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47. *Deformation~Fracture Relation in Earthquake Genesis and Derivation of Frequency Distribution of Earthquakes.*

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1. Introduction

The earthquake phenomena in a broad sense are kinematical processes which continue for a long period of time being accompanied by the crustal deformations on the earth's surface. An earthquake in a narrow sense is the radiation of seismic waves and is nothing but a fracturing phenomenon which takes place during the above kinematical processes. Therefore, in order to clarify the kinematical processes of earthquake genesis it is necessary to know certain relations which connect deformations, which are kinematical phenomena, and earthquake shock occurrence, which is a fracturing phenomenon. In other words, when one pursues the kinematical deformations and tries to know where, when and how big earthquake may occur, one should know certain relations which connect deformations and fractures. If this relation is called a deformation~fracture relation in earthquake genesis, the discovering of this relation will be one of the most basic problems in the theory of earthquake genesis.

The deformation~fracture relation in the earthquake genesis should be required to account for several important formulae which have been obtained empirically for the occurrence of earthquakes. They are, for example, (1) the Ishimoto-Iida's formula[1] for the frequency distribution of earthquakes with respect to the maximum amplitude of earthquake motion observed at a certain station in a certain long period, (2) the Gutenberg-Richter's formula[2] for the frequency distribution of earthquakes with respect to magnitude, (3) the Utsu-Seki's formula[3] for the relation between the aftershock area and the magnitude of the principal shock, and (4) the Omori's formula[4] or the improved Omori's formula[5] for

1) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 443-478.
2) B. GUTENBERG and C. F. RICHTER, *Seismicity of the Earth*, (Princeton Univ. Press, 1949).
3) T. UTSU and A. SEKI, *Zisin* [ii], **7** (1954), 233-240.
4) F. OMORI, *Jour. College of Science, Imp. Univ. Tokyo*, **7** (1894), 111-200.
5) T. UTSU, *Geophys. Mag.*, **30** (1961), 521-605.

the time variation of the frequency of occurrence of aftershocks. Since these formulae are nothing but very important properties of fracture occurrence of earthquake shocks, the above mentioned deformation$\sim$fracture relations should give theoretical explanations for these empirical formulae. Furthermore, the deformation$\sim$fracture relations in the earthquake genesis are desired to be such that, when they are introduced into the kinematics of deformations of earth's surface, they will generate the occurrence of earthquakes in a given deformation.

Hitherto, the Ishimoto-Iida's formula or the Gutenberg-Richter's formula has been regarded[6)] as one of the probability distribution functions of earthquakes with respect to the size of earthquakes. There have been very few arguments about the relation between their formula and deformation of the medium. Matuzawa[7)] and Mogi[8)] have given some physical interpretations for them, but still based upon the viewpoint of probability distribution. However, as Ishimoto[9)] and Kishinouye[10)] have clearly stated in their papers, and have been vividly proved by the recent Matsushiro earthquake swarms,[11)] the activity of earthquake swarm is, in nature, the result of crustal deformation of the earth's surface. Therefore, it will be very natural to think that the above empirical statistical formulae for the frequency distribution of earthquake occurrence are the results of crustal deformation and are nothing but reflections of the causal physical law of fracture and deformation.

In a previous paper,[12)] when a kinematical process of earthquake swarms was treated, a formula was used for the deformation$\sim$fracture relation in order to derive the time variation of earthquake number from a given kinematical deformation. There, only the total number of earthquakes were related to the deformation, and the size of the earthquake shocks has not been specified yet.

In order to account for the frequency distribution of earthquakes with respect to their sizes, another assumption should be added to the deformation$\sim$fracture relation.

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6) For examples, K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 831-853, T. UTSU, *Geophys. Bull. Hokkaido Univ.*, **17** (1966), 85-112, *ibid.*, **18** (1967), 53-69, Z. SUZUKI, *Zisin* [ii], **20** (1967), 134-140, Y. TOMODA, *Zisin* [ii], **7** (1954), 155-169, *ibid.*, **8** (1956), 190-204.

7) T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **19** (1941), 411-415.

8) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 831-853.

9) M. ISHIMOTO, *Zisin*, **9** (1937), 108-117.

10) F. KISHINOUYE, *Zisin*, **8** (1936) 590-591; *Bull. Earthq. Res. Inst.*, **15** (1937), 785-827.

