central area of the effective island. On the contrary, the induced currents flow close to the actual island when the period is short, so that stations near the edge of the island may be affected by the anomalous field to a large extent. The above consideration leads us to a conclusion that, on an island that upheaves steeply from deep ocean bottom, the island effect should best be observed even for fairly long periods. One of the best examples for such a case is Hawaii Island.

A less certain cause of the abnormal island effect would be the role of sea water permeating the interior of an island. When the rocks composing an island are highly porous and permeable, conductivity contrast between the island and surrounding sea would become small. In such a case, the island effect would possibly become less clear and irregular. Some evidence for such a situation has been reported on Oahu

16) T. RIKITAKE, "Electromagnetic Induction in a Perfectly Conducting Plate with a Circular Hole", *J. Geomag. Geoelec.*, **16** (1964), 31-36.

17) A. A. ASHOUR, "Electromagnetic Induction in Finite Thin Sheets", *Quart. J. Mech. Applied Math.*, **18** (1965), 74-86.

18) A. A. ASHOUR, "The Depth Variation of the Intensity of Current Induced in a Model Earth and Ocean, *Geophys. J.*, **17** (1969), 321-325.

Island[19] where the effect on geomagnetic variations is not quite the one expected from a simple theory. Nothing has been practically known, however, about the detailed influence of subsurface water on island effect.

In conclusion, it may be said that the present theory of island effect, in which the influence of self-induction is approximately taken into account, does not seem to account for the frequency dependent effect on $\Delta Z/\Delta H$ ratio of geomagnetic variation as observed on some islands. A possible cause of such frequency may be sought in the fact that the effective dimension of an island differs from variation to variation when the difference in the frequency is considered although no quantitative amount had as yet been taken account of. Klein[9] has reached a conclusion of a similar sort in relation to the interpretation of geomagnetic variations over Oahu Island.

19) S. ARAMAKI, Personal communication, (1968).

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## 13. 自己感応を考慮した場合の地磁気離島効果理論

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離島上で観測される地磁気短周期変化が，いわゆる離島効果を示すことは，最近の観測によつて確立された．磁場が北向きに変化する場合には，鉛直成分対水平成分比（$\Delta Z/\Delta H$）は通常島の北端で負，南端で正となる．しかし，たとえば八丈島の観測点でみられるように，$\Delta Z/\Delta H$ が変化の周期によつて変化し，その符号が変化する場合もある．従来の理論では自己感応の影響が省略されていたが，この論文では，近似的に自己感応の影響を議論し，$\Delta Z/\Delta H$ の周期依存性が説明できるか否かをみた．その結果自己感応の影響は比較的小さく，そのような依存性は説明不可能で，たとえば周期によつて海中に流れる電流の場所が異なるというような原因を考えざるを得ないという結果が得られた．