11) For examples, K. KASAHARA *et al.*, *Bull. Earthq. Res. Inst.*, **45** (1967), 225-239, *ibid.*, **46** (1968), 651-661, I. TSUBOKAWA *et al.*, *Bull. Earthq. Res. Inst.*, **45** (1967), 265-288, S. NAGUMO and K. HOSHINO, *Bull. Earthq. Res. Inst.*, **45** (1967), 1295-1311.

12) S. NAGUMO, *Bull. Earthq. Res. Inst.*, **44** (1966), 1623-1664.

The purpose of this paper is (1) to present the deformation~ fracture relation of earthquake genesis which will meet such requirements, and (2) to derive various formulae of frequency distribution of earthquakes from that relation.

2. Deformation~fracture relation in earthquake genesis

The deformation~fracture relations in earthquake genesis are assumptions at the present stage. The assumptions which will be adopted in this paper are as follows:

(I) The density of earthquake occurrence is assumed to be proportional to the curvature of the plastic bending deformation of the medium.

(II) The frequency distribution of earthquakes with respect to earthquake size is assumed to be proportional to the spectrum distribution of structural wave number of plastic deformation.

(III) The frequency distribution of earthquakes in a certain area is assumed to be proportional to the area of focal region.

The mathematical representation of the above assumptions are as follows:

For simplicity of representation, let us take the case of two-dimensional plastic bending deformation of a thin plate.

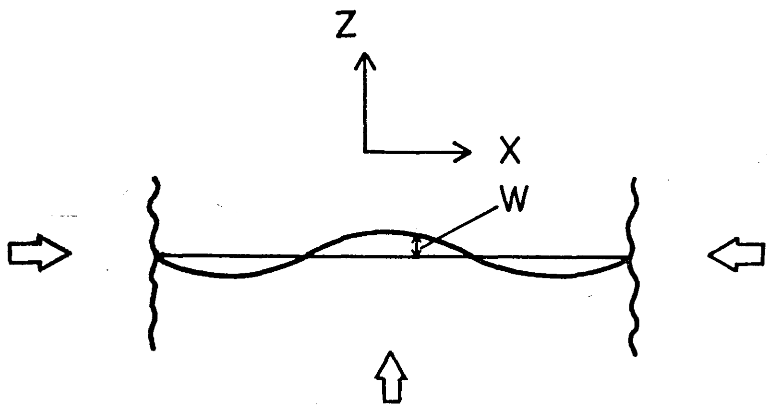

Fig. 1.

Take the coordinate as shown in Fig. 1. The number of earthquakes per unit length per unit thickness is denoted by $n(x)$, and is termed density of earthquake occurrence. The lateral displacement of bending deformation is denoted by $w(x)$.

The curvature of bending deformation is $(d^2w/dx^2)/\{1+(dw/dx)^2\}^{3/2}$, and is approximated by (d^2w/dx^2) if the deformation under consideration is limited to the early stage of deformation where the gradient of the lateral displacement, dw/dx, can be neglected with respect to 1.

Then, the first assumption is written as

$$n(x)=\frac{c}{b_0}\left|\frac{d^2w}{dx^2}\right| , \tag{1}$$

where c is a proportional constant of non-dimension, b_0 is a unit of dislocation having dimension of length.

This assumption is introduced by the analogy of the dislocation theory of metal. The occurrence of earthquake shock is nothing but the generation of crack in the earth's crust and mantle. According to the dislocation theory of crack production,[13] the production of a crack is caused by production, migration and piling up of moving dislocations. Therefore it will be natural to assume that density of cracks is proportional to the density of dislocations which are produced during plastic deformation. As regards the density of dislocation, it is known[14] that the density of edge dislocation produced by the plastic bending of the plate is proportional to the curvature of the deformation. This means that the number of cracks is larger in a deformation of sharp curvature and smaller in a gentle curvature.

The second assumption is written as

$$n(k)dk=n(a)da=n(E)dE=n(M)dM , \tag{2}$$

where $n(k)dk$, $n(a)da$, $n(E)dE$ and $n(M)dM$ are the frequency distributions of earthquakes, or the numbers of earthquake shock in a certain area and in a certain period of time with respect to the wave number k of the structure of deformation, amplitude a of seismic motion observed at a certain station, energy E of earthquake radiation energy and magnitude M of earthquake respectively.

This assumption implies that the size of earthquake is governed by the structural wave-length of the deformation, and is based upon the view that the larger earthquakes will be produced by the structure of longer wave-length.

The third assumption is written as

$$n(k)dk=S\cdot\bar{n}(k)dk , \tag{3}$$

where S is the area of the earthquake focal region and $\bar{n}(k)dk$ is the

13) A. N. STROH, *Proc. Roy. Soc.* **A**, **223** (1954), 404.

14) A. H. COTTRELL, *Theory of Crystal Dislocations* (Gorden and Breach, 1962), pp. 29-30.

density of frequency distribution of earthquake shocks, namely the number of earthquake shocks per unit area.

From these three basic assumptions we can obtain the expression of $n(k)dk$ as a function of deformation.

Since $\bar{n}(k)$ is given by the Fourier transform of $n(x)$,

$$\bar{n}(k)=\int_{-\infty}^{\infty} n(x)e^{-ikx}dx\ , \tag{4}$$

we obtain from the equation (3), (4) and (1),

$$n(k)dk=S\cdot\frac{c}{b_0}(-k^2)\,W(k)dk\ , \tag{5}$$

where $W(k)$ is the Fourier spectrum of the deformation $w(x)$,

$$W(k)=\int_{-\infty}^{\infty} w(x)e^{-ikx}dx\ . \tag{6}$$

Since the equation (1) gives the relation between the number of earthquakes and the deformation of the medium, it will be termed a deformation~fracture relation in earthquake genesis. However, the equation (1) does not contain the specification for the size of earthquakes. The equations (5) and (6) give the relation between the number of earthquakes and the deformation of the medium for the specified size of the structural wave number, which are related to the size of earthquakes by the equation (2). Thus, the equation (5) will be called a deformation~fracture relation in earthquake genesis having a specification for earthquake size.

3. Derivation of the Ishimoto-Iida's Formula

The Ishimoto-Iida's formula[15] states that the number of earthquakes $n(a)da$ which are observed at a certain station in a certain period of time and of which the maximum amplitudes are in the range of a and $a+da$ is expressed by the relation

$$n(a)da=Ka^{-m}da\ , \tag{7}$$

where K and m are constants. The value of m takes 1.8~2.0 in normal cases, and $m>2.0$ in volcanic earthquakes. Since the value m is not always an integer and varies from case to case, it is necessary to modify the equation (7) to such a non-dimensional form as

15) M. ISHIMOTO and K. IIDA, *loc. cit.*, 1).

$$n(a)da = K'(a/a_0)^{-m}(da/a_0) \ , \tag{8}$$

where K' is a non-dimensional constant and a_0 is a unit of length.

The Ishimoto-Iida's formula of equation (8) will be derived from the deformation$\sim$fracture relation of the equation (5), if we can know the relation between k and a. Next, let us consider this relation.

Radiation relation

The relation between the amplitude a of earthquake motion at a certain station and the structural wave number k of the deformation in the focal region is the radiation relation of seismic wave.

The radiation of seismic wave from various sources has been treated in the theory of seismic wave. In the model of bi-lateral faulting source in an infinite elastic medium the amplitude u of seismic motion at a sufficient distance r from the source is given by T. Hirasawa and W. Stauder[16] and takes the form of

$$u \propto b_0(WL/r^2) \ , \tag{9}$$

where b_0 is the amount of dislocation at the source, W and L are width and length of faulting respectively. This relation means that the amplitude of seismic motion at a distance r is proportional to the dislocation b_0 and the area of the faulting plane and is inversely proportional to the square of the distance. Also in the model of hollow sphere source in an infinite medium, the amplitude u of seismic motion at a sufficent distance r from the source is given, as shown by Sezawa and Kanai,[17] by the equation

$$u \propto (r_0 p_0/\mu)(r_0/r)^2 \ , \tag{10}$$

where r_0 is the radius of source sphere and p_0 is the pressure per unit area which is applied suddenly to the inner surface of the hollow sphere, μ is the rigidity of the medium. This relation means that the amplitude u is proportional to the surface area of the source sphere and is inversely proportional to the square of the distance.

From these results, it can be assumed that the amplitude a of seismic motion at a sufficently long distance r which is radiated from the sinusoidal plastic deformation, is given by

$$a = b_1(\mathscr{L}/r)^2 \ , \tag{11}$$

where b_1 is a constant which has dimension of length, $\mathscr{L}$ is the

16) T. HIRASAWA and W. STAUDER, *Bull. Seis. Soc. Ameri.*, **55** (1965), 237-262.
17) K. SEZAWA and K. KANAI, *Bull. Earthq. Res. Inst.*, **19** (1941), 151-161.

wavelength of the deformation. When the equation (11) is expressed by the wave number k by the relation $k=2\pi/\mathscr{L}$, it becomes

$$a=b_1(2\pi)^2(1/kr)^2 . \tag{12}$$

Equation (12) is the desired radiation relation which gives the relation between k and a.

Derivation of the Ishimoto-Iida's formula

By calculating dk as a function of da from the equation (12), and substituting dk into the equation (5), we have

$$n(k)dk=K_1\,|\,W(k)\,|\,(b_1/a)^{5/2}(1/b_1)da=n(a)da \tag{13}$$

where K_1 is a certain constant which includes parameters a, b_1 and A_0. The equation (13) implies that the functional form of $n(a)$ is governed by that of $W(k)$. This will be one of the most important properties. It will be seen in the equation (13) that in order to represent $n(a)da$ by a power function of a, $|\,W(k)\,|$ must be a power function of k. Therefore, we assume the power function representation for $W(k)$,

$$|\,W(k)\,|=(A_0/k_0)(k_0/k)^p , \tag{14}$$

where A_0 is a constant having the dimension of displacement, k_0 is the unit of wave number, and p is a non-dimensional constant.

Substituting equation (14) into equation (13), we have

$$n(a)da=K(b_1/a)^{1/2(5-p)}(da/b_1) , \tag{15}$$

where

$$K=S\cdot\frac{c}{b_0}\,\frac{k_0^3}{(k_0r)^{3-p}}(2\pi)^{3-p}(A_0/k_0) . \tag{16}$$

Equation (15) is the one which represents the frequency distribution of earthquakes as a power function of the maximum amplitude of seismic motion, and is nothing but the Ishimoto-Iida's formula.

By equating equation (15) to equation (8), we have the relation:

$$m=\frac{1}{2}(5-p) . \tag{17}$$

As is evident in equations (15) and (17), the exponent m of the Ishimoto-Iida's formula depends upon the expont p of the power function representation of the spectrum $W(k)$ of the deformation. The relations between m and p are shown in Table 1. For example, the values of m for $p=0, 1, 2$ are 2.5, 2.0 and 1.5 respectively.

Table 1. Relation of m and p. m: the coefficient of Ishimoto-Iida's formula $n(a)da=K(a)^{-m}da$. p: the exponent of spectrum $W(k)=(A_0/k_0)(k_0/k)^p$ of plastic deformation of the medium.

	$W(k)$	$n(a)da$	m
$p=0$	1	$K(b_1/a)^{2.5}da/b_1$	2.5
$p=1$	$1/k$	$K(b_1/a)^{2.0}da/b_1$	2.0
$p=2$	$1/k^2$	$K(b_1/a)^{1.5}da/b_1$	1.5

4. Derivation of $n(E)dE$

The frequency distribution $n(E)dE$ of earthquakes with respect to energy E will be directly derived from the deformation$\sim$fracture relation in earthquake genesis, the equation (5). In order to do this, we have to know the relation between k and E.

Since the energy of earthquake wave is thought to be proportional to the volume of the focal region, it will be quite natural to assume that the relation between the energy of siesmic waves and the structural wave number of the deformation in the focal region will be given by the equation

$$E=E_0(k_0/k)^3 , \tag{18}$$

where E_0 is the energy of seismic wave radiated from a unit volume, k_0 is the unit of wave number.

If we transform dk to dE by using equation (18), and also using equation (14), the equation (5) becomes

$$n(k)dk=K_E(E/E_0)^{-n}(dE/E_0)=n(E)dE , \tag{19}$$

where

$$K_E=(1/3)(Sc/b_0)(A_0/k_0)\cdot k_0^3 , \tag{20}$$

$$n=(1/3)(6-p) . \tag{21}$$

Equation (19) is the desired relation. The $n(E)dE$ is also represented by the power function of energy.

When the number of earthquakes of which energy is larger than E is denoted by $N(E)$, we have the relation

$$N(E)=\int_E^\infty n(E)dE=\frac{K_E}{(n-1)}(E/E_0)^{-n+1}\,. \tag{22}$$

5. Derivation of the Gutenberg-Richter's formula

Gutenberg and Richter[18] have presented the statistical formula for the frequency distribution of earthquakes with respect to magnitude in a form of

$$\log_{10} N(M)=A-bM \text{ or } N(M)=10^A 10^{-bM}\,, \tag{23}$$

where $N(M)$ is the number of earthquakes of which magnitude is M or greater, A and b are constants. This formula is also represented by its differential form of

$$dN=n(M)dM=10^a\cdot 10^{-bM}dM\,, \tag{24}$$

where

$$a=A+\log(b/\log_{10} e)\,.$$

Since the magnitude M is defined by the operational procedure, the direct derivation of the Gutenberg-Richter's formula from the deformation~fracture relation will not have much physical significance. The derivation will be made by the transformation from the frequency distribution with respect to energy as shown by Asada et al.[19]

As is well known,[20] the relation between energy E and magnitude M is represented by the equation

$$\log(E/e_0)=\alpha+\beta M\,, \tag{25}$$

where e_0 is the unit of energy and α, β are constants. Richter's final values[21] for α, β are

$$\alpha=11.8,\qquad \beta=1.5\,. \tag{26}$$

By using the equation (25), dE is transformed into dM. Thus the equation (19), which is $n(E)dE$, can be transformed into $n(M)dM$, and finally becomes

$$n(M)dM=10^a\cdot 10^{-bM}dM \tag{27}$$

18) B. GUTENBERG and C. F. RICHTER, *ibid.*, 2).
19) T. ASADA *et al.*, *Zisin* [ii], **3** (1950), 11-15.
20) C. F. RICHTER, *Elementary Seismology*, chap. 22, (W. H. Freeman, 1958).
21) C. F. RICHTER, *ibid.*, 20).

where

$$10^a = K_E(e_0/E_0)^{-n+1}10^{-\alpha(n-1)}\beta \log_e 10 , \tag{28}$$

$$b = \beta(n-1) = \beta(1-p/3) . \tag{29}$$

This means that the Gutenberg-Richter's formula is also derived from the deformation fracture relation in earthquake genesis.

In a special case of $p=1$ and $\beta=1.5$, we have the well-known relation[22)]

$$b = m-1 . \tag{30}$$

From equation (28), constant a has the meaning of

$$a = \log_{10} S + \log_{10}(A_0/k_0) + \log_{10}(c/b_0) + \text{const} . \tag{31}$$

The number of earthquakes $N(M)$ of which magnitude is M or greater is given by the relation

$$N(M) = \int_M^\infty n(M)dM = \frac{1}{b \log_e 10} 10^a \cdot 10^{-bM} . \tag{32}$$

6. Discussions

As seen in the preceding sections, the empirical statistical formula for the frequency distribution of earthquakes which represents very important properties of earthquake occurrence are derived directly from the deformation$\sim$fracture relation in earthquake genesis. This will mean that these statistical formulae are none other than reflections of crustal deformation in the focal region. This being the case, the functional forms and various constants which appear in these statistical formulae should have definite physical meaning. Now let us discuss such physical meaning.

1) *Power distribution*

As described in § 3, the functional form of the frequency distribution $n(a)da$ with respect to the maximum amplitude of seismic motion is governed by that of the spectrum $W(k)$ of deformation. The power function representation of $n(a)da$ is due to the fact that the spectrum of the deformation is represented by a power function. This means that, if the assumptions of the deformation and fracture in earthquake genesis are correct, the Ishimoto-Iida's formula suggests that the spectrum of the crustal deformation in the focal region will be represented

22) T. ASADA *et al.*, *loc. cit.*, 19).

by a power function, and, further, the usual empirical value of $m \fallingdotseq 2.0$ predicts that $p=1$ or $W(k) \propto 1/k$.

As regards the spectrum of the crustal deformation, Mizoue[23] has made an extensive study. According to his paper, it has been shown that the spectrum of the vertical deformation along the Japan island is almost inversely proportional to the wave number of the crustal deformation. His result is strikingly well in accordance with the theoretical prediction in this paper.

2) *Meaning of the Ishimoto-Iida's coefficent m.*

Since m is related to p by the equation (17), the so-called Ishimoto-Iida's coefficient m will represent the sharpness of the spectrum of deformation. The larger m value means a sharper spectrum. As regards the physical meaning of m, Mogi[24] has given a view that it represents the grade of heterogeneity of the medium, and a larger value of m corresponds to a higher heterogeneity. However, his view does not explain the reason why m takes the value $m \fallingdotseq 2.0$ in a usual case of deformation. His experimental result may be interpreted in such a way that heterogeneity of the materials of the model results in various localization of deformation, and causes the predominance of the spectrum components of smaller wave length.

3) *Meaning of coefficient b of the Gutenberg-Richter's formula*

As is shown in section 5, equation (30), b is related to m by the formula $b=m-1$. Therefore b has the same meaning of m and represents the sharpness of the spectrum of the deformation in the focal region.

4) *Meaning of K in the Ishimoto-Iida's formula*

The proportional constant K of the Ishimoto-Iida's formula is one of the most important constants for representing the grade of seismicity. According to equation (16), K is proportional to the area of the focal region, power function of focal distance r^{-2} for normal case $p=1$, and the strength (A_0/k_0) of the spectrum of the deformation.

5) *Meaning of a of the Gutenberg-Richter's formula*

As is shown by equation (31), a is a function of S, A_0/k_0, E_0 and other parameters.

6) *Physical law of frequency distribution*

From the above considerations, it will be natural to think that the

23) M. MIZOUE, *Bull. Earthq. Res. Inst.*, **45** (1967), 1019-1090.
24) K. MOGI, *loc. cit.*, 8).

Ishimoto-Iida's formula and the Gutenberg-Richter's formula for frequency distribution of earthquakes are of a causal physical law which relates the earthquake occurrence to the deformation of the focal region, and are not probability distribution functions, representing the probability of earthquake occurrence. These formulae are, of course, statistical formulae. However, a *statistical formula* will not always be the *probability distribution*.

As is well known there are many contradictions for interpreting these statistical formulae as probability distributions.

(1) If the Ishimoto-Iida's formula is the probability distribution density function, it should be equal to one when it is integrated in the whole range with respect to the random variable. As is evident, however, the power function representation for the probability distribution density function diverges when it is integrated, and contradicts to the requirement of the probability distribution function.

(2) The formula deviates from the actual observation in both ranges in the smaller and larger sizes of earthquakes.

(3) Theoretical considerations of the probability distribution with respect to energy based upon some proper a priori probability always result in an exponential form as is seen in the standard distribution of particles in the statistical mechanics.[25] This fact may show the difficulty of theoretical derivation of power function distribution for probability distribution.

7. Summary and Conclusions

Deformation$\sim$fracture relations in earthquake genesis are presented. Basic assumptions are that the number of earthquakes is proportional to the curvature of the plastic deformation of the medium and the size of earthquake is governed by the structural wave-length, the larger earthquake is produced by the structure of the longer wave-length.

It is shown that the Ishimoto-Iida's formula and the Gutenberg-Richter's formula for the frequency distribution of earthquakes are theoretically derived from the deformation$\sim$fracture relations in earthquake genesis. Main conclusions are as follows.

The functional forms of the Ishimoto-Iida's formula are the consequence of representing the spectrum of the deformation by the power function with respect to the structural wave number.

The coefficient m and b represent the sharpness of the spectrum of the deformation in the focal region.

25) R. B. LINSAY, *Concept and Method of Theoretical Physics*, Chap. 10, (Van Nostrand, 1951).

The value $m=2$ corresponds to the fact that the spectrum $W(k)$ of the deformation is inversely proportional to the wave number k.

The coefficients K and a are the functions of the area, strength of the spectrum of the deformation, and energy density of seismic wave radiation in the focal region.

It is thought that the statistical formulae such as the Ishimoto-Iida's formula and the Gutenberg-Richter's formula for the frequency distribution of earthquakes will represent the causal physical law, and will not be the probability distributions of earthquake occurrence.

Acknowledgement

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47. 地震発生における変形～破壊の関係式と地震の頻度分布の導出

地震研究所　南雲昭三郎

この論文では，地震発生における変形～破壊の関係式を提出し，それから石本・飯田公式，Gutenberg-Richter 公式として知られている地震の頻度分布を理論的に導くことを試みた.

変形～破壊に関する基本的仮定としては，地震の数は媒質の塑性変形の曲率に比例し，地震の大きさは構造波長で支配される，すなわち，より大きい地震はより長い波長成分の構造から発生するというものを採用した.

変形～破壊の関係式から地震の頻度分布を導いてみると，頻度分布に対して次のような物理的意味が見出された.

1) 石本・飯田公式および Gutenberg-Richter 公式の函数形は媒質の変形スペクトルを構造波数のベキ函数で表現したことに由来する.

2) 石本・飯田公式の係数 m, Gutenberg-Richer 公式の係数 b は震源域の変形スペクトルの鋭さを表現する.

3) $m=2$ という値は変形スペクトルが構造波数の逆数に比例する（構造波長に比例する）ということを意味する.

4) 石本・飯田公式の係数 K, Gutenberg-Richter 公式の係数 a は震源域の面積，変形スペクトルの強度，地震波輻射エネルギー密度等に依存する.

以上のことから，石本・飯田公式，Gutenberg-Richter 公式等の地震の頻度分布に関する統計式は，地震発生の確率分布を表わすものではなくて，因果的な物理法則を表わすものと考えられる.